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THREE DIMENSIONAL TRANSIENT ANALYSIS OF FGM RECTANGULAR SANDWICH PLATE SUBJECTED TO THERMAL LOADING

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1. INTRODUCTION

In recent years with extensive progresses in science and technology, an extensive research has been conducted dealing with materials with novel structures. FGM materials are a new kind of composites that have been used extensively in aerospace, nuclear, biomechanics, electronic and etc. One of the main applications of FGM materials is in high temperature environments. So a major part of researches in literature have been focused on thermal stress analysis, thermal buckling, fracture mechanics and optimization, for instance Senthil and Batra [1] investigated transient thermal analysis of rectangular FGM plates using three dimensional elasticity theory, Zhong [2] analyzed the three dimensional behavior of FGM plates using state space equations. Tounsi et al.[3] Provided a static analysis of functionally graded sandwich plate using modified trigonometric shear deformation theory, Alibeigloo [4] carried out an time dependent analysis on sandwich plates based on Lord-Shulman formulation, using generalized coupled thermoelasticity. In the present study, a three-dimensional transient analysis of functionally graded rectangular sandwich plate under thermal load has been investigated. The functionally graded material is graded in thickness direction and follows exponential distribution. The plate with simply supported boundary conditions is under transient thermal load on the upper surface. The solution process is analytical and semi-analytical for simply supported boundary conditions.

2. PROBLEM DESCRIPTION

The geometry of the plate and the position of the coordinate system is shown in Fig. 1. The surface layers are made of metal and ceramics whereas the mid layer is of functionally graded material (FGM) with exponential function distribution. It is noted that the plate is under an uniform transient thermal load applied on the upper face of the sandwich plate.

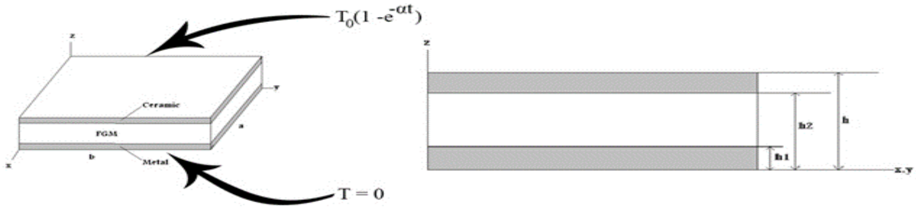


Fig. 1: Geometric diagram of sandwich plate.

Temperature Field

Heat distribution equation in the Cartesian coordinate system in General is:

$$k\frac{\partial^2 T}{\partial x^2}+k\frac{\partial^2 T}{\partial y^2}+\frac{\partial k}{\partial z}\frac{\partial T}{\partial z}+k\frac{\partial^2 T}{\partial z^2}=\rho c\frac{\partial T}{\partial t} \tag{1}$$

Properties of constituent material, Initial, and boundary conditions are:

$$b.\mathrm{c}:\begin{cases} T=0 & \mathrm{t=0} \\ T=0 & \mathrm{z=0} \\ T=T_0(1-e^{-\alpha t}) & \mathrm{z=h} \\ T=0 & \mathrm{x=o,a} \\ T=0 & \mathrm{y=o,a} \end{cases} \quad ,\{E,\rho,k,C\}=\left\{\hat{E},\hat{\rho},\hat{k},\hat{C}\right\}e^{\frac{p\times(z-h_1)}{h_f}} \tag{2}$$

And the ^ notation shows the properties of material and the P shows the index of material changes.

Now, with respect to the boundary conditions in x=0, a and y=0, b and consider the following Fourier expansion for temperature distribution and by applying Laplace transform and using Eq .2, the temperature for the FG layer and upper and lower layers of plate will be: (p2, p3, p4 shows the index of material changes.)

For FG layer:

$$T=\xi^{\frac{3-A}{2}}I(\frac{1}{2}\sqrt{1-2A+A^2+\frac{16w^2}{a^2}},\frac{2\xi}{a})C_1+\xi^{\frac{3-A}{2}}K(\frac{1}{2}\sqrt{1-2A+A^2+\frac{16w^2}{a^2}},\frac{2\xi}{a})C_2, A=3+\frac{2p_3}{p},$$

$$\mathrm{w}=\sqrt{{p_m}^2+{p_n}^2-a^2\left(\frac{1}{4}+\frac{p_3}{2p}\right)},\ \xi=b.e^{\frac{a}{2}z}, b=\sqrt{\frac{\hat{c}\hat{\rho}s}{\hat{k}}e^{\frac{-ph_1}{h_f}}}, a=\frac{p}{h_f}, p=p_2+p_4-p_3 \quad (3)$$

For lower and upper layer respectively temperature is:

$$T_m=C_3e^{\sqrt{K}z}+C_4e^{-\sqrt{K}z}, K={\mathrm{p}_m}^2+{p_n}^2+\frac{C_m\rho_m s}{k_m}, p_m=\frac{m\pi}{a}, p_n=\frac{n\pi}{b}$$
$$T_c=C_5e^{\sqrt{K'}z}+C_6e^{-\sqrt{K'}z}, K'={\mathrm{p}_m}^2+{p_n}^2+\frac{C_c\rho_c s}{k_c} \quad (4)$$

The parameters I and K are Bessel functions. Constants of (C1, C2, C3, C4, C5, and C6) are obtained from thermal boundary conditions.

