Supporting Information for: "Catchment drainage network scaling laws found experimentally in overland flow morphologies"

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Text S1: Experimental setup

We used a 2-m × 1-m erosion flume as shown in Figure S1. The total sand depth was 15 cm. Uncohesive sediments with a mean diameter of 0.53 mm (Table S1 and Figure S2) were placed in three successive 5-cm layers within the flume. Each layer was compacted by 30-min rainfall (droplet size of 3-7 mm) with an intensity of 10 mm h^{-1} followed by consolidation via a 600 kg m^{-2} weight. Then, the sediment was air dried for 48 h. Heterogeneous rainfall was generated by two sprinklers located 3 m above the sediment surface. The distribution of rainfall intensity is shown in Figure 1. The average rainfall was 85 mm h^{-1} with a Christiansen uniformity coefficient [*Christiansen*, 1942] of 26%. The rainfall was applied continuously except a 30-min break for each laser scan.

There was no drainage from the flume bottom and all surface flow was collected at a single, 4-cm wide outlet ($z = 0$), located at 6 cm above the base of the flume. The experiment started with a smooth surface. At the outlet, the initial elevation difference between the sediment surface and the flume outlet was 9 cm. The flow rate at the flume outlet is plotted in Figure S3. During the first 5 min, the sediments became saturated and the outlet flow rate was low. Afterwards, there was a rapid elevation drop near the outlet (blue areas in Figure 1) until about $t = 10$ min. Until this time, it was not possible to capture the flow rate due to high sediment concentrations at the outlet. Scanning started after 15 min when the flow rate became steady. At this stage, the sediments were saturated and the precipitation and discharge rates were equal. A 3D laser scanner (Konica Minolta Vivid 910), with about 4-mm horizontal resolution and accuracy of 0.1 mm in the vertical direction, was used to extract Digital Elevation Models (DEMs) at 0.25, 0.5, 1, 2, 4, 8 and 16 h. The scanner was calibrated using 20 fixed points on two bars along the flume's lateral walls. Eight individual scans were taken to cover the entire flume. Following registration and post-processing, each DEM produced was trimmed 2 cm from the side walls and 10 cm from the upstream wall.

Table S1: Characteristics of the sand used in the experiment.

Bulk density	Particle size (d) range	d_{50}	$d < 0.6\,mm$	$d > 2\,mm$
1584 (kg m^{-3})	0-6 mm	0.53 mm	70 (% mass)	12 (% mass)

Text S2: Further morphological features

The scaling relation between the slope and drainage area is plotted in Figure S6. Similarly to large scale river networks and channeled surfaces at the laboratory scale [e.g., *Sweeney et al.*, 2015], a power law describes the intermediate drainage areas of the overland flow (Figure 5). Figure S7 shows the curvature versus the product of drainage slope and area (slope-area). Similar to the catchment scale [*Perron et al.*, 2009], the average values of curvature are negative where the product of area and slope ($A|\nabla z|$) is low whereas positive curvature values are observed for increasing $A|\nabla z|$.

Text S3: Another experiment

In order to investigate the scaling laws at different slope and rainfall intensity, another experiment was carried out at 10% slope and average rainfall of 60 mm h^{-1} for a duration of 20 h. Again, the surface morphology remained unchanneled during whole experiment. The corresponding elevation field and the extracted networks are shown in Figures S8-S10. As shown in Figure S11, Hack's law was observed were the exponents were in the range of [0.51-0.55] during the experiment. The distributions of discharge, drainage area and upstream length follow the power laws: $P(Q > q) = q^{-\varphi}$, $P(A > a) = a^{-\beta}$ and $P(L > l) = l^{-\psi}$, with $\varphi = 0.49$, $\beta = 0.47$ and $\psi = 0.71$, respectively (Figure S12). Furthermore, the relation between the exponents are close to analytical results, $\beta = 1 - h$ and $\psi = \beta/h$, derived by *Maritan et al.* [1996]. These results show that irrespective of rainfall intensity and slope, the same scaling laws were generated.

