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NORMAL AND SHEAR ELASTIC FOUNDATION CONSTANTS FROM ELASTICITY AND THE EXTENDED HIGH-ORDER SANDWICH PANEL THEORY

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1. INTRODUCTION

One of the approaches that could be used to analyze a face/core debond is the elastic foundation approach. In this approach, the top debonded face is considered to rest on an elastic foundation, which is provided by the rest of the structure, i.e., the core and the bottom face. Such elastic foundation models have been used from the 70s for the study of crack propagation. Kanninen [1,2] used such a model for the study of the double cantilever beam (DCB) test specimen in a homogeneous material with the crack at mid-thickness. A "crude" approximation of the elastic foundation modulus, based on the half-thickness of the beam, as outlined in the Results section, was used. Williams [3] extended Kanninen's model by using the Timoshenko beam theory in a homogeneous material and he used a formula for the elastic foundation constant similar to Kanninen's [3] based on the thickness of the debonded layer. In a more recent paper on the subject, Li and Carlsson [4] used the elastic foundation analysis on the tilted sandwich debond specimen. In a sandwich section, which is non-homogeneous, with varying properties and geometry of the two faces and the core, it is sensible to assume that the elastic foundation constant will depend on the mechanical properties and thicknesses, etc of all the constituent layers. Thus, the objective of this research is to provide a formula that answers this question.

2. FORMULATION

Extended High Order Sandwich Panel Theory (EHSAPT) Solution

The details of the EHSAPT theory are in [5]. We also use the same notation as in [5]. Let us consider the loading configuration that would be most appropriate for the normal elastic foundation constant. The one that would most first come to mind is a single face loading, in which case the normal spring constant would be defined by the transverse displacement of the top face/core interface and the transverse normal stress at the same place, after subtracting the transverse displacement at the bottom bounding surface, i.e. from a relationship of the form

$$k_n(x)\ \left[w(x,c)-w(x,-c-f_b)\right]=b\sigma^c_{zz}(x,c) \tag{1a}$$

This relationship would, however, include the bending deformation of the bottom face and thus would not represent the pure transverse compression of the structure below the top face. A symmetric loading configuration would, however, create a reference surface of uniform zero transverse displacement (i.e. the bending deformation would be eliminated and it would be like resting on a "rigid" surface. This reference surface would be at the middle of the core for a symmetric sandwich construction. Then, the normal spring constant would be defined from

$$k_n(x)\ w(x,c)=b\sigma^c_{zz}(x,c) \tag{1b}$$

Furthermore, if we consider a simply supported sandwich panel under transversely applied symmetric sinusoidal loading i.e. of the form:

$$q^t\left(x\right)=q_0\sin\frac{\pi x}{a};\qquad q^b\left(x\right)=-q_0\sin\frac{\pi x}{a}; \tag{2}$$

then the seven differential equations of the EHSAPT become seven algebraic equations for the seven constants: $U_0^{t,c,b}$, $W_0^{t,c,b}$ and Φ_0^c. Eliminating the $\sin(\pi x/a)$ from both sides of the resulting equation gives an expression of a constant k_n:

$$k_n=\frac{b}{W_0^t}\left[-c^c_{13}\left(\frac{\pi}{a}U_0^t+\frac{f_t\pi^2}{2a^2}W_0^t\right)+c^c_{33}\left(-\frac{2}{c}W_0^c+\frac{3}{2c}W_0^t+\frac{1}{2c}W_0^b\right)\right] \tag{3}$$

The shear constant of the elastic foundation (shear spring constant) expresses the relationship between the axial displacement, u, and the transverse shear stress, τ_{xz}, at the face/core interface. In the same spirit as for the normal spring constant, we need to refer to a zero axial displacement reference surface (i.e., again as resting on "rigid" surface). This can be achieved with an anti-symmetric loading. The reference surface is at the middle of the core for a symmetric sandwich construction. The shear spring stiffness can then be found from the relationship

$$k_s(x)\left[u(x,c)+\left(c-z_{ref}\right)w^c_{,x}(x,c)\right]=b\tau^c_{xz}(x,c) \tag{4}$$

Eliminating the $\cos(\pi x/a)$ from both sides of the resulting equation gives a constant value of the shear spring constant as

$$k_s=c^c_{55}\left|\frac{\left[-2\Phi^c_0-\frac{1}{2c}U^b_0-\frac{2}{c}U^c_0+\frac{5}{2c}U^t_0+\frac{\pi f_b}{4ca}W^b_0+\left(1+\frac{5f_t}{4c}\right)\frac{\pi}{a}W^t_0\right]}{U^t_0+\frac{\pi f_t}{2a}W^t_0}\right| \tag{5}$$

Since the shear stress can be negative, the absolute value is taken in Eq. 5.