State Space Equation Extraction

Using stress-displacement relations and governing equations of motion in the absence of body forces the state-space equations are obtained.

Analytical and Semi-Analytical Solution for a Simply Supported Plate

In the case of analytical solution, the Laplace transform is used for time, the Fourier series are used for the length and width of the plate and the state-space method is used in thickness direction. For the cases of semi-analytical solution, the differential quadrature method is used. In this case, by applying differential quadrature method in length and width directions, and by making use of state-space method in thickness direction, semi-analytical solution is presented. First, the matrix of the state space is calculated for each layer, then the boundary conditions between the layers and the continuity of the stress and displacement are applied and the general state space matrix is obtained. Finally, the equations are solved using the stress –displacement relations of the upper and lower surface of the plate. Assumed that the plate is under transient temperature on the upper surface. The boundary conditions at the upper and lower layers of the plate are:

$$\sigma_z=0,\ \tau_{xz}=\tau_{yz}=0 \qquad z=0,\quad \sigma_x=0,\ V=W=0,\quad x=0,a$$
$$\sigma_z=0,\ \tau_{xz}=\tau_{yz}=0 \qquad z=h,\quad \sigma_y=0,\ U=W=0,\quad y=0,b \quad (5)$$

Vector δ and General solution of the state space equation for analytical and semi-analytical for point (i, j) respectively is

$$\frac{d\delta}{dz}=G\delta+KT,\ \delta=\left\{\bar{\sigma}_z\quad \bar{U}\quad \bar{V}\quad \bar{W}\quad \bar{\tau}_{xz}\quad \bar{\tau}_{yz}\right\}^T,\ \delta(z)=\frac{1}{\mu(z)}\left[\int_{z_0}^{z}\mu(z).k.\bar{\mathrm{T}}\mathrm{d}z+\delta(\mathrm{z}_0)\right],\ \mu(z)=e^{\int_{z_0}^{z}-Gdz} \quad (6a)$$

$$\frac{d\delta_{(i,j)}}{dz}=G_{(i,j)}\delta_{(i,j)}+kT,\ \delta_{(\mathrm{i,j})}=\left\{\sigma_{z_{(\mathrm{i,j})}},\mathrm{U}_{(\mathrm{i,j})},\mathrm{V}_{(\mathrm{i,j})},\mathrm{W}_{(\mathrm{i,j})},\tau_{\mathrm{xz(i,j)}},\tau_{\mathrm{yz}_{(\mathrm{i,j})}}\right\}^{\mathrm{T}} \quad (6b)$$

Now consider the continuity of the boundary conditions between layers and then by applying the boundary conditions upper and lower layers of the plate transient bending of the plate will complete:

3. RESULTS AND DISCUSSION

In this section, the results obtained by solving the problem are discussed. Generally, validation and convergence are investigated. Material properties of sandwich plate of the lower surface (Aluminum) and upper surface (Silicon carbide) of the structure are listed in table 1.

Table 1: Material properties of sandwich plate.

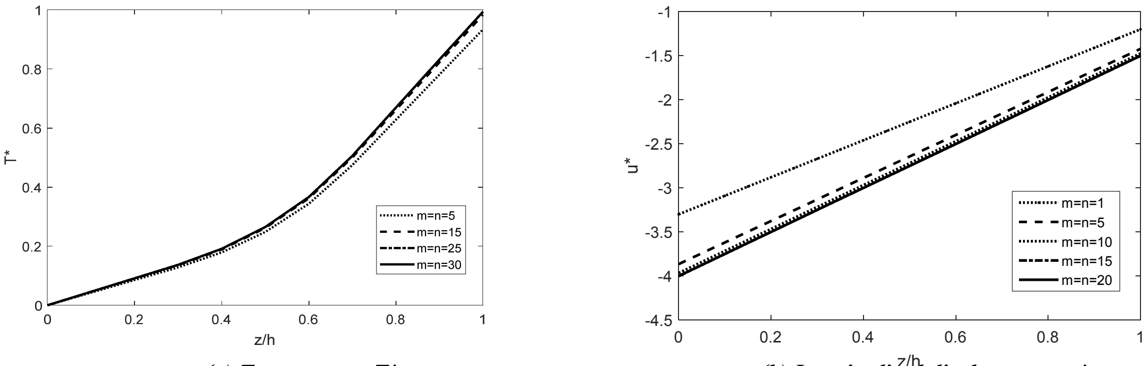
Material	K[W/(mK)]	C[J/(KgK)]	E (GPa)	α(1/K)	ρ(kg/m^3)
SiC	65	670	427	4.3E-6	3100
AL	233	896	70	23.4E-6	2707

