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The advent of the sharing culture and its effect on product pricing

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ABSTRACT

Empirical observations suggest that consumers' propensity towards sharing varies with culture and the individuals' socio-demographic characteristics. In an economy with overlapping generations of heterogeneous consumers, we study optimal dynamic selling by a durable-goods monopolist in equilibrium. Feasible dynamic pricing strategies include second-degree price discrimination offering intertemporal consumption bundles in the form of rental and/or purchase options. We find that as the population's peer-trade propensity increases, possibly due to a cultural shift from private ownership to collective consumption, the durable-goods monopolist's optimal strategy shifts from unbundling (offering exclusively rentals), via mixed bundling (offering the options of rental and purchase side-by-side), to pure bundling (offering purchase only). We show that an increase in peer-trade propensity has an ambiguous effect on the firm's profit. Cultural shifts from low to high peer-trade propensity may be delayed by a firm's attempts to artificially disable sharing markets by offering overly low rental rates. However, beyond a certain threshold of peer-trade propensity, the firm prefers a cultural transition to an access-based economy. The underlying reason is that the asset base of a sharing economy ultimately depends on the firm's output, so that a portion of the anticipated rents from sharing can be captured by the durable-goods monopolist.

1. Introduction

Fuelled by the emergence of peer-to-peer markets, sharing has been gaining in popularity all across the globe.[1] Current estimates for average growth in the sharing economy over the next decade are well in excess of 30% per annum (Yaraghi and Ravi, 2017). Yet at present, the participation in collaborative consumption proves to be fairly product-specific and tends to vary significantly across cultures. Some societies are more prone to adopting (or transitioning towards) a sharing culture than others. According to Nielsen (2014), while in North America less than half (43%) of the population is willing to borrow from others, in China nearly everyone (94%) would be happy to do so. Trust attitudes, government regulation and incentives, technological development, reliability of financial transactions, consumer confidence, as well as environmental awareness are among the factors that may affect a population's propensity towards sharing. In this paper, we examine the impact of "peer-trade propensity" on a firm's optimal strategy for offering products, be it for sale, for rent, or a combination thereof.

The fact that peer-to-peer transactions may decrease the perceived ex-ante value compared to standard retail transactions can be thought of as the result of a *propensity mismatch*. The degree of this mismatch is naturally subject to variations within the consumer population and across time. For example, consumers' propensity to share may well be correlated with socio-demographic data, including age, gender, income, and social status. A survey by PwC (2015) indicates that in the United States the consumer segments most excited about sharing are 18–24-year-old adults, households with an income between $50,000 and $75,000, as well as families with children under the age of 18. The *peer-trade propensity* (also referred to as *sharing propensity* in our context), which indexes a subjective transaction cost of retail versus peer-to-peer interactions, depends on the state of diffusion of sharing markets and concomitant word-of-mouth. Indeed, Vision Critical (2014) notes that people are increasingly willing to participate in sharing activities with about an equal split (in 2014) between recent users and prospective new users. The reluctance (or propensity mismatch) of engaging in sharing activities can be viewed *behaviorally*, as a mental burden of uncertainty and ambiguity, or else *economically*, as an actual transaction cost related to transportation and perceived risk. On the provider's side, the transactional risk stems from the moral hazard because the borrower's actions when using the provider's item are unobservable,

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[1] For the purposes of our analysis, we do not consider the reuse of corporate assets between different users as sharing *per se*, but rather as renting or leasing. Short-term contractual agreements of this kind are a component of product offerings by companies, created in part as a reaction to the existence of peer-to-peer sharing markets. The resulting coexistence (or not) of corporate purchase and rental options in the presence of sharing markets is precisely what we examine here.

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which may be mitigated by insurance contracts (Weber, 2014). On the borrower's side, perceived risks may include the quality of the item, credit checks, license and history checks, and hidden fees. On both sides of the peer-to-peer sharing market a propensity mismatch produces a frictional transaction cost. Hence, the consumers' attitude towards peer-to-peer sharing is an important factor that needs to be taken into account by the incumbent firms. In a recent survey conducted by the Boston Consulting Group (BCG, 2017), respondents from various countries were asked about their preferences for short-term borrowing from either peers or an established firm. The results vary by culture: while in the United States 48% of the respondents prefer professional suppliers, only 27% of German users share that attitude. The finding indicates that some societies embrace a peer-to-peer sharing culture more than others. This in turn creates an opportunity for incumbent firms to leverage consumers' peer-trade propensity, tailoring their product offerings accordingly.

Sharing intermediaries, manufacturers, and regulators can pro-actively influence a propensity mismatch by implementing trust systems, by providing insurance options or transaction guarantees, as well as by offering ways for consumers to gain experience and build reputations on both sides of the market. Anecdotal evidence suggests that established manufacturers of durable goods are already responding to the population's positioning towards sharing by introducing their own rental programs, in addition to the classic purchase options. *Car2Go* is a subsidiary of Daimler that offers rental services since 2008. BMW is operating *DriveNow* since 2011, and Audi launched the *Audi On Demand* in 2015, allowing consumers to rent an Audi instead of purchasing it. Hence, the prevailing peer-trade propensity can be expected to be time-varying and subject to a drift, which may be interpreted as a cultural shift. It is therefore of interest to examine the effect of increasing or decreasing peer-trade propensity on product offerings, the balance between retail as well as sharing transactions, and the resulting distribution of surplus in the system.

In this paper, we study a firm's menu design and pricing to target different consumer types. To the best of our knowledge, this study is the first to explicitly incorporate consumers' varying disposition to trade in a peer-to-peer sharing economy in the firm's problem of how to design product offerings, given the possibility of an active aftermarket for the consumption of its products. Using a game-theoretic approach, we show that the population's peer-trade propensity plays a significant role in the firm's optimal design of consumption-bundling options across periods. Introducing a rental option can be used as a defensive tool, for it may help to strategically deactivate the sharing market, provided consumers' peer-trade propensity is low—reflecting significant trust in the established retail brand. For intermediate sharing propensities, the firm's optimal strategy is to allow for the rental service and the sharing market to operate side-by-side. Although the firm's transaction volume decreases, it can compensate the loss by increasing both the rental rate and the purchase price. For higher sharing propensities, offering the rental service does not benefit the firm. The firm exclusively sells shareable goods and its activities effectively decouple from those of the sharing market. Overall, our results indicate that when sharing markets are disabled, a firm would benefit from lowering the population's peer-trade propensity. However, in the presence of an active sharing market, the seller may have strong incentives to help promote a cultural transition towards sharing.

1.1. Literature review

Trust as a "general willingness to trust others" was viewed by Rotter (1967) as "interpersonal trust," by Farris et al. (1973) as a "personality trait," and by Hofstede (1980) as subject to "cultural conditioning." Mayer et al. (1995) introduce the empirical construct of a "trust propensity" as a driver of trust—together with antecedent trustworthiness factors such as the trustee's ability, benevolence, and integrity, from the perspective of the trustor. Here we interpret consumers' peer-trade propensity (or sharing propensity) as trust propensity with respect to transactions on peer-to-peer sharing markets. Because perceived transactional risk increases in a lack of trust, there is a subjective transaction cost associated with the use of sharing markets; and this transaction cost decreases in the sharing propensity.

We examine here the effect of changes in peer-trade propensity on the design of consumption bundles. This "package selling" is much in the spirit of Adams and Yellen (1976) and Schmalensee (1984), only that in our case the consumption bundling is intertemporal, corresponding to renting (temporal unbundling), selling (temporal bundling), and the coexistence of these product offerings (mixed temporal bundling). In the presence of a sharing market, which in itself may be fragile due to liquidity constraints, the consumption bundling can be viewed as a possible defensive tool to disable or otherwise interact with the peer-to-peer exchange, as a function of the peer-trade propensity.[2]

Because of the dynamic aspect in the consumption bundling, our analysis also relates to the classic literature on durable-goods monopoly, in particular the choice of selling versus renting in view of strategically influencing the future product competition, for example by aftermarkets. Coase (1972) noted that a fundamental difficulty to commit in continuous time forces a monopolist to be in perfect competition with itself. Similar commitment problems, although moderated in discrete time, lead Bulow (1982) to conclude that a monopolist may be unable to commit to a high quality, thus producing artificial obsolescence endogenously. Both Bulow (1982) and Stokey (1981) suggest that leasing dominates selling for a durable goods monopolist, reducing the need for long-term price commitment. Moreover, renting tends to limit secondary markets (including those for sharing) and enables the durable-goods monopolist to better control the product lifecycle.[3] Desai and Purohit (1998) find that leasing is not necessarily more profitable than selling if the sold and leased items do not depreciate at the same rate, or the market is competitive. Bhaskaran and Gilbert (2005) investigate the effect of complementary products on selling and leasing strategies, allowing for a hybrid selling and leasing strategy. They show that the optimal fraction of leasing decreases in the degree of product complementarity until eventually it becomes best for the firm to restrict attention to pure selling. In line with this result, we obtain that peer-to-peer transactions complement the purchase option, and as the sharing propensity grows the firm's strategy shifts from mixed bundling to pure bundling. Chien and Chu (2008) showed that selling might be more profitable than leasing when the products exhibit network effects. Our results implicitly confirm this finding, for selling gains in comparison with renting as the size of the peer-to-peer sharing market increases.

[2] A frictional cost of participating in the sharing economy has been identified in various forms in the sharing literature. Varian (2000) showed that when the marginal costs of production exceed the "transactions cost of sharing" the firm, selling a smaller amount of its shareable goods at a higher price, may benefit from the existence of the sharing market. Weber (2014) considers the effect of an agency cost, reducing a lender's willingness to participate in a sharing market. Benjaafar et al. (2015) allow for frictional "inconvenience costs" incurred by renters that lead to higher ownership incentives, abstracting from the firm's optimal pricing decisions. Razeghian and Weber (2015) construct a diffusion model for peer-to-peer markets with costly entry and exit decisions, where the participants' adjustment costs determine in large part the speed of adoption of peer-to-peer collaborative consumption in the economy. Horton and Zeckhauser (2016) demonstrate that a "bringing-to-market" cost may increase sales revenue and cause transaction volumes on the sharing market to decrease. Jiang and Tian (2016) examine the effect of "moral-hazard cost" on product design and pricing, and show that such costs can have ambiguous effects on the firm's profit. Our notion of peer-trade propensity not only includes the various forms of frictional costs identified in the prior literature but also highlights consumers' preference for either type of provision, be it corporate or peer-based.

[3] Weber (2017) probes further possibilities of aftermarket control through smart devices and usage-contingent pricing.

Our analysis builds on Weber (2016) who uses an overlapping-generations model to study optimal pricing in the presence of sharing markets.[4] In contrast to extant models with aftermarkets this model avoids endpoint effects, such as those in the two-period durable-goods model by Bulow (1982) or in the finite-horizon two-sided sharing markets by Jiang and Tian (2016). Abhishek et al. (2018) compare monopoly selling and rental with a corporate-sponsored peer-to-peer market and find that when usage rates tend to be low, the firm prefers to jointly offer purchase and rental options without sharing. In our model, the peer-to-peer market can form and break down endogenously, so the firm's menu of product offerings becomes a strategic tool to control the existence of the peer-to-peer market. This and other earlier work did not consider the effect of variations in the population's sharing propensity. As a horizontally differentiating characteristic in the tradition of Hotelling (1929) the peer-trade propensity, which defines the consumers' mismatch cost for corporate and peer-trade transactions, turns out to be crucial for the composition of the firm's optimal menu of product offerings.

1.2. Outline

The paper is organized as follows. In Section 2, the model primitives in terms of consumers, firm, sharing market, and peer-trade propensity are introduced. Section 3 provides a detailed analysis of the dynamic equilibrium in steady-state, both in the presence and in the absence of an active peer-to-peer sharing market. It also contains remarks on the degree of competition between the sharing market and the firm, depending on the peer-trade propensity. Section 4 discusses the distribution of surplus and frictional losses in the economy, as a function of the sharing propensity. The results are illustrated by a numerical example. Section 5 concludes with practical implications and directions for further research.

2. Model

Consider a profit-maximizing monopolist who produces durable goods over an infinite time horizon, at a unit production cost $c > 0$. Each item becomes obsolete after two periods, so that product lifetime is in fact identical to the life span of the consumers in this economy, and no items can be inherited from previous generations.[5] Furthermore, each item can be used at most once in each period.