Elasticity Solution

The elasticity solution is given in detail in [6,7]. The analysis is done in complex variables. In this solution, the displacements in each layer are as follows:

$$u=\sum_{j=1,2,3,4}e^{\lambda_j z}a_j\cos px \tag{6a}$$

and

$$w=\sum_{j=1,2,3,4}\frac{c_{11}p^2-c_{55}\lambda_j^2}{\left(c_{13}+c_{55}\right)p\lambda_j}e^{\lambda_j z}a_j\sin px \tag{6b}$$

where p = π/a and λ_j are the four roots of the characteristic equation of each layer:

$$c_{33}c_{55}\lambda^4+p^2\left[\left(c_{13}+c_{55}\right)^2-c_{11}c_{33}-c_{55}^2\right]\lambda^2+p^4c_{11}c_{55}=0 \tag{6c}$$

The roots of the characteristic equation (Eq. 6c) are either all real or complex conjugates. Regarding the a_j constants in the previous equations, within each layer i, where $i = t, c, b$, there are four constants: a_{ij}, j = 1,2, 3, 4. Therefore, for the three layers, this gives a total of 12 constants to be determined. These are found from the continuity conditions at the faces/core interfaces and the traction conditions at the bounding surfaces

Again, eliminating the sin ($\pi x/a$) from both sides of the resulting equation, gives an expression of a constant k_n:

$$k_n=b\frac{\sum_{j=1,2,3,4}\left[-c^c_{13}p+c^c_{33}\frac{c^c_{11}p^2-c^c_{55}\lambda^2_{cj}}{\left(c^c_{13}+c^c_{55}\right)p}\right]e^{\lambda_{cj}c}a_{cj}}{\sum_{j=1,2,3,4}\frac{c^t_{11}p^2-c^t_{55}\lambda^2_{tj}}{\left(c^t_{13}+c^t_{55}\right)p\lambda_{tj}}e^{\lambda_{tj}c}a_{tj}} \tag{7}$$

The shear spring constant is defined in Eq.4. Assuming symmetric construction, for which $z_{ref} = 0$, and subsequently eliminating the cos ($\pi x/a$) from both sides of the resulting equation, gives the following expression of a constant k_s:

$$k_s=b\frac{\sum_{j=1,2,3,4}c^c_{55}\left[\lambda_{cj}+\frac{c^c_{11}p^2-c^c_{55}\lambda^2_{cj}}{\left(c^c_{13}+c^c_{55}\right)\lambda_{cj}}\right]e^{\lambda_{cj}c}a_{cj}}{\sum_{j=1,2,3,4}\left[1+c\frac{c^c_{11}p^2-c^c_{55}\lambda^2_{cj}}{\left(c^c_{13}+c^c_{55}\right)\lambda_{cj}}\right]e^{\lambda_{tj}c}a_{tj}} \tag{8}$$

3. RESULTS

Table 1 shows the elastic foundation "normal spring" constant from Elasticity (benchmark), EHSAPT and the Kanninen-type simple formula based on the core. It can be seen that the agreement between Elasticity and EHSAPT is excellent. There is only a case of moderate discrepancy, namely a 10% discrepancy in the isotropic homogeneous case, which is to be expected, considering that the EHSAPT is a sandwich panel theory and makes assumptions regarding the faces and the core. The Kanninen-type simple formula has a varying accuracy, which depends on the core type, and in all cases predicts a less stiff foundation.

Table 1: Elastic Foundation Constant – "normal spring", k_n, MPa. Comparison of Elasticity, EHSAPT and Kanninen-type formula.

	Elasticity	EHSAPT	Kanninen [core] bE_3^c/c
Aluminum Faces H100 Core	692.0	698.1 +0.88%	520 -24.8%
Carbon Epoxy Faces H100 Core	692.6	698.5 +0.85%	520.0 -24.9%
Aluminum Faces Balsa Wood Core	31,248.1	31,308.4 +0.19%	30,880.0 -1.18%
Aluminum Faces Honeycomb Core	1,220.8	1,221.5 +0.06%	1,200.0 -1.70%
Aluminun Faces Aluminum Core (homogeneous isotropic)	313,936.2	346,387.4 +10.3%	276,000.0 -12.1%

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