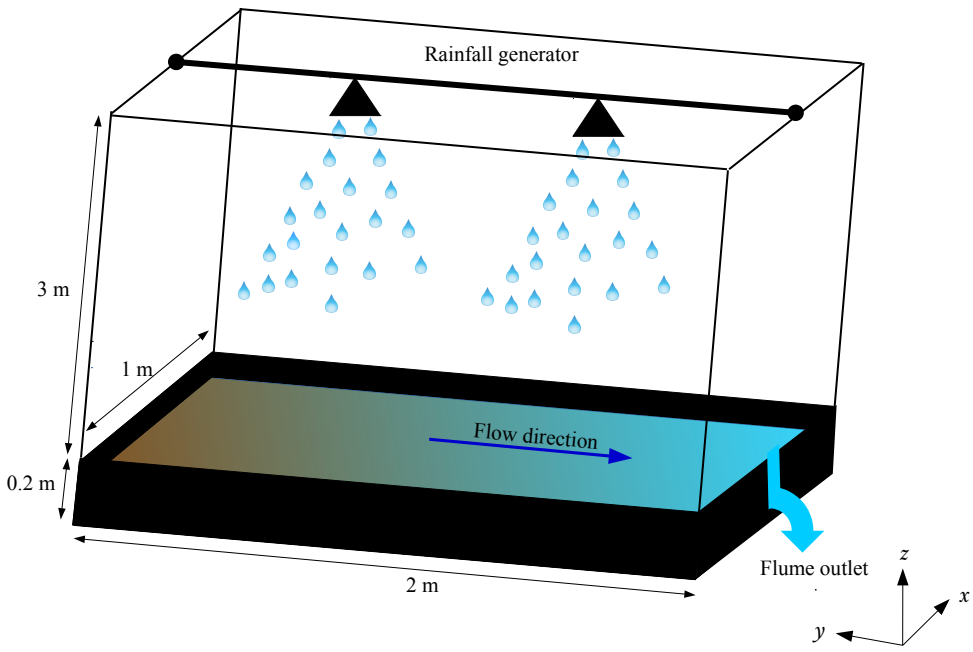

Figure S1: Schematic of the flume experiment. The flume width, W, was 1 m and its length, L, was 2 m. The sediment vertical elevation (z) was measured from the outlet ($z = 0$). The relief height (the maximum elevation) was 0.19 m. Each layer was compacted by 30-min rainfall (droplet size of 3-7 mm) with an intensity of 10 mm h^{-1} followed by consolidation via a 600 kg m^{-2} weight. Then, the sediment was air dried for 48 h.

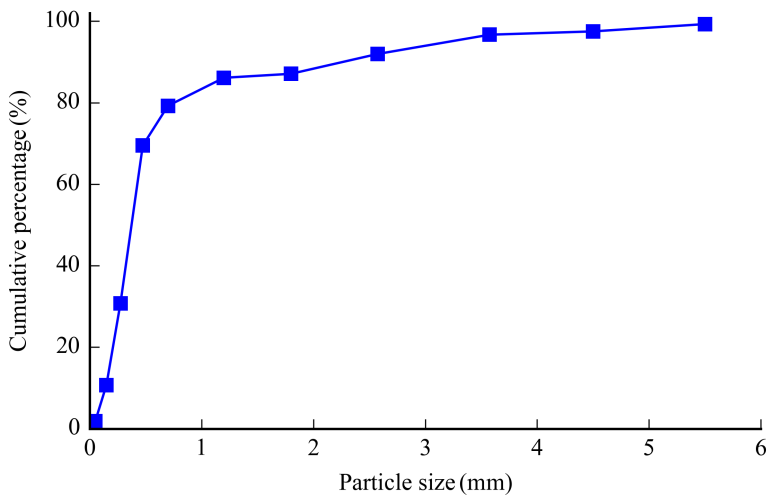

Figure S2: Particle size distribution of the sediment.

