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STRUCTURAL DESIGN OPTIMISATION OF RECTANGULAR HONEYCOMB CORE SANDWICH PANELS UNDER OUT-OF-PLANE LOADING

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1. INTRODUCTION

Sandwich systems are often used in applications where enhanced structural performance is required at minimal weight. Offshore deck systems stand to benefit from sandwich construction not only in relation to improved resistance to localised equipment loads and other distributed loads but also due to favourable manufacturing and assembly procedures.

Minimising weight, which is particularly important for offshore applications, requires a relatively accurate design method to be established, and to increase the applicability of sandwich panels, a design approach based on a practical methodology is required. Achieving a design methodology naturally evolves from an ability to assess candidate solutions, which requires an accurate and practical analysis method and a comprehensive understanding of all possible failure modes.

This paper proposes a methodology for optimising rectangular steel sandwich panels with a rectangular honeycomb core under combined uniformly distributed loads and patch loads. The analysis of sandwich panels is based on the first-order shear theory for isotropic sandwich plate bending [1] as well as the sandwich concept, where it is assumed that top and bottom plates resist bending moments and the core resists the shear forces. When compared to the more complex equations for orthotropic sandwich plate bending derived by Robinson [2], accurate results for panels with similar core shear stiffness in the x- and y-directions can be achieved. Moreover, isotropic sandwich plate bending equations allow for a relatively accurate optimisation sequence to run without recalculating the internal forces for every iterative step.

The internal forces obtained from the aforementioned analysis are used to generate the constraints of a structural optimisation problem to minimise weight, which is solved using the Method of Moving Asymptotes (MMA) [3]. These constraints are generated considering the various failure modes in the domain of the panel, including material yielding, plate buckling and deformation control.

The establishment of this methodology, based on trigonometric series for analysis and on a gradient-based method for optimisation, enables the optimal design of rectangular sandwich panels to be established, leading to the minimum weight panel that fulfils all the design constraints.

2. METHODOLOGY

Failure Modes

It is important to consider all possible failure modes to achieve a systematic performance assessment method at a local level. The critical failure modes considered for rectangular honeycomb core sandwich panels are face yielding, face intercellular buckling, core compressive yielding, core compressive buckling, core shear yielding, which includes punching shear, and core shear buckling. The deformation of the panel is also controlled.

Steel structures yield according to the von Mises yield criterion. According to the sandwich concept, the top and bottom plates are under a general stress state, while the plates of the core are under a pure shear stress state. In this study, the interaction of compression and shear stresses within the core is neglected.

For plate buckling failure modes, the critical buckling stress σ_{cr} can be generally expressed as:

$$\sigma_{cr} = k \cdot \frac{\pi^2 \cdot E}{12(1-v^2)} \cdot \left(\frac{t}{b}\right)^2 \qquad (1)$$

where E is Young's modulus, set to 210 GPa, t is the plate thickness, v is the Poisson ratio, b is a dimension of the plate, and k is the buckling coefficient that is defined by the loading and support conditions along the plate boundaries.

Analysis

Consider a rectangular sandwich plate of sides a and b, simply supported on all edges. The deformed shape of a sandwich plate, which can be decomposed in partial deflections of bending $w_b(x,y)$ and shear $w_s(x,y)$ deformation [1], can be respectively computed by double trigonometric series according to:

$$w(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{p_{mn} \cdot (1-v^2) \cdot sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}}{D \cdot \pi^4 \cdot \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{p_{mn} \cdot sin\frac{m\pi x}{a} sin\frac{n\pi y}{b}}{S \cdot \pi^2 \cdot \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]} \qquad (2)$$

where D is the sandwich plate flexural stiffness, S is the core shear stiffness, and p_{mn} are the coefficients of the double Fourier expansion that are respectively described for an uniformed distributed load (UDL) and a patch load (PL) according to:

$$p_{mn,UDL} = \frac{4p_0}{\pi^2 mn} \cdot [1-\cos(m\pi)] \cdot [1-\cos(n\pi)]; \quad p_{mn,PL} = \frac{16p_0}{\pi^2 mn} \sin\frac{m\pi\xi}{a} \sin\frac{n\pi\eta}{b} \sin\frac{m\pi u}{2a} \sin\frac{n\pi v}{2b} \tag{3}$$

where p_0 is the applied pressure, and ξ, η, u and v describe the location and size of the patch.
The internal forces in the domain of the panel can be easily derived from the deflected shape using similar double trigonometric series. Load combinations can be considered by superimposing the effects of each load, calculated separately, since the internal forces are assumed to be recovered accurately based on linear analysis.

