CFT data and spontaneously broken conformal invariance

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We derive consistency conditions for the CFT data, which systems with exact but spontaneously broken conformal invariance must satisfy.

I. INTRODUCTION

The potential phenomenological applications of scale (SI) and conformal invariance (CI) in particle physics [1,2] and cosmology [3–5] have been pointed out long ago. In the past years there has been a resurgence of interest in this direction, see for example [6–32].

Both of these symmetries, apart from forbidding the presence of dimensionful parameters in the action, also constrain heavily the observables (correlation functions) of a theory.[1] Usually, in unitary theories with symmetry-preserving vacuum no distinction is made between SI and CI, since the presence of the former implies the latter (in flat spacetime) [33–36].

Note that in interacting scale and conformal field theories (CFTs) with symmetric vacuum there is no particle interpretation. As a result, a crucial ingredient when it comes to utilizing SI or CI for constructing theories that stand a chance of being phenomenologically viable, is to require that they be spontaneously broken. Now, contrary to what happens when the invariance under dilatations is linearly realized, SI need not imply CI [34]. In this case the ground state is degenerate, as it contains a nontrivial flat direction which is parametrized by the massless dilaton. This in turn forces the cosmological constant to be zero and at the same time enables the theory to accommodate massive excitations.

To the best of our knowledge, there has not been an attempt to study generic theories exhibiting spontaneously

broken SI or CI without a known explicit Lagrangian formulation. In this paper, we will provide a set of conditions that should be fulfilled by theories with exact, but spontaneously broken CI. More specifically, we will derive relations on the CFT data = {operator dimensions, operator product expansion(OPE) coefficients} in the broken phase, which are *universal and independent of the specifics of a system*. It should be noted that to study CFTs as a whole and not case by case, we will not rely on a particular microscopic description; rather we will be working solely with the OPE and correlators.

II. OPE AND SPONTANEOUS BREAKING OF CONFORMAL SYMMETRY

Let us start by assuming that the conformal symmetry is spontaneously broken at a certain mass scale v—the vacuum expectation value (vev) of the order parameter. As we already mentioned, this is an important requirement if we wish for CI to be a guiding principle for building realistic theories. We are going to illustrate that by employing the OPE, it is possible to infer many general properties of any unitary system that possesses a flat direction along which the conformal symmetry is spontaneously broken. More specifically, we will establish a set of consistency conditions that need to be satisfied.

Our starting point is the OPE of two scalar primary operators, which reads[2];

$$\mathcal{O}_i(x) \times \mathcal{O}_j(0) \sim \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}} \mathcal{O}_k + \cdots. \qquad (2.1)$$

Here c_{ijk} are the OPE coefficients, $|x| = \sqrt{x^\mu x_\mu}$, $\mathcal{O}_l \equiv \mathcal{O}_l(0)$, $\Delta_{ijk} \equiv \Delta_i + \Delta_j - \Delta_k$, with Δ_l the dimensions, and

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[1]Obviously, conformal symmetry is more restrictive than scale symmetry.

[2]It should be noted that, in principle, a theory might contain more than one operator with the same scaling dimension.

the ellipses stand for operators with nonzero spin, as well as descendants. For later convenience, let us stress that no implicit summation over Latin indices is assumed.

Consider a (unitary) four-dimensional CFT in which the conformal group is spontaneously broken to its Poincaré subgroup, $SO(4, 2) \rightarrow ISO(3, 1)$.[3] This might happen, for instance, when some of the (scalar) operators of the theory acquire a nonzero vev. To put it differently, there exists a Poincaré-invariant ground state, which we denote by $|0\rangle$, such that

$$\langle 0|\mathcal{O}_i|0\rangle \equiv \langle \mathcal{O}_i \rangle = \xi_i v^{\Delta_i} \neq 0, \qquad (2.2)$$

where ξ_i's are dimensionless parameters (and v carries dimension of mass).

From (2.1), we find that when the OPE is sandwiched between the symmetry-breaking vacuum $|0\rangle$, it yields

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle \sim \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}}\langle \mathcal{O}_k \rangle = \sum_k \frac{c_{ijk}}{|x|^{\Delta_{ijk}}}\xi_k v^{\Delta_k}. \quad (2.3)$$

Clearly, the only terms which survive and thus contribute to the two-point function are the scalar operators. If the symmetry was not broken, then only the unit operator ($\mathbb{1}$) would be allowed to acquire nonvanishing vev. Since $\Delta_{\mathbb{1}} = 0$, the expression above would boil down to

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle \sim \frac{\delta_{ij}}{|x|^{\Delta_i+\Delta_j}}, \qquad (2.4)$$

as it should.