Consumers. As in Weber (2016), we use an overlapping-generations model to describe the consumers in this economy. At any period of time $t \in \{0, 1, 2, \ldots\}$, a new generation of agents is born, the size of which is normalized to 1 (without any loss of generality). Each consumer generation lives for two periods, termed *early consumption phase* $\mathscr{C}^0$ and *late consumption phase* $\mathscr{C}^1$, respectively. As noted before, agents cannot inherit units from a past generation, nor pass items to a future generation (see footnote 5). Because the intensity of demand for products varies across time and among agents, consumers in this economy are heterogeneous with respect to their probability of need. At each period, a given agent finds himself in either a "high" or a "low" need state for the item produced by the monopolist. Let $\tilde{s}^j$ be an agent's need state in his consumption phase $j \in \{0, 1\}$ with realizations in $\mathscr{S} = \{0, 1\}$, where the realization $s^j = 0$ corresponds to the low-need state and $s^j = 1$ to the high-need state. An agent's type $\theta \in \Theta = \{\theta_L, \theta_H\}$, where $0 < \theta_L < \theta_H < 1$, denotes the i.i.d. probability with which he finds himself in the high-need state at any given time,[6] i.e.,

$$P(\tilde{s}^j = 1) = \theta.$$

When an agent is born, he learns his type θ and observes his need state s^0 in the early consumption phase $\mathscr{C}^0$. For simplicity, we assume that all consumers derive the common monetary value v of a unit consumption. Hence, an agent's utility function $u: \mathscr{S} \to \mathbb{R}$ maps the subjective realization of a need state $s \in \mathscr{S}$ to a willingness to pay, so

$$u(s) = sv,$$

where $v > 0$ is an agent's consumption value in case of a high need for the item.[7] The type distribution is fully described by $\lambda \in [0, 1]$, which denotes the seller's belief that a given agent of random type $\tilde{\theta}$ (with realizations in Θ) has a high probability of need:

$$P(\tilde{\theta} = \theta_H) = \lambda. \qquad (=1 - P(\tilde{\theta} = \theta_L))$$

Without loss of generality, the number of consumers in each generation is normalized to 1. Hence, the fraction of high-type consumers in the total population (of size 2, corresponding to two generations of size 1 each) is 2λ. All agents have the same discount factor $\delta \in [0, 1]$; furthermore, they are risk-neutral and rational (i.e., expected-payoff-maximizing) decision-makers given all information available to them.

Firm. In an attempt to capture as much consumer surplus as possible, the monopolist provider of the good pursues a second-degree price-discrimination strategy, offering a consumption menu , containing both *rental service* and *purchase* as options. Any option is characterized by a tuple

$$(\rho, q) \in \mathscr{M} \triangleq \{(\phi, 1), (r, 2)\},$$

where ρ is the price and q is the duration of service offered by the monopolist.[8] The rental service is offered at price ϕ and is valid for consumption in one period. The purchase option is available at price r and can be used over the buyer's remaining lifetime. The purchase premium,

$$\pi = r - \phi,$$

is the surcharge that a consumer pays to obtain unlimited usage rights.

Sharing Market. To exchange usage rights during their lifetime, consumers can transact on a sharing market, provided the latter is "active" (in the sense that total transaction volume is positive). If an active sharing market exists, non-owners in either consumption phase ($\mathscr{C}^0$ or $\mathscr{C}^1$) may borrow from other agents in the sharing market, in addition to their outside options of renting or buying the product from the firm. The firm's menu of product offers is of key importance for the demand in the sharing market. Conversely, the suppliers in the sharing market are product owners who in the current period have no need for the item, having thus experienced the realization $s^1 = 0$ in their late consumption phase ($\mathscr{C}^1$). The sharing price p matches supply with demand in the secondary peer-to-peer market.

[4] Razeghian and Weber (2017) extend the overlapping-generations model to allow for product-durability decisions, but they do not consider any transaction costs for dealing with the incumbent and/or a peer-to-peer sharing market.

[5] If product lifetime does not match the agents' lifetime, a stationary solution will need to rebalance inflows and outflows of products in any given agent generation. For example, if the product lifetime is three periods, then items produced at time t can be used up to time $t + 2$, and in each period all three product vintages are available in equal numbers (unless products fail prematurely). One would then need to consider the possibility of purchase not only from firms, but also from agents in their late consumption phase who bought a new item in their early consumption phase. Larger product lifetimes amount to an effective decrease in production cost and a schedule of product prices conditional on the remaining periods of life, provided that this information could be credibly communicated between potential trading partners. Long lifetimes may lead to incentives by the supplier to artificially limit product durability; see Razeghian and Weber (2016) for a related model.

[6] While consumers' types are persistent, need realizations are uncorrelated across the two consumption phases.

[7] To keep the analysis tractable, all agents have consumption value v; thus, the agents' expected utility corresponds to the standard utility representation by Mussa and Rosen (1978)—with likelihood of need taking on the role of marginal utility.

[8] Since consumers live for only two periods, the contract durations on offer need not exceed 2.

Peer-Trade Propensity ("Sharing Propensity"). Depending on the general sentiment in the agent population, consumers collectively experience a peer-trade propensity, which we refer to as "sharing propensity" in the context of our analysis. As the BCG (2017) survey suggests, peer-to-peer rentals are not always preferred to services offered by a firm, with significant differences in sharing propensity across cultures. The peer-trade propensity mismatch proposed in this model corresponds to a (perceived) transportation cost for agents that transact on the sharing market. Moreover, there are also cost burdens (mental or physical) on consumers who use the firm's services. As a representation we employ the standard linear city by Hotelling (1929) where the monopolist and the sharing market are located at the two ends of a line segment $\mathscr{X} = [0, l]$ of fixed length $l > 0$. All consumers exhibit a common sharing propensity $x \in \mathscr{X}$;[9] higher values of x represent higher willingness to engage in sharing activities. To get access to the firm, agents with sharing propensity x are subject to the linear transportation cost τx, where τ is the unit peer-trade mismatch cost.[10] Similarly, accessing the sharing market costs $(l - x)\tau$ to the agents. For simplicity, we normalize the distance between the firm and the sharing market by setting $l = 1$, so the firm and the sharing market are located at $x = 0$ and $x = 1$, respectively. We further assume that $v \geqslant \tau$, to rule out the uninteresting situations where consumers are *a priori* not interested in consumption. Fig. 1 illustrates locational arrangement of the sharing propensity relative to the location of firm and sharing market. As alluded to in Section 1, the location x captures the general status of a population's (possibly evolving) peer-trade propensity.

Remark 1. The "transportation cost" τ can be interpreted in various ways. For example, as in Varian (2000), it could include the actual cost of travelling to each market, or alternately, the willingness to pay to avoid the perceived inconvenience from engaging in transactions with the respective counterparties. In addition, the transportation cost may include any taxes imposed by a regulator and levied on consumers for accessing either market. In this last view, the firm, the sharing market, and the regulator may all play active roles in shaping the sharing propensity x, in the medium to long run.

3. Equilibrium analysis

As noted in the previous section, the consumer population is stationary and features overlapping generations. At each time consumers of either consumption phase make their choices. Driven by the recurring nature of these decisions at the population level, all else equal, aggregate consumption levels must be stationary, which implies the optimality of stationary product offerings, and thus also stationary price levels in an active sharing market. However, at the level of an individual consumer, there is no reason for choice behavior to be stationary, i.e., invariant over the two consumption phases. Because of rational expectations (Muth, 1961), each consumer is able to anticipate conditions over his two consumption phases and can use backward-induction to optimize his choices. To analyze the equilibrium behavior in the economy we use the concept of subgame-perfect Nash equilibrium by Selten (1965): for the consumer this means backward-induction of individual choice, whereas for the infinite-lifetime firm this means acting according to a one-stage deviation principle.[11] We start by considering the agents' consumption decisions using backward-induction for a given generation, born at time $t \geqslant 1$.

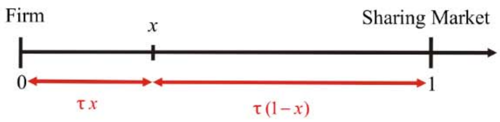

Fig. 1. Sharing propensity and the cost of mismatch.

Renting vs. Borrowing in $\mathscr{C}^1$. Non-owners in their late consumption phase, conditional on a realized need, may choose any of the available options to get access to a product, including rental, purchase, or sharing. Nevertheless, the end-of-horizon effect implies that it is never optimal for non-owners to choose the purchase option $(r, 2)$ and incur the surcharge $\pi \geqslant 0$, given that they would not be able to take advantage of more than a single period of authorized product use. Hence, a non-owner of type $\theta \in \Theta$, with sharing propensity $x \in [0, 1]$, would *either* choose to rent at the price ϕ *or* to borrow at the price p. Specifically, in the high-need state, the rental service is preferred if and only if

$$v - \phi - \tau x \geqslant v - p - (1 - x)\tau,$$

or equivalently:

$$x \leqslant \frac{1}{2}\left(1 + \frac{p - \phi}{\tau}\right). \tag{1}$$

For any given prices p and ϕ, as long as the sharing propensity x is small enough, by condition (1) non-owners do prefer to rent from the firm rather than getting access to the product on a sharing market. By contrast, when the sharing propensity is high, so condition (1) is violated, non-owners prefer the sharing market as the channel to satisfy their need for the product through peer-to-peer access. Hence, a type-θ non-owner's expected payoff becomes[12]

$$\overline{U}(\theta;x) = \theta[v - \min\{p + (1 - x)\tau, \phi + \tau x\}]_+.$$

Remark 2. If $\phi - p \geqslant \tau$, then the firm's rental service is never used by the consumers, independent of their peer-trade propensity $x \in \mathscr{X}$. Indeed, even for $x = 0$, i.e., for the lowest possible sharing propensity, the high retail price justifies a costly mismatch with the sharing market. When condition (1) fails to hold, the rental service, even if offered by the firm, is not able to attract customers and effectively cannot compete with the peer-to-peer market; see Section 3.2 for additional details.

Lending vs. Using in $\mathscr{C}^1$. An owner derives utility from his product in his late consumption phase only if he finds himself in need for the item ($s^1 = 1$) for the second time in his life. In the low-need state, the utility of consumption is $u(0) = 0$; however, the sharing possibility can generate revenue for the owners. For the latter, sharing is attractive only if the subjective transaction cost is compensated by the sharing price:

$$p - (1 - x)\tau > 0. \tag{2}$$

Hence, the expected indirect utility of an owner of type $\theta \in \Theta$ with sharing propensity $x \in \mathscr{X}$ in his late consumption phase is

$$V(\theta;x) = \theta v + (1 - \theta)[p - (1 - x)\tau]_+.$$

Note that owners supply their unused resources on the market only if $x > 1 - p/\tau$. Otherwise, the peer-to-peer market collapses.

Remark 3. In equilibrium, the owner's payoff from consumption necessarily exceeds the net profit obtained from sharing, i.e.,

$$p - (1 - x)\tau \leqslant v. \tag{3}$$

If the preceding inequality is not satisfied for any $x \in \mathscr{X}$, then

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[9] The model complexity increases combinatorially in the number of different sharing propensities and becomes complicated even in the case of just two different sharing propensities, without a noticeable increase of insights.

[10] Throughout the text we use "propensity-mismatch cost" and "transportation cost" interchangeably.

[11] Given any positive per-period discount rate the one-stage deviation principle is applicable for the firm's recurring infinite-horizon optimization problem because the per-period payoffs are bounded (see, e.g., Fudenberg and Tirole (1991), Thm. 4.2).

[12] $[z]_+ \triangleq \max\{0, z\}$, for all $z \in \mathbb{R}$.

necessarily $v < p$. That is, no agent is willing to borrow from the sharing market, which must result in its collapse. Therefore, for the sharing market to remain liquid, inequality (3) must be satisfied. In the case of equality, we assume that the agents prefer not to share, for example because of an infinitesimal transaction cost or residual benefit of private use.

As a result of Eqs. (1)–(3), the equilibrium sharing price in an active sharing market satisfies the following *liquidity condition*:

$$(1-x)\tau < p \leqslant \min\{v-(1-x)\tau,\ \phi+(2x-1)\tau\}. \tag{4}$$

The lower bound on the sharing price ensures that suppliers obtain a nonnegative payoff, so it is individually rational for them to participate. The upper bound provides a similar kind of rationality for renters: it guarantees them a nonnegative payoff and ensures that they are not strictly better off by abandoning the sharing market and renting from the firm. As soon as the sharing price drops to its lower bound, the sharing market effectively shuts down. Next we consider the cases where the sharing markets are inactive and active separately, and provide a characterization of positive liquidity in the sharing market.

3.1. *Sharing shutdown*

We start by analyzing the case where the sharing market is inactive. Without loss of generality, the unpopularity of sharing implies an equilibrium sharing price p equal to its lower bound, so owners are no longer willing to share, and the sharing market dries up. By the liquidity condition (4),

$$p \equiv p(x) = (1-x)\tau, \tag{5}$$

whence the peer-to-peer market is effectively disabled. In the late consumption phase, non-owners in the high-need state have the sole option of renting from the firm at price ϕ. The following individual-rationality constraint ensures that any renter obtains a nonnegative payoff:

$$\phi \leqslant v - \tau x. \tag{6}$$

Subject to that constraint, a non-owner's expected utility,

$$\overline{U}(\theta;x) = \theta(v-\phi-\tau x), \tag{7}$$

when renting an item from the firm is nonnegative. Without sharing, owners in the late consumption phase derive utility only from self-consumption. Hence, an owner of type θ obtains the expected utility

$$\overline{V}(\theta;x) = \theta v. \tag{8}$$

Purchase Decision in $\mathscr{C}^0$. We now examine which consumer types are willing to invest in purchasing. The following incentive-compatibility constraint ensures that purchasers benefit more from buying than from renting, i.e., from choosing the menu-item specifically designed for them:

$$v - r - \tau x + \delta\overline{V}(\theta;x) \geqslant v - \phi - \tau x + \delta\overline{U}(\theta;x). \tag{9}$$

The preceding constraint implies that buyers do not consume the rental option and determines which consumer types (if any) belong to the purchasers group.

