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Nonlinear PD plus sliding mode control with application to a parallel delta robot

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In this paper, a hybrid nonlinear proportional-derivative-sliding mode controller (NPD-SMC) is developed for the trajectory tracking of robot manipulators. The proposed controller combines the advantage of the easy implementation of NPD control and the robustness of SMC. The gains of PD control are tuned on-line in order to increase the convergence rate, whereas the SMC term is introduced to reject the external disturbances without requiring to know the system dynamics. The stability of the NPD-SMC is proved using Lyaponuv theorem, and it is demonstrated that the tracking error and the tracking error rate converge asymptotically to zero. Experiments are carried out on the parallel Delta robot to illustrate the effectiveness and robustness of the proposed approach. It is also shown the superiority of the NPD-SMC control over the NPD control and PD-SMC control.

K e y w o r d s: nonlinear PD, sliding mode control, stability, delta robot, trajectory tracking

1 Introduction

The utilisation of robot manipulators has been increased during the last decades thanks to their high quality of manufacture. Meanwhile, much research has been conducted to solve the trajectory tracking problem. Usually, the conventional PD/PID controllers are used due to their easy implementation and acceptable performances. However, PD/PID failed to track the desired trajectory for high-dynamic movement where strong nonlinearity and coupling have an important effect. To increase the robustness of PD/PID, nonlinear gains tuned on-line using the tracking error and its time derivative were introduced instead of the fixed gain [1, 2]. Nevertheless, the robustness range of NPD/NPID is limited by controller structure, high dynamic movement, and external disturbances. To overcome this shortcoming, many control strategies have been proposed [3–7]. Fuzzy logic control was implemented successfully in some robots [8]. However, the choice of the linguistic rules and guaranteeing the system stability still remain a challenging issue. Moreover, the industrial implementation is not an easy task. In many adaptive control [6], the dynamic model is supposed a premultiplied of two separate linear unknown parameters with known nonlinear functions. However, finding this separation cannot always be systematic, especially for complex systems. Sliding mode control is knowing by its ability to decrease the tracking error with a wide range of robustness for nonlinear systems subject to model uncertain and disturbances. The drawback of SMC is that require to know the system dynamic, as a consequence, the implementation can be challenging and priced. Another issue represents in the chattering phenomena which can damage the actuators. In order to benefit from the SMC robustness and the easy implementation of PD control, a hybrid PD plus SMC (PD-SMC) schemes have been proposed in some works. In [9] the authors developed PD-SMC to solve the trajectory tracking problem of robot manipulators with an application to a two-link SCARA robot, where PD control is applied in the reaching phase and the semicontinuous sliding mode control is applied in the sliding mode phase. However, semicontinuous sliding mode phase requires the knowledge of the exact dynamic which could be a challenging issue. In [10] an adaptive sliding mode control with PID tuning was proposed for uncertain systems. However, the controller is complex due to the requirement of the dynamic model. Moreover, there is no experimental validation. In [11], a PD-SMC was developed for trajectory tracking control of robot manipulators where some tuning rules were also discussed through simulation. However, the inertia matrix, the Coriolis and centrifuge matrix and the gravity vector were considered constant, as a consequence, the convergence analysis cannot be generalised to the other robot manipulators such as the parallel Delta robot.

Delta robot is a parallel manipulator invented by R. Clavel [12] in order to execute pick and place operations at a high dynamic movement. Many control strategies have been implemented on the Delta robot such as, PD plus feedforward control [13], a linear robust H-infinity control that was synthesised around an operating point [14] and an adaptive control that was developed for uncertain model in [15]. However, the drawback of the above-mentioned works is that knowing the exact dynamic model is crucial to satisfy the required performances.

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This paper proposes a NPD-SMC to solve the trajectory tracking problem of robot manipulators. In the proposed controller a nonlinear PD control replaced the fixed gains PD control in order to increase the convergence rate and a SMC is introduced in order to increase the robustness range and to reject the disturbances. Unlike [9, 11], the proportional and derivative gains are adapted on-line using the tracking error and its time derivative respectively. Unlike, [11] the dynamic model matrices are not constant but depend on the joints coordinate. Convergence analysis of the proposed approach is driven using Lyapunov technique. Experiments are carried out on a parallel Delta robot and a comparative study between the NPD, the PD-SMC and the NPD-SMC is provided in order to show the effectiveness and the high tracking performances of this latter.

Throughout this paper, the norm of vector x is defined as $\|x\| = \sqrt{x^\top x}$, and the norm of matrix A is defined as follows: $\|A\| = \sqrt{\lambda_{max}(A^\top A)}$, where $\lambda_{max}(\cdot)$ indicate the maximum eigenvalue of matrix $(\cdot)$.

2 Problem formulation

Consider a rigid robot manipulator with a n-degree of freedom (DOF) described by following dynamic equation

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + w(t) = \tau \tag{1}$$

where $q \in \mathbb{R}^n$ is the generalised joint vector, $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is a vector resulting from Coriolis and centrifugal forces, $G(q) \in \mathbb{R}^n$ is the gravity torque vector, $\tau \in \mathbb{R}^n$ is the control input vector containing the torques to be applied at each joint and $w(t) \in \mathbb{R}^n$ is the vector contain external disturbances.

