European Journal of Physics

PAPER

A teaching showcase unveils the links between special relativity and the birth of quantum physics

To cite this article: G Margaritondo 2019 *Eur. J. Phys.* **40** 035402

IOP Publishing European Journal of Physics
Eur. J. Phys. **40** (2019) 035402 (8pp) https://doi.org/10.1088/1361-6404/ab025a

A teaching showcase unveils the links between special relativity and the birth of quantum physics

G Margaritondo

Faculté des Sciences de Base, Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, Switzerland

E-mail: giorgio.margaritondo@epfl.ch

Received 12 September 2018, revised 4 January 2019
Accepted for publication 28 January 2019
Published 13 March 2019

Abstract

Quantum effects scale up with the frequency of electromagnetic waves. Therefore, modern synchrotron sources of high-frequency x-rays—that can be visited by students either personally or online—offer an opportunity for a captivating introduction to basic quantum notions. We discuss here how they also unveil fascinating and often neglected links between quantum physics and special relativity, and in particular their role in the original thinking of Albert Einstein.

Keywords: photon, relativity, synchrotron, x-rays, Einstein, photoemission

(Some figures may appear in colour only in the online journal)

1. Introduction

The deep and often adversarial relation between relativity and quantum mechanics is an intriguing subject [1] that deserves more attention, also at the undergraduate teaching level. The popular scientific literature occasionally presents the conflict between general relativity (the theory of gravitation) and quantum notions—with ramifications to string theories. However, undergraduate teaching largely ignores the links between special relativity and the birth of quantum mechanics. This is regrettable since such links were a key element in the advent of quantum notions.

The discussion of this issue, within the general introduction of quantum concepts to undergraduate students [2, 3], can now profit from an excellent 'showcase': synchrotron radiation [4]. Students can visit synchrotron facilities either personally or through their excellent websites [5]. What they discover provides direct evidence of quantum properties and of their relations to special relativity, at a level ranging from elementary to more advanced but still easy to grasp.

The first example is almost trivial. Many synchrotron experiments use single-photon detectors, so the students can witness the quantisation of electromagnetic radiation by watching the flashes produced by individual photons arriving on the fluorescent screen. Such observations are also possible with lower-energy photons, as proposed by a number of websites [6]. Other properties, however, make high-photon-energy synchrotron radiation more effective for revealing quantum effects.

For example, while visiting a beamline dedicated to synchrotron photoemission experiments the students can directly observe from the spectra energy quantisation effects in molecules and solids—and grasp in general the quantum aspects of the photoelectric effect. The teachers can also seize the opportunity to dispel the common historical misconception that Einstein derived the existence of photons from the photoelectric effect [7]. In truth, he arrived at it by analysing the entropy of a volume full of electromagnetic radiation [8]—and then *predicted* the yet-to-be-observed properties of photoelectrons.

We shall now discuss other examples of how synchrotron radiation can assist undergraduate introduction to quantum notions—including their links to special relativity.

2. The Doppler effect

Einstein's photon hypothesis raises an intriguing question [2]. How could a young man outside academia find the courage to propose the most revolutionary scientific notion of all times, against the universally accepted evidence of the wave nature of light? How could he dare to interpret as reality a formal similarity between light and an ideal gas?

The answer can be found in a revealing sentence in Einstein's 1905 special relativity article [9]: '*Es ist bemerkenswert, daß die Energie und die Frequenz eines Lichtcomplexes sich nach demeselben Gesetze mit dem Bewegungszumstande des Beobachters ändern*' ('*It is remarkable that the energy and the frequency of a light complex vary with the state of motion of the observer in accordance with the same law*'). He was specifically referring to the equivalence of the relativistic equations for the Doppler effect and for the transformation of the energy of light.

This reveals that Einstein did not work at his 1905 papers separately from each other, in a somewhat schizophrenic fashion. Instead, he kept an eye on the links between them. Most likely, it was the relation between frequency (Doppler) and energy transformations that helped him to find the courage to propose light quanta as real things.

