Modelling competition in demand-based optimization models

Stefano Bortolomiol
Virginie Lurkin Michel Bierlaire

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne

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Outline

1. Motivation
2. Modelling the problem
3. Current status of the research

1 Motivation

2 Modelling the problem

3 Current status of the research

Competition in transportation

- Competition is often present in the form of oligopolies (regulations, limited capacity of the infrastructure, barriers to entry, etc.).
- Deregulation often led to oligopolistic markets.
    - Airlines
    - Railways
    - Buses

Trending topic

TRANSPORTS FLIXBUS ET EUROBUS S'ALLIENT POUR DESSERVIR LA SUISSE

Les deux compagnies de bus Flixbus et Eurobus se sont mises d'accord pour démarrer le cabotage en Suisse à partir du 10 juin. C'est une concurrence accrue pour les CFF.

PAR PASCAL SCHMUCK
ZURICH
05.06.2018

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ARTICLES EN RELATION

- Gros succès de Flixbus en Suisse
- Flixbus épinglé pour cabotage

Flixbus s'implante en Suisse. A partir du 10 juin, la compagnie allemande de bus desservira les trajets St-Gall-Aéroport de Genève, Coire-Sion, Coire-Aéroport de Zurich et Bâle EuroAirport-Lugano. Elle s'associe avec Eurobus, la plus grande entreprise de bus en Suisse, révèle le *Blick*.

Réforme de la **SNCF** : à quoi va ressembler la suite après le vote du ...
LCI - 5 hours ago
Après son vote par l'Assemblée en avril, puis par le Sénat le 5 juin, le projet de loi de **réforme** de la **SNCF** doit faire l'objet d'une commission ...

Réforme de la **SNCF**. Faut-il vous préparer à une poursuite de la ...
Ouest-France - 6 Jun 2018

Contre la **réforme** ferroviaire, les cheminots envahissent le siège de la ...
Le HuffPost - 5 Jun 2018

SNCF : le Sénat a voté le projet de **réforme** ferroviaire
Franceinfo - 6 Jun 2018

Vote au Sénat de la **réforme** de la **SNCF** : la grève n'est pas finie
In-Depth - La Tribune.fr - 6 Jun 2018

Le Sénat vote la **réforme** de la **SNCF**
In-Depth - Le Figaro - 5 Jun 2018

How to study competitive transport markets?

- Modelling demand
- Modelling supply
- Modelling competition

Modelling demand

- Each customer chooses the alternative that maximizes his/her utility.
- Customers have different tastes and socioeconomic characteristics that influence their choice.

Modelling supply

- Operators take decisions that optimize their objective function (e.g. revenue maximization).
- Decisions can be related to pricing, capacity, frequency, availability ...
- Decisions are influenced by:
  - The preferences of the customers
  - The decisions of the competitors

Modelling competition

- We consider non-cooperative games.
- We aim at understanding the Nash equilibrium solutions of such games, i.e. stationary states of the system in which no competitor has an incentive to change its decisions.

1 Motivation

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Modelling the problem

Starting point:
MILP for the demand-based optimization problem for one operator (Pacheco et al. (2017)).

The goal:
MILP that models the non-cooperative multi-leader-follower game played by operators and customers.

The framework

Three-level framework: customers, operators and market.

1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.

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2. **Operators level**: a mixed integer linear program can maximize any relevant objective function.

The framework

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1. **Customer level**: discrete choice models take into account preference heterogeneity and model individual decisions. These can be integrated in a MILP by relying on simulation to draw from the distribution of the error term of the utility function.
2. **Operators level**: a mixed integer linear program can maximize any relevant objective function.
3. **Market level**: Nash equilibrium solutions are found by enforcing best response constraints.

The framework: customer level

- For all customers $n \in N$ and all alternatives $i \in I$, R draws are extracted from the error term distribution, each corresponding to a different behavioral scenario. For each $r \in R$ we have:

$$U_{inr} = \beta_{in} p_{in} + q_{in} + \xi_{inr}$$

where p_{in} is a variable endogenous to the optimization model, β_{in} is the corresponding parameter, q_{in} is the exogenous term and ξ_{inr} is the error term.