The actual dynamics (1) can be written using the nominal model in the following formulation

$$M_n(q)\ddot{q} + C_n(q,\dot{q})\dot{q} + G_n(q) + d(t) = \tau\,. \tag{2}$$

The expression of the lumped disturbances $d(t)$ is given as follows

$$d(t) = \Delta M(q)\ddot{q} + \Delta C(q,\dot{q})\dot{q} + \Delta G(q) + w(t) \tag{3}$$

where Δ indicates the modelling error.

The following properties, common to robot manipulators are considered [16–18]:

(P1) The inertia matrix $M_n(q)$ is symmetric, positive and satisfies

$$0 < \beta_1 \le \|M_n(q)\| \le \beta_2 \tag{4}$$

where β_1, and β_2 are known positive constants.

(P2) The norm of the Coriolis-centrifugal matrix and the gravity vector is bounded as follows

$$\|C_n(q,\dot{q})\| \le K_c\|\dot{q}\|,\ \|G_n(q)\| \le K_G,\ \forall\, q,\ \dot{q} \in \mathbb{R}^n \tag{5}$$

where K_c and K_G are a known positive constants.

(P3) The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew symmetric, hence

$$q^\top(\dot{M}(q) - 2C(q,\dot{q}))q = 0 \quad \forall q \in \mathbb{R}^n. \tag{6}$$

Throughout this paper, the following reasonable assumptions are made:

Assumption 1. The robot velocity is bounded by a known constant V_m, such that

$$\|\dot{q}\| \le V_m\,. \tag{7}$$

Assumption 2. The lumped disturbances are bounded by a constant l_d, such that

$$\|d(t)\| \le l_d\,. \tag{8}$$

Assumption 3. The desired trajectory $q_d(t)$ up to its nth derivative are continuous, bounded and available, for instance

$$\|\ddot{q}_d(t) + \lambda\dot{q}_d(t)\| \le \alpha \tag{9}$$

where α and λ are positive constants.

Remark 1. Assumption 1 is very common in real time application where many real systems are subjected to state and input constraints due to the safety requirements and the physical limitations of the actuators. As a result, the velocity joints can be bounded by a known constant. This assumption has been considered in many works such as [19, 20].

Remark 2. Assumption 2 is very reasonable for real-time application where due to the safety requirements and the tracking error tolerance, the disturbances are always bounded. This assumption has been considered in many real-time applications such as mobile wheeled inverted pendulum [21].

The control objective of this paper is to design a robust control law for system (2) such that the tracking position error and the tracking velocity error can converge asymptotically to zero with a simple implementation scheme.

3 Proposed NPD-SMC

3.1 Controller design

The proposed control law is expressed as

$$\tau(t) = K_p(\tilde{q})\tilde{q} + K_d(\dot{\tilde{q}})\dot{\tilde{q}} + H sgn(\dot{\tilde{q}} + \lambda\tilde{q}) \tag{10}$$

and

$$K_p(\tilde{q}) = \mathrm{diag}\{K_{p_1}(\tilde{q}_1), \ldots, K_{p_n}(\tilde{q}_n)\} \tag{11}$$

$$K_d(\dot{\tilde{q}}) = \mathrm{diag}\{K_{d_1}(\dot{\tilde{q}}_1), \ldots, K_{d_n}(\dot{\tilde{q}}_n)\} \tag{12}$$

where

$$K_{p_i}(\tilde{q}_i) = \begin{cases} K_{p0}|\tilde{q}_i|^{\alpha_1 - 1}, & |\tilde{q}_i| > \delta_1,\ i = 1, \dots, n, \\ K_{p0}\delta_1^{\alpha_1 - 1}, & |\tilde{q}_i| \le \delta_1, \\ K_{\mathrm{pmax}}, & K_{p_i}(\tilde{q}_i) \ge K_{\mathrm{pmax}}, \\ K_{\mathrm{pmin}}, & K_{p_i}(\tilde{q}_i) \le K_{\mathrm{pmin}}, \end{cases} \tag{13}$$

$$K_{d_i}(\dot{\tilde{q}}_i) = \begin{cases} K_{d0}|\dot{\tilde{q}}_i|^{\alpha_2 - 1}; & |\dot{\tilde{q}}_i| > \delta_2,\ i = 1, \dots, n, \\ K_{d0}\delta_2^{\alpha_2 - 1}, & |\dot{\tilde{q}}_i| \le \delta_2, \\ K_{\mathrm{dmax}}, & K_{d_i}(\dot{\tilde{q}}_i) \ge K_{\mathrm{dmax}}, \\ K_{\mathrm{dmin}}, & K_{d_i}(\dot{\tilde{q}}_i) \le K_{\mathrm{dmin}}. \end{cases} \tag{14}$$

Index i indicates the ith row of a vector. The expression of $\tilde{q}$ and $\dot{\tilde{q}}$ is

$$\tilde{q} = q_d - q\,, \qquad \dot{\tilde{q}} = \dot{q}_d - \dot{q}$$

where, q_d and $\dot{q}_d$ represent the desired joint position and the desired joint velocity respectively. The gains K_{p0}, K_{pmax}, K_{pmin}, K_{d0}, K_{dmax}, K_{dmin} and H are positive constant.