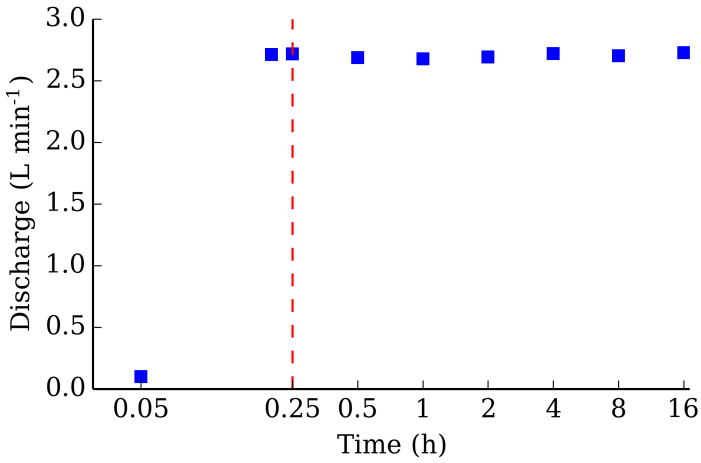

Figure S3: Measured water flow rate at the flume outlet.

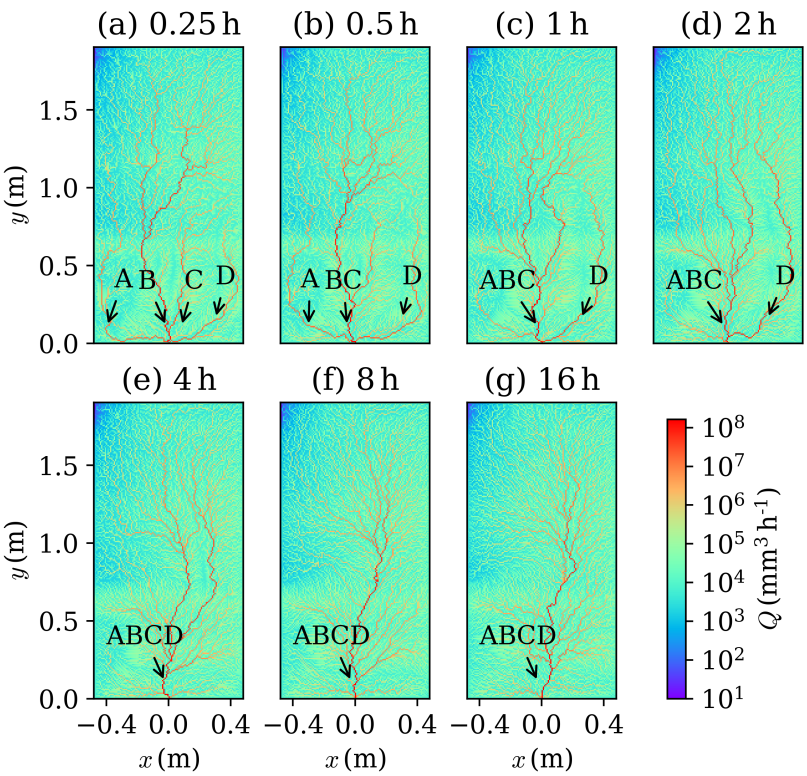

Figure S4: Drainage discharge distribution determined using the D8 algorithm and the measured morphologies and the rainfall intensity (Figure 1). In spite of the heterogeneous rainfall intensity, the generated network and its evolution are very similar to the drainage area network (Figure 2). As expected, the maximum calculated discharge at the outlet (2.709 L min^{-1}) is consistent with the measured value at the steady state (Figure S2).

