Parameter Determination of Sensor Stochastic Models under Covariate Dependency

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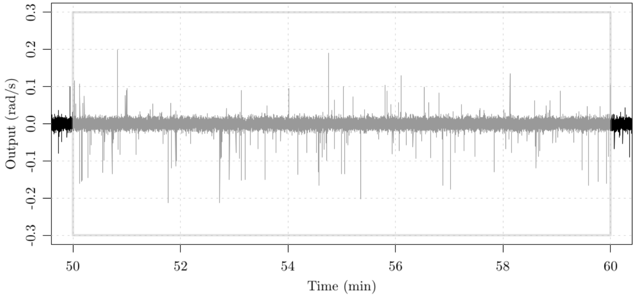
Menu

- ➤ Noise Characterization via Allan / Wavelet Variance
- ➤ Generalized Method of Wavelet Moments (GMWM)
- ➤ GMWM Extensions for Covariate Dependencies

Analyzing a Signal

Types of Signals

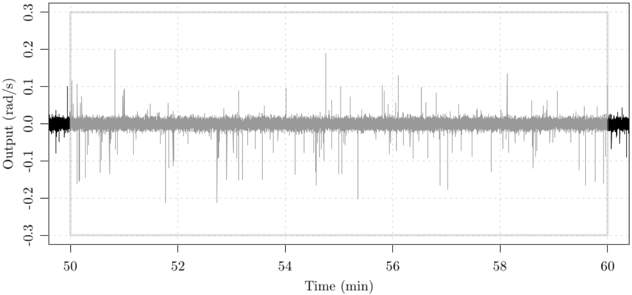
- Oscillator
- Accelerometer
- Gravimeter
- Gyroscope



Analyzing a Signal

Types of Signals

- Oscillator
- Accelerometer
- Gravimeter
- Gyroscope
- Stochastic characteristics of time series: $y_t, t \in \mathbb{N}$



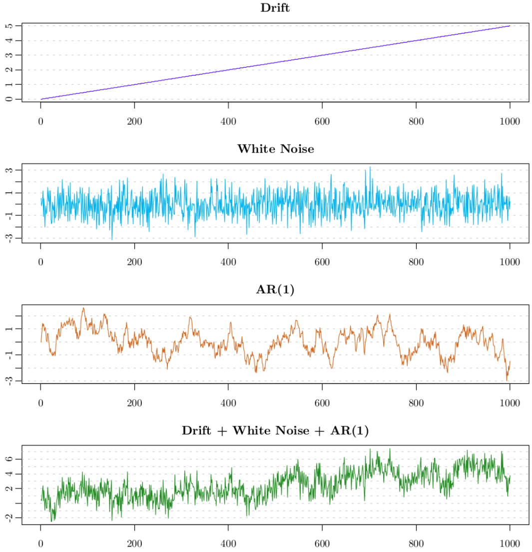
Analysis of an Error Signal

State-space model

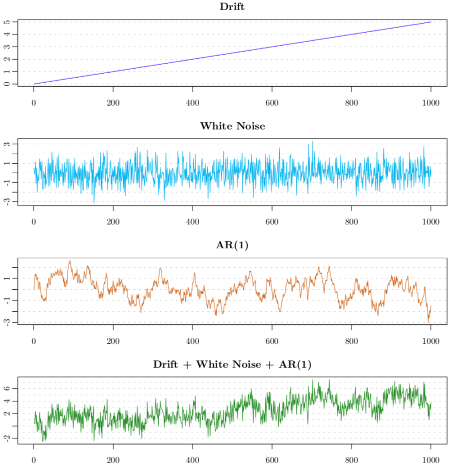
$$y_t = \omega t + u_t + \varepsilon_t$$
$$\varepsilon_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\text{WN}})$$
$$u_t = \phi u_{t-1} + \eta_t$$
$$\eta_t \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_{\text{AR(1)}})$$
$$\omega, \phi \in \mathbb{R}$$