Let us now insert a complete set of states in the left-hand side of (2.3), i.e.

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \sum_N \langle 0|\mathcal{O}_i(x)|N\rangle\langle N|\mathcal{O}_j(0)|0\rangle. \quad (2.5)$$

In the limit $x \rightarrow \infty$, due to the cluster decomposition principle [37,38], we will pick up only the vacuum state. As a result, the above asymptotes to

$$\lim_{x\rightarrow\infty}\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle = \langle \mathcal{O}_i\rangle\langle \mathcal{O}_j\rangle = \xi_i\xi_j v^{\Delta_i+\Delta_j}. \quad (2.6)$$

We can then conclude that the two-point function (2.3) formally yields

$$\xi_i\xi_j = \lim_{z\rightarrow 0}\sum_k c_{ijk}\xi_k z^{\Delta_{ijk}}, \qquad (2.7)$$

where we introduced $z \equiv (v|x|)^{-1}$. This constitutes the first relation that the CFT data must satisfy. It is quite natural to expect that the OPE will also contain operators whose scaling dimension Δ_k is larger than $\Delta_i + \Delta_j$. In such a case,

and since there is no *a priori* reason for their corresponding OPE coefficients to vanish, it is obvious that $\Delta_{ijk} < 0$; consequently, $z^{\Delta_{ijk}}$ will appear in the denominator of the consistency condition (2.7). The presence of such terms implies that the infrared limit ($z \rightarrow 0$) should be taken only after the series have been summed. It should be stressed that whether this procedure is mathematically well defined or not depends on the convergence of the series in the above equation. Unfortunately, it is not known if this is the case; the results of [39,40] (see also [41]), according to which the conformal OPE indeed converges are not applicable here, for they were derived for CFTs with unbroken vacuum. Nevertheless, in what follows we will assume that the series on the right-hand side of (2.7) is convergent at least in some finite domain of z, say at $z \gtrsim \mathcal{O}(1)$, and that the result of summation can be analytically continued to $z \rightarrow 0$.

Interestingly, the consistency relation (2.7) has been presented previously by El-Showk and Papadodimas [42], in the context of finite temperature effects on CFTs. Generally speaking, a rather natural next step for our considerations would be to relax the requirement of having a Lorentz-invariant vacuum. If the ground state preserves spatial rotations only, this situation would be similar to what happens in thermal CFTs, e.g. in high-temperature QCD [43,44]. We leave this for future work.

Coming back from this small digression, we note that, in principle, we can go ahead and use the OPE to construct higher-order scalar correlators. However, they will not provide us with further information.[4] To make this point more clear, let us consider for instance the three-point function,

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\mathcal{O}_k(z)\rangle \sim \sum_l \frac{c_{ijl}}{|x|^{\Delta_{ijl}}}\langle \mathcal{O}_l(0)\mathcal{O}_k(z)\rangle + \cdots,$$
$$(2.9)$$

where, as before, we use the ellipses to denote terms involving the derivatives of the operators, etc. Following the previous logic, once we insert complete sets of states between the three operators, it is apparent that the above—upon using (2.6)—boils down to (2.7) in the deep infrared. It is not difficult to see that this behavior persists in higher-order scalar functions.

Let us note in passing that the low-energy domain of the theory contains *only one Goldstone boson* π associated with the breaking of (SI and) CI, even though the number of broken generators is five in total (one related to dilatations and four to special conformal transformations).

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[3]We will work exclusively in four-dimensional Minkowski spacetime.

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[4]Three-point functions that involve two scalars and, for example, the dilatational current J_μ, i.e.

$$\langle O_i(x)O_j(0)J_\mu(y)\rangle, \qquad (2.8)$$

effectively reduce to the Ward identities and might give extra but more complicated constraints, since these will involve double limits.