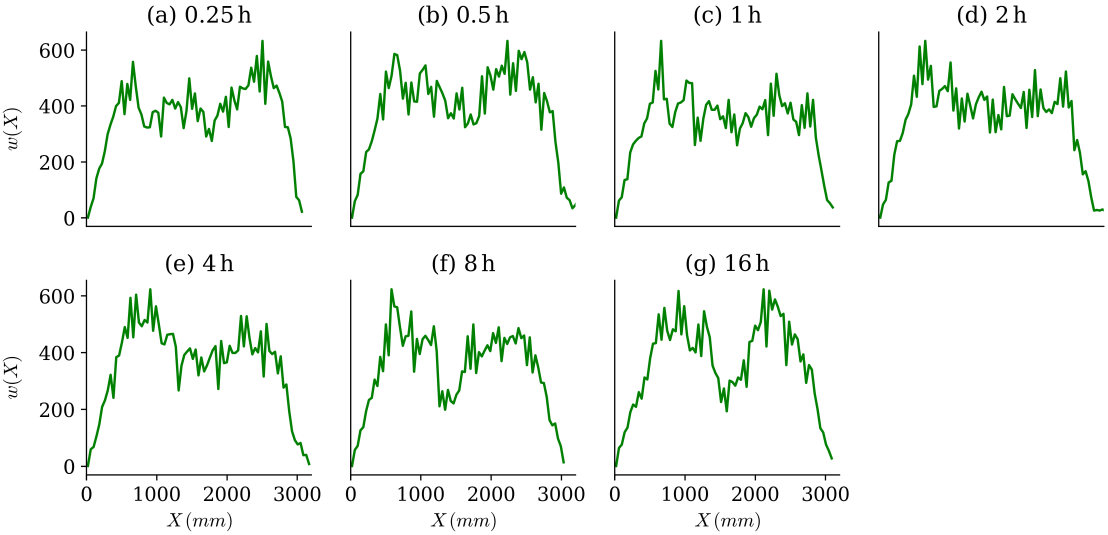

Figure S5: Width function (w) of the drainage area (and discharge) network (Figures 2 and S3). The value X is the distance of each point from the flume outlet (along the flow paths of the network) and $w(X)$ is the number of points with the same distance (X) from the network outlet. The width function of the network dynamically changed during the morphology evolution.

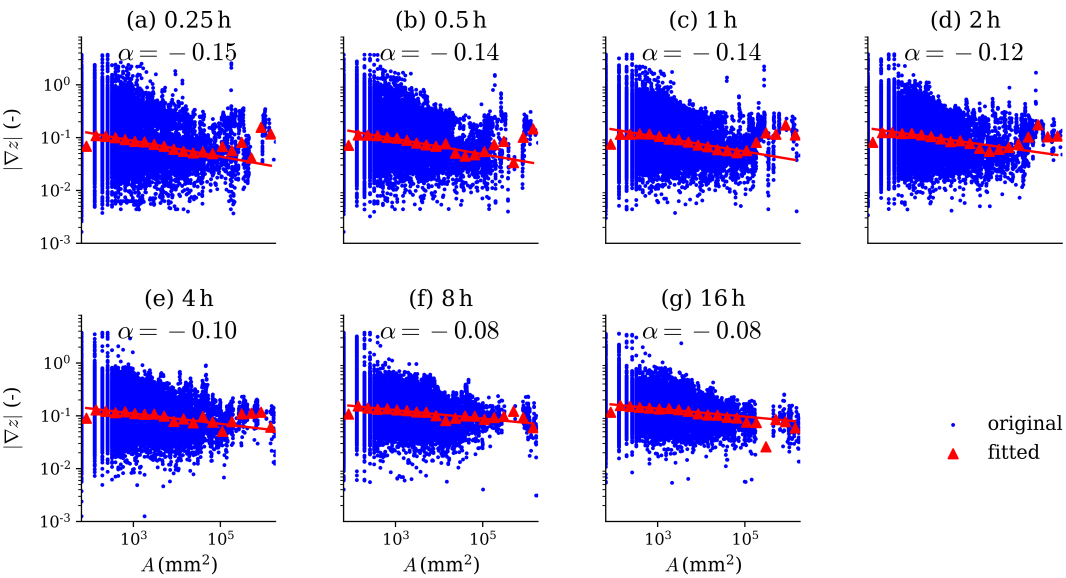

Figure S6: Slope versus drainage area. A power law describes the intermediate drainage areas.

