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Recent improvements of the Critical Shear Crack Theory for punching shear design and its simplification for code provisions

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Abstract

In 1960, Kinnunen and Nylander presented a mechanical approach for calculating the punching resistance of slab-column connections. That approach accounted for the response of a slab sector and considered failure to occur on the basis of the strains developing in the soffit of the slab. The model constituted a major step forward in the understanding of the punching phenomenon, being the most advanced mechanical theory at that time. Based on similar considerations, the Critical Shear Crack Theory (CSCT) was proposed by Muttoni and Schwartz in 1991 considering also the behaviour of a slab sector but defining the capacity to carry shear forces as a function of the opening and roughness of a crack developing in the shear-critical region. This approach has shown to be physically-consistent, allowing to account for the influence of both rotations and shear deformations on the punching strength. In this paper, the historical development of the CSCT and of its theoretical model is reviewed, highlighting a number of recent improvements and findings. It is also shown that suitable closed-form design expressions for code provisions can be derived from the general mechanical model. Finally, the different possible approaches for design and assessment according to the CSCT are discussed and compared.

Keywords: Critical Shear Crack Theory, Punching shear, Members without transverse reinforcement, Mechanical model, Design expressions

1. Introduction

The high complexity of the phenomena involved in punching shear failures of concrete slabs required significant research efforts before rational design approaches appeared. Amongst the first rational punching models, the one presented by Kinnunen & Nylander in 1960 constituted a significant step forward with respect to the empirical formulae used at that time. According to Kinnunen & Nylander (1960), shear is carried by an inclined strut and punching failures occur for a load at which a given value of the tangential strain in the soffit of the slab develops. This failure criterion, relating the deformation and the load-carrying capacity, was defined based on experimental results. For the slab response, the authors considered a simplified model for a slab sector, allowing to calculate both the punching strength and the rotation (deformation capacity) at failure.

Based on the pioneer ideas of Kinnunen & Nylander (1960) and extending its mechanical approach, the Critical Shear Crack Theory (CSCT) was first developed for punching failures of slab-column connections without shear reinforcement (Muttoni & Schwartz 1991). The theory was later extended to failures in shear of one-way slabs without shear reinforcement (Muttoni 2003) and to punching failures in two-way slabs equipped with shear reinforcement (Fernández Ruiz & Muttoni 2009). Thereafter, several studies on the CSCT demonstrated its applicability to other cases (as membrane effects and continuous slabs (Einpaul, Fernández Ruiz & Muttoni 2015), prestressed slabs (Clément & al. 2014), or concentrated loads in linearly supported slabs (Natário & al. 2014)). The theory has also been shown to be simple for its use in practice and grounds currently the shear and punching shear provisions of the Swiss Code for Concrete Structures (SIA 262, 2013) and the punching provisions of *fib*'s Model Code 2010. Furthermore, this theory is also used in the current draft of the shear and punching shear provisions for the next generation of Eurocode 2 (prEN 1992-1-1:2018-04, 2018).

The objective of the present work is to present a brief historical review of the development of the CSCT for punching failures of structural members without shear reinforcement. This review will focus on its mechanical model, highlighting the fundamentals of the theory and explaining the most recent theoretical improvements introduced in the theory. Finally, it is also explained how the theory can be simplified in order to obtain simple expressions, yet accurate and general, for punching shear design.

2. Historical development of the Critical Shear Crack Theory for punching

At its origin, the CSCT was developed in the frame of the revision of the punching design provisions for the Swiss Code for Concrete Structures in the 1990's (SIA 162, 1993), where some principles of the theory (Muttoni & Schwartz 1991) were used to prepare the punching provisions of the Draft Code Proposal (Muttoni 1985). According to the CSCT, the punching strength and its associated deformation capacity at failure can be calculated by intersecting the load-deformation relationship of a slab with a punching failure criterion. This approach is presented in Figure 1(a) for a slender slab, where the deformation of the slab is characterized by its rotation.