The Doppler effect is also one of the foundations of synchrotron radiation [4]. Indeed, the production of high-frequency x-rays is partly due to the Doppler shift caused by the relativistic longitudinal velocity of the emitting electrons. And the extreme angular collimation of the emission is a consequence of the Doppler angular 'aberration'. In this context, it is intriguing that Einstein even hinted in 1905 the extreme intensity of synchrotron radiation [9]: '*für einen Beobachter, der sich mit der Geschwindigkeit c einer Lichtquelle näherte, diese Lichtquelle unendlich intensive erscheinen müßte*' ('*to an observer approaching a source of light with the velocity c, this source must appear of infinite intensity*'). He was specifically referring to the relativistic transformation of I' (intensity in the source reference frame) into I (intensity in the observer reference frame):

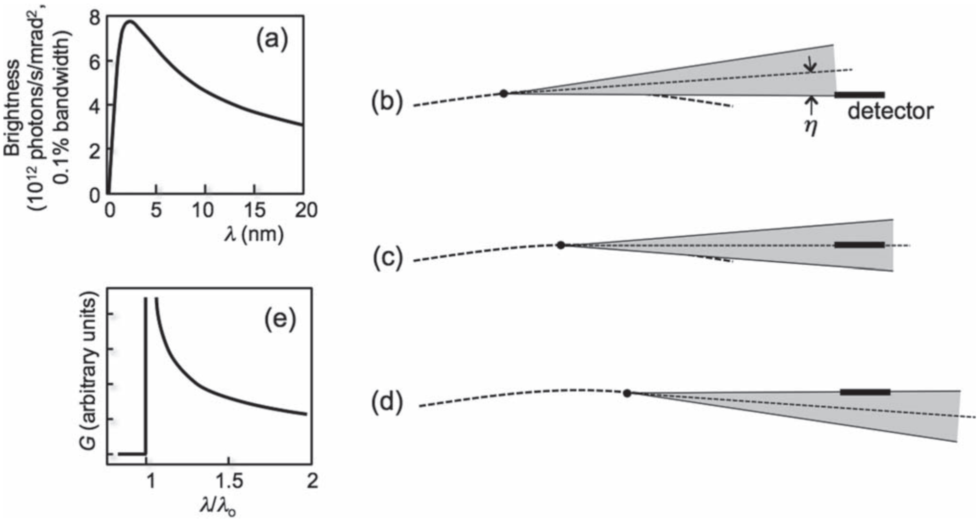

Figure 1. (a) The synchrotron bending magnet spectrum is sharply asymmetric [12]. (b)–(d) This asymmetry is explained [12] by the directional Doppler shift, which during the illumination of the detector by the radiation cone changes the detected wavelength; (b) the detection begins at an angle equal to half the detection cone amplitude, η; after passing through the longitudinal direction (c), it ends (d) at the same angle as (b). The corresponding direction-dependent Doppler effect produces the asymmetric (unbroadened) calculated lineshape shown in (e) [12]. Reproduced with permission of the International Union of Crystallography [12]. © International Union of Crystallography.

$$I = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} I'. \tag{1}$$

Paradoxically, the role of the Doppler effect in synchrotron radiation was partially neglected or even misunderstood for a long time. Only recently was the confusion with the (non-existing) role of time dilation clarified [10, 11]. And we just proposed [12] a new elementary model of the bending magnet spectrum that fully unveils the role of the directional Doppler effect.

Previous simplified models described the bending magnet spectrum [13] by first finding the ‘central’ emitted wavelength with a combination of the relativistic cyclotron theory and of the longitudinal Doppler equation. Then, they derived the broad bandwidth from the duration of the detected radiation pulse and from the Fourier theorem. This logic was not totally wrong, but missed one essential feature [14] of the bending magnet emission: its extreme spectral asymmetry (figure 1(a)), with a sharp cutoff at short wavelengths and a long tail to high wavelengths.