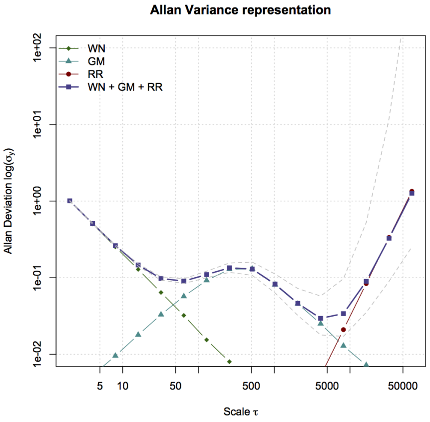
Noise parameters

$$\boldsymbol{\theta} = (\omega, \phi, \sigma^2_{\text{AR(1)}}, \sigma^2_{\text{WN}})$$



Analysis of an Error Signal via Allan Variance





Challenges

Existing methods:

- "Graphical" Allan Variance
  - Limited to a few models
  - (Proven as) not consistent in general
  - "Inefficient" (non-automated)
- MLE (with EM algorithm)
  - Computationally intensive
  - Diverges with "complex" models

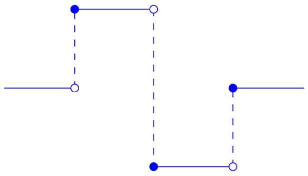
Need of an estimator

- Complex time-series model
- Computational efficiency
- Robust to outliers

Generalized Method of Wavelet Moments

Main idea

- Usage of the Wavelet Variance (WV)
- Filter of the signal with the Wavelet Function
- Exploitation of the relationship existing between a model $\boldsymbol{\theta}$ and its WV $\boldsymbol{\nu}(\boldsymbol{\theta})$ (i.e. **mapping** $\boldsymbol{\theta} \mapsto \boldsymbol{\nu}(\boldsymbol{\theta})$).
- "Inverse" this mapping by minimizing some discrepancies between empirical (i.e. observed WV/AV $\hat{\boldsymbol{\nu}}$) and the theoretical WV for a model $\boldsymbol{\nu}(\boldsymbol{\theta})$.



GMWM Estimator

Definition

Solution of the following optimization problem with weighting matrix $\mathbf{\Omega}$

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \; \left(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\right)^T \mathbf{\Omega} \left(\hat{\boldsymbol{\nu}} - \boldsymbol{\nu}(\boldsymbol{\theta})\right)$$

Identifiable

$$\boldsymbol{\nu}(\boldsymbol{\theta}_1) = \boldsymbol{\nu}(\boldsymbol{\theta}_2) \quad \text{iff} \quad \boldsymbol{\theta}_1 = \boldsymbol{\theta}_2$$