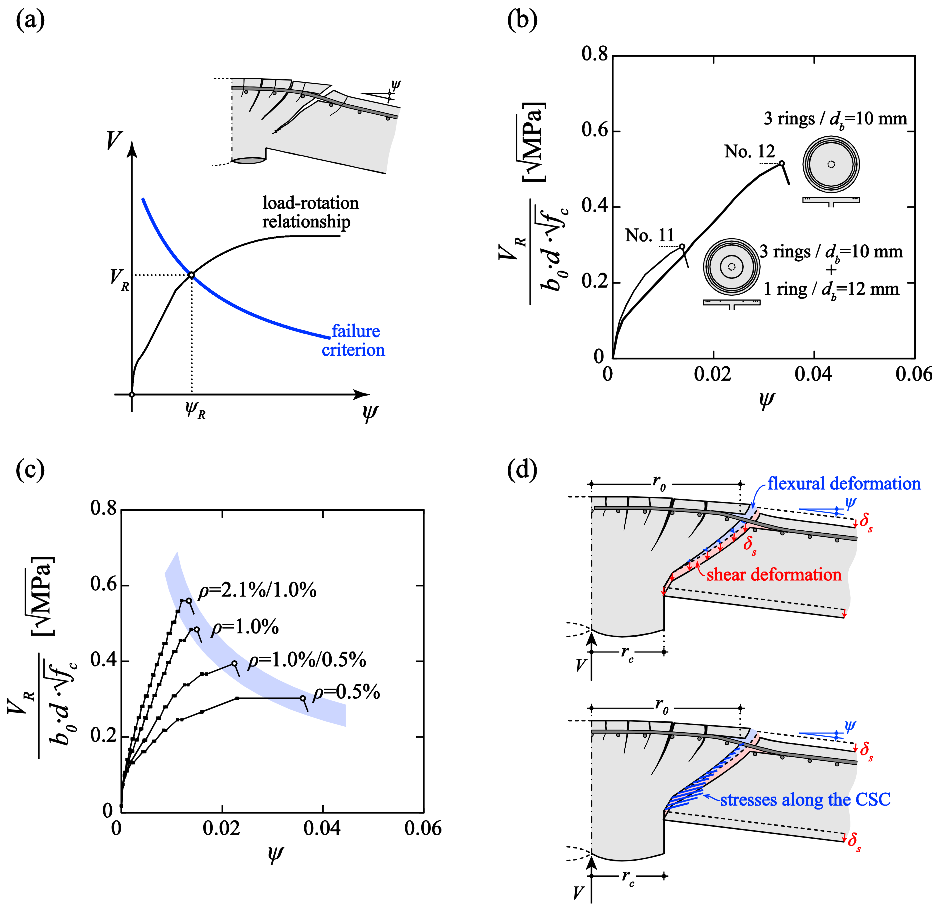

Figure 1. (a) Punching strength given by intersection of load-rotation curve and failure criterion; (b) experimental results of Bollinger (1985) with ring reinforcements (adapted from Muttoni (2008)); (c) experimental results of Kinnunen & Nylander (1960): load-rotation relationships for different flexural reinforcement ratios (adapted from Muttoni (2008)); (d) theoretical principles of the mechanical model of the CSCT (adapted from Muttoni, Fernández Ruiz & Simões (2017)).

2.1 Slab response

The slab response can be calculated based on different approaches. An analytical load-deformation relationship can be computed based on the equilibrium conditions of a slab sector, provided that a simplified kinematics (compatibility conditions) and sectional response (moment-curvature relationship) are adopted (Muttoni 2008). This approach can also be numerically performed allowing the use of refined moment-curvature relationships and the consideration of advanced kinematics (e.g. Guandalini 2005; Einpaul, Fernández Ruiz & Muttoni 2015). Another possible approach, becoming increasingly popular for cases with complex geometries or for the assessment of existing structures, consists on the calculation of the load-deformation relationship by means of finite elements (e.g. Belletti & al. 2015).

For design purposes, Muttoni (2008) proposed a simplified formulation of the load-rotation relationship based on the analytical formulation (see the complete derivation in Muttoni & al. (2013)):

$$\psi = k_m \frac{r_s}{d} \frac{f_y}{E_s} \left(\frac{V}{V_{flex}} \right)^{3/2} \tag{1}$$

where r_s is the radial distance between the axis of the column and the location of the zero radial bending moment, d is the effective depth, f_y is the yielding strength, E_s is the modulus of elasticity, V is the applied load and V_{flex} is the flexural capacity. The parameter k_m is a factor whose value can be taken as 1.2 when a refined estimate of the previous physical parameters is known and 1.5 otherwise (Muttoni & al., 2013).

2.2 Failure criterion

Differently to the model of Kinnunen & Nylander (1960) and other approaches inspired on it (Broms, 1990; Hallgren, 1996), the failure criterion of the CSCT is not based on the tangential strains developing in the soffit of the slab, but it is related to the opening of a crack with flexural origin developing in the shear-critical region. The main principles of the failure criterion of the CSCT for punching were already introduced by Muttoni & Schwartz (1991) (see also Muttoni (2003, 2008)):

(1) a critical shear crack (CSC) develops in the vicinity of the column, disturbing the concrete strut carrying shear and consequently influencing the punching strength. This principle was originally grounded on the tests of Bollinger (1985), Figure 1(b), which showed that the development of a crack in the shear-critical region (enforced by placing tangential reinforcement for these tests) decreases the punching strength;

(2) the punching strength decays with increasing opening of the CSC. This principle was evidenced by the experimental results of Kinnunen & Nylander (1960) which indicated that specimens with smaller reinforcement ratios failed at significantly larger deformations but at lower load levels, refer to Figure 1(c).

The failure criterion of the CSCT can thus be calculated on the basis of the two previously stated ideas. The punching strength associated to a given crack opening can be evaluated by integration of the stresses developing along the CSC, which may be calculated for an adopted location, shape and kinematics. Since an analytical integration of stresses is not suitable for design purposes, Muttoni & Schwartz (1991) proposed a semi-empirical failure criterion for slender slabs based on the assumption that the opening of the CSC can be correlated to the product of the rotation (ψ) times the effective depth (d) as $w \propto \psi \cdot d$ (SI Units [N, mm]):

$$V_R = \frac{b_0 \cdot d \cdot \sqrt[3]{f_c}}{1 + (\psi \cdot d / 4)^2} \tag{2}$$

where V_R is the punching strength, b_0 the control perimeter at $d/2$ from the column edge and f_c the cylinders concrete compressive strength.

This expression was later refined to account for the influence of the size of the aggregate on the punching strength. This consideration is grounded on the interlock model of Walraven (1981) and considers that, for the same opening of a CSC, a larger amount of shear forces can be transferred for rougher surfaces (characterized by larger aggregate sizes). On that basis, Muttoni (2003,2008) proposed the following failure criterion (SI Units [N, mm]):

$$V_R = \frac{0.75 \cdot b_0 \cdot d \cdot \sqrt{f_c}}{1+15 \cdot \psi \cdot d / d_{dg}} \tag{3}$$

Where the parameter d_{dg} refers to the roughness of the crack. This roughness parameter was proposed to be estimated as d_{dg} = 16 [mm] + d_g (d_g being the maximum aggregate size, consistently with the formulation originally proposed by Vecchio & Collins (1986)). Recent refinements of the theory (Cavagnis, Fernández Ruiz & Muttoni 2018; Muttoni, Fernández Ruiz & Simões 2018) have shown that more consistent values of this parameter may be obtained by considering the influence of concrete strength in the roughness (fracture surface developing through the aggregates (e.g. Collins & Kuchma, 1999)) and the limited influence of the size of the aggregate on the shear strength (Sherwood, Bentz & Collins 2007), yielding to the following expression (SI Units [N, mm]):

$$d_{dg} = 16 + d_g \cdot \min\left(\left(60 / f_c\right)^2, 1\right) \leq 40 \text{ mm} \tag{4}$$

Despite its simplicity, the failure criterion of Eq. (3) has shown to be robust and physically-consistent (Muttoni, 2008, Muttoni & Fernández Ruiz 2017). In the last years, a number of theoretical works have been published (Muttoni, Fernández Ruiz & Simões 2018; Simões, Fernández Ruiz & Muttoni 2018) describing a comprehensive mechanical model based on the CSCT principles and whose direct integration of the stresses developing at the CSC allows justifying the shape of this failure criterion. These works are briefly presented and discussed in the following section.

