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NONLINEAR BUCKLING AND POST-BUCKLING BEHAVIOR OF SANDWICH BEAMS/WIDE PANELS

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1. INTRODUCTION

The advantages in strength, stiffness and overall response of the sandwich structure can be realized only if it is stabilized against buckling. The compressibility of the core significantly affects the stability response and contributes to the local instability phenomenon. Therefore, despite the global buckling (Euler buckling), very common in ordinary beams and plates, wrinkling, also known as local buckling, may occur in sandwich structures. Wrinkling manifests itself as short-wave buckling of the face sheets.

In the literature, the sandwich theories, especially high order theories, have been applied to investigate the stability behavior of sandwich structures, e.g., High-order Sandwich Panel Theory (HSAPT) [1-3], Extended High-order Sandwich Panel Theory (EHSAPT) [4, 5]. Perhaps due to the complexity of the stability of sandwich structures, assumptions are usually adopted to simplify modelling and solving efforts, i.e., the anti-plane assumption [1-3] in the core, and membrane pre-buckling state [4, 5]. The anti-plane assumption assumes the axial rigidity of the core is negligible since its magnitude is usually about two to three orders of magnitude smaller than that of the faces. Although the validation of anti-plane assumption is approved for the static and dynamic behavior, there is no clear evidence that the effect caused by the axial rigidity of the core is negligible to the stability response. As a compound structure containing layers made of different materials, the sandwich structure may also have a non-membrane pre-buckling state even when subjected to axial compressive loading. Therefore, it is of crucial importance to investigate the stability property of sandwich panels comprehensively without applying these simplifications.

The Extended High-order Sandwich Panel Theory (EHSAPT) accounts for the axial, transverse, and shear rigidity of the core. It shows high accuracy and yields identical displacement and stress to the elasticity for both static behavior and dynamic response. Therefore, a general and comprehensive investigation about the stability of sandwich panels is carried out based on the EHSAPT. The weak form governing equations are formulated based on EHSAPT-based finite element [6]. Both faces and core undergo large displacements with moderate rotations.

2. MATHEMATICAL FORMULATION

The stability analysis is based on the equilibrium equations of sandwich panels, which are derived from the nonlinear EHSAPT. The high stiffness thin faces follow the Euler-Bernoulli assumptions and the thick core has a high order displacement pattern. In the core, the axial displacement and transverse displacement are assumed to be a third order polynomial and a second order polynomial in terms of the transverse coordinate, z, respectively. The geometric nonlinearity is considered in both faces and core, and the Lagrange strain is used as the kinematic relation. In faces, only the axial normal strain is considered as the result of the Euler-Bernoulli assumption,

$$\varepsilon_{xx}^{t,b} = u_{,x}^{t,b} + \frac{1}{2}\left(u_{,x}^{t,b}\right)^2 + \frac{1}{2}\left(w_{,x}^{t,b}\right)^2 \tag{1}$$

and in the core, the nonlinear axial normal strain, transverse normal strain, and shear strain are,

$$\varepsilon_{xx}^{c} = u_{,x}^{c} + \frac{1}{2}\left(u_{,x}^{c}\right)^2 + \frac{1}{2}\left(w_{,x}^{c}\right)^2 \tag{2a}$$

$$\varepsilon_{zz}^{c} = w_{,z}^{c} + \frac{1}{2}\left(u_{,z}^{c}\right)^2 + \frac{1}{2}\left(w_{,z}^{c}\right)^2 \tag{2b}$$

$$\gamma_{xz}^{c} = u_{,z}^{c} + w_{,x}^{c} + u_{,x}^{c}u_{,z}^{c} + w_{,x}^{c}w_{,z}^{c} \tag{2c}$$

Together with the EHSAPT-based element, which has 10 DOFs at each node, the weak form nonlinear governing equations are obtained from the variational principle of total potential energy. It is given as,

$$\left\{\mathbf{G}\left(\left\{\mathbf{U}\right\},\lambda\right)\right\} = \left[\mathbf{K}\left(\left\{\mathbf{U}\right\}\right)\right]\left\{\mathbf{U}\right\} - \lambda\left\{\mathbf{R}\right\} = \left\{\mathbf{0}\right\} \tag{3}$$

where $[\mathbf{K}(\{\mathbf{U}\})]$ is the stiffness matrix, $\{\mathbf{U}\}$ is the displacement vector, $\{\mathbf{R}\}$ is the unit load vector and λ is a load scalar. Since the geometric nonlinearity is considered, the stiffness matrix $[\mathbf{K}(\{\mathbf{U}\})]$ depends on the deformation, $\{\mathbf{U}\}$. Eq. 3 gives the governing equations of sandwich panels in arbitrary equilibrium state, including the pre-buckling state, buckling state, and post-buckling state. Therefore, it is used to determine the critical load and buckling mode, and will also be used to obtain the post-buckling response.