Variables α_1 and α_2 refer to the nonlinearity, while δ_1 and δ_2 refer to the threshold of the error and its time derivative respectively.

Remark 3. The nonlinear gains (13) and (14) indicate that a small gain will be used for a small error and a large gain for a large error. On the other hand, a small gain will be used for a small change of the error a large gain for a large change of the error.

Remark 4. As it can be seen from the controller law, a very important aspect of the NPD-SMC reside in its easy implementation and integration in the industrial robots.

3.2 Convergence analysis

The convergence of the proposed NPD-SMC is guaranteed by the following theorem.

Theorem. *Consider the system* (2) *under the control law* (10)*, assumption* (A1–A3) *and satisfies properties* (P1-P3)*. Then the tracking position error and the tracking velocity error converge asymptotically to zero, ie,* $\lim_{t\to\infty}\tilde{q}(t) = \lim_{t\to\infty}\dot{\tilde{q}}(t) = 0$*, provided that the control gains are chosen such that*

$$H > \alpha\beta_2 + K_c V_m^2 + \lambda\beta_2 V_m + K_G + l_d\,, \tag{15}$$

$$2\lambda K_{\mathrm{pmin}} > 2\lambda\mu + 2\lambda^2\mu + \lambda K_{\mathrm{dmax}}\,, \tag{16}$$

$$2K_{\mathrm{dmin}} > 2\lambda\mu + 2\mu + \lambda K_{\mathrm{dmax}} \tag{17}$$

where, $\mu = \max\|C_n(q(t), \dot{q}(t))\|$*, for all* $t \in \mathbb{R}^+$.

Proof. To simplify the notation, let

$$M_n(q) = M_n,\ \ C_n(q,\dot{q}) = C_n,\ \ G_n(q) = G_n,\ \ d(t) = d,$$
$$K_p(\tilde{q}(t)) = K_p,\ \ K_d(\dot{\tilde{q}}(t)) = K_d\,.$$

Define a Lyapunov function as follows

$$V(\tilde{q}(t), \dot{\tilde{q}}(t)) = \frac{1}{2}(\dot{\tilde{q}} + \lambda\tilde{q})^\top M_n(\dot{\tilde{q}} + \lambda\tilde{q}) + \int_0^{\tilde{q}} \sigma^\top K_p(\sigma)d\sigma \tag{18}$$

Function $\int_0^{\tilde{q}} \sigma^\top K_p(\sigma)d\sigma$ is positive definite as shown in [1]. Also, that above integral is radially unbounded with respect to $\tilde{q}$ and this implies $\int_0^{\tilde{q}} \sigma^\top K_p(\sigma)d\sigma \to \infty$ as $|\tilde{q}| \to \infty$. By differentiating V with respect to time, we obtain

$$\dot{V} = (\dot{\tilde{q}} + \lambda\tilde{q})^\top M_n(\ddot{\tilde{q}} + \lambda\dot{\tilde{q}}) + \frac{1}{2}(\dot{\tilde{q}} + \lambda\tilde{q})^\top \dot{M}_n(\dot{\tilde{q}} + \lambda\tilde{q}) + \tilde{q}^\top K_p(\tilde{q})\dot{\tilde{q}} \tag{19}$$

The differentiation of $\int_0^{\tilde{q}} \sigma^\top K_p(\sigma)\mathrm{d}\sigma$ is made via the Leibniz rule for differentiation of integrals.

By substituting (2) into (19), we get

$$\dot{V} = (\dot{\tilde{q}} + \lambda\tilde{q})^\top M_n\big[\ddot{q}_d - M_n^{-1}(\tau - C_n\dot{q} - G_n - d) + \lambda\dot{\tilde{q}}\big] + \frac{1}{2}(\dot{\tilde{q}} + \lambda\tilde{q})^\top \dot{M}_n(\dot{\tilde{q}} + \lambda\tilde{q}) + \tilde{q}^\top K_p\dot{\tilde{q}}\,. \tag{20}$$

Using property (P3), (20) becomes

$$\dot{V} = (\dot{\tilde{q}} + \lambda\tilde{q})^\top M_n\big[\ddot{q}_d - M_n^{-1}(\tau - C_n\dot{q} - G_n - d) + \lambda\dot{\tilde{q}}\big] + (\dot{\tilde{q}} + \lambda\tilde{q})^\top C_n(\dot{\tilde{q}} + \lambda\tilde{q}) + \tilde{q}^\top K_p\dot{\tilde{q}}\,. \tag{21}$$

Substituting the control law (10) into (21) yields

$$\dot{V} = (\dot{\tilde{q}} + \lambda\tilde{q})^\top\big[M_n\ddot{q}_d - K_p\tilde{q} - K_d\dot{\tilde{q}} - Hsgn(\dot{\tilde{q}} + \lambda\tilde{q}) + C_n\dot{q} + G_n + d + M_n\lambda\dot{\tilde{q}} + C_n(\dot{\tilde{q}} + \lambda\tilde{q})\big] + \tilde{q}^\top K_p\dot{\tilde{q}} \tag{22}$$