Lemma 1. *In the absence of sharing, an agent of type $\theta \in \Theta$ with sharing propensity $x \in \mathscr{X}$ opts for the purchase option $(r, 2)$ if*

$$\theta \geqslant \frac{\pi}{\delta(\phi+\tau x)} \triangleq \underline{\theta}\left(\phi, \pi; x\right).$$

This result implies that by strategically setting rental price ϕ and purchase premium π, the firm can control whether only high types, both types, or neither would invest in purchasing the product.

3.1.1. *Optimal Menu Design (without Sharing)*

The sharing market may be inactive either in a natural or an induced manner. If the population's sharing propensity is so low that potential suppliers and/or borrowers expect no surplus from sharing, the peer-to-peer market cannot form. By the liquidity condition (4) the sharing market is naturally "choked off" if

$$x \leqslant \max\left\{1-\frac{v}{2\tau},\ \frac{2}{3}-\frac{\phi}{3\tau}\right\}. \tag{10}$$

The threshold for this "natural sharing shutdown" depends on the consumption value v, the firm's rental rate ϕ, as well as on the transportation cost τ. For a given propensity $x \in [0, 1)$, sharing is not feasible when the transportation cost is very high compared to the consumption value. In addition, the sharing market also does not form when $x \in [0, 2/3)$ and the rental price ϕ is small enough, so

$$1-\frac{v}{2\tau} < x \leqslant \frac{2}{3}-\frac{\phi}{3\tau}, \tag{11}$$

leading to an "induced sharing shutdown." Thus, the firm may be able to use its rental rate ϕ as a lever to strategically deactivate the peer-to-peer market. The next result characterizes the regions in (x, τ)-space where the sharing market is active or else experiences a (natural/induced) shutdown; see Fig. 2.

Lemma 2 (*Sharing Shutdown*). *Assume that $0 \leqslant \phi \leqslant v$, and let $\underline{x} \triangleq 1 - v/(2\tau)$, $\underline{\underline{x}} \triangleq (2/3) - \phi/(3\tau)$.*

(i) *For $x \leqslant \max\{\underline{x}, \underline{\underline{x}}\}$ (or equivalently, $\tau \geqslant \max\{\phi/(2-3x), v/(2(1-x))\}$), there is a "natural" sharing shutdown, and the firm's profit-maximizing rental price is $\phi \equiv \phi(x) = v - \tau x$.*
(ii) *For $\underline{x} < x \leqslant \underline{\underline{x}}$ (or equivalently, $\phi/(2-3x) \leqslant \tau < v/(2(1-x))$), there is an "induced sharing shutdown," and the firm's profit-maximizing rental price is $\phi \equiv \phi(x) = (2-3x)\tau$.*
(iii) *For $x > \underline{\underline{x}}$ (or equivalently, $0 < \tau < \phi/(2-3x)$), the sharing market remains active.*

When the consumers' sharing propensity is small ($x \leqslant \underline{x}$), the firm is able to increase the rental price such that renters are left with zero surplus, while the sharing market still experiences a natural shutdown. However, as the sharing propensity grows, the firm's only tool to defeat the peer-to-peer market is to lower the rental price and offer a more

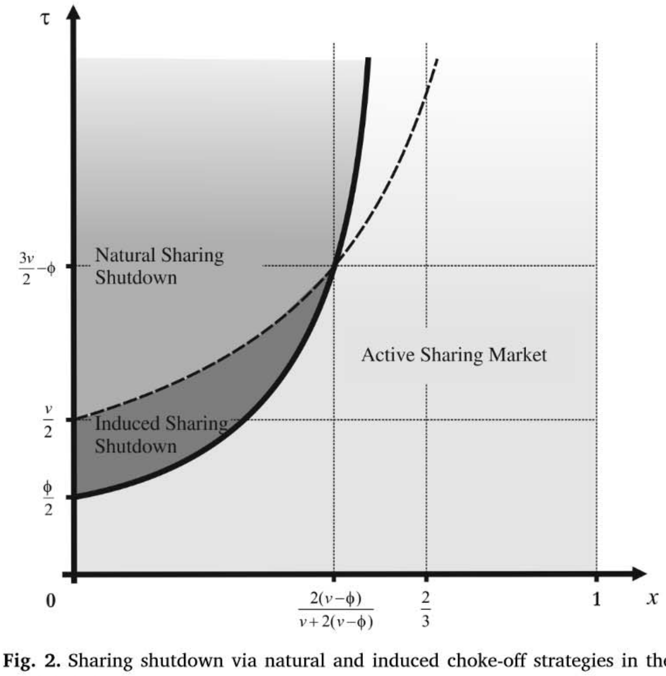

Fig. 2. Sharing shutdown via natural and induced choke-off strategies in the (x, τ)-space.

attractive option to the consumers. In this situation, the individual-rationality constraint (6) does not bind, and renters enjoy a strictly positive gain. For higher sharing propensities ($x \geqslant 2/3$), the firm is no longer able to induce a sharing shutdown. We will see below that the firm may benefit from coexistence with a sharing market more than from a costly price war so as to choke off peer-to-peer transactions. In other words, for intermediate sharing propensities, where agents do not have a strong preference for getting a good from the firm or from a peer, the competition between the monopolist and the sharing market works in favor of the consumers.

By optimally setting the purchase premium π, the firm is able to pursue second-degree price discrimination by using one of the following three different consumption-bundling strategies: *pure rental*, *high-end selling* & *rental*, or *mass selling* & *rental*. The type distribution of the agents and the cost structure together determine the monopolist's optimal menu design.

Remark 4. Renting does not have the same marginal cost as selling. Indeed, each item that is not sold but rented out can be offered on the market for two periods. Hence, if the demand for the rental service in steady state is $D_\phi \in [0, 2]$ units, the firm needs to produce $D_\phi/2$ units in each period. This is equivalent to incurring a per-period cost of $c/2$ for offering an item for rent.

In what follows, we characterize the three possible pricing and menu-design strategies, subject to the fact that the sharing market has been shut down.

Pure Rental. If the purchase premium is high, so

$$\theta_H \leqslant \underline{\theta}(\phi, \pi; x), \tag{12}$$

then by La. 1 purchasing is not attractive to any consumer segment.[13] This is equivalent to eliminating the purchase option, and offering exclusively rentals to consumers in either generation who find themselves in the high-need state. The monopolist's profit from employing this strategy is

$$\Pi_0(x) = 2((1-\lambda)\theta_L + \lambda\theta_H)\left(\phi(x) - \frac{c}{2}\right), \tag{13}$$

where $\phi(x) \in \{v - \tau x, (2-3x)\tau\}$ is the firm's optimal rental price, as specified by La. 2.

High-End Selling & Rental. If the firm's pricing strategy features an intermediate purchase premium, such that

$$\theta_L \leqslant \underline{\theta}(\phi, \pi; x) \leqslant \theta_H,$$

then the purchase option $(r, 2)$ is exclusively designed for consumers of type θ_H in their early consumption phase. In equilibrium, it is optimal for the firm to increase the purchase premium such that the incentive-compatibility constraint (9) binds for the high-type consumers. That is,

$$\pi \equiv \pi(x) = \delta\theta_H(\phi + \tau x). \tag{14}$$

The high-type non-owners in $\mathscr{C}^1$, as well as the low-type agents in both $\mathscr{C}^0$ and $\mathscr{C}^1$ are served by the rental service. The profit obtained from employing this strategy is

$$\Pi_1(x) = \lambda\theta_H\left(r(x) - c\right) + \left(\lambda\theta_H\left(1-\theta_H\right) + 2\left(1-\lambda\right)\theta_L\right)\left(\phi(x) - \frac{c}{2}\right), \tag{15}$$

where, by virtue of La. 2 and Eq. (14), the firm's prices are such that

$$(\phi(x), \pi(x)) \in \{(v - \tau x, \delta\theta_H v), ((2-3x)\tau, 2\delta\theta_H(1-x)\tau)\}. \tag{16}$$

Mass Selling & Rental. When the menu design is such that

$$\underline{\theta}(\phi, \pi; x) \leqslant \theta_L,$$

purchasing is the optimal decision for *any* agent in $\mathscr{C}^0$, as long as they find themselves in need of the item. The monopolist optimally increases the purchase premium such that the incentive-compatibility constraint (9) binds for agents of type θ_L, so

$$\pi \equiv \pi(x) = \delta\theta_L(\phi + \tau x). \tag{17}$$

While this pricing strategy leaves the low-type agents with zero expected surplus, the high-type buyers obtain a positive *information rent*. The rental service serves only the non-owners in their late consumption phase ($\mathscr{C}^1$). The firm's profit is therefore

$$\Pi_2(x) = \left(\lambda\theta_H + \left(1-\lambda\right)\theta_L\right)(r(x) - c) + \left(\lambda\theta_H\left(1-\theta_H\right) + \left(1-\lambda\right)\theta_L\left(1-\theta_L\right)\right)\left(\phi(x) - \frac{c}{2}\right), \tag{18}$$

and, by virtue of La. 2 and Eq. (17), the optimal pricing strategy requires that

$$(\phi(x), \pi(x)) \in \{(v - \tau x, \delta\theta_L v), ((2-3x)\tau, 2\delta\theta_L(1-x)\tau\}. \tag{19}$$

One might conjecture that under certain conditions, the monopolist may be willing to shut down the rental service and offer only the purchase option $(r, 2)$. However, the following result shows that when sharing is not feasible, such a strategy is dominated and thus never optimal for the monopolist. The underlying reason is that the firm can always generate additional revenue by also renting to the non-owners in the older generation ($\mathscr{C}^1$).

Lemma 3. *If the sharing market is inactive, disabling the rental service is a dominated strategy.*

The optimality of each of the three aforementioned menu-design strategies depends on several factors. To simplify our discussion, we first normalize costs and benefits relative to the unit cost of mismatch (or transportation cost) as follows:

$$(\hat{c}, \hat{v}) \triangleq (c/\tau, v/\tau).$$

The "pure rental" threshold is defined as

$$x_1 = \min\left\{\left(\frac{1-\theta_H}{2\theta_H}\right)\hat{c} + \left(1-\delta\right)\hat{v}, \underline{x}\right\}; \tag{20}$$

below this threshold it is optimal for the firm to exclusively offer rentals. By omitting the purchase option, the firm is able to avoid the quantity discount and capture the consumer's entire surplus. This strategy is compatible with the optimal leasing strategy of a durable-goods monopolist, discovered in the stream of literature on selling versus renting, by Coase (1972) and Bulow (1982) among others. As the sharing propensity increases, the firm is compelled to lower the rental price, and consequently its revenue decreases. At the threshold x_1, the firm finds mixed bundling more profitable, offering the rental and purchase options side-by-side. Note that the purchase option could be designed in a way that targets only high-type agents, or both groups of consumers. The results show that mixed bundling starts with high-end selling, and as the sharing culture develops, it transitions to mass selling. Let

$$\ell \triangleq \frac{(1-\lambda)\theta_L}{\lambda\theta_H} \tag{21}$$

be the likelihood-ratio of consumption which measures how the per-period aggregate consumption is divided between the high- and low-type agents. Furthermore, let

$$\Delta\theta \triangleq \theta_H - \theta_L \tag{22}$$

denote the difference between the high- and low-type agents. The "high-end selling" threshold is defined as

[13] By setting $\pi = v$, Eq. (12) is automatically satisfied.

$$x_2 = \min\left\{\left(\frac{1-\theta_L}{2\theta_L}\right)\hat{c} + \left(1-\delta+\frac{\delta\Delta\theta}{\ell\theta_L}\right)\hat{v},\ \underline{x}\right\}; \tag{23}$$

below this threshold high-end selling is a dominant strategy. As the sharing propensity passes this threshold, the firm switches to the mass-selling strategy.

Proposition 1. *Let $\underline{x}$, x_1, and x_2 be the propensity thresholds defined by Eqs. (10), (20), and (23). When sharing is not feasible, then*

(i) for all $x \in [0, x_1]$, the monopolist's optimal strategy is "pure-rental," where the optimal rental option is $(v - \tau x, 1)$;
(ii) for all $x \in [x_1, x_2]$, the monopolist's optimal strategy is "high-end selling & rental," where the optimal consumption menu consists of $(v - \tau x, 1)$ and $((1 + \delta\theta_H)v - \tau x, 2)$;
(iii) for all $x \in [x_2, \underline{x}]$ the monopolist's optimal strategy is "mass selling & rental," where the optimal consumption menu consists of $(v - \tau x, 1)$ and $((1 + \delta\theta_L)v - \tau x, 2)$; and
(iv) for all $x \in [\underline{x}, 1]$, an induced sharing choke-off may or may not be optimal.

As $\underline{x} \to 1^-$, sharing is not feasible for any $x \in \mathscr{X}$. For example, this situation may occur when the transportation cost is high compared to the consumption value. For example, for highly personalized goods such as hygiene items, the high mismatch cost deters agents from peer-to-peer sharing. A similar effect can be observed when the agents' perceived consumption value decreases, towards the end of a fashion cycle or due to technological obsolescence.

Remark 5. If $x_2 < \underline{x}$, offering "high-end selling & rental" with sharing shutdown is a dominated strategy for all $x \in \mathscr{X}$. This is because the payoff from choke-off "high-end selling & rental" is smaller than the payoff from natural "high-end selling & rental," and at $x = \underline{x}$, this strategy is already no longer optimal. Similarly, when $x_1 < \underline{x}$, offering "pure rental" with sharing shutdown is a dominated strategy for all $x \in \mathscr{X}$. At $x = \underline{x}$, the payoff from "pure rental" with sharing shutdown is smaller than the payoff from natural "pure rental," and applying this strategy would not be optimal for higher values of x.