From this analysis, the following quantities are computed: maximum equivalent von Mises bending moment, M_{vM}, for face yielding, obtained from the general planar von Mises stress combined with the sandwich effect; maximum bending moment in x- and y-directions at critical locations, M_x and M_y, for intercellular buckling; maximum shear forces in x- and y-directions, $V_{ed,x}$ and $V_{ed,y}$, for core shear yielding and buckling; maximum compressive stress, C_{ed}, for compressive yielding and buckling; the constants K_{bend} and K_{shear}, for serviceability, which are equal to $D \cdot w_{b,max}$ and $S \cdot w_{s,max}$, obtained from Eqs. 2 and 3.

Optimisation

At this stage, the MMA algorithm for optimisation can be established to minimize the weight function, given by:

$$W = 7850 \cdot \left[t_{f,top} + t_{f,bot} + h \cdot \left(\frac{t_{wx}}{l_x} + \frac{t_{wy}}{l_y} \right) \right] \tag{4}$$

where $t_{f,top}$, $t_{f,bot}$, h, t_{wx}, l_x, t_{wy} and l_y are the geometric variables that define the rectangular honeycomb core sandwich panel to be optimised, representing the thickness of the top and bottom plates, height of the panel and thicknesses and cell sizes in the x- and y-directions, respectively. The steel yield strength is represented by f_y, set to 235 MPa.
The problem constraints are generated from the aforementioned failure modes. Face yielding for the top plate is given by:

$$h \cdot t_{f,top} \cdot f_y \geq M_{vM} \tag{5}$$

Core shear buckling in the x-direction, for the case where h is greater than l_y, is given by:

$$\left(5.35 + 4 / \left(\frac{h}{l_y} \right)^2 \right) \cdot \frac{{t_{wx}}^3}{{l_y}^2} \geq V_{ed,x} \cdot \frac{12(1-v^2)}{\pi^2 E} \tag{6}$$

Deflection control, assuming the maximum allowable displacement to be 1/360 of the smaller span, is given by:

$$E \cdot \frac{t_{f,top} \cdot t_{f,bot} \cdot h^2}{t_{f,top} + t_{f,bot}} / K_{bend} + G \cdot h \cdot \min\left(\frac{t_{wx}}{l_x}, \frac{t_{wy}}{l_y} \right) / K_{shear} \geq \frac{360}{min(a,b)} \tag{7}$$

Other constrains include face yielding of the bottom plate, face intercellular buckling, for which contribution of M_{xy} is neglected, core compressive yielding, core compressive buckling and core shear yielding.

3. RESULTS

A rectangular sandwich panel with an area of 5400×9000 mm^2 under a combination of distributed and patch loads is analysed and optimised using the proposed methodology. To improve the convergence of the method, the failure modes are rearranged to the following form:

$$\frac{A}{R} - SF \leq 0 \tag{8}$$

where, A stands for actions, R for resistance and SF for the target safety factor for the optimal panel. In this example, the target safety factors for all the failure models are kept as 1.0. A schematic view of the example is presented in Fig. 1(a), an illustrative example of a map of internal forces, obtained from the double Fourier series, is presented in Fig. 1(b) for the bending moment in the x-direction, and the detailed nonlinear FE model of the optimal panel is presented in Fig. 1(c), displaying the compressive stress maps in the x-direction.

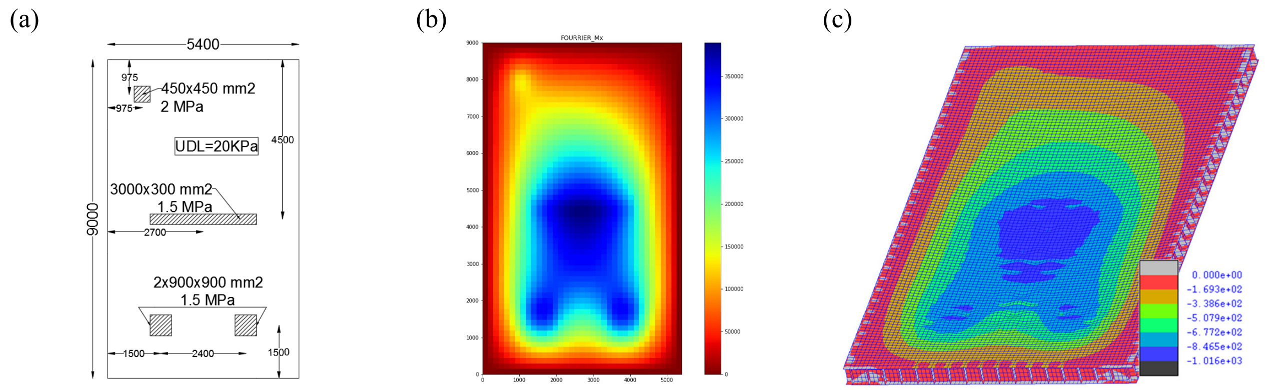

Fig. 1: (a) Description of load case (b) Example of stress map (c) Detailed FE Model.