3 Validation of the failure criterion of the CSCT for punching failures

3.1 The theoretical principles of the CSCT

A refined calculation of the failure criterion can be carried out on the basis of the theoretical principles of the CSCT. According to this theory, a tangential crack with flexural origin develops in the shear-critical region and propagates in an inclined manner due to the presence of shear forces (Muttoni, 2008). As previously discussed, the development of this crack in the shear-critical region disturbs the concrete strut carrying the shear force (Muttoni, 2008). Its location, shape and kinematics are thus critical for the punching strength. According to the principles of the theory, the failure criterion can be calculated adopting an appropriate location, shape and kinematics of the CSC (for instance based on experimental observations).

A first refined calculation of the failure criterion of the CSCT was presented by Guidotti (2010), who considered some simplified assumptions for the location and shape of the CSC (straight and inclined at 45°), as well as for the governing shear-transfer actions (aggregate interlocking and residual tensile strength). Some refinements were later introduced by Clément (2012). A more complete mechanical model was recently presented by Simões, Fernández Ruiz & Muttoni (2018). This approach is briefly presented in the following and some of the main results are used to discuss on the suitability of the analytical failure criterion of the CSCT.

3.2 Direct calculation of the failure criterion according to Simões, Fernández Ruiz & Muttoni (2018)

3.2.1 Basic assumptions from the CSCT

The main hypotheses of the refined calculation of the failure criterion presented by Simões, Fernández Ruiz & Muttoni (2018) are based on the principles of the CSCT and grounded on experimental evidences (Figure 2(a)):

- Tangential cracks develop on the tension side due to the flexural response of the slab. The principal flexural cracks are considered to have an approximately constant spacing;

- A CSC, corresponding to a tangential crack with flexural origin, propagates through the height of the slab in an inclined manner due to the shear forces (Muttoni, 2008). According to Muttoni, Fernández Ruiz & Simões (2018), this crack can be assumed to have two different phenomenological behaviours: localized and smeared cracking on the tension and compression side, respectively;
- The kinematics of the CSC consists of flexural and shear deformations (as already stated by Guidotti (2010) and in accordance with the experimental results of Clément (2012));
- The location of the CSC is considered to be variable, in accordance with the experimental results of Guandalini, Burdet & Muttoni (2009) and to the interpretation of crack patterns by Simões, Fernández Ruiz & Muttoni (2018).

3.2.2 Regions of the slab and shape of the critical shear crack

As shown in Figure 2(b), the inner portion of the slab (inside r_0), where primary flexural cracks develop, deforms following a spherical shape (already considered by Kinnunen & Nylander (1960)). The outer portion of the slab, corresponding to slab sectors divided by the formation of radial cracks, deforms following a conical shape (as also originally considered by Kinnunen & Nylander (1960)). Finally, a deformable region is considered below the neutral axis in the vicinity of the column (shaded triangular area in Figure 2(b)). This region accommodates the negative radial displacements associated to the flexural deformations, in analogy with the considerations of Kanelloupolos (1986) for beams in bending (compressive displacements smeared in a given region). The CSC thus defines the transition between the inner and outer regions of the slab (on the tension side) or between the inner and deformable regions (on the compression side).

Based on experimental observations and theoretical considerations (Braestrup & al., 1976; Yankelevsky & Leibowitz, 1999), Simões, Fernández Ruiz & Muttoni (2018) adopted a third-degree parabola for the geometry of the CSC and considered additionally a variable location of the CSC at the height of the flexural reinforcement (based on a nominal level of shear stress).

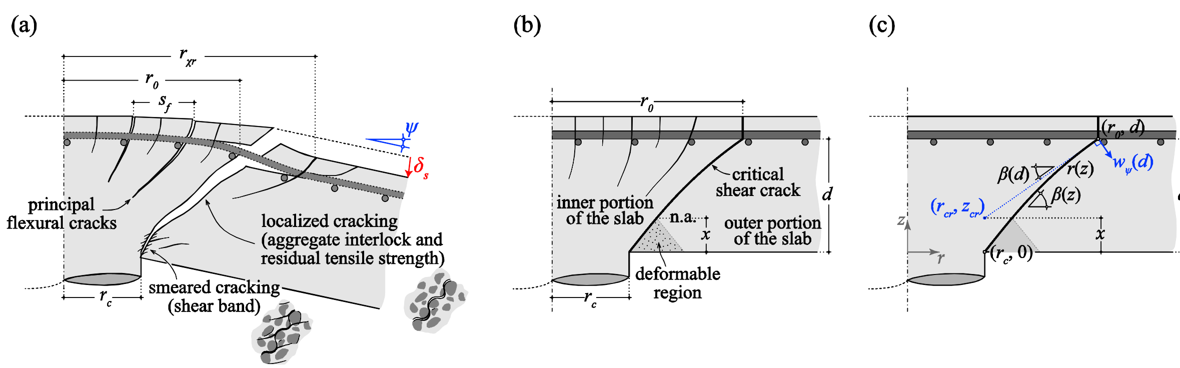

Figure 2. Mechanical model of Simões, Fernández Ruiz & Muttoni (2018): (a) main assumptions; (b) different regions of the slab; (c) hypotheses for geometry of the critical shear crack and location of the centre of rotation associated with flexural deformations; figure adapted from Simões, Fernández Ruiz & Muttoni (2018).