The thresholds x_1 and x_2 depend on the model parameters. The following result describes the corresponding comparative statics.

Lemma 4.

(i) x_1 is nondecreasing in c, and nonincreasing in δ and θ_H. x_1 is first increasing and then decreasing in v, with the maximum achieved at

$$v_0 \triangleq \max\left\{0, \frac{\theta_H(2+\hat{c}) - \hat{c}}{\theta_H(3-2\delta)}\right\}\tau.$$

(ii) x_2 is nondecreasing in c, θ_H, λ, and $\Delta\theta$, and nonincreasing in θ_L and ℓ. It is nondecreasing in δ as long as

$$\ell \leqslant \frac{\Delta\theta}{\theta_L};$$

finally, it is first increasing and then decreasing in v, with the maximum achieved at

$$v_1 \triangleq \max\left\{0, \frac{\theta_L(2+\hat{c}) - \hat{c}}{3\theta_L - \delta(2\theta_L - \Delta\theta)}\right\}\tau.$$

For a discussion of the preceding result, assume that the intervals $[0, x_1]$, $[x_1, x_2]$, and $[x_2, \underline{x}]$ are all nondegenerate. As the consumption value v increases (or the unit transportation cost τ decreases), the interval $[0, x_1]$ increases in size. Customers' net willingness to pay increases, which allows the firm to extract the full surplus, thus avoiding quantity discounts. Note that at the same time $\underline{x}$ decreases. Hence, the pure-rental region grows relative to the entire natural sharing-shutdown region. When x_1 hits the threshold $\underline{x}$ and pure rental is the firm's only strategy in the absence of sharing, the no-sharing region starts to decrease in the consumption value. An increase in the consumption value also increases x_2, which is the threshold for "high-end selling." This diminishes the size of the interval $[x_2, \underline{x}]$ where "mass selling & rental" is optimal. Consumers' higher willingness to pay allows the firm to serve more consumers with rental service and offer fewer purchase options.

Similar effects effect may be observed when the production cost c increases. As it is always less costly to satisfy the rental demand in a given period (compared to the same number of purchase units), as a consequence of a cost increase the "pure rental" region increases and the "mass selling & rental" region shrinks, until the two thresholds hit $\underline{x}$. Note that the production cost c does not affect $\underline{x}$. This is because the threshold is determined by the agents' willingness to share or borrow, and their sharing decisions are unaffected by the production cost.

θ_H decreases the threshold x_1 and increases x_2, i.e., the region in which "high-end selling & rental" is optimal becomes larger. The firm is able to increase its profits by augmenting the purchase premium that is adjusted for the high-type consumers. An increase in θ_L or the type difference $\Delta\theta$ does not affect the pure rental threshold, but diminishes x_2. The interval where "mass selling & rental" is optimal grows in size. It becomes more profitable to include the low-type agents in the purchasing pool. Similarly, when the number of low-type agents in the economy $(1 - \lambda)$ increases, low-type agents exert externalities on the other consumer segment, so high-type agents can benefit from a positive information rent. As λ grows, x_2 approaches $\underline{x}$, the information rent vanishes, and the firm never finds it optimal to design the purchase option in a way that is appealing to both types. The high-type consumers benefit from the existence of low-type consumers in the economy.

As consumers become more patient and δ increases, they are willing to invest more in purchasing, resulting in a smaller "pure rental" region. The effect of an increase in δ on the two regions of "high-end selling & rental" and "mass selling & rental" is ambiguous. Depending on the two types as well as their proportions, each region may increase or decrease in size.

3.2. Active sharing

If the peer-to-peer market is active, either sharing is strictly preferred by all consumers, or the agents are indifferent between borrowing at price p and renting at price ϕ. Otherwise, the sharing market is not attracting any consumers, and consequently, the sharing price drops to $p = (1 - x)\tau$, as discussed in Section 3.1. Here, we concentrate on the case where $p > (1 - x)\tau$ and characterize the changes in the firm's optimal menu-design as the sharing propensity grows.

In the late consumption phase, non-owners in need can either borrow from the sharing market or rent from the firm. The following individual-rationality constraint ensures that in the presence of sharing, the borrowers not only get a nonnegative payoff from borrowing but also (at least weakly) prefer to engage in a sharing transaction:

$$v - p - (1 - x)\tau \geqslant [v - \phi - \tau x]_+. \tag{24}$$

The expected utility of a non-owner is

$$\overline{U}(\theta; x) = \theta(v - p - (1 - x)\tau). \tag{25}$$

When sharing is welcome, an owner of type $\theta \in \Theta$ consumes his asset in the need state $s^1 = 1$ and supplies it on the sharing market in the need state $s^1 = 0$. Hence, the expected utility of an owner in $\mathscr{C}^1$ is

$$\overline{V}(\theta; x) = \theta v + (1 - \theta)(p - (1 - x)\tau). \tag{26}$$

We now examine which consumer segments are willing to use the purchase option when accessing the product on a sharing market is also possible.

Purchase Decision in $\mathscr{C}^0$. If an agent prefers peer-to-peer borrowing to the rental service in $\mathscr{C}^1$, he does so in the early consumption-phase $\mathscr{C}^0$. If the individual-rationality constraint (24) is satisfied, by nonnegativity of the utility function $\bar{U}(\theta;x)$ we get

$$v - \phi - \tau x + \delta\bar{U}(\theta;x) \leqslant v - p - (1-x)\tau + \delta\bar{U}(\theta;x),$$

which implies that the expected utility of borrowing outweighs the expected utility of renting, even in the first period. In other words, the optimal choice between rental and sharing is time-invariant. It follows that potential purchasers need to only compare the costs and benefits of the purchase option with the ones of accessing the peer-to-peer market. In this regard, the following incentive-compatibility constraint needs to ensure that the targeted purchasers do not have any incentive to use other offers available on the market. In other words, the purchase option needs to be designed in way that expected benefits of ownership exceeds the expected gains of non-ownership for purchasers:

$$v - r - \tau x + \delta\bar{V}(\theta;x) \geqslant v - p - (1-x)\tau + \delta\bar{U}(\theta;x), \tag{27}$$

where $\bar{U}(\theta;x)$ and $\bar{V}(\theta;x)$ are specified in Eqs. (25) and (26). The consumer types that satisfy the incentive-compatibility constraint (27) belong to the group of purchasers. This leads to a purchase criterion for the agents, formulated below.

Lemma 5. *In the presence of an active sharing market, an agent of type $\theta \in \Theta$ with sharing propensity $x \in \mathscr{X}$ chooses the menu item $(r, 2)$ if*

$$\theta \geqslant \frac{1}{2} + \frac{r - p(1+\delta) + (2x-1)\tau}{2\delta(1-x)\tau} \triangleq \bar{\theta}\left(r;x\right).$$

The preceding purchase criterion plays a critical role in the monopolist's optimal menu design. By adjusting its intertemporal pricing strategy, the firm can target the consumer segments that generate the highest profits.

3.2.1. Optimal menu design (with sharing)

If no purchase option is offered by the firm, the sharing market is forced to be inactive, as discussed in Section 3.1. Hence, the "pure rental" strategy is not compatible with the coexistence of the sharing market and can therefore be omitted from the analysis at this point. The set of potentially viable strategies includes "high-end selling & rental" and "mass selling," to be characterized next.

High-end Selling & Rental. Under this strategy, the purchase option $(r, 2)$ is designed such that

$$\theta_L \leqslant \bar{\theta}(r;x) \leqslant \theta_H.$$

While high-type consumers are willing to invest in buying, low-type consumers always prefer sharing. As a result, the expected demand for sharing is

$$\bar{D} = 2(1-\lambda)\theta_L + \lambda(1-\theta_H)\theta_H,$$

where the first term accounts for the low-type agents in the high-need state from both generations, and the second term represents the high-type non-owners who happen to be in need only in their second consumption phase ($\mathscr{C}^1$). The sharing supply is provided by the high-type owners who find themselves in the low-need state in the second period. That is,

$$S = \lambda\theta_H(1-\theta_H).$$

Clearly $\bar{D} \geqslant S$, meaning that the demand for sharing always exceeds the supply. In equilibrium, the competition between non-owners pushes the sharing price to the upper bound in the liquidity condition (4):

$$p = \min\{v - (1-x)\tau, \phi + (2x-1)\tau\}.$$

The dependence of the sharing price on ϕ creates an opportunity for the firm to attract a fraction of the demand for sharing. By offering a sufficiently low rental price ϕ, such that

$$p = \phi + (2x-1)\tau, \tag{28}$$

the monopolist renders the consumers indifferent between borrowing and renting, given the cost of propensity mismatch.

Remark 6. If $x \geqslant 1/2$, then the nominal sharing price is greater than the nominal rental price, and peer-to-peer borrowing is (at least weakly) preferred by the consumers. There is anecdotal evidence that in real life the nominal price of sharing services is not the only determinant of consumption. For example, in the not unlikely case where Uber's multipliers cause the price of a shared ride to surpass taxi fares, the sharing service continues to receive steady use.

Knowing that in equilibrium, no agent is strictly better off by borrowing or renting, the firm optimally increases the rental price up to the point where the entire renters' surplus is squeezed out. That is,

$$v - p(\phi;x) - (1-x)\tau = v - \phi - \tau x = 0.$$

This determines the optimal rental price,

$$\phi \equiv \phi(x) = v - \tau x. \tag{29}$$

Consequently, the equilibrium sharing price is

$$p \equiv p(x) = v - (1-x)\tau. \tag{30}$$

At the prices specified in Eqs. (29) and (30) the sharing market clears, so

$$S = D = \lambda(1-\theta_H)\theta_H.$$

The remaining portion of the demand ("residual demand"),

$$D_r = \bar{D} - D = 2(1-\lambda)\theta_L,$$

is served by the firm's rental service.

Remark 7. At the prices specified in Eqs. (29) and (30) consumers employ a mixed strategy, whereby they choose the firm or the sharing market with probabilities α and $1-\alpha$, respectively, with

$$\alpha = \frac{D_r}{D_r + D} = \frac{2\ell}{2\ell + (1-\theta_H)}.$$

Under a "high-end selling & rental" strategy, the optimality requires the firm to increase the purchase premium π up to the point that the incentive-compatibility constraint (27) binds for the θ_H consumers. Substituting Eqs. (29) and (30) into La. 5 determines the optimal purchase premium,

$$\pi(x) = \delta(v - 2(1-\theta_H)(1-x)\tau). \tag{31}$$

The following lemma compares the optimal purchase premiums, with or without sharing, under the high-end selling strategy.

Lemma 6. *Assume that $\underline{x} > 0$, such that a region with natural sharing choke-off exists. Under the "high-end selling & rental" strategy, the purchase premium is strictly higher with sharing than without.*

This last result implies that the firm is able to charge a higher purchase premium when the peer-to-peer market is active. Although the firm loses revenue from its rental service (due to the competition with the sharing market), its sales do increase overall. In other words, the sharing market plays a dual role for the firm: while the sharing service substitutes one product (rental service), it also complements the other (purchase option). In the presence of sharing, the high-end selling strategy yields a profit of

$$\Pi_3(x) = \lambda\theta_H\left(\phi(x) + \pi(x) - c\right) + 2\left(1-\lambda\right)\theta_L\left(\phi(x) - \frac{c}{2}\right), \tag{32}$$

where $\phi(x)$ and $\pi(x)$ are given by Eqs. (29) and (31). Compared to the no-sharing profit in Eq. (15), the firm benefits from an increase in the sales revenue, thanks to the "sharing premium" (Weber, 2016).

Nevertheless, the company's revenue from the rental service strictly decreases. The resulting effect of the sharing market on the firm's profit is therefore ambiguous.

Mass Selling. Purchasing is the dominant strategy for all young agents if

$$\bar{\theta}(r;x) \leqslant \theta_L.$$

Access via the sharing market is sought only by non-owners in their late consumption phase. This includes all agents who happened to be in the high-need state for the first time in their late consumption phase. The demand for sharing is

$$D = \lambda(1-\theta_H)\theta_H + (1-\lambda)(1-\theta_L)\theta_L.$$

The sharing supply consists of all owners who bought and used the item in the first period, but do not need it in the second period. That is,

$$S = \lambda(1-\theta_H)\theta_H + (1-\lambda)(1-\theta_L)\theta_L.$$

Clearly, under the mass-selling strategy $S = D$ regardless of the model parameters. The sharing market is sustainable at any viable sharing price p, which satisfies the liquidity condition (4).

In numerous sharing platforms, the price is determined by the suppliers. Examples include AirBnB, BlaBlaCar, TURO, and Eloue, among others. When this is the case and the renters are price takers, it is in the suppliers' best interest to raise the sharing price to the upper bound in relation (4), so,

$$p = \min\{v - (1-x)\tau, \phi + (2x-1)\tau\}. \tag{33}$$

Note that setting any rental price $(2-3x)\tau < \phi \leqslant v - (2-3x)\tau$ is not a viable pricing strategy in equilibrium. In fact, the sharing suppliers can react by further cutting down the price such that they can attract all the demand. If the firm sets $\phi = (2-3x)\tau$, the sharing market is choked off in an induced manner as discussed earlier. Here, we concentrate on the case where the firm shuts down its own rental service (by choosing $\phi \in (v-(2-3x)\tau, \infty)$). It sells exclusively to all consumers at the nominal purchase price $r(x)$. By Eq. (33), the sharing market clears at

$$p(x) = v - (1-x)\tau.$$

The monopolist increases the purchase price such that the incentive-compatibility constraint (27) binds for the low-type agents. By La. 5, that implies

$$r \equiv r(x) = v - \tau x + \delta(v - 2(1-\theta_L)(1-x)\tau). \tag{34}$$

The following lemma compares the purchase price in a mass-selling strategy with sharing versus without sharing.