- In each scenario, customers choose the alternative with the highest utility:

$$w_{inr} = 1 \text{ if } U_{inr} = \max_{j \in I} U_{jnr}, \text{ and } w_{inr} = 0 \text{ otherwise}$$

- Over multiple scenarios, the probability of $n \in N$ choosing $i \in I$ is given by:

$$P_{in} = \frac{\sum_{r \in R} w_{inr}}{R}$$

The framework: operators level

- We assume that an operator $k \in K$ can decide on price p_{in} and availability y_{in} of each alternative $i \in C_k$ for all customers $n \in N$.
- Stackelberg game: the operator (the leader) knows the best response of the customers ("collective" follower) to all strategies.
- Objective function to be maximized by operator k:

$$V_k = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{in} w_{inr}$$

The framework: market level

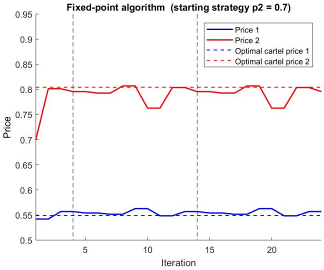
- The payoff of an operator also depends on the strategies of the competitors
- Let's define as X_k the set of strategies that can be played by operator $k \in K$
- Condition for Nash equilibrium (best response constraints):

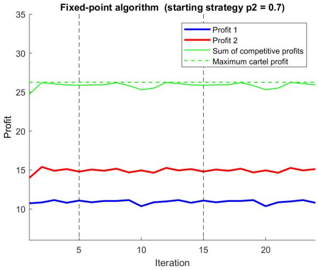
$$V_k = V_k^* = \max_{x_k} V_k(x_k, x_{K \setminus \{k\}}) \qquad \forall k \in K$$

- Nash (1951) proves that every finite game has at least one mixed strategy equilibrium solution

A fixed-point iteration method

- Sequential algorithm to find Nash equilibrium solutions of a two-player game:
  - Initialization: one player selects an initial feasible strategy.
  - Iterative phase: operators take turns and each plays its best response pure strategy to the last strategy played by the competitor.
  - Termination criterion: either a Nash equilibrium or a cyclic equilibrium is reached.





A fixed-point iteration method

- The algorithm reproduces the behavior of two or more operators that do not know the competitors' objective function.
- Different initial strategies can lead to different equilibria.
- There is no guarantee that a pure strategy Nash equilibrium exists or that it is unique.

MILP formulations

Pure strategies:

- Each operator $k \in K$ chooses a pure strategy from a finite set S_k.
- Number of pure strategy solutions of the game: $|S| = \prod_{k \in K} S_k$.
- For each solution $s \in S$ we can derive a payoff function V_{ks} for each operator $k \in K$.
- If $s \in S$ includes only best response strategies for all operators, then it is a pure strategy Nash equilibrium for the finite game.

Mixed strategies:

- Operator k chooses a mixed strategy from the finite set S_k, i.e. a vector of probabilities p_{s_k} associated to all pure strategies s_k in S_k, such that $\sum_{s_k \in S_k} p_{s_k} = 1$.

Customer level

Customer constraints:

$$\sum_{i \in I} w_{inrs} = 1 \qquad \forall n \in N, \forall r \in R, \forall s \in S \qquad (1)$$

$$w_{inrs} \leq y_{inrs} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (2)$$

$$y_{inrs} \leq y_{ins} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (3)$$

$$y_{ins} = 0 \qquad \forall i \in I, \forall n \in N : i \notin C_n, \forall s \in S \qquad (4)$$

$$\sum_{n \in N} w_{inrs} \leq C_i \qquad \forall i \in I \setminus \{0\}, \forall r \in R, \forall s \in S \qquad (5)$$

$$C_i(y_{ins} - y_{inrs}) \leq \sum_{m \in N : L_{im} < L_{in}} w_{imrs} \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R, \forall s \in S \qquad (6)$$

$$\sum_{m \in N : L_{im} < L_{in}} w_{imrs} \leq (C_i - 1) y_{inrs} + (n - 1)(1 - y_{inrs}) \qquad \forall i \in I \setminus \{0\}, \forall n \in N, \forall r \in R, \forall s \in S \qquad (7)$$

$$U_{inrs} = \beta_{in} p_{ins} + q_{in}^d + \xi_{inr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (8)$$

$$lbU_{nr} \leq z_{inrs} \leq lbU_{nr} + M_{U_{nr}} y_{inrs} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (9)$$

$$U_{inrs} - M_{U_{nr}}(1 - y_{inrs}) \leq z_{inrs} \leq U_{inrs} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (10)$$

$$z_{inrs} \leq U_{nr} \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (11)$$

$$U_{nr} \leq z_{inrs} + M_{U_{nr}}(1 - w_{inrs}) \qquad \forall i \in I, \forall n \in N, \forall r \in R, \forall s \in S \qquad (12)$$