3.2.3 Kinematics and resulting displacement field along the critical shear crack

Simões, Fernández Ruiz & Muttoni (2018) considered that the displacements along the CSC result from the vector sum of the flexural and shear deformations (grounded on the experimental observations of Clément (2012) and consistently with the work of e.g. Guidotti (2010)). As depicted in Figure 3(a), the displacements associated with the flexural deformation results from a rotation ψ_{CSC} around a centre of rotation (with coordinates (r_{CR},z_{CR})). The rotation at the CSC may thus be calculated by diving the total rotation by the total number of cracks (considering an equal distribution of the rotation in the principal flexural cracks; as assumed by Guidotti (2010)). The centre of rotation associated with the flexural deformation is considered to be located at the edge of the support area (in accordance with the experimental results of Clément (2012)) and at the height of the neutral axis (as adopted by e.g. Hallgren (1996)). The flexural deformation at the CSC is followed prior to failure by a shear deformation (see Figure 3(b)), characterized by a constant shear displacement along the CSC (δ_s) with a minimum angle with respect to it (γ_0). The displacement field along the CSC is eventually calculated as the vector sum of the flexural and shear deformations, refer to Figure 3(c) and to Simões, Fernández Ruiz & Muttoni (2018) for further details.

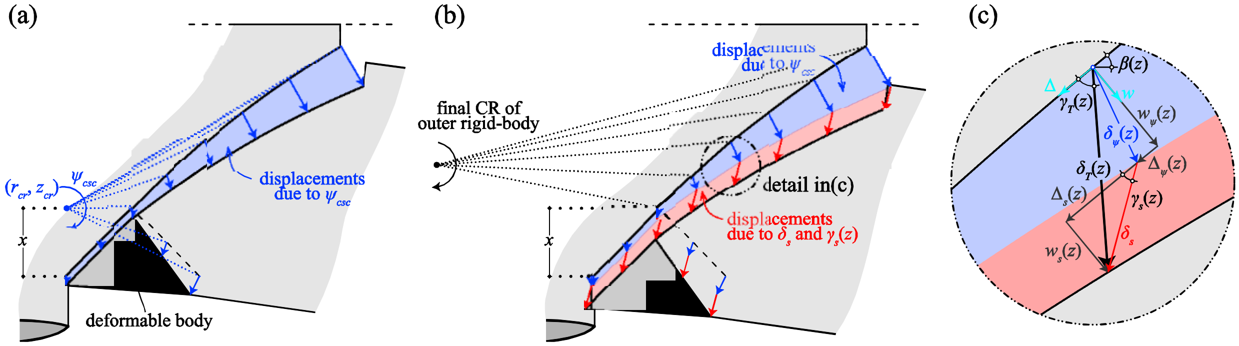

Figure 3. Calculated displacement field along the critical shear crack: (a) flexural deformation; (b) vector sum of flexural and shear deformation; (c) calculation of associated displacements.

3.2.4 Resulting stresses along the critical shear crack

For the calculation of the stresses along the CSC, Simões, Fernández Ruiz & Muttoni (2018) considered two distinct regions with different phenomenological behaviours (as suggested by Muttoni, Fernández Ruiz & Simões (2018)): a region with localized cracking on the tension side and a region with smeared cracking on the compression side, refer to Figure 4. The stresses in the region with localized cracking are calculated combining the approach of Cavagnis, Fernández Ruiz & Muttoni (2018) for the aggregate interlock engagement stresses ($\sigma_{agg,0}$, $\tau_{agg,0}$) with the approach of Hordijk (1992) for the residual tensile stresses (σ_{fct}). In the region of smeared crack (σ_{sb} and τ_{sb} in Figure 4), the concept of shear band used by Jensen (1975) was combined with a realistic strain-stress relationship representing the response of the concrete in this region. For that purpose, the triaxial behaviour of concrete presented by Guidotti, Fernández Ruiz & Muttoni (2011) was simplified in order to obtain a simple strain-stress relationship accounting for the effects of biaxial compression (Kupfer, Hilsdorf & Rusch 1969, effect associated with the tangential bending) and strain-softening (Vecchio & Collins 1986; effect resulting from the presence of tensile strains). With respect to the transition between the localized and smeared cracking regions, Simões, Fernández Ruiz & Muttoni (2018) adopted three simplified displacement-based criteria based on the experimental findings of Jacobsen, Olesen & Poulsen (2012).

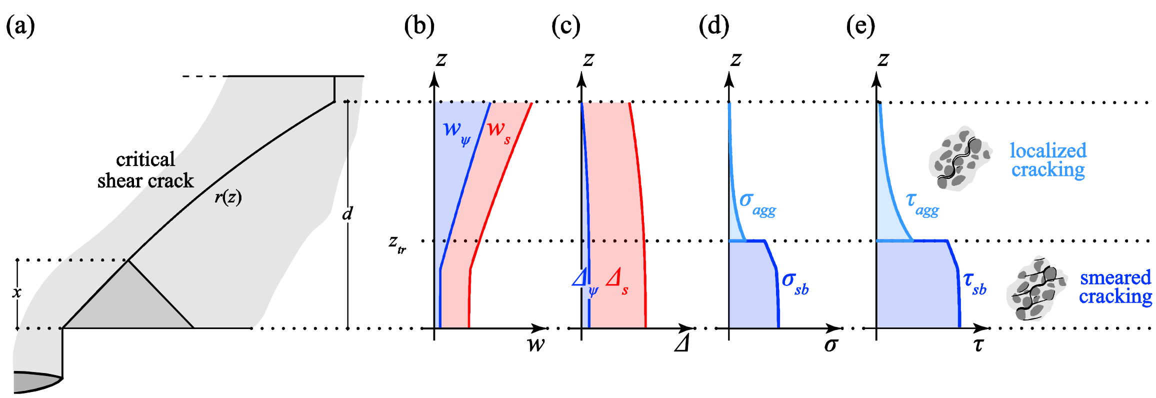

Figure 4. Calculated displacements and stresses: (a) geometry; displacement (b) normal and (c) parallel to the CSC; (d) normal and (e) shear stresses.