Lemma 7. *Assume that $\underline{x} > 0$. Under the "mass selling" strategy,*

(i) *the purchase price is strictly higher with sharing than without it.*
(ii) *the surcharge is strictly increasing in x.*

When the sharing market is operating, the firm asks for a surcharge on each unit sold. Interestingly, La. 7 implies that the surcharge is increasing in the sharing propensity $x \in \mathscr{X}$. As the consumers' tendency towards sharing grows, consumers are willing to make bigger payments, so that they can participate in the sharing activities, later. Note that under the mass-selling strategy with sharing, the rental service is inactive and the firm's only source of revenue comes from selling. This yields the profit

$$\Pi_4(x) = (\lambda\theta_H + (1-\lambda)\theta_L)(r(x) - c), \tag{35}$$

where $r(x)$ is determined in Eq. (34).

Recall that the peer-to-peer market is active only if the firm does not find it beneficial to artificially shut it down. Let

$$\underline{\Pi}(x) = \max\{\Pi_0(x), \Pi_1(x), \Pi_2(x)\}$$

be the firm's net gain from employing an induced choke-off strategy when $\phi = (2-3x)\tau$. Then the sharing market is active if

$$\underline{\Pi}(x) \leqslant \min\{\Pi_3(x), \Pi_4(x)\}.$$

We define the sharing threshold $\bar{x} \in [\underline{x}, 2/3]$ as the sharing propensity, above which sharing is strictly preferred, i.e.,

$$\underline{\Pi}(\bar{x}) = \min\{\Pi_3(\bar{x}), \Pi_4(\bar{x})\}. \tag{36}$$

The "high-end selling" threshold in presence of sharing is defined as

$$x_3 = \min\left\{1, \max\left\{\bar{x}, \frac{2\delta(1+\zeta(1-\theta_L)) + \zeta(1-\delta)\hat{v}}{2\delta(1+\zeta(1-\theta_L)) + \zeta}\right\}\right\}, \tag{37}$$

where $\zeta = \ell/\Delta\theta$. When the sharing propensity is $x \in [\bar{x}, x_3]$, high-end selling is the firm's optimal strategy. The firm offers both the rental service and the purchase option and targets different consumer segments with different menu-items. Above this threshold, the rental service is not available and the firm exclusively offers purchase options to all consumers. The following proposition formally characterizes the optimality of the two regimes.

Proposition 2. *If the sharing market is active, then*

(i) *for all $x \in [\bar{x}, x_3]$, the firm's optimal strategy is "high-end selling & rental," where the optimal menu is comprised of $(v - \tau x, 1)$ and $((1+\delta)v - \tau x - 2\delta(1-\theta_H)(1-x)\tau, 2)$;*
(ii) *for all $x \in [x_3, 1]$, the firm's optimal strategy is "mass selling," where the rental service is inactive and the optimal purchase option is $((1+\delta)v - \tau x - 2\delta(1-\theta_L)(1-x)\tau, 2)$.*

By Prop. 2, in the advent of a cultural transition, sharing begins by high-end selling. For intermediate sharing propensities, only high-type users are willing to purchase, and both the sharing market and the rental service coexist and serve their own customers. As the sharing propensity passes the threshold x_3, sharing becomes mainstream. All consumers are willing to invest in ownership such that they can benefit from the additional lifetime revenue from the assets.

Lemma 8. *x_3 is nonincreasing in δ, ℓ, and θ_L and nondecreasing in $\hat{v}$, λ, θ_H, and $\Delta\theta$.*

With sharing it is $x_3 < 1$, provided that $(1-\delta)\hat{v} < 1$. Mass selling becomes optimal, at least for some x, if consumers are patient enough to perceive the net present value of future sharing-related activities as significant. By La. 8, a decrease in δ or an increase in $\hat{v}$ make the mass selling region grow in size. When δ is small, consumers are not willing to pay high purchase premiums. Hence, the optimal strategy is to offer a mixed bundle of high-end selling & rental for a larger interval of sharing propensities. It is less likely that the firm finds it optimal to shut down the rental market because the revenue stream generated from this service does not depend on the consumers' discount factor δ. This is also the case when $\hat{v}$ is large. As the normalized consumption value increases, the revenue from the rental service increases and it is more likely that the firm prefers to keep this service functional.

The size of the mass-selling interval, $[x_3, 1]$, also increases as the low-type agents' usage frequency θ_L or their likelihood-ratio ℓ of consumption increases. When the low-type agents have a higher willingness to pay, or their population increases, it is more likely for the firm to target them as purchasers at lower sharing propensities. On the other hand, when the high-type agents' usage frequency θ_H or their proportion λ in the population increases, the mass selling region shrinks.

3.3. Market structure

The firm's optimal selling-versus-renting strategy as specified by Props. 1 and 2 divides the propensity-space $\mathscr{X}$ into two main regions. This effectively characterizes the market structure as a function of the sharing propensity. Indeed, when consumers' sharing propensity is small, the firm enjoys monopoly power as the sharing market remains inactive. As the sharing propensity grows, the firm and the P2P market engage in a competition, with either offering imperfect substitutes for the other.

The menu design takes into account the consumers' self-interested choice behavior. Purchase decisions are taken in the early consumption phase ($\mathscr{C}^0$) when $s^0 = 1$. Those agents who decided to purchase take sharing decisions in their late consumption phase ($\mathscr{C}^1$) when $s^1 = 0$. The possible market structures and the agents' corresponding actions are discussed next.

1. Firm's Monopoly. For all $x \in [0, \bar{x})$, sharing is not viable and the firm obtains monopoly rents. All owners in $\mathscr{C}^1$ who find themselves in the low-need state do nothing with their idle capacity. The rental service is always active. The consumption menu is designed such that

- for all $x \in [0, x_1)$, all non-owners of type $\theta \in \Theta$ rent in $\{\mathscr{C}^0, \mathscr{C}^1\}$.
- for all $x \in [x_1, x_2)$, high-type (θ_H) non-owners purchase in $\mathscr{C}^0$ and rent in $\mathscr{C}^1$. Low-type (θ_L) non-owners rent in $\{\mathscr{C}^0, \mathscr{C}^1\}$.
- for all $x \in [x_2, \underline{x})$, all non-owners of type $\theta \in \Theta$ purchase in $\mathscr{C}^0$ and rent in $\mathscr{C}^1$.

2. (Imperfect) Competition between Firm and Sharing Market. For all $x \in [\bar{x}, 1]$, the sharing market coexists and competes with the firm. All owners in $\mathscr{C}^1$ supply their unused items on the market. The available consumption offers are such that

- for all $x \in [\bar{x}, x_3)$, renting, borrowing, and purchasing options are available on the market. High-type non-owners purchase in $\mathscr{C}^0$. High-type non-owners in $\mathscr{C}^1$ and low-type non-owners in $\{\mathscr{C}^0, \mathscr{C}^1\}$ use a mixed strategy whether to rent or borrow.
- for all $x \in [x_3, 1]$, the rental service is inoperative. All non-owners of type $\theta \in \Theta$ purchase in $\mathscr{C}^0$ and borrow in $\mathscr{C}^1$.

Fig. 3 shows the optimal actions of heterogeneous agents of type $\theta \in \Theta$ during their lifespan, for a case where all specified regions are nondegenerate. Note that for the generation in $\mathscr{C}^1$, the specific type $\theta \in \Theta$ does not play a role in decision making. The end-of-horizon effect requires the agents to consider only their current need state $s^1 \in \{0, 1\}$ and ignore the future.

Depending on the market structure, the second-degree price discrimination is optimally performed to achieve the highest profit. Let

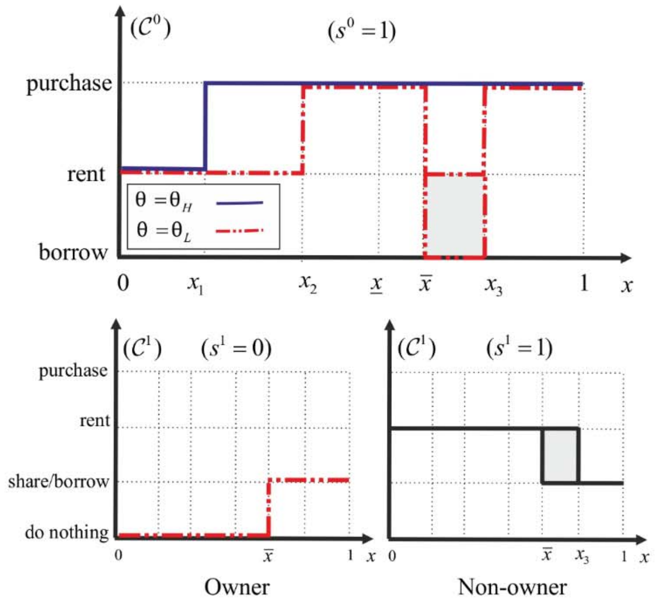

Fig. 3. Agents' optimal actions in their early and late consumption phases.

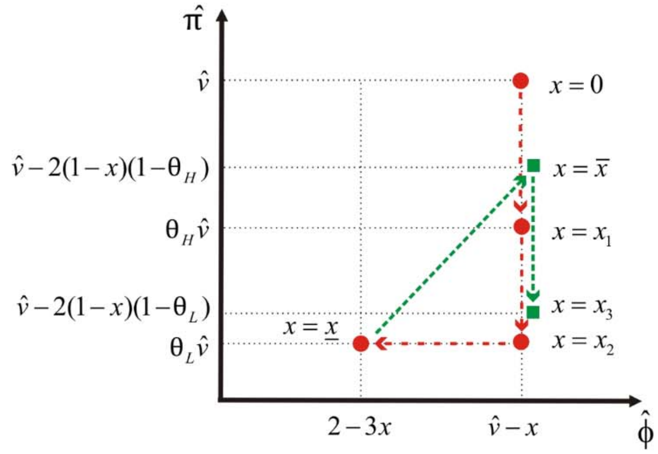

Fig. 4. Optimal pricing strategy in terms of $(\hat{\phi}, \hat{\pi})$.

$\hat{\phi} = \phi/\tau$ and $\hat{\pi} = \pi/\tau$ be the normalized rental price and the normalized purchase premium, respectively. Note that the pair $(\hat{\phi}, \hat{\pi})$ is sufficient to describe the optimal pricing strategy. Fig. 4 summarizes the results in Props. 1 and 2 and Las. 6 and 7 in the $(\hat{\phi}, \hat{\pi})$-space. The red dots represent the optimal pricing strategy under the firm's monopoly, and the green squares represent the optimal second degree price discrimination when there is monopolist competition between the firm and the peer-to-peer market. The emergence of the peer-to-peer market creates a nonmonotonicity in the firm's pricing strategy. When there is no sharing, the normalized rental price is constant and designed to extract the consumers' maximum willingness to pay in all regimes, except for the induced sharing choke-off when $x \in [\underline{x}, \bar{x}]$. When the sharing market starts to operate, the rental price jumps back to its constant value.[14]

Note that the rental price and the purchase premium tend not to change at the same time, in any of the optimality regimes. The purchase premium changes while the rental price stays constant or vice versa. When there is no sharing, $\hat{\pi}$ jumps downward as the optimal regime changes. However, when the sharing market starts to operate, the firm benefits from an upward jump in the purchase premium, and the premium attains the highest amount, at which selling is still feasible. As the sharing market matures and the optimal regime further changes at $x = x_3$, the purchase premium falls back down in such a way that peer-to-peer transactions are embraced by all agents in the economy.

Optimal Consumption Bundling. A durable product can be viewed as a bundle of consumption opportunities (Jiang and Tian, 2016). In this regard, the firm's optimal portfolio design amounts to the problem of finding the best intertemporal consumption-bundling strategy. By and large, the firm's choices consist of *pure bundling*, *mixed bundling*, and *unbundling*.

Pure bundling requires the firm to offer an integral consumption bundle to all customers. This is equivalent to exclusively offering purchase options. The individual components of the bundle, i.e., single-use rental options are not separately available to the customers. By mixed bundling, the firm offers the integral bundle (here, the purchase option), and at the same time, customers have the option to separately select each component of the bundle (the per-period rental service). When the firm pursues an unbundling strategy, it decouples the individual elements of the bundle and offers its products exclusively via rentals.

Fig. 5 depicts the firm's optimal consumption-bundling strategy as a function of x. As the economy's sharing propensity grows, the firm's

[14] In equilibrium, with inactive rental market for $x \in [x_3, 1]$, only the sum $\hat{r} = \hat{\phi} + \hat{\pi}$ matters for consumers, and all $(\hat{\phi}, \hat{\pi})$ that satisfy Eq. (34) are implementable. In Fig. 4, the chosen pair contrasts the two market structures.