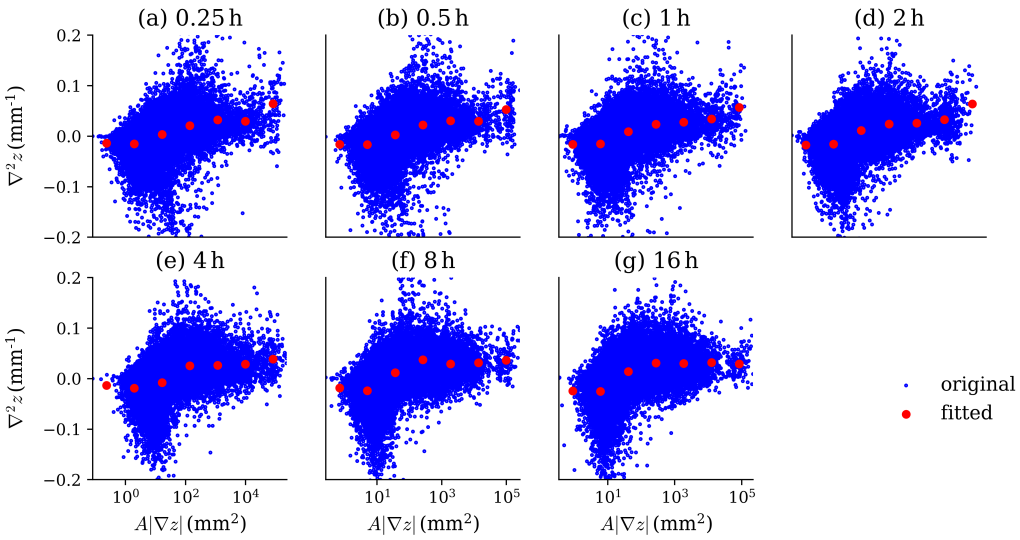

Figure S7: Curvature versus the product of drainage area and slope. The average values of curvature are negative where the product of area and slope ($A|\nabla z|$) is low whereas positive curvature values are observed as $A|\nabla z|$ increases.