This fact does not depend on the details of the symmetry-breaking mechanism and has been studied extensively in the literature; the interested reader is referred to [45–49] for further details. With this in mind, the next step is to consider the implications of having π in the spectrum. Assuming that the theory contains no other massless scalar fields apart from the dilaton, we expect that the vacuum amplitude of two operators will contain a pole at vanishing virtuality [37,38].[5] This becomes evident by inserting into the scalar two-point function the "resolution of unity," which at the IR due to the domination of the dilaton can be approximated by

$$1 \underset{x \to \infty}{\sim} |0\rangle\langle 0| + \int \frac{d^3\vec{p}}{2p_0(2\pi)^3} |\pi(p)\rangle\langle\pi(p)|. \qquad (2.10)$$

A straightforward computation reveals that indeed

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle \underset{x \to \infty}{\sim} \langle\mathcal{O}_i\rangle\langle\mathcal{O}_j\rangle - \frac{\langle 0|\mathcal{O}_i|\pi\rangle\langle\pi|\mathcal{O}_j|0\rangle}{|x|^2}. \qquad (2.11)$$

Let us define the matrix element of the operator $\mathcal{O}_i$ between the vacuum and the dilaton as[6]

$$\langle 0|\mathcal{O}_i|\pi\rangle = f_i v^{\Delta_i - 1}, \qquad (2.13)$$

with f_i a dimensionless coupling. Consequently, it is easy to see that (2.11), upon using (2.6) and (2.3), leads to the second consistency condition

$$f_i f_j = \lim_{z \to 0}\left[\frac{1}{z^2}\left(\xi_i \xi_j - \sum_k c_{ijk} \xi_k z^{\Delta_{ijk}}\right)\right]. \qquad (2.14)$$

It should be noted that Eq. (2.7) follows from Eq. (2.14). It is evident that the above includes an infinite number of new parameters, f_i. As we will now show, these can in general be fixed with the use of the Goldstone theorem. Let us note that even if a conserved current J_μ associated with dilatations exists,[7] this need not be invariant under translations; therefore, the conventional proof (e.g. [38,50]) might not be applicable here. The way out is to work directly with the (improved) energy-momentum tensor $T_{\mu\nu}$ [51].[8]

Lorentz invariance and $\partial_\mu T^{\mu\nu} = 0$ dictate that the matrix element of $T_{\mu\nu}$ between the vacuum and the dilaton π, be of the following form,[9]

$$\langle 0|T_{\mu\nu}(0)|\pi(p)\rangle = \frac{1}{3} f_\pi v p_\mu p_\nu, \qquad (2.15)$$

with f_π the dimensionless dilaton decay constant and the factor of 1/3 was added for later convenience. It can then be shown that the expectation value of the commutator between the energy-momentum tensor and an operator reads

$$\langle [T_{\mu\nu}, \mathcal{O}_i]\rangle = \frac{i}{3} f_\pi f_i v^{\Delta_i} \partial_\mu \partial_\nu G(x), \qquad (2.16)$$

where $G(x)$ is the (massless) Green's function. Provided that $J_\mu \equiv x^\nu T_{\mu\nu}$, then the charge associated with dilatations can be written as a particular moment of the energy-momentum tensor,

$$D = \int d^3\vec{x} x^\mu T_{0\mu}, \qquad (2.17)$$

while, by definition,

$$\langle [D, \mathcal{O}_i]\rangle = i\xi_i \Delta_i v^{\Delta_i}. \qquad (2.18)$$

Consequently, from the relations (2.16)–(2.18), it easily follows that

$$f_i = \frac{\xi_i \Delta_i}{f_\pi}. \qquad (2.19)$$

We observe that (2.7), (2.14) and (2.19) constitute a system of equations for ξ_i and Δ_i. If a nontrivial solution exists, this can serve as an indication that the CFT data describes a system that exhibits the symmetry breaking pattern $SO(4,2) \to ISO(3,1)$.

At this point, we would like to turn our attention to the energy-momentum tensor, an operator of particular importance as far as CFTs are concerned. The relevant for our considerations terms in the two-point correlator of $T_{\mu\nu}$ with itself are

$$\langle T_{\mu\nu}(x)T_{\lambda\sigma}(0)\rangle = \sum_k \mathcal{T}_{\mu\nu\lambda\sigma} \frac{\xi_k v^{\Delta_k}}{|x|^{8-\Delta_k}}. \qquad (2.20)$$

To keep the expression short, we introduced the most general Lorentz-covariant structure consistent with the symmetries of the energy-momentum tensor (see also [54,55]),