Then (22) can be rewritten as

$$\dot{V} = (\dot{\tilde{q}} + \lambda\tilde{q})^\top\big[M_n(\ddot{q}_d + \lambda\dot{q}_d) - K_p\tilde{q} - K_d\dot{\tilde{q}} - Hsgn(\dot{\tilde{q}} + \lambda\tilde{q}) + (C_n - M_n\lambda)\dot{q} + G_n + d + C_n(\dot{\tilde{q}} + \lambda\tilde{q})\big] + \tilde{q}^\top K_p\dot{\tilde{q}}\,. \tag{23}$$

Therefore

$$\begin{aligned}\dot{V} \le{}& (\dot{\tilde{q}} + \lambda\tilde{q})^\top(-K_p\tilde{q} - K_d\dot{\tilde{q}} + C_n(\dot{\tilde{q}} + \lambda\tilde{q})) + \tilde{q}^\top K_p\dot{\tilde{q}} \\ &+ \|\dot{\tilde{q}} + \lambda\tilde{q}\|\|M_n(\ddot{q}_d + \lambda\dot{q}_d)\| + \|\dot{\tilde{q}} + \lambda\tilde{q}\|\|C_n\dot{q}\| \\ &+ \|\dot{\tilde{q}} + \lambda\tilde{q}\|\|M_n\lambda\dot{q}\| + \|\dot{\tilde{q}} + \lambda\tilde{q}\|\|G_n + d\| \\ &+ \|\dot{\tilde{q}} + \lambda\tilde{q}\|(-H)\,.\end{aligned} \tag{24}$$

We can obtain based on properties (P1–P2) and assumptions (1–3)

$$\dot{V} \le (\dot{\tilde{q}} + \lambda\tilde{q})^\top(-K_p\tilde{q} - K_d\dot{\tilde{q}} + C_n(\dot{\tilde{q}} + \lambda\tilde{q})) + \tilde{q}^\top K_p\dot{\tilde{q}} + \|\dot{\tilde{q}} + \lambda\tilde{q}\|(\alpha\beta_2 + K_c V_m^2 + \lambda\beta_2 V_m + K_G + l_d - H)\,. \tag{25}$$

Fig. 1. The delta robot

Table 1. Geometric and dynamic parameters

Parameter	Description	Value
L_A	Length of upper arm	0.380 m
L_B	Length of forearm	0.205 m
m_n	Mass of the travelling plate	0.042 kg
m_{br}	Mass of the upper arm	0.098 kg
m_{fb}	Masses of the forearms	0.028 kg
m_c	Mass of the elbow	0.015 kg

We have

$$(\dot{\tilde{q}}+\lambda\tilde{q})^\top(-K_p\tilde{q}-K_d\dot{\tilde{q}}+C_n(\dot{\tilde{q}}+\lambda\tilde{q}))+\tilde{q}^\top K_p\dot{\tilde{q}}\le -\lambda K_{pmin}\|\tilde{q}\|^2-K_{dmin}\|\dot{\tilde{q}}\|^2+\|\tilde{q}\|\|\lambda K_d\|\|\dot{\tilde{q}}\|+\lambda^2\mu\|\tilde{q}\|^2 + \mu\|\dot{\tilde{q}}\|^2+2\lambda\mu\|\tilde{q}\|\|\dot{\tilde{q}}\|\,. \tag{26}$$

Then, we can obtain

$$(\dot{\tilde{q}}+\lambda\tilde{q})^\top(-K_p\tilde{q}-K_d\dot{\tilde{q}}+C_n(\dot{\tilde{q}}+\lambda\tilde{q}))+\tilde{q}^\top K_p\dot{\tilde{q}}\le -\lambda K_{pmin}\|\tilde{q}\|^2-K_{\mathrm{dmin}}\|\dot{\tilde{q}}\|^2+\|\tilde{q}\|(\lambda K_{\mathrm{dmax}})\|\dot{\tilde{q}}\|+\lambda^2\mu\|\tilde{q}\|^2 + \mu\|\dot{\tilde{q}}\|^2+2\lambda\mu\|\tilde{q}\|\|\dot{\tilde{q}}\|\,. \tag{27}$$

Considering the fact that

$$\|\tilde{q}\|\|\dot{\tilde{q}}\|\le\frac{\|\tilde{q}\|^2}{2}+\frac{\|\dot{\tilde{q}}\|^2}{2} \tag{28}$$

leads to the following upper bound

$$(\dot{\tilde{q}}+\lambda\tilde{q})^\top\big(-K_p\tilde{q}-K_d\dot{\tilde{q}}+C_n(\dot{\tilde{q}}+\lambda\tilde{q})\big)+\tilde{q}^\top K_p\dot{\tilde{q}}\le \big(-\lambda K_{pmin}+\frac{\lambda K_{dmax}}{2}+\lambda\mu+\lambda^2\mu\big)\|\tilde{q}\|^2 + (-K_{\mathrm{dmin}}+\frac{\lambda K_{\mathrm{dmax}}}{2}+\lambda\mu+\mu)\|\dot{\tilde{q}}\|^2. \tag{29}$$