In addition to the resulting stresses along the CSC, also the dowel action of the flexural reinforcement was included as potential shear-transfer action (V_{DA}). This was performed using a similar approach to one followed by Einpaul (2016), consisting on the combination of different works on the topic (Rasmussen 1963; Millard & Johnson 1984; Fernández Ruiz, Plumey & Muttoni 2010; Fernández Ruiz, Mirzaei & Muttoni 2013; Randl 2013; Cavagnis, Fernández Ruiz & Muttoni 2018).

3.2.5 Calculation of the punching strength and associated deformation capacity

Following the approach of the CSCT, the punching strength and associated deformation capacity are calculated by intersection of the load-rotation relationship describing the slab response and the calculated failure criterion. In this work, the load-rotation relationship is calculated as suggested by Muttoni (2008), by integration of a quadri-linear moment-curvature relationship accounting for tension-stiffening effects.

On the other hand, the punching strength (V_c) associated to a given state of deformations (considering both rotations and shear deformations) is given by the sum of the different shear-transfer actions as follows:

$$
\begin{aligned}
V_c(\psi,\delta_s) = {} & \overbrace{2\cdot\pi\cdot\int_0^{z_{tr}}\frac{r(z)}{\sin\big(\beta(z)\big)}\cdot\Big[\tau_{sb}(z)\cdot\sin\big(\beta(z)\big)+\sigma_{sb}(z)\cdot\cos\big(\beta(z)\big)\Big]dz}^{\text{shear-transfer due to smeared cracking}} \\
& + \overbrace{2\cdot\pi\cdot\int_{z_{tr}}^{d}\frac{r(z)}{\sin\big(\beta(z)\big)}\cdot\Big[\tau_{agg}(z)\cdot\sin\big(\beta(z)\big)+\sigma_{agg}(z)\cdot\cos\big(\beta(z)\big)\Big]dz}^{\text{shear-transfer due to localized cracking}} \\
& + \overbrace{V_{DA}}^{\text{dowel-action}}
\end{aligned}
\tag{5}
$$

The failure criterion is eventually calculated by maximizing the punching strength (increasing the shear deformation) for a given level of rotation.

3.3 Brief discussion on the results of the refined mechanical model

Figure 5(a) depicts the results of the refined mechanical model of Simões, Fernández Ruiz & Muttoni (2018) for an investigated case where the flexural reinforcement ratio is the only parameter varied. The results of the model are in accordance to the well-known influence of the flexural reinforcement ratio on the punching strength, yielding larger influences for lower values of this parameter. Figure 5(b) shows also the punching strength represented as a function of the calculated rotation at failure (multiplied by the effective depth and divided by the reference value of the roughness of the CSC) corresponding to the case investigated in Figure 5(a). A clear trend of decreasing punching strength with increasing rotation at failure results, in accordance with experimental results (refer for instance to the results of Kinnunen & Nylander (1960) shown in Figure 1(c)). In addition, also the contributions of the different shear-transfer actions (smeared cracking representing the direct strut action, localized cracking corresponding to the aggregate interlock action and dowel action of the flexural reinforcement) are represented in Figure 5. The results show that the relative contribution of localized cracking to the punching strength is higher for failures associated with large rotations (corresponding to low flexural reinforcement ratios). This is justified by the larger crack openings, which decrease the extent of the region of the CSC under smeared cracking conditions. Figure 5(b) also shows that the contribution of all shear-transfer actions decrease with increasing rotations at failure. Again, this result is justified by the larger crack openings occurring in these cases, limiting the development of aggregate interlock stresses, softening the concrete and leading to yielding of the flexural reinforcement (thus limiting the potential dowelling action of the reinforcement).

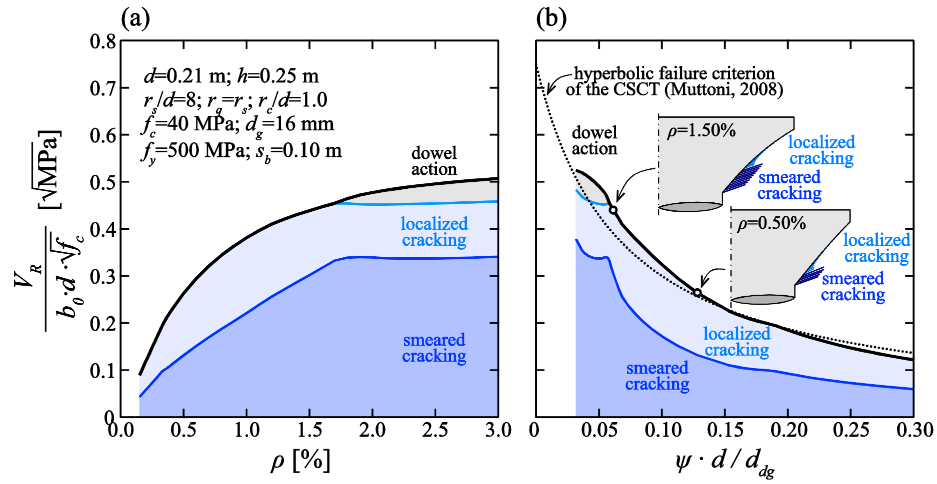

Figure 5. Numerical results of the punching strength calculated with the refined mechanical model as a function of the: (a) flexural reinforcement ratio; (b) calculated rotation at failure.

To investigate the accuracy of the refined mechanical model, Simões, Fernández Ruiz & Muttoni (2018) compared it against a database of experimental results and individual tests series. The comparison revealed a fine and consistent agreement between theoretical and experimental results. Moreover, the refined mechanical model was shown to consistently capture the influence of the main parameters on the punching strength.

As also shown in Simões, Fernández Ruiz & Muttoni (2018), when the numerical results of the normalized punching strength are depicted as a function of the calculated normalized rotation, all results remain within a narrow band, refer to Figure 6(a). The rather limited width of this band justifies that the calculation of the punching strength by integration of the stresses along the CSC (refined calculation of the failure criterion) may be replaced by the use of a single analytical failure criterion without a significant loss of accuracy. As shown in Figure 6(a), the hyperbolic failure criterion of the CSCT (Muttoni 2008) fairly well agrees with the band where all results concentrate.