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$$\mathcal{T}_{\mu\nu\lambda\sigma} = \mathrm{a}_k^T A_{\mu\nu\lambda\sigma} + b_k^T B_{\mu\nu\lambda\sigma}$$
$$+ \frac{1}{|x|^2}(c_k^T C_{\mu\nu\lambda\sigma} + d_k^T D_{\mu\nu\lambda\sigma}) + e_k^T \frac{E_{\mu\nu\lambda\sigma}}{|x|^4}, \qquad (2.21)$$

where

$$A_{\mu\nu\lambda\sigma} = \eta_{\mu\nu}\eta_{\lambda\sigma},$$
$$B_{\mu\nu\lambda\sigma} = \eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda},$$
$$C_{\mu\nu\lambda\sigma} = \eta_{\mu\nu}x_\sigma x_\lambda + \eta_{\lambda\sigma}x_\nu x_\mu,$$
$$D_{\mu\nu\lambda\sigma} = \eta_{\lambda\mu}x_\nu x_\sigma + \eta_{\mu\sigma}x_\nu x_\lambda + \eta_{\nu\sigma}x_\mu x_\lambda + \eta_{\nu\lambda}x_\mu x_\sigma,$$
$$E_{\mu\nu\lambda\sigma} = x_\mu x_\nu x_\lambda x_\sigma, \qquad (2.22)$$

and $\eta_{\mu\nu}$ is the Minkowski metric. From the vanishing of the divergence

$$\partial^\mu \langle T_{\mu\nu}(x)T_{\lambda\sigma}(0)\rangle = 0, \qquad (2.23)$$

we obtain

$$2d_k^T - (5-\Delta_k)c_k^T - (8-\Delta_k)\mathrm{a}_k^T = 0,$$
$$c_k^T - (4-\Delta_k)d_k^T - (8-\Delta_k)b_k^T = 0,$$
$$(10-\Delta_k)(c_k^T + 2d_k^T) + (5-\Delta_k)e_k^T = 0, \qquad (2.24)$$

which must hold for each k.

In addition, since $T_\mu^\mu = 0$, it immediately follows that for all k's

$$16\mathrm{a}_k^T + 8b_k^T + 8c_k^T + 4d_k^T + e_k^T = 0, \qquad (2.25)$$

where we used (2.20)–(2.22). Note that from the above algebraic equations we can express four of the coefficients in terms of one, say a_k^T. Once this is effectuated, we find that the two-point function (2.20) boils down to

$$\langle T_{\mu\nu}(x)T_{\lambda\sigma}(0)\rangle = \sum_k \tilde{\mathcal{T}}_{\mu\nu\lambda\sigma}\frac{\tilde{\alpha}_k \mathrm{a}_k^T \xi_k v^{\Delta_k}}{|x|^{8-\Delta_k}}, \qquad (2.26)$$

with $\tilde{\alpha}_k = (10 - 8\Delta_k + \Delta_k^2)^{-1}$, while

$$\tilde{\mathcal{T}}_{\mu\nu\lambda\sigma} = \alpha_k^{(1)}A_{\mu\nu\lambda\sigma} - \alpha_k^{(2)}B_{\mu\nu\lambda\sigma}$$
$$+ \frac{1}{|x|^2}(\alpha_k^{(3)}C_{\mu\nu\lambda\sigma} + \alpha_k^{(4)}D_{\mu\nu\lambda\sigma}) - 2\alpha_k^{(5)}\frac{E_{\mu\nu\lambda\sigma}}{|x|^4}, \qquad (2.27)$$

and we have defined

$$\alpha_k^{(1)} = 1, \qquad \alpha_k^{(2)} = 20 - 12\Delta_k + \frac{3}{2}\Delta_k^2,$$
$$\alpha_k^{(3)} = (8 - \Delta_k)\Delta_k, \qquad \alpha_k^{(4)} = 40 - 17\Delta_k + \frac{3}{2}\Delta_k^2,$$
$$\alpha_k^{(5)} = 80 - 18\Delta_k + \Delta_k^2. \qquad (2.28)$$

As a sanity check, if the symmetry were linearly realized, then only the unity would contribute. In such a case, Eqs. (2.26)–(2.28) dictate that up to irrelevant numerical factors

$$\langle T_{\mu\nu}(x)T_{\lambda\sigma}(0)\rangle_{\text{unbroken}}$$
$$\propto \mathrm{a}_{\mathbb{1}}^T \left(\frac{1}{2}(I_{\mu\lambda}I_{\nu\sigma} + I_{\nu\lambda}I_{\mu\sigma}) - \frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\sigma}\right)\frac{1}{|x|^8}, \qquad (2.29)$$

with $I_{\mu\nu} \equiv \eta_{\mu\nu} - 2x_\mu x_\nu/|x|^2$, and $\mathrm{a}_{\mathbb{1}}^T$ an overall coefficient setting the scale of the two-point function.[10]