As a result (25) becomes

$$\dot{V}\le\|\dot{\tilde{q}}+\lambda\tilde{q}\|\big(\alpha.\beta_2+K_cV_m^2+\lambda.\beta_2.V_m+K_G+l_d-H\big) + \big(-\lambda K_{\mathrm{pmin}}+\frac{\lambda K_{\mathrm{dmax}}}{2}+\lambda\mu+\lambda^2\mu\big)\|\tilde{q}\|^2 + (-K_{\mathrm{dmin}}+\frac{\lambda K_{\mathrm{dmax}}}{2}+\lambda\mu+\mu)\|\dot{\tilde{q}}\|^2. \tag{30}$$

By using (15), (16) and (17) it is obvious that

$$\dot{V}\le 0\,. \tag{31}$$

Therefore, $\dot{V}$ is negative definite. Since V is positive definite and $\dot{V}$ is negative definite, Lyapunov's method can guarantees that the tracking error and its time derivative are globally uniformly asymptotically stable [22].

We conclude that

$$\lim_{t\to\infty}\dot{\tilde{q}}=0\,, \tag{32}$$

$$\lim_{t\to\infty}\tilde{q}=0 \tag{33}$$

which concludes the proof. As it can be noticed, the convergence analysis indicates that there exist positive gains allow us to obtain the asymptotic convergence.

R e m a r k 5. It is worth noting that the sign function used in the proposed control (10) might lead to the chattering phenomenon in control inputs. To reduce this phenomenon in practical applications, saturation function can be introduced instead. As a consequence, the tracking error converges to a domain around zero with a smooth control signal.

4 Experimental studies

In this section, we present the experimental results obtained by applying the NPD-SMC on the Delta robot illustrated in Fig. 1.

The Delta robot has 4 DOF, three translations respectively in the directions x, y and z and the rotation Rz, of the mobile plate around the axis z. The inverse dynamic model of the Delta robot is developed based on the principle of virtual work [13], where:

$$M(q)=I_b+m_{nt}J^\top J\,,\quad C(q,\dot{q})=J^\top m_{nt}\dot{J}\,,$$
$$G(q)=-\tau_{Gn}-\tau_{Gb}\,.$$

For the detailed expressions of the Jacobian, m_{nt},τ_{Gn} and τ_{Gb} please refer to [13]. The parameters of the geometrical and dynamic models are detailed in Table 1.

The experimental implementation is carried out on a Core Duo PC, running at 2,8 GHz under Windows XP and a real-time extension (RTX) from IntervalZero. The robot actuators are brushed DC motors (minertia motor mini series UGTMEM-03LB2), with a belt-driven transmission of ratio $r_g=12$.

The control algorithms were programmed in C language and execute at a sampling time of 1 ms.

The controller gains are selected as: $K_{p0}=2.2$, $K_{d0}=0.5$, $K_{pmin}=1.2$, $K_{\mathrm{pmax}}=3.0$, $K_{\mathrm{dmin}}=0.3$, $K_{\mathrm{dmax}}=0.7$, $\lambda=0.5$, $\alpha_1=0.4$, $\alpha_2=0.6$, $\delta_1=0.2$, $\delta_2=0.7$,