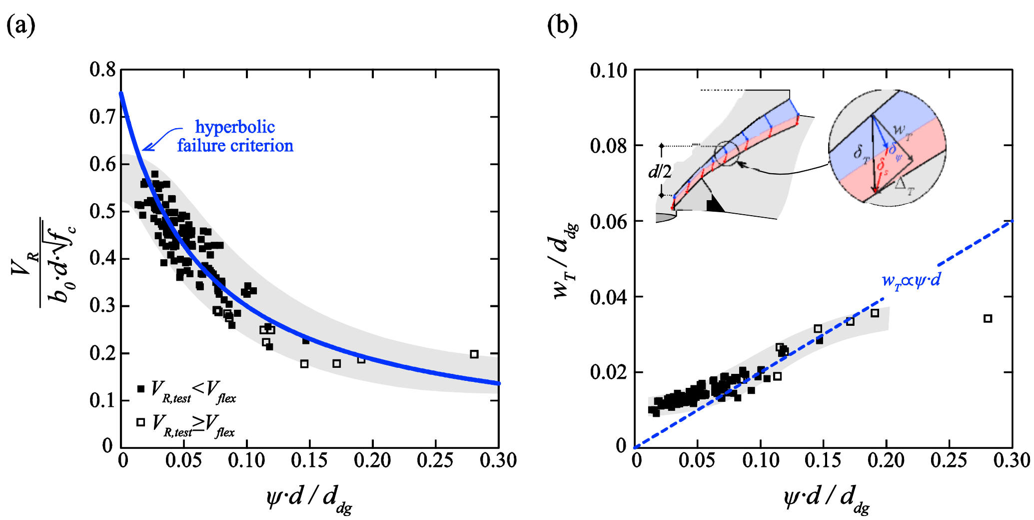

Figure 6. Results of the refined mechanical model corresponding to the tested specimens of a database with 133 specimens (database used by Simões, Fernández Ruiz & Muttoni (2018), dots represent calculated values): (a) normalized punching strength as a function of normalized rotation; (b) normalized crack opening at *d*/2 from the soffit of the slab as a function of the normalized rotation; figure adapted from Simões, Fernández Ruiz & Muttoni (2018).

Another interesting result from the refined model confirming the validity of the simplified assumptions for derivation of the analytical failure criterion of the CSCT (Eq. (3)) is also shown in Figure 6(b). That figure plots the normalized crack opening at failure (calculated at $d/2$ from the soffit of the slab) as a function of the normalized rotation for all the investigated experimental tests (Simões, Fernández Ruiz & Muttoni (2018)). The results of the detailed model confirm that the opening of the CSC is correlated to the product of the rotation times the effective depth (following an almost linear relationship), validating thus the original assumption of Muttoni & Schwartz (1991).

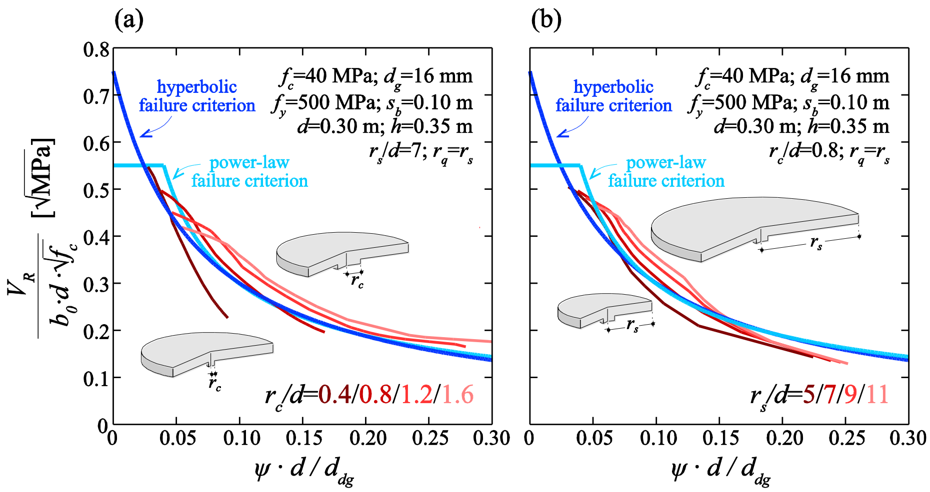

Figure 7. Parametric study of the normalized punching strength as a function of the normalized rotation based on the refined mechanical model (Simões, Fernández Ruiz & Muttoni 2018) and comparison with failure criteria of the CSCT (Muttoni, 2008; Muttoni, Fernández Ruiz & Simões 2018) by varying: (a) column radius-to-effective depth ratio; (b) slab radius-to-effective depth ratio.

Finally, by performing parametric studies with the refined mechanical model, the validity of the analytical failure criterion of the CSCT can also be validated for cases other than those corresponding to the experimental tests. Some results are for instance shown in Figure 7, where different numerically calculated failure criteria (by varying the flexural reinforcement ratio) are presented corresponding to different (a) columns sizes and (b) slenderness. The results depicted in Figure 7 show that, for the investigated cases, all failure criteria remain again within a narrow band and are well represented by the hyperbolic failure criterion, thus confirming the validity of the analytical failure criterion of the CSCT (Eq. (3)).

4 Simplification of the mechanical model of the CSCT for design and assessment

Practical design for punching according to the CSCT is usually performed by considering the simplified load-rotation relationship of Eq. (1) and the hyperbolic failure criterion (with characteristic values and partial safety factors, refer to Muttoni & al. (2013)). This formulation is very convenient for design of new structures as the punching strength can be checked directly by determining the rotation associated to the acting load and verifying that the punching strength corresponding to this rotation is larger than the acting load. However, when the failure load (defined as the intersection between the load-rotation curve and the failure criterion) needs to be calculated, an iterative procedure is usually required, which can be time-consuming.