If we now consider the low-energy limit, we end up with the following constraints on a_k^T:

$$\lim_{z\to 0}\sum_k \alpha_k^{(n)}\tilde{\alpha}_k \mathrm{a}_k^T \xi_k z^{8-\Delta_k} = 0, \quad n = 1,\dots,5. \qquad (2.30)$$

Yet another set of conditions relating the OPE coefficients of $T_{\mu\nu}$ can be obtained by considering its interaction with the dilaton. This practically amounts to plugging (2.10) into the left-hand side of the correlator (2.26). A (long but) straightforward computation gives

$$\lim_{z\to 0}\sum_k \alpha_k^{(n)}\tilde{\alpha}_k \mathrm{a}_k^T \xi_k z^{2-\Delta_k} = C^{(n)}f_\pi^2, \quad n = 1,\dots,5, \qquad (2.31)$$

with

$$C^{(1)} = -C^{(2)} = \frac{8}{9}, \quad C^{(3)} = C^{(4)} = -\frac{16}{3}, \quad C^{(5)} = -\frac{64}{3}. \qquad (2.32)$$

Like it happened before, Eq. (2.30) is a consequence of (2.31).

Before moving to the conclusions, let us for completeness discuss briefly what can be deduced by considering the OPE of (translationally invariant) vector operators. The OPE contains, among others, the following terms:

$$V_\mu^i(x) \times V_\nu^j(0) \supset \sum_k \left(\mathrm{a}_{ijk}^V \eta_{\mu\nu} + b_{ijk}^V \frac{x_\mu x_\nu}{|x|^2}\right)\frac{\mathcal{O}_k}{|x|^{\Delta_{ijk}}}. \qquad (2.33)$$

By taking the average in the symmetry-breaking vacuum, we see that in the IR ($x \to \infty$) the left-hand side must be zero due to Lorentz invariance. As a consequence, we

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[10]Note that instead of the symbol $\mathrm{a}_{\mathbb{1}}^T$, C_T is most commonly used in the literature.

obtain a set of constraints that the OPE coefficients a_{ijk}^V and b_{ijk}^V—provided that they are not trivial—should satisfy

$$\lim_{z\to 0}\sum_k a_{ijk}^V \xi_k z^{\Delta_{ijk}} = 0, \qquad \lim_{z\to 0}\sum_k b_{ijk}^V \xi_k z^{\Delta_{ijk}} = 0.$$

(2.34)

As before, $z = (v|x|)^{-1}$. Note that for higher-spin operators, owing to the fact that for $z\to 0$ (equivalently, $x\to\infty$) their vev's must also vanish, a similar type of relations can be obtained; see for example the ones we presented for the energy-momentum tensor.

III. CONCLUSIONS

In the present short paper, we reported on constraints that the CFT data should satisfy when a system possesses a flat direction, and thus, exhibits nonlinearly realized conformal invariance. Our considerations are very general, for they only require knowledge of the operator spectrum of a theory, their corresponding anomalous dimensions and the OPE coefficients. What remains to be seen is if the relations we presented can be used to identify the allowed regions of the phase portrait of CFTs with spontaneously broken symmetry; the main challenge is the convergence of the OPE and whether it is possible to analytically continue it into the infrared regime.

An ideal testing ground for our findings would be theories such as $\mathcal{N} = 4$ super Yang-Mills, which is known to possess exact but spontaneously broken conformal invariance at the Coulomb branch. It would be very interesting to confront our results with the ones for the full spectrum of the anomalous dimensions for this theory [56–58].

Although a bit tangent to this paper, let us mention that on the phenomenological side, spontaneously broken scale and conformal symmetries have served as a guiding principle for constructing realistic theories able to describe our Universe from its very early stages up until the present day. In their context, the hierarchy and cosmological constant problems can be viewed from a fresh perspective. Their potential resolution might be achieved under certain extra assumptions on the UV dynamics, such as the absence of new particle thresholds between the electroweak and Planck scales [8,59,60] (see also [61] for an implementation of this idea in grand unified theories).

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