The directional Doppler effect is the main cause of the asymmetry [12]. From the relativistic invariance of the wave phases (rather than from time dilation, as it is frequent done obtaining the same result, but in a conceptually wrong way), one can derive [11] the Doppler wavelength equation:

$$\lambda = \frac{\left(1 - \frac{v}{c}\cos\theta\right)}{\sqrt{1 - \frac{v^2}{c^2}}}\lambda' = \gamma\left(1 - \frac{v}{c}\cos\theta\right)\lambda', \tag{2}$$

linking the wavelengths λ and λ' in the observer and source frames, and the angle θ of the source-observer direction with respect to the longitudinal (source motion) direction. For the longitudinal direction, equation (2) becomes

$$\lambda = \frac{\left(1 - \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}}\lambda' = \frac{\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)}{\sqrt{1 - \frac{v^2}{c^2}}\left(1 + \frac{v}{c}\right)}\lambda'$$

$$\lambda = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 + \frac{v}{c}\right)}\lambda' = \frac{1}{\left(1 + \frac{v}{c}\right)\gamma}\lambda', \tag{3}$$

which in the relativistic case of synchrotron radiation gives

$$\lambda \approx \frac{1}{2\gamma}\lambda'. \tag{4}$$

Equation (2) implies in general that the wavelength increases with θ. As illustrated in figures 1(b)–(d), this means [12] that the wavelength captured by a point-like detector changes during the motion of an electron through a bending magnet, and for most of the time is larger than the 'central' wavelength corresponding only to the longitudinal direction (figure 1(c)). This causes [12] the aforementioned spectral asymmetry, as shown by the theoretical (unbroadened) spectrum in figure 1(e).

For teaching purposes, the above logic can be inverted as follows. The asymmetric extension to long wavelengths of the measured bending magnet spectrum indicates that, as illustrated by figures 1(b)–(d), the Doppler shift must cause a wavelength increase with θ. Furthermore, the direct experience with the 'flashes' at two-dimensional fluorescent photon detectors shows that the radiation consists of particles. Einstein's photon hypothesis [8] links the energy of such particles to the frequency. To further verify this hypothesis, one can check if energy and frequency behave indeed in a similar way.

Now, special relativity shows that the energy of an electromagnetic wave transforms from the observer frame to the source frame as

$$E' = \gamma\left(1 - \frac{v}{c}\cos\theta\right)E, \tag{5}$$

(where the second term in the parenthesis corresponds to the contribution of the momentum of the radiation). Equation (5) is the reciprocal transformation of the wavelength, equation (2), and therefore the same transformation as that of the frequency. This is specifically consistent with the directional wavelength Doppler shift required to explain the asymmetry of the bending magnet spectrum. The photon hypothesis is thus doubly validated.

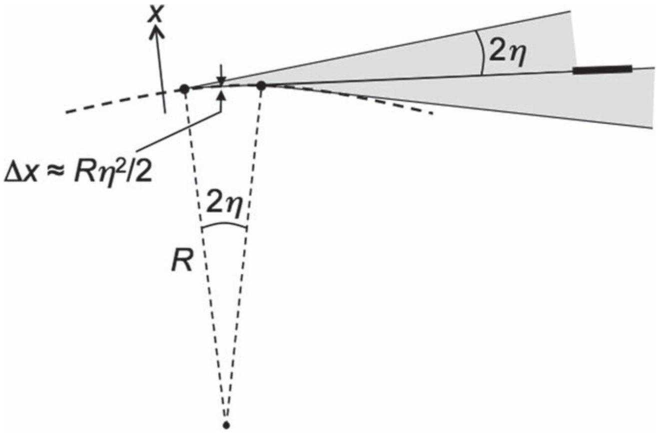

Figure 2. Evaluation of the position and momentum uncertainties in the transverse direction x during the illumination of the detector by the bending magnet synchrotron radiation cone.