Operator and market level (Pure strategies)

Find $s \in S$ such that $e_s = 1$

$s.t.$

Equilibrium constraints:

$$e_s \geq \sum_{k \in K} x_{ks} - (|K| - 1) \qquad \forall s \in S \qquad (13)$$

$$e_s \leq x_{ks} \qquad \forall k \in K, \forall s \in S \qquad (14)$$

Operator constraints:

$$V_{ks} = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \qquad \forall k \in K, \forall s \in S \qquad (15)$$

$$V_{ks} \leq V_{kt}^{max} \qquad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \qquad (16)$$

$$V_{kt}^{max} \leq V_{ks} + M_r(1 - x_{ks}) \qquad \forall k \in K, \forall s \in S_k, \forall t \in S_k^C \qquad (17)$$

$$\sum_{s \in S} x_{ks} = \left| S_k^C \right| \qquad \forall k \in K \qquad (18)$$

Operator and market level (Mixed strategies)

Find $p_{s_k}, b_{s_k}, r_{s_k}, V_{s_k}, V_k$ such that... or $\max \sum_{k\in K} V_k$ or...

$s.t.$

MILP mixed-strategy Nash:

$$\sum_{s_k \in S_k} p_{s_k} = 1 \qquad \forall k \in K \qquad (19)$$

$$V_{s_k} = \sum_{s_k^C \in S_k^C} p_{s_k^C} V_k(s_k, s_k^C) \qquad \forall k \in K, \forall s_k \in S_k \qquad (20)$$

$$V_k \geq V_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (21)$$

$$r_{s_k} = V_k - V_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (22)$$

$$p_{s_k} \leq 1 - b_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (23)$$

$$r_{s_k} \leq M b_{s_k} \qquad \forall k \in K, \forall s_k \in S_k \qquad (24)$$

Pure strategy payoffs:

$$V_k(s_k, s_k^C) = \frac{1}{R} \sum_{i \in C_k} \sum_{n \in N} \sum_{r \in R} p_{ins} w_{inrs} \qquad \forall k \in K, \forall (s_k, s_k^C) \in S \qquad (25)$$

Numerical example: pure strategy equilibria

(a) Game with 1 pure strategy Nash equilibrium

Payoff matrix of player 1

S1 \ S2	*0,70*	*0,73*	*0,76*	*0,79*	*0,82*	*0,85*
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,53	10,49	**10,60**	**10,60**	10,60	10,60	10,60
0,56	**10,53**	10,42	10,53	**10,86**	**11,20**	11,20
0,59	10,27	10,03	9,80	9,91	10,62	**11,45**
0,62	10,04	9,80	9,42	9,42	9,42	9,92
0,65	9,62	9,36	8,84	8,45	8,71	8,58

Payoff matrix of player 2

S1 \ S2	*0,70*	*0,73*	*0,76*	*0,79*	*0,82*	*0,85*
0,50	14,00	14,45	14,74	14,69	**14,76**	14,62
0,53	14,00	14,45	14,74	**15,01**	14,60	14,45
0,56	14,00	14,60	14,74	**14,85**	14,76	14,28
0,59	14,00	14,60	15,05	**15,48**	15,09	14,45
0,62	14,00	14,60	15,20	15,48	**15,91**	15,81
0,65	14,00	14,60	15,20	15,80	15,91	**16,32**

(b) Game with no pure strategy Nash equilibrium

Payoff matrix of player 1

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	10,00	10,00	10,00	10,00	10,00	10,00
0,52	10,40	10,40	10,40	10,40	10,40	10,40
0,54	**10,80**	**10,80**	10,80	10,80	10,80	10,80
0,56	10,42	10,53	**10,86**	**11,09**	**11,20**	11,20
0,58	9,74	9,86	10,09	10,44	10,67	**11,37**
0,60	9,60	9,60	9,72	10,08	10,44	10,68