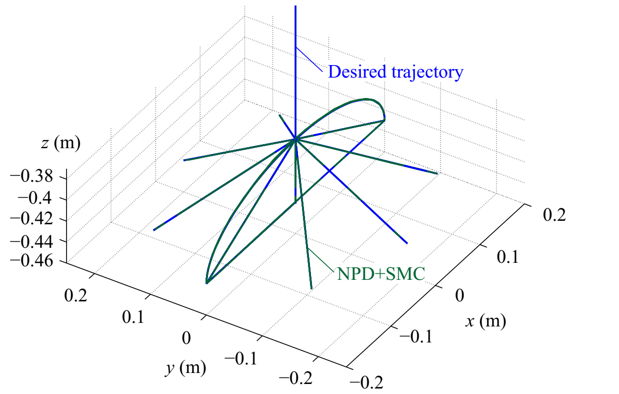

Fig. 2. The operational trajectory tracking under the proposed controlle

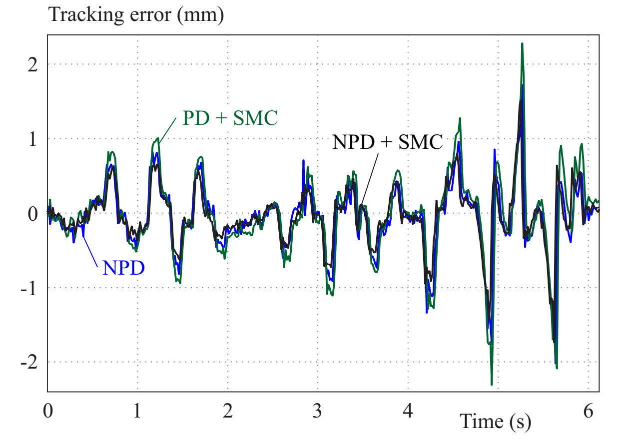

Fig. 3. Experimental tracking error in x-axis for case 1

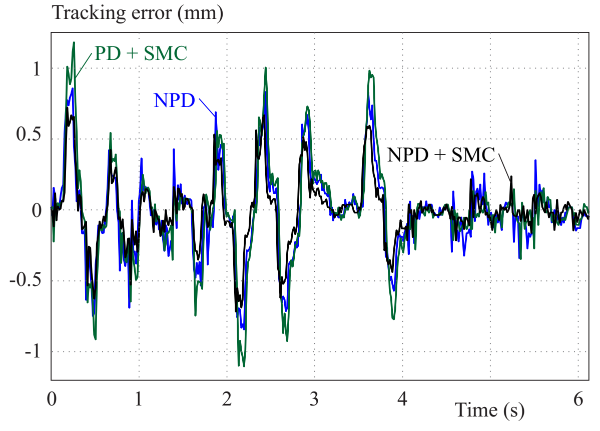

Fig. 4. Experimental tracking error in y-axis for case 1

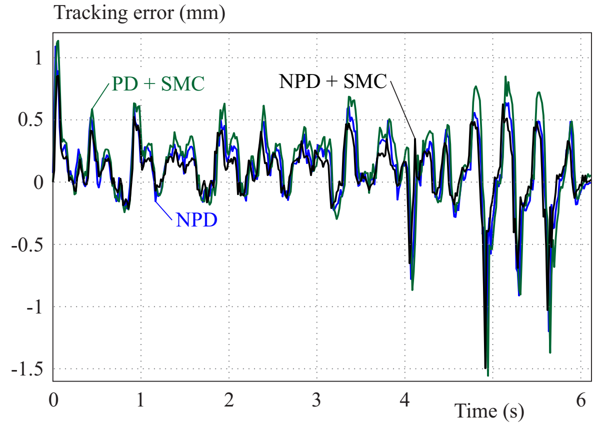

Fig. 5. Experimental tracking error in z-axis for case 1

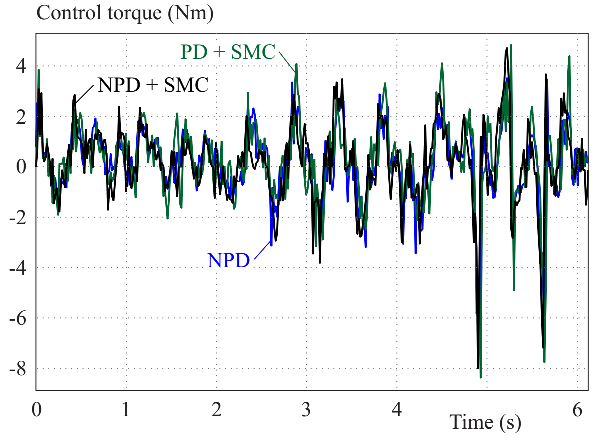

Fig. 6. Experimental torque of joint 1 for case 1

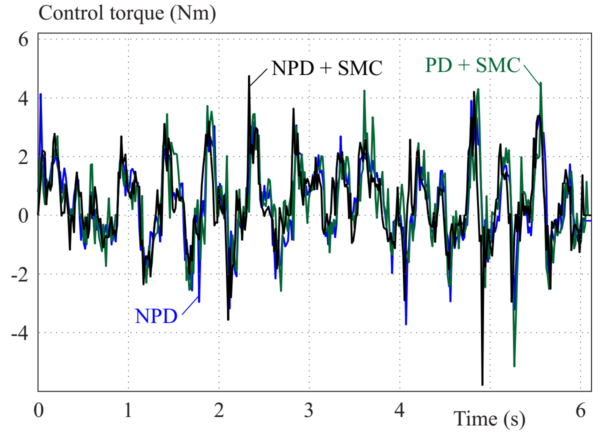

Fig. 7. Experimental torque of joint 2 for case 1

Table 2. Tracking performance for case 1

Controller	NPD [1]	PD+SMC [11]	NPD+SMC
$RMSE_x$ (m)	4.3980×10^{-4}	5.5766×10^{-4}	3.7585×10^{-4}
$RMSE_y$ (m)	2.9400×10^{-4}	3.5341×10^{-4}	2.2682×10^{-4}
$RMSE_z$ (m)	2.9315×10^{-4}	3.6034×10^{-4}	2.9430×10^{-4}
$RMSE$ (m)	6.0481×10^{-4}	7.5214×10^{-4}	5.0485×10^{-4}

and $H = 2$. The gains K_{p0} and K_{d0} are selected for the PD-SMC controller.