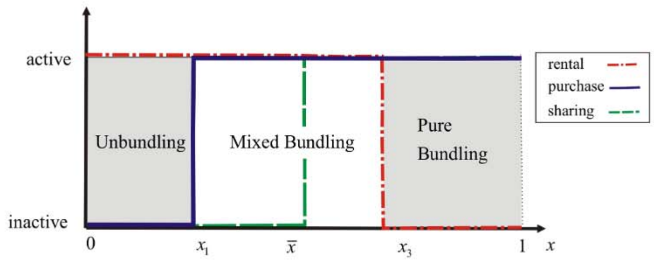

Fig. 5. Optimal consumption bundling.

strategy shifts from unbundling (pure rental), via mixed bundling (including the purchase option and the rental service in the menu) to pure bundling (exclusively selling). When unbundling, the firm is able to extract the consumers' full surplus by charging their willingness to pay for each unit, omitting any quantity discounts. When the sharing propensity is high, while the economy experiences a cultural shift towards collaborative consumption, the firm finds pure bundling optimal. By exclusively offering purchase options, the firm encourages peer-to-peer transactions, while enjoying a revenue growth thanks to an increasing sharing premium. When consumers' peer-to-peer trading propensity are located between these two extreme scenarios, mixed bundling becomes optimal for the firm. The latter strategy may be optimal irrespective of whether the sharing market is active or not.

4. Economic impact of sharing culture

We now analyze the impact of shifts in consumers' sharing culture in terms of changes in their peer-trade propensity. For this, we first characterize the dependence of the firm's profit on $x \in \mathscr{X}$. We then introduce a numerical example, which is also used to show the dependence of the consumers' surplus, their gains from trade, and ultimately the frictional losses in the economy caused by the persistent propensity mismatch.

4.1. Monopoly profit

As long as sharing propensity is fairly small, the firm can economically justify defensive behavior against the possibility of sharing, and thus opt to disable the aftermarket by offering rentals at sufficiently low rates. As this becomes more and more difficult with increasing sharing propensity, profits must decrease as consumers become more fond of collaborative consumption. However, once the existence of the sharing market can no longer be prevented, the firm benefits from further increases in the consumers' disposition to use peer-to-peer trading. The following result summarizes the dependence of the firm's profit on the sharing propensity $x \in \mathscr{X}$.

Lemma 9.

(i) *For all $x \in [0, \bar{x}]$, the firm's profit is strictly decreasing in x.*
(ii) *For all $x \in [\bar{x}, x_3]$, the firm's profit is increasing in x if*

$$\delta \geqslant \frac{1 + 2\ell}{2(1 - \theta_H)} \triangleq \delta_1.$$

(iii) *For all $x \in [x_3, 1]$, the firm's profit is increasing in x if*

$$\delta \geqslant \frac{1}{2(1 - \theta_L)} \triangleq \delta_0.$$

Provided the consumers are patient enough to enjoy the benefits of sharing and joint consumption, the firm can increase its profits by increasing the purchase premium and extracting some of the consumers' added surplus. This result is in line with the earlier findings by Razeghian and Weber (2016). By offering loans and ancillary financial services, the firm can effectively increase the consumers' discount factor. Through the provision of such services the firm benefits from the cultural transition towards sharing and collaborative consumption. Clearly, $\delta_1 \geqslant \delta_0$. As the sharing propensity increases and the firm moves from mixed bundling to pure bundling, it can effectively downgrade financial aid while also enjoying a net increase in profits as the population transitions towards sharing.

4.2. Numerical example

As noted in Las. 4 and 8, the discount rate, production cost, and consumption likelihood are the key determinants of the propensity thresholds (e.g., x_1, x_2, x_3). In response to changes in these parameters, the strategic propensity-regimes may grow, shrink, or completely vanish. In what follows, we use a numerical example to illustrate the results for the most interesting case where all six regions in the propensity space $\mathscr{X}$ exist.

Parameter Values. While the precise parameter values are immaterial, we assume—to fix ideas—that the (common) per-period discount factor is $\delta = 0.9$ and the marginal cost of production is $c = 0.1$. Furthermore, we assume that the consumers' subjective probabilities of need are $\theta_L = 0.4$ and $\theta_H = 0.5$, respectively. Finally, the firm believes that a given agent is of high type with probability $\lambda = 0.6$. The unit transportation cost is assumed not greater than the consumption value, but—for simplicity—equal $v = \tau = 0.6$. As a result, all agents find consumption appealing in time of need (because the propensity-mismatch costs τx and $\tau(1 - x)$ cannot exceed the consumption value v, independent of the sharing propensity $x \in \mathscr{X}$). A low marginal production cost and a low dispersion of usage rates *per se* do not favor a company's preference for the presence of a sharing market, as noted by Weber (2016) and Abhishek et al. (2018), respectively. In other words, parameters are chosen to slightly favor the corporate option. A discount rate that is not too low embeds a concern for the future both in consumers and the firm, emphasizing the importance of rational expectations for the model outcomes.

Comparative Statics. Fig. 6 shows the firm's optimal profit when the discount factor is $\delta = 0.9$ and the production cost equals $c = 0.1$. For all $x \leqslant \underline{x} = 0.5$, the peer-to-peer market naturally does not form. For an intermediate propensity region $x \in [0.5, 0.52]$, the firm artificially deactivates the sharing market by lowering the rental price. For higher sharing propensities, the peer-to-peer market is functioning. The

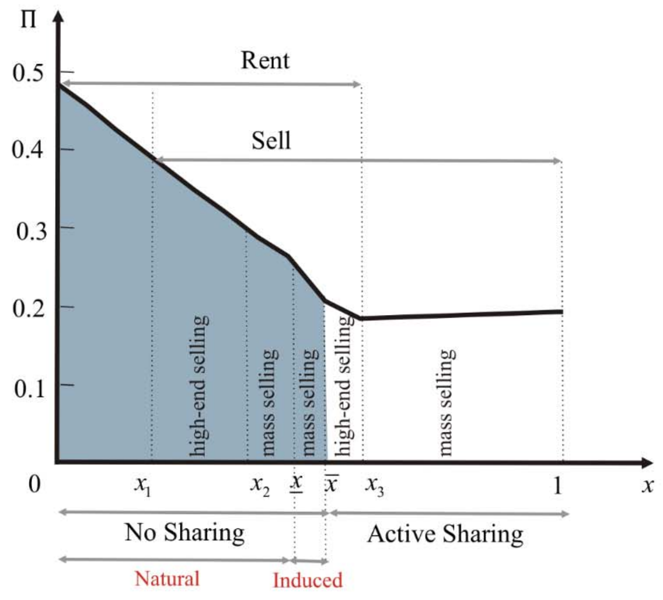

Fig. 6. Optimal selling strategy and profit.

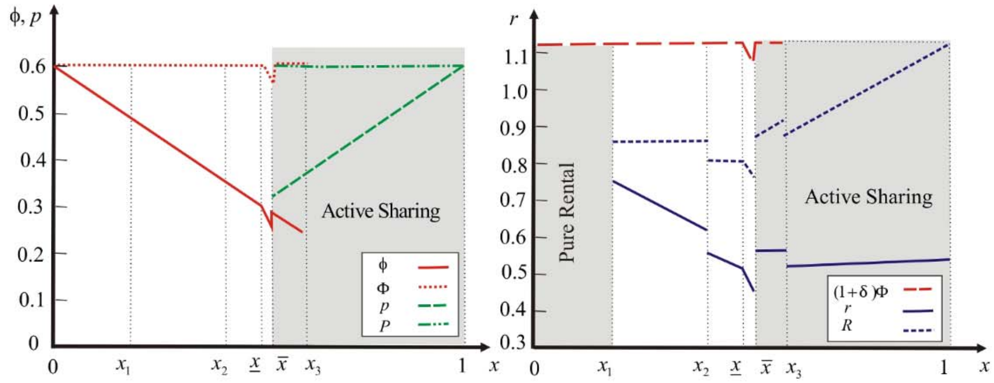

Fig. 7. Nominal and effective prices.

optimal bundling strategy for each interval is indicated in the figure.

Fig. 7 shows how the nominal prices ϕ and r change as the sharing propensity grows. As depicted in the left panel of the figure, the nominal rental price is strictly decreasing for all $x \in [0, \bar{x}]$, as the firm has to adjust its pricing to the growing cost of propensity mismatch. At the propensity threshold $x_1 = 0.18$, the firm opens up the purchase option exclusively for high-type customers. The nominal purchase price is seen in the right panel of Fig. 7. As the cost of propensity mismatch grows and the revenue from rental service further decreases, the firm adjusts its strategy at $x_2 = 0.41$ by opening the sales market to *all* consumers. If sharing was not possible such that $\underline{x} = \bar{x} = 1$, then mass selling would be the optimal strategy for all $x \geqslant x_2$. As the sharing propensity passes this threshold, the firm is able to maintain the mixed bundling of mass selling & rental only by artificially reducing both the rental and purchase prices in order to deactivate the peer-to-peer market. However, this is optimal for the firm only when the sharing propensity x lies in the interval $[0.5, 0.52]$. For values of x in the interval $[0.52, 0.6)$, the firm can mitigate the impact of the sharing market, using an upgraded purchase option that suits only the high-type users. This allows the firm to increase the rental price, too. The price increase in this region compensates for the demand decrease. The transition to mass selling happens once again at $x_3 = 0.6$ when the revenue from the rental service becomes negligible. Although the sharing price decreases compared to high-end selling, the increased demand for the purchase option compensates for the revenue loss from the rental service.

Cultural Shift. When the sharing propensity is low enough such that the peer-to-peer market does not function, the firm's profit constantly decreases in the sharing propensity. This is due to the fact that the mismatch cost is endured by the firm. As x increases, the monopolist is obligated to lower the prices in order not to lose its customers. In the absence of sharing, the firm might want to invest in a cultural transition towards trust in its established brand, such that the consumers' mismatch-cost is perceived to be smaller. Equivalently, lowering the tax burden and/or the transportation cost on the consumers who use the firm's products and services can have the same effect. For example, the firm may want to offer additional services such as home-delivery and home pick-up for its rental service, in an attempt to reduce the consumers' transportation cost. However, when the consumers' sharing propensity is high, the firm may in fact benefit from the presence of the peer-to-peer market. The firm's profit may be increasing in x. As shown in Fig. 6, in our example the firm achieves its highest profit when $x = 0$. The profit constantly decreases for all $x \in [0, 0.6]$ and the total profit drop amounts to 61%. The profit then continuously increases and at $x = 1$, the firm's profit is 5% higher than its value at $x = 0.6$.

4.3. Consumer surplus, gains from trade, and frictional losses

While the firm obtains the nominal prices ϕ or r from each unit provided, the consumers' perceived access/acquisition costs are in fact higher and include their subjective transaction cost as frictional losses. This also holds for the sharing market. The nominal sharing price p is, in fact, smaller than the total cost paid by the borrowers. We define an *effective* price as the sum of the corresponding nominal price and the concomitant propensity-mismatch cost. With this, the "effective" sharing, rental, and purchase prices are given by:

$$[\Phi, R, P] = [\phi, r, p] + \tau[x, x, 1 - x]. \tag{38}$$

A comparison of nominal and effective prices is shown in Fig. 7. When the sharing market is inactive, i.e., when $x \in [0, \bar{x}]$, the effective price is seen as either constant or decreasing by the consumers. Note that the consumers' valuation differs from their maximum willingness to pay. For example, for a one-time consumption, the valuation is v, while the willingness to pay is no more than $v - \tau x$. This implies that although the nominal prices decrease, consumers always face effective prices Φ and R which are equal to their valuation unless their sharing propensity implies an induced choke-off regime, i.e., $x \in (\underline{x}, \bar{x})$. However, when the sharing market is present, the effective purchase prices always grow in x. The underlying reason is the simultaneous growth of the sharing price p, which creates opportunities to compensate the surcharge. Meanwhile, the nominal purchase price may or may not increase.

Remark 8. In the presence of an active sharing market, the effective purchase price grows at a faster speed than the nominal price:

$$R'(x) = r'(x) + \tau \geqslant r'(x).$$

As the sharing propensity grows, consumers pay an increasing amount to get access to the purchase option. The existence of the peer-to-peer market *per se* allows the firm to pass a fraction of the propensity-mismatch cost on to the purchasers. This is in contrast to the case of no sharing, where the burden of the transportation cost is entirely on the firm. In the case of active sharing, buyers are willing to accept this extra transaction cost, expecting to compensate the loss with the additional revenue from sharing transactions.

Note that second-degree price discrimination requires quantity discounts on the purchase option. That is, the consumers' spending on a purchase option must not exceed the cost of renting in all periods. Otherwise, no agent is willing to invest in purchasing. Since the consumers in total pay a sum equal to the effective prices, the quantity discount must satisfy

$$R < (1 + \delta)\Phi. \tag{39}$$

That is, the effective purchase price must be strictly less than the total discounted effective rental price. Fig. 7 shows that the firm chooses the nominal price ϕ and r such that for all consumers purchasing is always cheaper than renting in both periods.