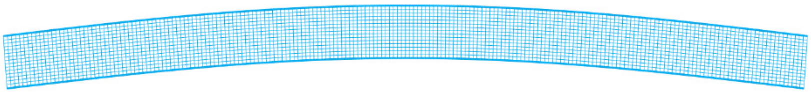
In the buckling analysis, the pre-buckling state is determined via a nonlinear static analysis. Thus, the pre-buckling state is not necessary to be a membrane state. The global buckling and wrinkling are not treated separated, and thus, the interaction between the global buckling and wrinkling can be captured. The critical load is calculated by solving an eigen-value problem and the corresponding eigen-vector gives the buckling mode.

The post-buckling response of sandwich structures can be regarded as an extension of the geometric nonlinear static behavior. Therefore, it is obtained from the weak form nonlinear governing equations. The path following procedure together with branch switching technique or imperfections is considered to capture the bifurcation phenomenon.

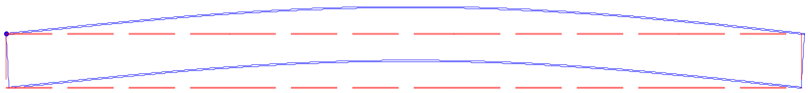
3. NUMERICAL EXAMPLES

Several sandwich panels with different lengths subjected to axial compressive loads are considered to study the stability properties. To investigate the effect of the axial rigidity of the core, additional numerical examples are considered by setting the core axial rigidity to be a very small amount. It shows that the core axial rigidity, although its magnitude is about two to three orders smaller than that of the faces, has a significant effect upon the stability response of sandwich panels. Although the core axial rigidity is usually negligible in the static analysis, it does affect the critical load, buckling mode, and the corresponding post-buckling response.

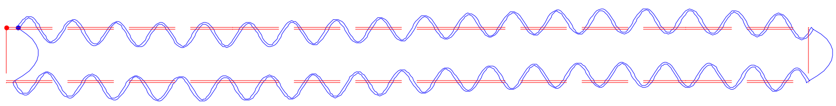
Fig. 1 plots the first buckling mode shape of a sandwich panel of 300 mm length. Fig. 1(a) is the mode shape given by the commercial FEA software ADINA, which is the same as the result given by the present approach, shown in Fig. 1(b). Fig. 1(c) is the mode shape when neglecting the core axial rigidity (by setting E_1^c=52.0×10^{-5} MPa instead of E_1^c=52.0 MPa). It is seen that the wrinkling appears as the first buckling mode when the core axial rigidity is neglected. This wrinkling mode shape agrees with the results given in the literature [3], in which the core axial rigidity was neglected.



(a)



(b)



(c)

Fig. 1: First buckling mode of a=300 mm sandwich panel: (a) ADINA; (b) EHSAPT, E_1^c=52.0 MPa; (c) EHSAPT, E_1^c =52.0×10^{-5} MPa.

Both global buckling and local wrinkling are observed in the numerical examples. The examples also show that the sandwich panels have non-membrane pre-buckling state, and the buckling mode may have a non-uniform sine wave pattern.

The post-buckling analysis reveals that the sandwich panels have different nonlinear post-buckling response when it shows global buckling or local wrinkling. Although sandwich panels may have similar global buckling shapes as ordinary beams, as shown in Fig. 1(b), they cannot retain the stable post-buckling response for a long duration as ordinary beams do. When global buckling occurs, the post-buckling response is stable at the beginning. However, with the growing of the deformation, sandwich panels may be destabilized due to the imperfections and localized effects. When wrinkling occurs, sandwich panels lose their load carrying capability immediately, and the required axial compressive load level for further deformation is lower than that needed to maintain current deformation.

The present study gives a comprehensive investigation on the nonlinear buckling and post-buckling response of sandwich panels. With several numerical examples, some conclusions and new finds are given.

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