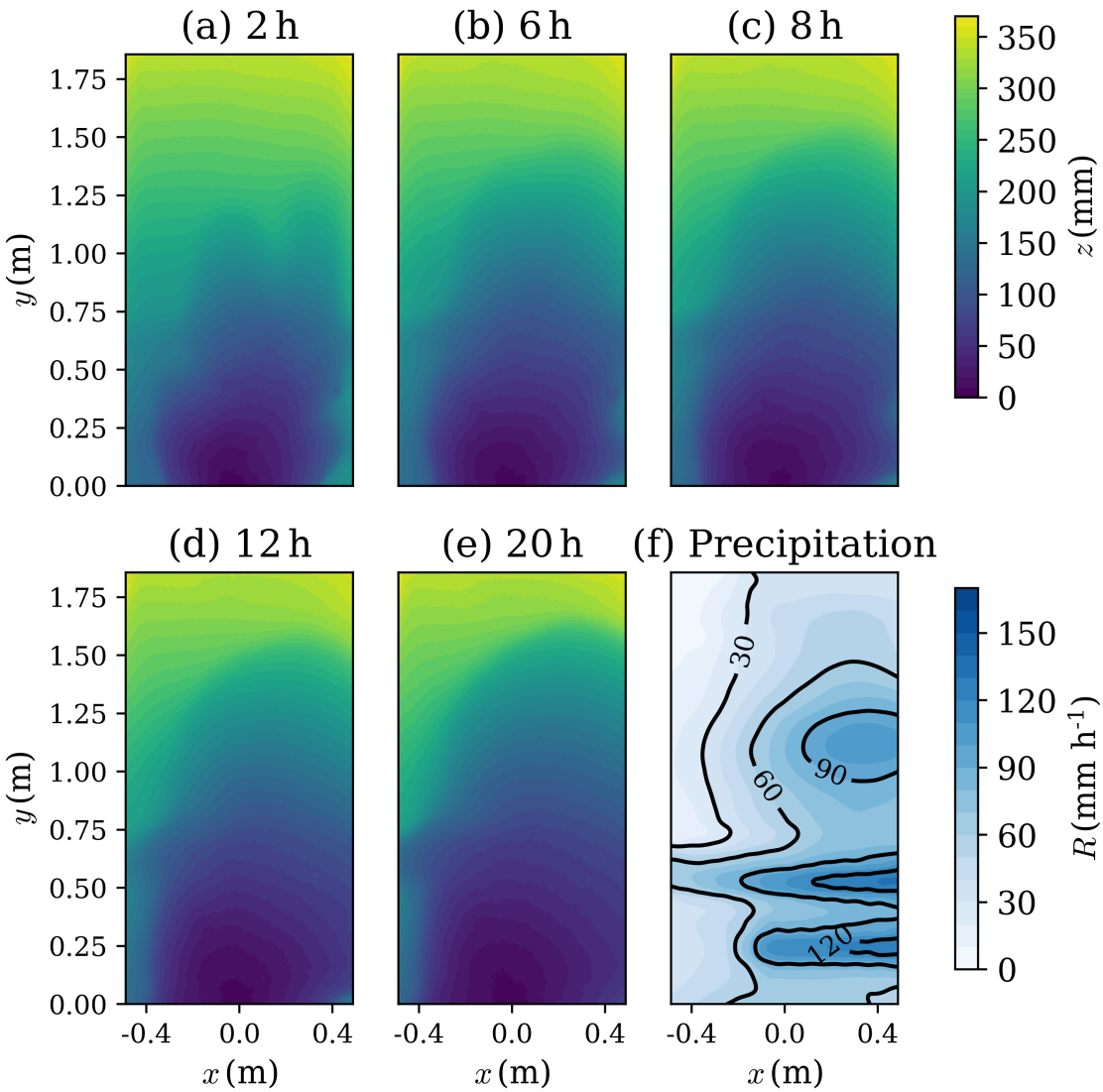

Figure S8: Elevation field at 10% slope and 60 mm h^{-1} average rainfall.