As discussed by Muttoni, Fernández Ruiz & Simões (2018), it is not possible to derive a closed-form expression for the punching strength by combining Eq. (1) and Eq. (3). For that purpose, Muttoni & Fernández Ruiz (2017) suggested that the hyperbolic failure criterion of the CSCT may be suitably replaced by a power-law failure criterion as follows:

$$V_R = V_{Rc,0} \cdot \left(\frac{d_{dg}}{25 \cdot \psi \cdot d} \right)^{2/3} \leq V_{Rc,0} \tag{6}$$

where $V_{Rc,0}$ represents the maximum achievable punching strength (which can be approximated to $V_{Rc,0} = 0.55 \cdot b_0 \cdot d \cdot \sqrt{f_c}$ for design purposes) and corresponds to a failure mechanism governed by shear deformations (Muttoni, Fernández Ruiz & Simões 2018). As discussed in Simões, Fernández Ruiz & Muttoni (2018), refer to Figure 7, this power-law criterion provides also consistent estimates of the failure criterion when compared to the refined mechanical model. In addition, such failure criterion has the advantage of allowing the analytical derivation of closed-form expressions for the punching strength and deformation capacity (Muttoni and Fernández Ruiz, 2016; Muttoni, Fernández Ruiz & Simões 2018).

As shown by Muttoni & Fernández Ruiz (2017), a closed-form expression for the punching shear design can be obtained by combining Eqs. (6) (power-law failure criterion) and (1) (simplified load-rotation relationship), leading to the following relationship:

$$V_{Rc} = \sqrt{V_{Rc,0} \cdot V_{flex}} \cdot \left(\frac{d_{dg}}{25 \cdot k_m \cdot r_s} \cdot \frac{E_s}{f_y} \right)^{1/3} \leq V_{Rc,0} \tag{7}$$

Eq. (7) considers explicitly the flexural strength of the slab on the punching strength, as well as the slenderness and size effects or the concrete and reinforcement types. The influence of each parameter depends both upon the adopted failure criterion and load-rotation relationship (potential refinements on these laws may slightly influence the closed-form formula). By adopting some additional considerations, Eq. (7) can be further simplified. This work was performed by Muttoni, Fernández Ruiz & Simões (2018) by considering that:

$$V_{flex} = a \cdot m_R \qquad \text{where} \qquad m_R = k_1 \cdot d^2 \cdot (\rho \cdot f_y)^{k_2} \cdot f_c^{1-k_2} \tag{8}$$

with k_1=0.75 and k_2=0.9. Introducing Eq. (8) in Eq. (7), rounding some exponents and considering that E_s=200´000 MPa, Eq. (7) finally leads to:

$$V_{Rc} = k_b \cdot \left(100 \rho \cdot f_c \cdot \frac{d_{dg}}{r_s} \cdot \right)^{1/3} \cdot b_0 \cdot d \leq 0.55 \cdot b_0 \cdot d \cdot \sqrt{f_c} \tag{9}$$

where k_b refers to a shear-gradient enhancement factor, calculated as follows:

$$k_b = \sqrt{8 \cdot a \cdot \frac{d}{b_0}} \geq 1 \tag{10}$$

It can be noted that this latter parameter tends to one (decreases) when the control perimeter length-to-effective depth ratio increases (cases physically closer to one-way slab responses).

5 Discussion on the different approaches for punching design and assessment

According to what is presented in this paper, different approaches may be used to apply the CSCT. The choice of the approach to be used for a given case is mainly related to the needs discussed in Figure 8 and summarized in the following:

- *Research:* Very detailed analyses can be performed for research purposes, providing detailed information on the stress field and associated displacement field of a member. The refined mechanical model can be used to evaluate the punching strength of complex cases as well as to assess the applicability, to validate or to propose improvements of the analytical failure criterion of the CSCT (e.g. Simões, Fernández Ruiz & Muttoni (2018)). This criterion can be used in combination with a refined load-rotation relationship to compare the theory against experimental results (Muttoni 2008).

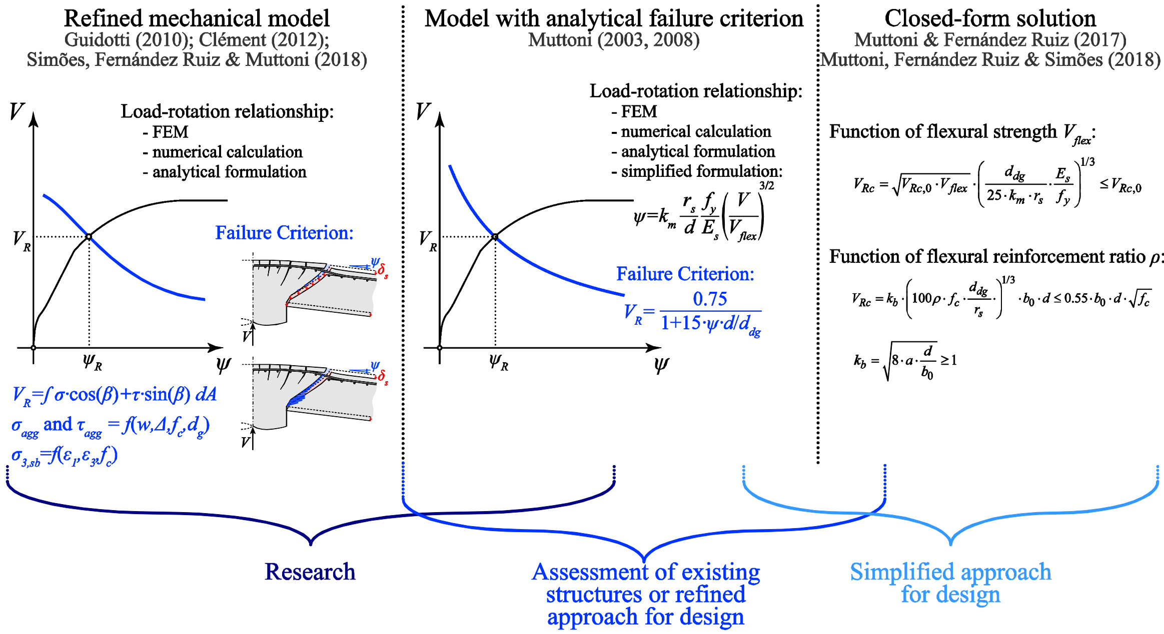

Figure 8. Different approaches for using the CSCT for design and assessment.