3. The uncertainly principle

Some synchrotron radiation properties are linked to the Heisenberg principle, another pillar of quantum physics. Specifically, the principle can be used for an alternate demonstration of the angular collimation, not using special relativity as the standard one [4]—and unveiling additional links between special relativity and quantum physics.

Consider again bending magnet radiation: as shown in figures 1(b)–(d) and 2, the narrow light cone illuminates a point-like detector only for a short period of time. How narrow is the cone? As mentioned above, its angular width 2η is normally derived [4] from the relativistic Doppler aberration, obtaining $2/\gamma$ and therefore

$$\eta \approx \frac{1}{\gamma}. \tag{6}$$

Let us use instead the uncertainly principle. The lower limit of the product of the uncertainties in measuring the position and the momentum of synchrotron radiation photons in the transverse direction x is

$$\Delta x \Delta p_x \approx \frac{h}{4\pi}. \tag{7}$$

Now, the momentum magnitude is h/λ, and its uncertainty is related to the momentum rotation angle during the detector illumination—which equals the width of the illumination cone, 2η. We shall assume that

$$\Delta p_x \approx 2\eta \frac{h}{\lambda}. \tag{8}$$

On the other hand, Δx corresponds to the transverse displacement during the illumination. Referring to figure 2, for small values of η:

$$\Delta x \approx (R\eta)\eta/2 = R\eta^2/2, \tag{9}$$

where R is the curvature radius of the electron trajectory. Equations (7)–(9) give

$$\frac{R\eta^3 h}{\lambda} \approx \frac{h}{4\pi}, \tag{10}$$

or

$$\frac{R\eta^3}{\lambda} \approx \frac{1}{4\pi}. \tag{11}$$

The relativistic theory of cyclotron motion [4, 12] shows that the central bending magnet wavelength is

$$\lambda \approx \frac{2\pi cm}{2\gamma^2 eB}, \tag{12}$$

and the curvature radius is

$$R \approx \frac{\gamma cm}{eB} = \frac{\gamma^3\lambda}{\pi}. \tag{13}$$

Combined together, equations (11) and (13) give

$$\eta \approx \frac{0.63}{\gamma}, \tag{14}$$

close to the result (equation (6)) obtained from the Doppler effect, i.e. from special relativity (the difference in the numerical factor is due to our use of maximum spreads to approximate the uncertainties rather than of smaller, e.g. Gaussian, spreads). Thus, once again, quantum aspects and relativistic properties appear intriguingly related to each other.

Note, however, that this does not imply that the full theory of synchrotron radiation can be solely derived from quantum properties. On the contrary, it requires the use of both special relativity and quantum physics. The reason is that photons are quantum objects and also, being massless, ultra-relativistic entities. The ultimate framework of a rigorous and complete treatment of synchrotron radiation, as of all electromagnetic phenomena, is the relativistic quantum field theory.

The cases discussed above merely show that some specific synchrotron radiation features can be linked to quantum notions. Other examples could be cited, some of which are quite subtle and therefore probably not useful for undergraduate teaching, but still interesting.

We note, in particular, the case of the ‘brightness’ or ‘brilliance’ of synchrotron radiation, which is related to the deviations of the positions and of the velocity directions of individual electrons with respect to the standard trajectory—the so-called ‘emittance’ of the accelerator. Such deviations have several causes, one of them being the quantised nature of synchrotron emission and its consequent stochastic character, the so-called ‘quantum excitation’ mechanism [15].

4. Practical teaching plan

Can the facts discussed above be practically used for real teaching? This is a reasonable question, since they are rather subtle and constitute a radical departure from the standard way to present the first steps into quantum physics.