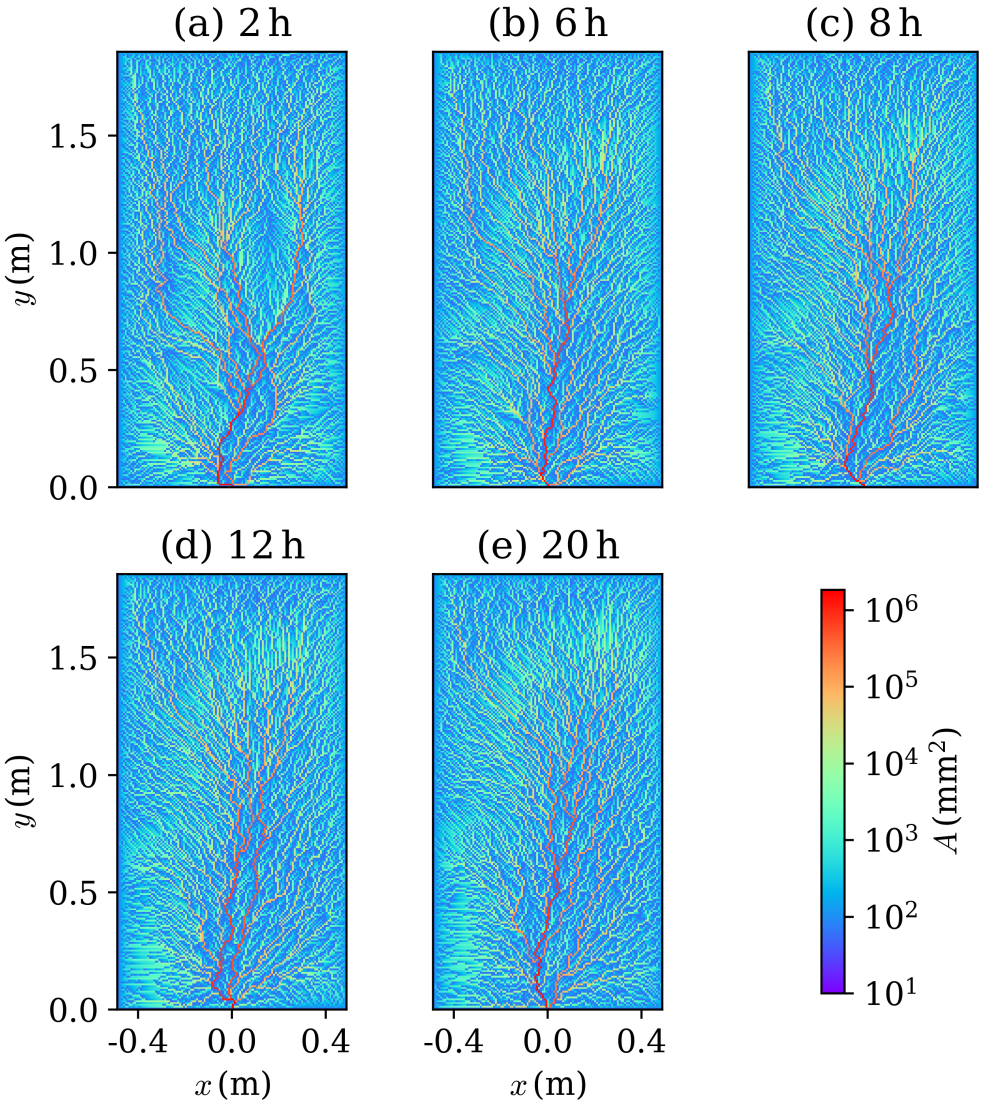

Figure S9: Drainage area at 10% slope and 60 mm h^{-1} average rainfall.

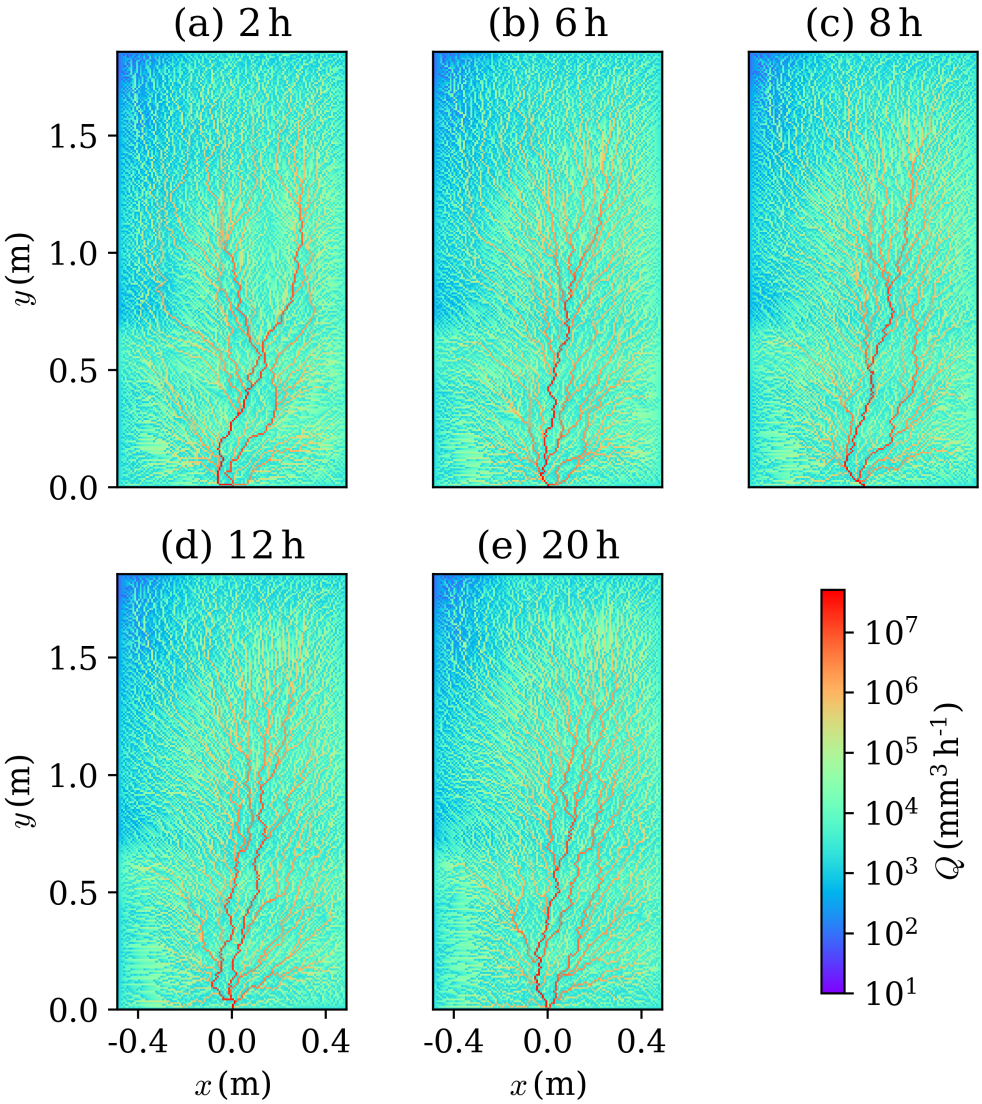

Figure S10: Discharge flux at 10% slope and 60 mm h^{-1} average rainfall.