- *Assessment of existing structures or design of complex structures*: The assessment of existing structures can be performed using the closed-form solutions as a function of the flexural capacity of the slab (Muttoni & Fernández Ruiz 2017). For complex structures, the assessment or even the design may require the combination of load-rotation relationships calculated with linear or non-linear finite element methods and of the analytical failure criterion (considering characteristic values and partial safety factors) (see Muttoni 2008). The procedure required in this latter case approaches the one used in research to evaluate the punching strength of tested specimens.

- *Simplified design of new structures*: The design of new structures according to the CSCT may be performed using the closed-form design expression as a function of the flexural resistance (Eq. (7)) or of the flexural reinforcement ratio (Eq. (9), Muttoni, Fernández Ruiz & Simões 2018). The latter constitutes a very simple approach which remains valid for a wide range of cases (provided that the value of $a=V_{flex}/m_R$ is suitability estimated).

Figure 9 depicts the punching strength obtained with the different approaches for an investigated case with varying (a) flexural reinforcement ratio, (b) concrete compressive strength and (c) slab radius-to-effective depth ratio. The results show that all approaches lead to similar and consistent results.

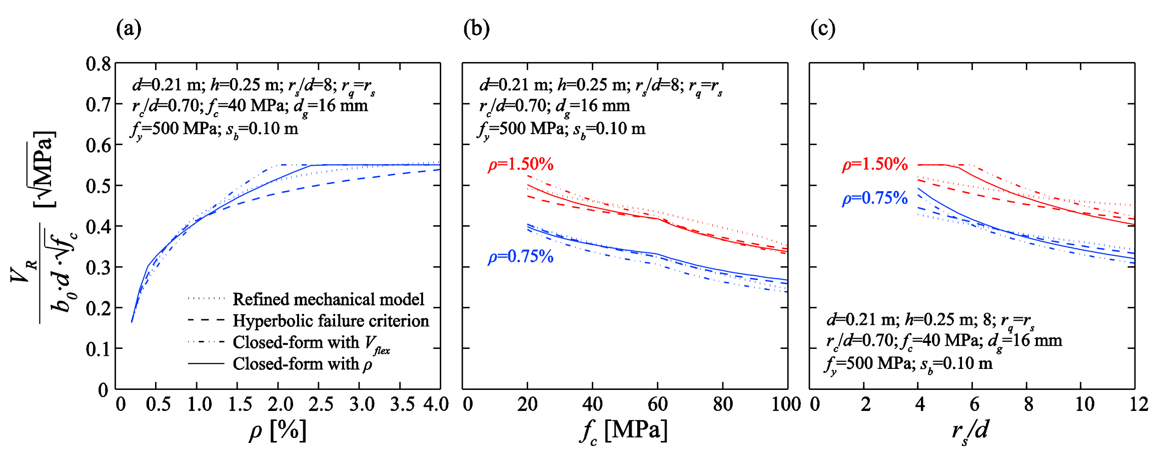

Figure 9. Comparison of the results of the refined mechanical model with the different approaches of the CSCT for a given case and as a function of the: (a) flexural reinforcement ratio; (b) concrete compressive strength; (c) slenderness-to-effective depth ratio.

6 Conclusions

This work presents the fundamental theoretical aspects of the Critical Shear Crack Theory (CSCT) as well as its recent developments. The main conclusions of this paper are the following:

1. The CSCT is a mechanical model that extends the ideas of the approach by Kinnunen & Nylander (1960). It is nevertheless based on a different consideration with respect to the failure criterion, considering that failure is governed by the formation and propagation of a critical shear crack (CSC; tangential crack with flexural origin developing in the shear-critical region);
2. Failures in punching occur when the shear demand at the CSC equals its capacity to transfer shear stresses. This capacity depends on the opening of the CSC and thus on the level of deformation of the member. The punching strength and its associated deformation capacity can thus be calculated by intersecting the load-deformation relationship of a slab and a failure criterion defining the maximum shear strength associated to a given state of deformations. The failure criterion may be calculated by integration of the stresses developing at the CSC by considering a realistic shape of the CSC and kinematics of the slab as well as suitable constitutive material laws;
3. The CSCT shows that the increase of rotations at failure decrease the contributions of all shear-transfer actions. This is a result of the larger crack openings which reduce the aggregate interlocking stresses, softens the concrete and increases the level of utilization of the flexural reinforcement (yielding of the reinforcement limiting the shear-transfer by dowel action)
4. A parametric study based on a refined model shows that the calculation of the failure criterion by integration of stresses can be replaced by the use of an analytical failure criterion without a significant loss of accuracy;
5. The mechanical model of the CSCT for punching may be applied following different approaches. A refined approach, to be applied for research purposes or for complex structures, includes the calculation of the intersection of a suitable load-rotation relationship (by means of refined analytical expressions or numerical analyses) and a failure criterion calculated by integration of the stresses at the CSC. A simpler approach, suited for the design of new structures and assessment of conventional structures, consists on the calculation of the punching strength by means of analytical expressions defining the failure criterion and load-rotation relationship or even by using closed-form design expressions (providing directly the intersection of these relationships)
6. The refined approach of the CSCT has the advantage to be general and applicable to complex cases. On the other hand, the application of analytical expressions for punching design (as intersection of a load-rotation curve and an analytical failure criterion or alternatively by using closed-form expressions) within the framework of the CSCT has the advantage to be very simple. When applied to conventional cases (regular rectangular flat slabs) for which the closed-form expressions are suited, all possible approaches yield similar and consistent results.

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