To evaluate the performance of the controllers, the root mean square error (RMSE) of the end-effector position is selected as a performance index. Its expression is given by

$$RMSE = \sqrt{RMSE_x^2 + RMSE_y^2 + RMSE_z^2} \tag{34}$$

with

$$RMSE_x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - x_{d_i})^2} \tag{35}$$

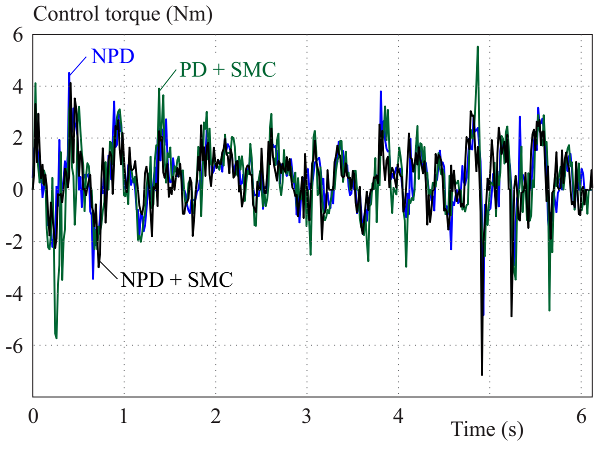

Fig. 8. Experimental torque of joint 3 for case 1

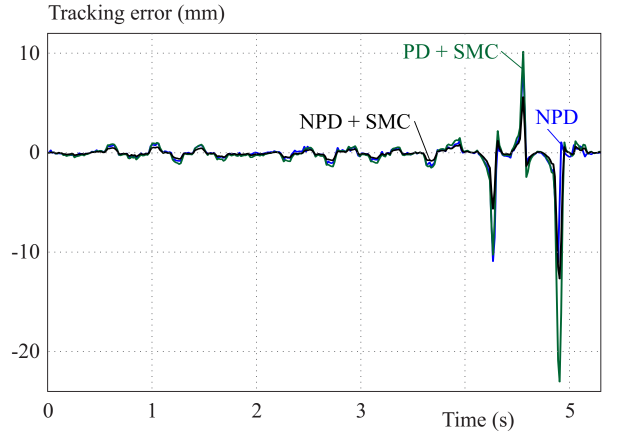

Fig. 9. Experimental tracking error in x-axis for case 2

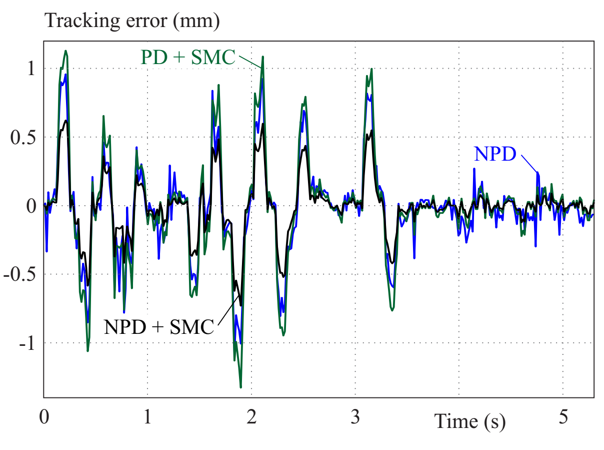

Fig. 10. Experimental tracking error in y-axis for case 2

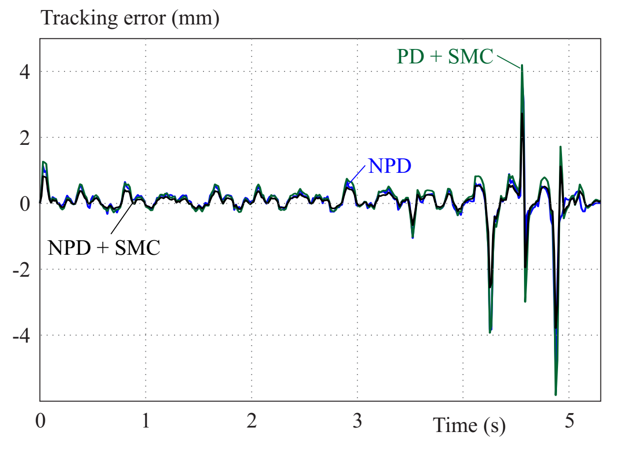

Fig. 11. Experimental tracking error in z-axis for case 2

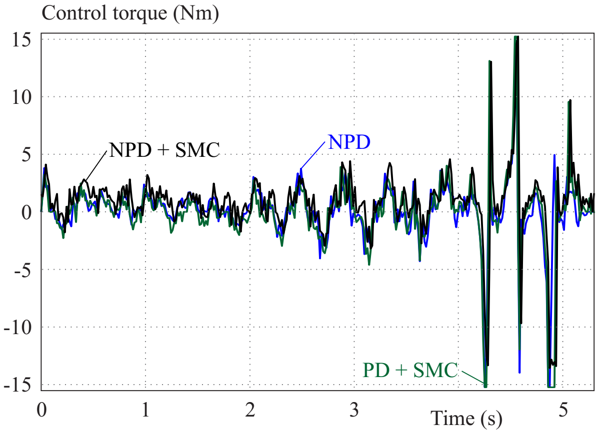

Fig. 12. Experimental torque of joint 1 for case 2

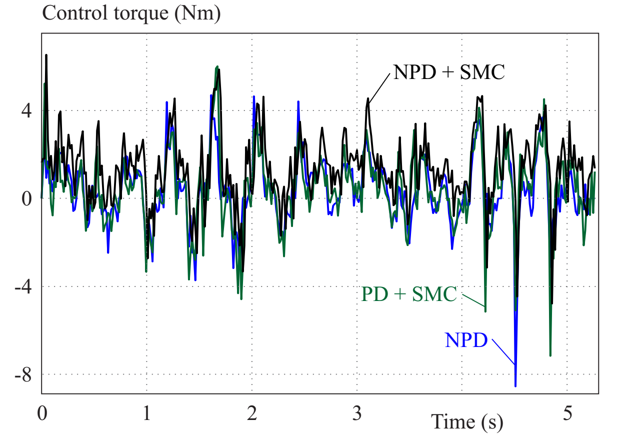

Fig. 13. Experimental torque of joint 2 for case 2

where x_{d_i} is the desired trajectory, x_i is the actual response, and n is the number of samples.

The desired operational trajectory is a combination of semi-elliptic realised with a parabolic position profile and straight lines as described in Fig. 2, where the starting point is $(0, 0, -0.37)$ m and the ending point is $(0, 0, -0.43)$ m. The experiments are given in two cases.

Case 1. This case is considered as a nominal one where the desired trajectory has a maximum velocity of 7.5 m/s, and a maximum acceleration of $15\,\mathrm{m/s^2}$. In order to show the effectiveness and the tracking performances of the proposed approach, the NPD-SMC (10), the NPD and the PD-SMC are implemented.

Figure 2 presents the trajectory tracking in the operational space. Figures 3–5 depict the tracking error in the direction x, y and z under the proposed NPD-SMC, the NPD proposed in [1], and the PD-SMC proposed in [11]. It is obvious that the proposed controller has better track-

Table 3. Tracking performance for case 2

Controller	NPD [1]	PD+SMC [11]	NPD+SMC
$RMSE_x$ (m)	1.4292×10^{-3}	2.3538×10^{-3}	1.2946×10^{-3}
$RMSE_y$ (m)	3.0272×10^{-4}	3.6037×10^{-4}	2.1189×10^{-4}
$RMSE_z$ (m)	5.5946×10^{-4}	6.6297×10^{-4}	4.6069×10^{-4}
$RMSE$ (m)	1.5643×10^{-3}	2.4718×10^{-3}	1.3903×10^{-3}

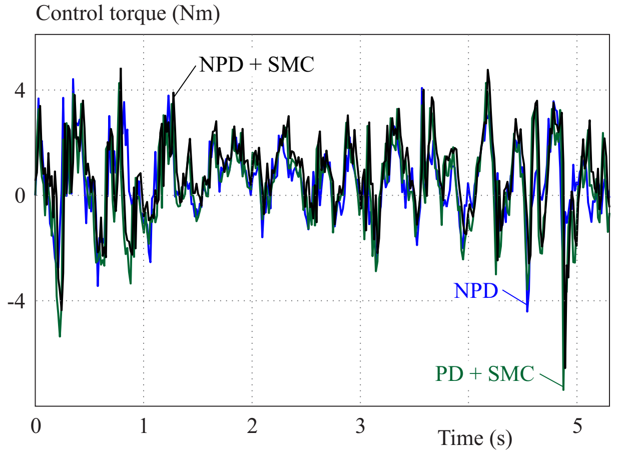

Fig. 14. Experimental torque of joint 3 for case 2