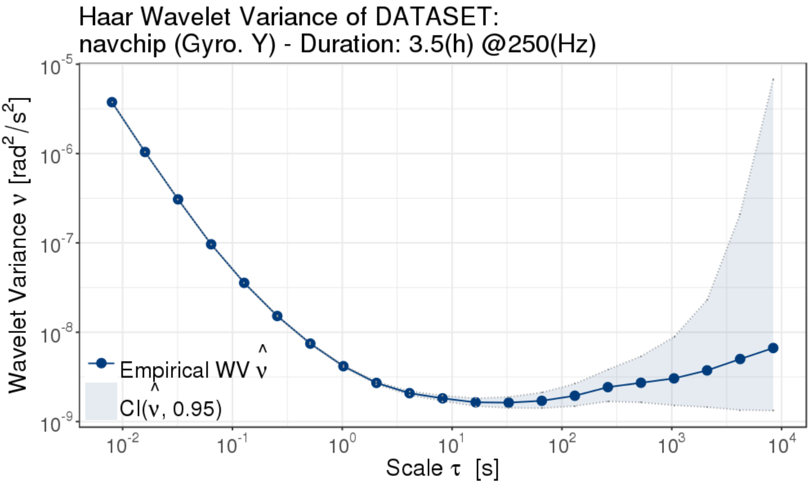
Consistent

$$\hat{\boldsymbol{\theta}} \xrightarrow{\text{p}} \boldsymbol{\theta}_0$$

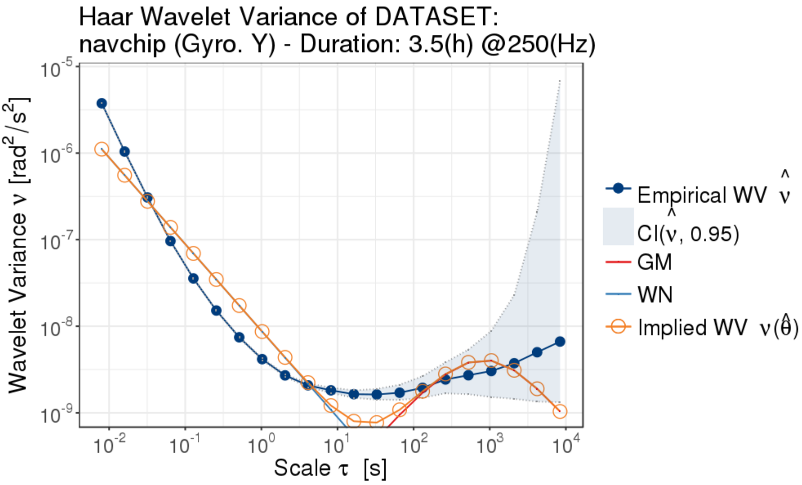
Asymptotically Normal

$$\sqrt{T}\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right) \underset{T \to \infty}{\xrightarrow{\text{p}}} \mathcal{N}\left(\mathbf{0}, \mathbf{\Sigma}\right)$$

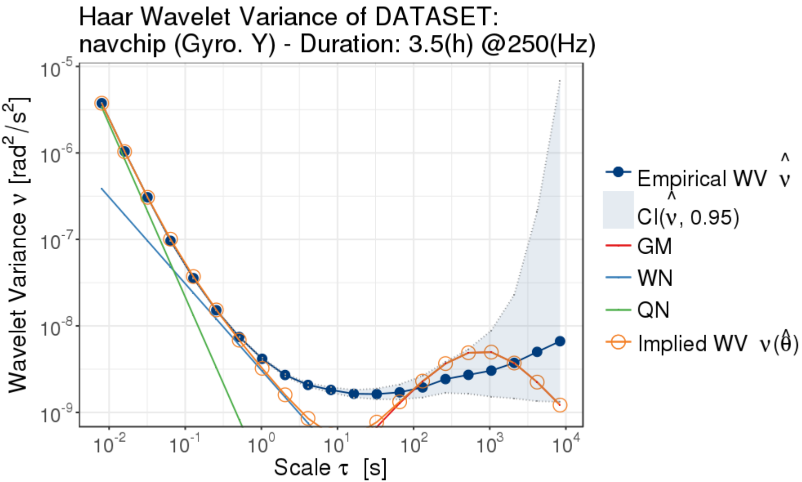
GMWM Example: Empirical WV



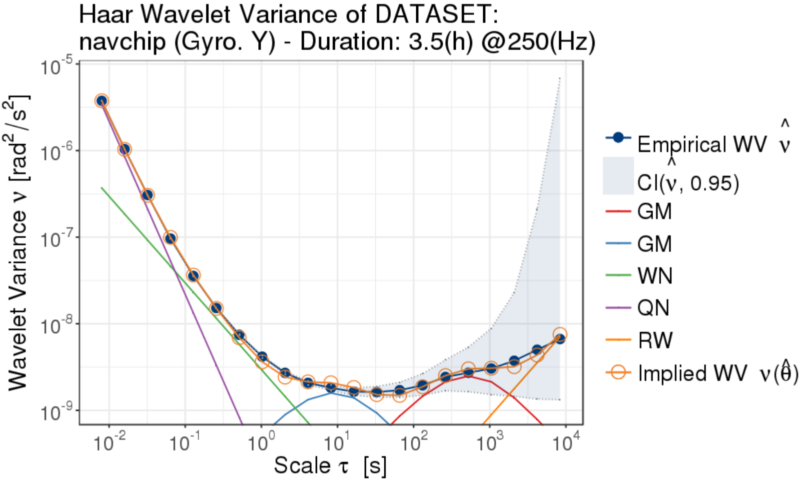
GMWM Example: incomplete model



GMWM Example: incomplete model cont.