Besides, nondimensionalization of parameters is performed as follows:

$$t^*=\frac{t}{t_r}, \mathrm{t_r}=1s, T^*=\frac{T}{T_i}, {U_{ij}}^*=\frac{U_{ij}}{ph}, {\sigma_{ij}}^*=\frac{\sigma_{ij}}{pk^*}, \bar{\mathrm{z}}=z/\mathrm{h}, \tau=\alpha t_r, a/b=1, a/h=10, T_0=200k$$

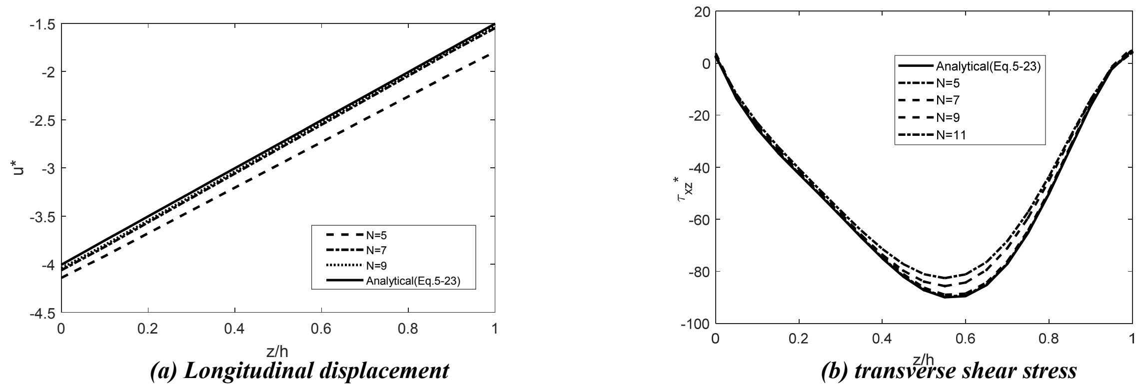
Where $p=\alpha_O\times T_0(\alpha_O=10^{-6}\frac{1}{k}, k^*=1GPa)$ is valid for thermal load.

In the presented analysis, the number of terms in Fourier series is increased to converge the result of series to the final solution. In Figs .2(a) and 2(b), temperature convergence, Longitudinal displacement are illustrated respectively versus number of terms in Fourier series. In Figs. 3(a) and 3(b), the results of analytical solution and differential quadrature method for longitudinal displacement and transverse shear stress are illustrated. It can be observed that for longitudinal displacement and transverse shear stress for 9×9 and 11×11 nodes, the illustrated figures are so close which indicates acceptable convergence. By comparing the semi-analytical solution using differential quadrature method and the results obtained by analytical method, it can be inferred that the semi-analytical solution provides accurate results.



(a) Temperature T* ***(b) Longitudinal displacement u****

Fig. 2: Convergence of distribution, temperature, longitudinal displacement and shear stress in analytical solution.



(a) Longitudinal displacement ***(b) transverse shear stress***

Fig. 3: Convergence of distribution Longitudinal displacement and shear stress in semi-analytical solution and comparison with analytical solution at t=60s.

In order to provide validation, the results for scaled rectangular sandwich plate under thermal loading with simply supported are calculated and compared for a single layer plate according to the properties which are mentioned in reference [1]. As can be observed from Table. 2, the present results are in acceptable agreement with reference results. The differences between the values of displacement and shear stress are caused by adopting different methods for solving the problem

Table 2: Comparison of result transverse displacement obtained from analytical solution with the reference [1].

Time	0	0.25	0.5	0.75	1	1.25	1.5
Present	0	0.311	0.322	0.324	0.323	0.322	0.321
Ref [1]	0	0.318	0.32	0.32	0.32	0.32	0.32

CONCLUSIONS

The obtained results indicate that Rapid convergence is achieved in differential quadrature method, and the obtained results are in acceptable agreement with the results of analytical method.

REFERENCES

[1] Vel, S.S., Batra, R, “Three-dimensional analysis of transient thermal stresses in functionally graded plates”, Int J Solids Struct, 2003; 40:7181-7196.

[2] Liu, W., Zhong, Z., “Three-dimensional analysis of simply supported functionally graded plate with arbitrary distributed elastic modulus”, Tsinghua Sci Technol, 2009; 14:58-63 .

[3] Tounsi, A., Houari, M.S.A., Benyoucef, S, “A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates”, Aerosp Sci Technol, 2013; 24: 209-220.

[4] Alibeigloo, A., “Three dimensional coupled thermoelasticity solution of sandwich plate with FGM core under thermal shock”, Compos Struct, 2017; 177:96-103.