We now examine how the consumer surplus depends on the population's sharing propensity, the presence of a peer-to-peer market, and strategic decisions of the profit-maximizing firm. At each period, the consumers' aggregate surplus is the sum of the benefits of both generations that live in that period. That is,

$$\mathrm{CS}(x) = \mathrm{CS}^0(x) + \mathrm{CS}^1(x).$$

The surplus may stem either from self-consumption, or the additional revenue obtained from the sharing activities. Let $D_\phi^0 \in [0, 1]$ and $D_\phi^1 \in [0, 1]$ be the stationary demands for the rental service offered by the firm in the early and late consumption phases, respectively. Similarly, let $D^0 \in [0, 1]$ and $D^1 \in [0, 1]$ be the stationary demands on the sharing market in $\mathscr{C}^0$ and $\mathscr{C}^1$, respectively. We also define $D_r \in [0, 1]$ as the stationary demand for the purchase option. The demand for the purchase option consists of the demand coming from high- and low-type users, such that [15]

$$D_r = D_{rH} + D_{rL}.$$

Among the young generation, consumers either rent from the firm, borrow from the sharing market, purchase the item, or do nothing. Note that the consumers who do not engage in any transactions necessarily gain zero surplus. Hence, the benefits of the young agents in aggregate amounts to

$$\mathrm{CS}^0(x) = D^0(v - P(x)) + D_\phi^0(v - \Phi(x)) + D_r(v - R(x)). \tag{40}$$

The first term corresponds to the net benefit of the borrowers on the sharing market. The second and third terms describe the benefits of renters and buyers, respectively. The aggregate surplus of the older generation is

$$\begin{aligned}\mathrm{CS}^1(x) = {} & D^1(v - P(x)) + D_\phi^1(v - \Phi(x)) + (D_{rH}\theta_H + D_{rL}\theta_L)v \\ & + (D_{rH}(1 - \theta_H) + D_{rL}(1 - \theta_L)(p(x) - (1 - x)\tau).\end{aligned} \tag{41}$$

The first and second terms correspond to the benefits of the borrowers and renters, respectively. The third term describes the surplus of the owners who consume the product themselves. Note that for this consumer segment, the purchase price is already a sunk cost and they enjoy a positive surplus without incurring any cost. The fourth term corresponds to the owners who supply their unused capacity on the sharing market. Note that by Eq. (30), the sharing price in equilibrium is such that the users of such a service are left with zero surplus for all $x \in [\bar{x}, 1]$. Hence,

$$D^0(v - P(x)) = D^1(v - P(x)) = 0,$$

and the terms can be omitted from Eqs. (40) and (41).

The total frictional losses ("transaction cost") $T(x)$ correspond to the sum of all propensity-mismatch costs levied on the consumers. The aggregate transaction cost for the two consumer generations, at any given time period, is:

$$T(x) = T^0(x) + T^1(x).$$

$T^0(x)$ is paid by the younger agents and includes all travels either to the firm or the rental market. That is,

$$T^0(x) = D^0(1 - x)\tau + (D_\phi^0 + D_r)\tau x. \tag{42}$$

For the older agents, $T^1(x)$ includes transportation costs to both the firm and the sharing market, so

$$T^1(x) = [D^1 + D_{rH}(1 - \theta_H)\theta_H + D_{rL}(1 - \theta_L)\theta_L](1 - x)\tau + D_\phi^1\tau x. \tag{43}$$

Note that the propensity-mismatch cost to the sharing market is paid both by borrowers and lenders.

Fig. 8 depicts the firm's profit, the consumer surplus, and the frictional losses for the example introduced in Section 4.2. When the sharing propensity $x \in [0, x_1]$ is so low that pure rental is the optimal

[15] The purchase option is attractive only to consumers in $\mathscr{C}^0$. Hence, the superscript is omitted for ease of notation.

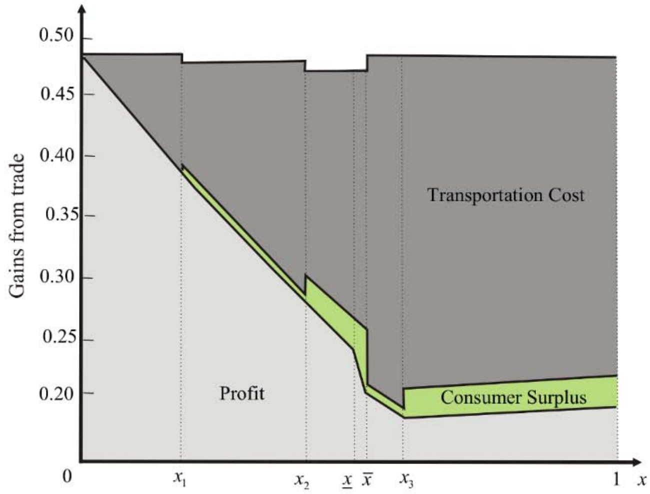

Fig. 8. Distribution of the gains from trade.

strategy, consumers gain zero surplus. The firm extracts all of the consumers' surplus by unbundling the consumption and charging the maximum willingness to pay per usage. This happens when the population finds their preferences closest to the firm or when the sharing services are heavily taxed. When $x \in [x_1, x_2]$, under the high-end selling strategy, the purchase option is designed to leave young buyers with zero expected surplus. Nevertheless, for owners among the older generation, the price paid in the first period is "water under the bridge," and they obtain a strictly positive surplus in their late consumption phase ($\mathscr{C}^1$). Consumer surplus increases, as the firm undertakes a mass-selling strategy. In $x \in [x_2, \underline{x}]$, the optimal purchase option is designed to extract the surplus only from the low-type buyers, which allows the high-type buyers to enjoy a net positive gain from purchasing. This results in a positive jump in the consumer surplus at $x = x_2$. All owners (of both types) enjoy a positive net benefit in their late consumption phase contingent on the realization of a high-need state. The transportation cost has its lowest value in this interval, since both types enjoy the quantity discount and the number of travels to the firm is at its minimum.

Consumer surplus is the highest when the firm applies an induced choke-off strategy when $x \in [\underline{x}, \bar{x}]$. The prices that disable the sharing market are smaller than the consumers' actual willingness to pay, which works in favor of the population. As the firm has to sacrifice some of its gains to please the consumers, its profit rapidly decreases in x, and the optimality of the induced choke-off does not last for a large interval of sharing propensities.

When a sharing market becomes active for $x \in [\bar{x}, x_3]$, all three services, namely renting, sharing, and purchasing, coexist. At $x = \bar{x}$, the population experiences a large decrease in the aggregate surplus. The introduction of the sharing market allows the firm to increase the prices, and leaves consumers with a smaller surplus. Yet, both suppliers and borrowers are still not close enough to the sharing market to enjoy a significant surplus. For $x \in [\bar{x}, x_1]$, the transaction cost takes its highest value, since all consumer segments (buyers, renters, suppliers, and borrowers) incur fairly large propensity-mismatch costs to access either the firm or the sharing market.

For all $x \in [x_3, 1]$, consumer surplus starts to increase. High-type agents gain a positive information rent as a result of the mass-selling strategy. Due to the presence of the peer-to-peer market, all owners (of both types) gain a positive surplus from their own usage or the additional income they gain by supplying their items on the sharing market. Suppliers in this region gain a significant surplus, since their transportation cost is relatively much smaller than the price they charge. The transportation cost decreases, as the number of travels to the firm decreases. Although the number of travels to the sharing market increases, the cost is negligible due to the consumers' proximity to this market.

The gains from trade in this economy are the sum of the consumer surplus, firm's profit, and the transportation cost, i.e.,

$$\mathrm{GT}(x) \triangleq \Pi(x) + \mathrm{CS}(x) + T(x) = D_r\left(v - c + v\cdot\mathbb{1}_{\{x\geqslant\bar{x}\}}\right) + D_\phi\left(v - \frac{c}{2}\right) + \left(D_{\mathrm{rH}}\theta_H + D_{\mathrm{rL}}\theta_L\right)v. \tag{44}$$

When the sharing market is not active, whether naturally or artificially, the gains from trade are piecewise constant. When the sharing market becomes active, the gains from trade go up and achieve their highest value. This is formalized in the following result.

Lemma 10.

(i) *Without sharing, gains from trade are piecewise constant.*
(ii) *With sharing, gains from trade are constant:*

$$\mathrm{GT}(x) \equiv \left(\lambda\theta_H + \left(1 - \lambda\right)\theta_L\right)\left(v - \frac{c}{2}\right).$$

(iii) *Gains from trade with sharing are higher than without sharing.*

As shown in Fig. 8, the gains from trade under each strategy are constant. When there is no sharing, the consumer surplus is also constant, demonstrating that the firm is burdened with the total cost of mismatch. When the sharing market opens up, the society as a whole enjoys an increase in the gains from trade. The economy benefits from the sharing transactions and social welfare increases as a result of cultural transitions towards sharing. Without sharing, the firm and the population have conflicts of interest whether or not to move towards sharing. However, in the presence of sharing, the firm is able to pass some of the burden of the transaction cost to the consumers. Under certain conditions, when the discount factor is large, both the firm's profit and the consumer surplus increase in x and cultural transition towards sharing is in all players' interest.

5. Conclusions

5.1. Managerial implications

The consumers' sharing propensity modulates an incumbent's strategic response to an emerging sharing market. The firm can leverage society's attitude towards collaborative consumption and design its product portfolio to maximize the extraction of consumer surplus. This paper is the first to explicitly consider the population's sharing propensity in the context of a firm's optimal intertemporal bundling strategy. Given an economy with overlapping generations of consumers, the firm optimizes its menu design in the form of offering rental and/or purchase options. In our classical Hotelling linear-city framework, higher sharing propensities amount to a shift of the propensity-mismatch cost from the incumbent firm to favoring collaborative consumption, thus rendering the peer-to-peer market increasingly attractive.

Strategic Interaction with the Sharing Market. For the design of an effective intertemporal bundling strategy the firm needs to take into account the consumers' cultural readiness for sharing. Indeed, the degree of strategic interaction with the peer-to-peer market critically depends on their sharing propensity. The sharing market remains inactive as long as the incumbent firm offers a rental service as a sufficiently cheap substitute for peer-to-peer access. If sharing propensity is low (e.g., associated with significant trust in the established brand), the firm's rental service and the sharing market effectively engage in Bertrand competition with horizontally differentiated products and asymmetric (marginal) costs. And while the firm can distribute their unit costs for rentals over the product lifetime, suppliers on the sharing market cover the consumers' propensity-mismatch cost for sharing transactions. Thus, for low sharing propensities, the peer-to-peer market may be naturally choked off, so the firm captures the entire demand. For intermediate sharing propensities, the firm has an interest to artificially induce a sharing choke-off by decreasing its rental rates, so the peer-to-peer market remains unattractive for consumers. However, as such predatory pricing tends to imply a significant profit loss, the firm will eventually try to cooperate with the sharing market, in the sense that it will seek profits from selling goods to eventual suppliers on the sharing market, including a commensurate "sharing premium" as surcharge. In other words, when customers' attitudes have sufficiently shifted towards collaborative consumption and the existence of a sharing market is a foregone conclusion, the firm finds it optimal to "go with the flow" and promote sharing. The incumbent firm and the sharing market engage in monopolistic competition, and consumers perceive the rental service and the sharing market as imperfect substitutes.

Optimal Product Mix. Cultural transitions from private ownership to collective consumption shape the firm's portfolio design. As the sharing propensity grows, the firm switches from unbundling (exclusively renting), via mixed bundling (renting and selling), to pure bundling (exclusively selling).[16] The propensity threshold for unbundling increases in the firm's production cost. Indeed, as the cost of production increases, the attractiveness of unbundling goes up, since it is more cost-effective to rent rather than sell. On the other hand, when consumers become more patient (corresponding to an increase in the discount factor), the unbundling threshold decreases. The revenue from the purchase option makes it more appealing to offer both options at the same time. Mixed bundling happens when sharing and rental each has their own customers. The mixed-bundling threshold decreases in the low-type agents' consumption likelihood. When a large enough fraction of aggregate consumption comes from low-type agents, it is more profitable to target them as purchasers and switch to mass selling. The mixed-bundling threshold is also increasing in the discount factor. Consumers' increased patience has a positive effect on the sales price, and therefore also on the firm's bottom-line.

Although the quantity of the firm's transactions decreases in total, the emergence of a peer-to-peer sharing market allows the firm to increase both the purchase premium and the rental price, and potentially gain from sharing. Somewhat counterintuitively, we find that an increase in sharing propensity has an ambiguous (U-shaped) effect on the firm's profit. While an active sharing market tends to decrease the firm's rental revenues, it also tends to increase the firm's product sales. Thus, peer-to-peer sharing acts as an imperfect substitute for the firm's rental option and, at the same time, it complements the firm's purchase option. Consumer patience turns out to be an important discriminant for the upswing of the purchase premium in the presence of sharing: the firm's profit increases in the sharing propensity, provided that consumer patience (as measured by the discount factor) is large enough. Thus, by artificially increasing the discount factor, say by providing loans to the buyers, the firm could further increase its profits in the presence of a cultural transition towards sharing.