ing error than the other two controllers. More precisely, the NPD-SMC improved the RMSE around 16 % compared to the NPD and 33 % compared to the PD-SMC. On the other hand, the torques of the three controllers have similar variation and amplitude as it can be seen in Figs. 6–8. Table 2 shows the RMSE results of the trajectory tracking experiment of the NPD-SMC, NPD, and PD-SMC.

Case2. In order to study the efficiency of the controller at high cadence movement, a similar experiment is carried out where the same trajectory used before will be executed with a maximum velocity of 10 m/s and a maximal acceleration of $20\,\text{m/s}^2$.

The results of the experiment are presented from Fig. 9 to 14. It is clear that the proposed controller still able to provide better performances and decrease the tracking errors during the whole motion process, where the NPD-SMC improves the RMSE around 11 % compared to the NPD and 44 % compared to the PD-SMC. Moreover, the control torque stays at an admissible value. Table 3 summarises the performance results, at high dynamic movement. One can conclude that at high dynamic movement the performance improvement is more obvious compared to the PD-SMC, and less obvious compared to the NPD.

4 Conclusion

In this work, a NPD-SMC has been proposed for the trajectory tracking of robot manipulators. In order to increase PD control convergence rate, the proportional and derivative gains have been tuned on-line using the tracking error and its time derivative respectively. Also, to benefit from the SMC robustness, only the Signum function has been introduced. The main advantage of the NPD-SMC is its effectiveness and easy implementation that make it highly suitable for industrial applications. Asymptotic convergence of the tracking error and the tracking error rate has been proved using Lyapunov method. Experiments have been carried out on a parallel Delta robot under an accelerations of $15\,\text{m/s}^2$ and $20\,\text{m/s}^2$. It is shown that the proposed approach can obtain superior performance by comparing it with the NPD controller and the PD-SMC controller. On the other hand, the control torques have similar amplitude and variation.

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