Detailed nonlinear Finite Element models are developed in ADAPTIC [4] for verifying the results obtained from proposed approach. These models are built with 9-noded co-rotational shell elements [5] to discretise both the core and the facings while a triaxial elastoplastic material model is used for mild steel.

The main outcomes of the analysis are the maximum internal forces on the domain of the panel, which are presented in Table 1a and serve as input parameters for the optimisation sequence. The outcomes of the optimisation algorithm are the dimensions of the optimal panel and are presented in Table 1b, while the safety factors for the optimal panel are presented in Table 1c. It can be observed that several failure modes have a safety factor close to 1.0. The optimal panel is confirmed using exhaustive search in the vicinity of the solution.

Table 1: Presentation of results: (a) Material properties and internal forces (b) Optimal panel (c) Safety factors.

(a)

M_{vM} [N.mm/mm]	407000
M_x [N.mm/mm]	389000
M_y [N.mm/mm]	287000
$V_{ed,x}$ [N/mm]	467
$V_{ed,y}$ [N/mm]	452
C_{ed} [MPa]	2.02
K_{bend} [N.mm^2]	9.2e11
K_{shear} [N]	5.2e5

(b)

$t_{f,top}$ [mm]	6.17
$t_{f,bot}$ [mm]	4.97
h [mm]	348.1
t_{wx} [mm]	3.0
l_x [mm]	303.3
t_{wy} [mm]	2.97
l_y [mm]	303.3

(c)

Yielding Top	1.241
Yielding Bot	1.000
Shear yielding X	1.000
Shear yielding Y	1.022
Compressive yielding	2.289
Intercellular buckling	1.000
Shear buckling X	1.000
Shear buckling Y	1.000
Compressive buckling	1.621
Deformation control	1.000

The nonlinear response of the optimal panel (see Fig. 1(c)) is presented in Fig. 2.

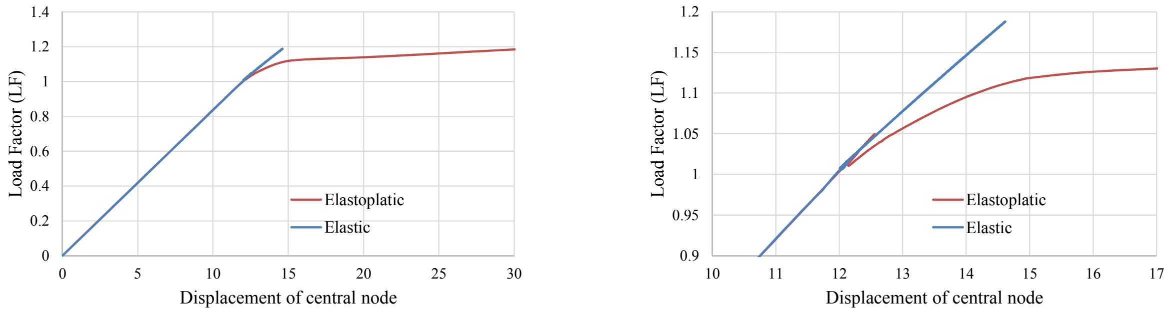

Fig. 2: Nonlinear response of the optimal panel.

From the deformed shape, it can be established that the critical failure mode is shear yielding and buckling on the core plates below the 900×900 mm^2 patch loads, at a load factor LF=1.01. The fact that this load factor is close to 1.0 indicates that the interaction between different failures modes is not significant. The results from an identical model with linear elastic material are shown so as to identify material yielding, which occurs at LF=1.02. At LF=1.0, the maximum displacement is 13.10 mm, complying with the serviceability limit of 15.0 mm (i.e. 5400mm/360). From the analysis of the post-failure response, the panel displays residual capacity to resist the applied loading.

4. CONCLUDING REMARKS

A methodology for design optimisation of rectangular honeycomb sandwich panels based on first-order shear theory for isotropic sandwich plate bending for structural analysis and the MMA algorithm for structural optimisation is proposed. Verification is undertaken against results of detailed nonlinear finite element analysis. The method is shown to provide relatively accurate results, rendering it useful for the development of an optimisation method for offshore deck systems composed of all-steel sandwich panels.

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