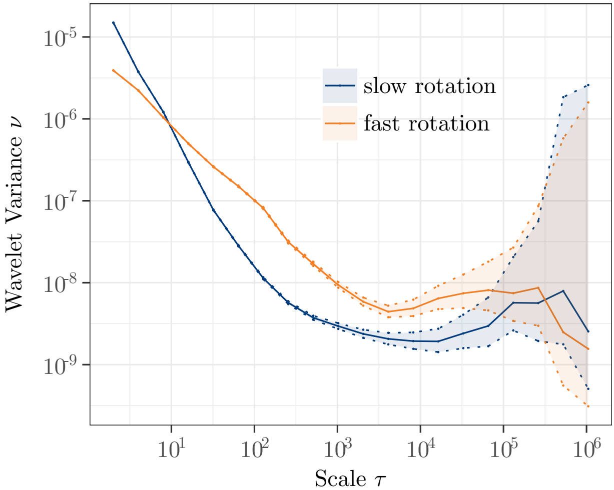


GMWM Example: complete model



GMWM Example: covariate influence

MEMS IMU Gyroscope rotating at $30\,°/s$ and $360\,°/s$



Extension for Covariate Dependency

Definition

- external process: $X_t, t \in \mathbb{N}$

  ➤ previous example: rotational speed
- *White Noise* process:

$$V_t \quad \overset{iid}{\sim} \mathcal{N}\left(0, \gamma^2\right)$$

- *Auto-Regressive* process of order 1:

$$\begin{aligned} u_t &= \phi \; u_{t-1} + \varepsilon \\ \varepsilon &\overset{iid}{\sim} \mathcal{N}\left(0, \eta^2\right) \end{aligned}$$

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- *Auto-Regressive* process of order 1:

$$u_t|X_t = \phi_t\, u_{t-1} + \varepsilon_t, \quad \phi_t = h(\varphi_1 + \varphi_2 X_t),$$

$$\varepsilon_t|X_t \overset{iid}{\sim} \mathcal{N}\left(0, \eta_t^2\right), \qquad \eta_t^2 = k\left(\upsilon_1 + \upsilon_2 X_t\right)$$

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Extended parameter vector

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\varsigma}^T & \boldsymbol{\varphi}_1^T & \dots & \boldsymbol{\varphi}_d^T & \boldsymbol{\upsilon}_1^T & \dots & \boldsymbol{\upsilon}_d^T \end{bmatrix}^T \in \boldsymbol{\Theta}$$

Extension

Dynamic GMWM estimator

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \boldsymbol{\Theta}}{\text{argmin}} \, \frac{1}{K} \sum_{k=1}^{K} \left\| \hat{\boldsymbol{\nu}}_k - \boldsymbol{\nu}(\boldsymbol{\theta}, c_k) \right\|^2_{\hat{\boldsymbol{\Omega}}_k}$$

c_k explains the covariate influence on the WV of bin k

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Dynamic GMWM estimator

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c_k explains the covariate influence on the WV of bin k

And the properties?

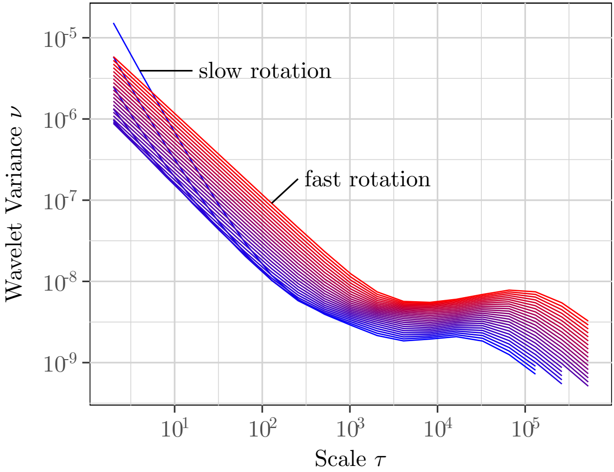
Identifiable ✓

Consistent ✓

Asymptotically Normal ✓

GMWM Example: covariate influence

MEMS IMU Gyroscope stochastic noise as a function of rotational speed



Conclusions

Properties

- Numerically stable
- Computationally efficient
- Covariate Dependency

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- Numerically stable
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And more...

- Implementation
  - opensource package in statistical tool *R*
  - online webbrowser tool on *ggmwm.smac-group.com*
- Proofs in an upcoming publication
- Application examples
  - rotational dynamics dependency: IEEE/ION PLANS 2018
  - temperature dependency: to be published with proofs

Thank you

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- samuel.orso@unige.ch
- `https://github.com/SMAC-group/GMWM`

References

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