We believe that the answer can be positive if a careful teaching plan is adopted. The teaching logic can be the following:

1. Einstein's photon hypothesis of 1905 was the real first step of quantum physics, since the previous contribution by Planck was an ad hoc assumption, not a revolutionary departure from classical physics.
2. In 1905, Einstein was simultaneously working at the photon hypothesis and at special relativity. His comment on the Doppler effect [9] (the equation should be explicitly presented to the students) reveals the conceptual interaction between these issues. Such relations constitute, since 1905, a fascinating subject.
3. Quantum effects in electromagnetic radiation scale up with the frequency, thus they are particularly evident for x-rays. There are now powerful synchrotron x-ray sources that can be used to observe them.
4. A single-photon counter in operation should be shown at this point to the students—either directly or with a video file—enabling them to 'see' the photons. If possible, the observation of photoemission experiments should follow, illustrating quantised core levels and the frequency/energy cutoff.
5. The derivation in the second part of section 2 should then be presented, linking again the Doppler effect and the photon hypothesis.
6. Next, the need for the uncertainty principle to guarantee the self-consistency of quantum physics should be discussed. Then, the derivation of section 3 should be presented.
7. The final discussion should stress the differences between special relativity and general relativity, often obscured by their names [10]. In particular, it should be noted that the links to quantum physics unveiled by the synchrotron radiation only concern special relativity, whereas the conflict of quantum mechanics with general relativity is a different issue and a major and very fascinating open question.

This didactic strategy minimises formalism and is directly based on practical observations. It can be, therefore, attractive for bright students, and much closer to the historical facts than the 'standard' introduction to quantum physics. Furthermore, it reveals fascinating but often neglected issues at the borderline between quantum physics and special relativity.

Acknowledgments

This work was supported by the Ecole Polytechnique Fédérale de Lausanne (EPFL).

ORCID iDs

G Margaritondo https://orcid.org/0000-0002-1243-7686

References

[1] Anderson E 2017 *The Problem of Time: Quantum Mechanics Versus General Relativity* (Berlin: Springer) and the references therein

[2] Margaritondo G 1995 Quantization, Doppler shift and invariance of the speed of light: some didactic problems and opportunities *Eur. J. Phys.* **16** 169–71

[3] Margaritondo G 2003 A historically correct didactic first step in the quantum world: stressing the interplay of relativity, thermodynamics and quantum physics *Eur. J. Phys.* **24** 15–9

[4] Margaritondo G 2017 A success story over six decades *Riv. Nuovo Cimento* **40** 411–71 https://www.sif.it/riviste/sif/ncr/econtents/2017/040/09

[5] There are many excellent web sites presenting synchrotron facilities, so it is difficult to select only a few of them. For a first virtual visit, we like, this one: https://synchrotron-soleil.fr/en/videos/vr-360deg-visit-soleil

[6] See, for example, the flashes caused by individual photons in the web movie https://youtube.com/watch?v=NaEo517NDXo

[7] https://en.wikipedia.org/wiki/Annus_Mirabilis_papers

[8] Einstein A 1905 Über einen die erzeugung und verwandlung des lichtes betreffenden heuristischen gesichtspunkt *Ann. Phys.* **17** 132–48

[9] Einstein A 1905 Zur Elektrodynamik bewegter Körper *Ann. Phys.* **17** 891–921

[10] Rafelski J 2017 *Relativity Matters: From Einstein's EMC2 to Laser Particle Acceleration and Quark-Gluon Plasma* (Berlin: Springer)

[11] G Margaritondo G and J Rafelski J 2017 The relativistic foundations of synchrotron radiation *J. Synchrotron Radiat.* **24** 898–901

[12] Margaritondo G 2018 The simple physics of the bending magnet spectrum *J. Synchrotron Radiat.* **25** 1271–6

[13] Margaritondo G 1988 *Introduction to Synchrotron Radiation* (Oxford, New York: Oxford University Press)

[14] Mobilio S, Boscherini F and Meneghini C 2015 *Synchrotron Radiation Basics, Methods and Applications* (Berlin: Springer)

[15] Huang Z and Ruth R D 1998 Laser-electron storage ring *Phys. Rev. Lett.* **80** 976–80