Payoff matrix of player 2

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	14,70	**14,78**	14,69	14,74	14,28	14,62
0,52	14,70	**15,09**	14,85	14,58	14,61	14,45
0,54	14,85	14,94	**15,17**	14,74	14,44	14,45
0,56	14,85	**14,94**	14,85	14,90	14,61	14,28
0,58	15,00	15,09	**15,17**	15,07	15,11	14,45
0,60	15,00	15,25	**15,48**	15,39	15,27	14,30

Payoff matrices for two games with different support strategies. Best response payoffs are in bold. Equilibrium payoffs are in blue.

Numerical example: mixed strategy equilibria

Payoff matrices of player 1 and player 2

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*	p_1	V_1
0,50	10,00	10,00	10,00	10,00	10,00	10,00	*0*	*10,00*
0,52	10,40	10,40	10,40	10,40	10,40	10,40	*0*	*10,40*
0,54	10,80	10,80	10,80	10,80	10,80	10,80	*0.27*	*10,80*
0,56	10,42	10,53	10,86	11,09	11,20	11,20	*0.73*	*10,80*
0,58	9,74	9,86	10,09	10,44	10,67	11,37	*0*	*10,05*
0,60	9,60	9,60	9,72	10,08	10,44	10,68	*0*	*9,70*

S1 \ S2	*0,75*	*0,77*	*0,79*	*0,81*	*0,83*	*0,85*
0,50	14,70	14,78	14,69	14,74	14,28	14,62
0,52	14,70	15,09	14,85	14,58	14,61	14,45
0,54	14,85	14,94	15,17	14,74	14,44	14,45
0,56	14,85	14,94	14,85	14,90	14,61	14,28
0,58	15,00	15,09	15,17	15,07	15,11	14,45
0,60	15,00	15,25	15,48	15,39	15,27	14,30
p_2	*0*	*0.19*	*0.81*	*0*	*0*	*0*
V_2	*14.85*	*14.94*	*14.94*	*14.86*	*14.56*	*14.33*

Figure: Game with mixed strategy Nash equilibrium

Discussion

- The model requires finite strategy sets (enumeration), therefore the problem is solvable with small solution spaces only.
- The assumption of a finite game requires price discretization.
- Formulation 1: all pure strategy Nash equilibria of the game can be found, if they exist.
- Formulation 2: among the mixed strategy Nash equilibria, it is possible to select one by choosing a relevant objective function, e.g. total welfare maximization.

1 Motivation

2 Modelling the problem

3 Current status of the research

A MILP model for the fixed-point problem

- The fixed-point iteration method stops when the same strategies are played in two consecutive iterations.
- What if we can write a MILP model to minimize the "difference" in strategies between two consecutive iterations?

A MILP model for the fixed-point problem

- A solution for a two-operator problem: (x_1, x_2)
- Optimization problem for operator 1:

$$x_1^* = \arg\max_{x_1} V_1(x_1, x_2, (x_{cust}))$$

- Optimization problem for operator 2:

$$x_2^* = \arg\max_{x_2} V_1(x_1, x_2, (x_{cust}))$$

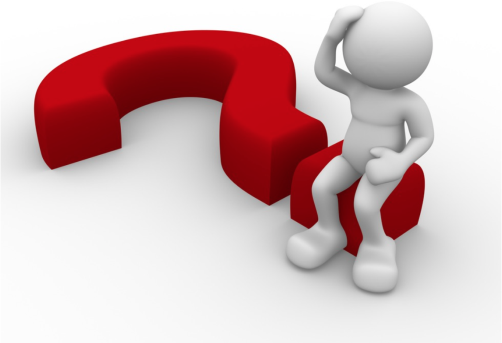
- Fixed-point problem:

$$\min_{x_1, x_2, x_1^*, x_2^*} \|x_1^* - x_1\| + \|x_2^* - x_2\|$$

Future work

- Implement and test the MILP model for the fixed-point problem.
- Efficient search for equilibria in the solution space.
- Investigation of the concept of Nash equilibrium region for real-life applications.

Questions?



Stefano Bortolomiol

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne (EPFL)

Email: stefano.bortolomiol(at)epfl.ch