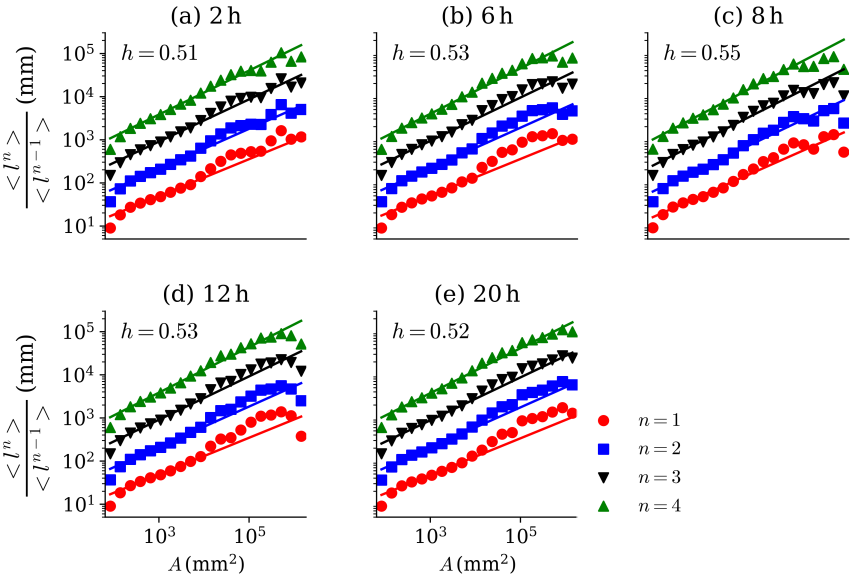

Figure S11: Plot of Hack's law ($l = A^{\alpha}$) at 10% slope and 60 mm h^{-1} average rainfall. The curves of higher moments ($n > 1$) are shifted for the purpose of visualization.

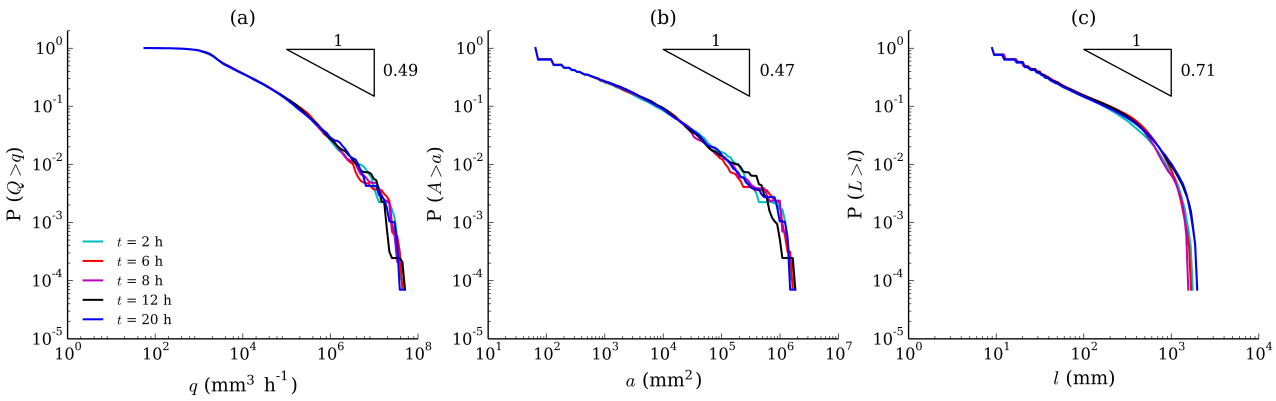

Figure S12: Scaling relation for the distribution of discharge (a), drainage area (b) and length (c) at 10% slope and 60 mm h^{-1} average rainfall.

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