5.2. Regulatory perspectives

The model also has implications for the possibilities of regulatory interventions on the market structure. While the sharing propensity is

[16] With increasing sharing propensity, as accessing the firm becomes more costly, the firm is compelled to offer quantity discounts by enabling the purchase market (where the purchase premium over a one-period rental is significantly below the price of an additional rental period). The quantity discount is at first offered solely to the high-type agents (high-end selling), and then to the entire population (mass selling).

intrinsic to consumers, it may be influenced by regulatory interventions, such as imposing consumption taxes or changing the perceived cost of propensity mismatch (e.g., through a campaign for sharing). Imposing consumption taxes on the firm's product offerings amounts to subsidies for sharing and brings consumers closer to the sharing market. This, in fact, may benefit society as a whole and result in a marked increase of the gains from trade. Moreover, if consumers are patient enough, then not only the size of the "transactional pie" increases but also both firm and customers benefit from a transition towards sharing. This is in contrast to the fact that when sharing markets are disabled, the monopolist would prefer to lower the population's peer-trade propensity. Yet, when an active sharing market is a given, the firm is left with strong incentives to help promote a cultural transition towards sharing. Hence, regulatory intervention in the form of promoting the sharing economy can be beneficial to all players.[17]

5.3. Limitations and extensions

The model developed here is a simplification of a more complex reality. It highlights the role of the sharing propensity in shaping the firm's portfolio of product offerings. Several interesting extensions to this work may be of interest in future research. To reduce the problem complexity, we assumed that the entire population possesses a common attitude towards sharing. A natural extension would be to include various consumer types in the economy with respect to the sharing propensity. When the attitudes of the various consumer groups are close, the results stay qualitatively the same. However, when the dispersion in the sharing propensity across consumer groups is large, the emerging interpersonal arbitrage opportunities present an interesting topic for further investigation.

Another useful extension will be to endogenize horizontal (brand) and vertical (quality) product features offered in each menu. In our model, the firm restricts attention to determining solely the prices for the various modes of intertemporal consumption, and all consumers have the same per-period consumption value—no matter if access to the product is obtained via renting, sharing, or buying the product. However, the firm may gain by adjusting the product range and quality. For example, while owners stick to the product that they buy, renters may choose from a portfolio of different products in each period. Furthermore, the per-period consumption utility obtained may depend on the source of access.

Finally, in our linear-city representation of the sharing propensity, the absolute positions of firm and sharing market are fixed and static, and only the propensity-mismatch (transportation) cost is considered for potential changes. An interesting extension would be to study a dynamic version of this Hotelling model. For example, by investing in marketing activities, the firm may be able to affect consumers' perceived cost of mismatch, thus increasing or decreasing competition with the sharing market. More broadly, the firm's optimal investment in actively managing consumers' perceptions seems to be a fruitful area for further research.

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Appendix A. Analytical details

Proof of Lemma 1. Substituting Eqs. (7) and (8) in the incentive-compatibility constraint (9) and rearranging the terms yields that $\delta\theta(\phi + \tau x) \geqslant \pi$, as claimed. □

Proof of Lemma 2. The sharing market is naturally choked off if $x \leqslant (2/3) - \phi/(3\tau) \leqslant 1 - v/(2\tau)$ or if $x \leqslant 1 - v/(2\tau) < (2/3) - \phi/(3\tau)$, while there is an induced sharing shutdown if $1 - v/(2\tau) < x \leqslant (2/3) - \phi/(3\tau)$. Equivalently, $\tau \geqslant \max\{\phi/(2 - 3x), v/(2(1 - x))\}$ must hold for a natural choke-off, and $\phi/(2 - 3x) \leqslant \tau < v/(2(1 - x))$ for an induced sharing shutdown. Intuitively, induced choke-off obtains only when the low rental rate is really the only factor that causes the sharing market to fail.

In the absence of sharing, the firm can increase the rental price $\phi(x)$, as a function of x, at most until the individual-rationality constraint (6) becomes binding, so

$$\phi(x) \leqslant v - \tau x.$$

In the case of an induced choke-off, in addition to the last inequality $\phi(x) \leqslant (2 - 3x)\tau$ needs to hold, which implies that

$$\phi(x) \leqslant \min\{v - \tau x, (2 - 3x)\tau\}.$$

Hence, for an induced choke-off, which also requires that $x > 1 - v/(2\tau)$, the highest rental rate is

$$\phi(x) = (2 - 3x)\tau.$$

Conversely, in the case of a natural choke-off, from the above conditions for $x \leqslant 1 - v/(2\tau)$, in combination with the individual-rationality constraint, we have

$$(3v/2) - \tau \leqslant \phi(x) \leqslant \min\{(2 - 3x)\tau, v - \tau x\} = v - \tau x,$$

which implies that

$$\phi(x) = v - \tau x.$$

For $0 < \tau < \phi/(2 - 3x)$, there is no sharing shutdown, and the sharing market remains active. We have therefore established the veracity of all claims in parts (i)–(iii), which completes the proof. □

Proof of Lemma 3. Given a decision to shut down the rental service, the firm's strategy set contains three feasible courses of action: selling to young high-type agents, selling to young agents of either type, and selling to both generations. By targeting the young consumers, the constraint (9)

[17] This is analogous to the finding by Razeghian and Weber (2016) that the ensured existence of a sharing market provides strong incentives for a monopolist to increase product durability.

becomes

$$v - r - \tau x + \delta\theta v \geqslant 0.$$

If the incentive-compatibility constraint (9) binds only for the high-type agents, the purchase price is $r(x) = (1 + \delta\theta_H)v - \tau x$. At this price, the firm's optimal profit is

$$\lambda\theta_H[(1 + \delta\theta_H)v - \tau x - c]_+ \leqslant \Pi_1(x); \tag{45}$$

where $\Pi_1(x)$ is given in Eq. (15). If the incentive-compatibility constraint (9) binds for the low-type agents, so all consumers in need prefer to purchase, i.e, if $r(x) = (1 + \delta\theta_L)v - \tau x$, the firm nets

$$(\lambda\theta_H + (1 - \lambda)\theta_L)[(1 + \delta\theta_L)v - \tau x - c]_+ \leqslant \Pi_2(x), \tag{46}$$

where $\Pi_2(x)$ is specified in Eq. (18). If the monopolist chooses to sell to both generations, the purchase price drops to

$$r(x) = v - \tau x,$$

and the firm's profit would be

$$(\lambda\theta_H + (1 - \lambda)\theta_L + \lambda\theta_H(1 - \theta_H) + (1 - \lambda)\theta_L(1 - \theta_L))[v - \tau x - c]_+ \leqslant \Pi_2. \tag{47}$$

This concludes the proof. □

Proof of Proposition 1. If the sharing market is naturally chocked off, by La. 2, the sharing propensity is less than $1 - v/(\tau 2)$ and the rental price is $r(x) = v - \tau x$. The firm's profit from a pure rental strategy outweighs its profit from pursuing a high-end selling & rental strategy if

$$\Pi_0(x) \geqslant \Pi_1(x),$$

where $\Pi_0(x)$ and $\Pi_1(x)$ are specified in Eqs. (13)–(15). It follows that pure rental strategy dominates high-end selling & rental (in a natural sharing shutdown) if

$$x \geqslant \left\{\frac{(1 - \theta_H)c}{2\tau\theta_H} + \frac{(1 - \delta)v}{\tau}, \underline{x}\right\} \equiv x_1.$$

Similarly, for all $x \leqslant \underline{x}$, when the sharing market is naturally inactive, the firm's profit from a high-end selling & rental strategy is greater than mass selling & rental if

$$\Pi_1(x) \geqslant \Pi_2(x),$$

where $\Pi_1(x)$ and $\Pi_2(x)$ are specified in Eqs. (13)–(15), and $r(x) = v - \tau x$. It follows that high-end selling & rental is feasible and yields a higher profit if

$$x \geqslant \left\{\frac{\delta v\lambda\theta_H(\theta_H - \theta_L)}{(1 - \lambda)\theta_L^2\tau} + \frac{(1 - \theta_L)c}{2\theta_L\tau} + \frac{(1 - \delta)v}{\tau}, \underline{x}\right\} \equiv x_2.$$

Furthermore, it is easy to show that

$$\frac{(1 - \theta_H)c}{2\tau\theta_H} \leqslant \frac{(1 - \theta_L)c}{2\tau\theta_L} + \frac{\delta v\lambda\theta_H(\theta_H - \theta_L)}{(1 - \lambda)\theta_L^2\tau},$$

since the first term on the RHS is strictly greater than the one on the LHS, and the second term on the RHS is nonnegative. It follows that $x_1 \leqslant x_2$. Hence, for all $x \in [0, x_1]$:

$$\Pi_0(x) \geqslant \Pi_1(x) \geqslant \Pi_2(x),$$

which yields part (i). For all $x \in [x_1, x_2]$,

$$\Pi_1(x) \geqslant \max\{\Pi_0(x), \Pi_2(x)\}.$$

This completes the proof of part (ii). For all $x \in [x_2, \underline{x}]$,

$$\Pi_2(x) \geqslant \Pi_1(x) \geqslant \Pi_0(x),$$

which implies part (iii). By La. 2, the rental price in the induced choke-off strategy is $(2 - 3x)\tau$, which is strictly smaller than $v - \tau x$, for all $x \leqslant 1 - v/(2\tau)$. Hence, for all $x \leqslant \underline{x}$, the induced choke-off strategy is suboptimal. Applying La. 2 again, the induced choke-off strategy leads to a negative payoff for all $x > 2/3$, which indicates that it is not optimal in that region. Indeed, this strategy may be optimal only on the interval $x \in [\underline{x}, 2/3]$. This yields part (iv). □

Proof of Lemma 4.

(i) By Eqs. (10) and (20), the inequality $x_1 \leqslant \underline{x}$ holds if

$$\left(\frac{1 - \theta_H}{2\theta_H}\right)\hat{c} + \left(1 - \delta\right)\hat{v} \leqslant 1 - \frac{\hat{v}}{2},$$

or equivalently if

$$\hat{v} \leqslant \frac{\theta_H(2 + \hat{c}) - \hat{c}}{\theta_H(3 - 2\delta)} \triangleq \hat{v}_0 \quad \left(=v_0\tau\right).$$

If $\hat{v} \geqslant \hat{v}_0$, then by definition $x_1 = 1 - v/2\tau$, which is decreasing in v. We now examine the case where $\hat{v} < \hat{v}_0$. Differentiating x_1 with respect to v yields

$$\frac{\partial x_1}{\partial v} = \frac{1-\delta}{\tau} \geqslant 0,$$

indicating that the threshold x_1 is locally increasing in v. Hence, the maximum is achieved at $v = \hat{v}_0\tau$, as claimed. Comparative statics with respect to other parameters only requires analysis for the case where $\hat{v} < \hat{v}_0$. For all $\hat{v} \geqslant \hat{v}_0$, by definition $x_1 = \underline{x}$ and the maximum remains unaffected by changes in parameters other than v and τ. Differentiating x_1 with respect to c yields

$$\frac{\partial x_1}{\partial c} = \frac{1-\theta_H}{2\theta_H\tau} \geqslant 0,$$

as claimed. Differentiating x_1 with respect to δ gives

$$\frac{\partial x_1}{\partial \delta} = -\hat{v} \leqslant 0.$$

And finally, by differentiating x_1 with respect to θ_H:

$$\frac{\partial x_1}{\partial \theta_H} = -\frac{\hat{c}}{2\theta_H^2} \leqslant 0.$$

This completes the proof of part (i).

(ii) By Eqs. (10) and (20), the inequality $x_2 \leqslant \underline{x}$ holds if

$$\left(\frac{1-\theta_L}{2\theta_L}\right)\hat{c} + \left(1 - \delta + \frac{\delta\Delta\theta}{\ell\theta_L}\right)\hat{v} \leqslant 1 - \frac{\hat{v}}{2}.$$

Rearranging the terms yields

$$\hat{v} \leqslant \frac{\theta_L(2+\hat{c}) - \hat{c}}{3\theta_L - \delta(2\theta_L - \Delta\theta)} \triangleq \hat{v}_1 \quad \left(=v_1\tau\right).$$

If $\hat{v} \geqslant \hat{v}_1$, then $x_2 = \underline{x} = 1 - v/2\tau$, which is decreasing in v. If $\hat{v} < \hat{v}_1$, the derivative of x_1 with respect to $\hat{v}$ is

$$\frac{\partial x_2}{\partial v} = \frac{1-\delta}{\tau} + \frac{\tau\delta\Delta\theta}{\ell\theta_L} \geqslant 0.$$

Hence, the maximum is achieved at $v = v_1$, as claimed. Similar to the reasoning provided in part (i), comparative statics with respect to the other variables can be characterized by focusing on the case where $\hat{v} < \hat{v}_1$. Differentiating x_2 with respect to c, θ_L, $\Delta\theta$, and ℓ yields

$$\frac{\partial x_2}{\hat{c}} = \frac{1-\theta_L}{2\theta_L\tau} \geqslant 0 \geqslant -\frac{\hat{c}}{2\theta_H^2} = \frac{\partial x_2}{\partial \theta_L}$$

and

$$\frac{\partial x_2}{\partial \Delta\theta} = \frac{\delta\hat{v}}{\theta_L\ell\theta_L} \geqslant 0 \geqslant -\frac{\delta\hat{v}\Delta\theta}{\theta_L\ell^2} = \frac{\partial x_2}{\partial \ell},$$

respectively. Furthermore, differentiating x_2 with respect to λ provides

$$\frac{\partial x_2}{\partial \lambda} = \frac{\partial x_2}{\partial \ell}\frac{\partial \ell}{\partial \lambda} = \frac{\delta\hat{v}\Delta\theta}{\ell^2\lambda^2\theta_H} \geqslant 0,$$

while differentiating x_2 with respect to θ_H gives

$$\frac{dx_2}{d\theta_H} = \frac{\partial x_2}{\partial \ell}\frac{\partial \ell}{\partial \theta_H} + \frac{\partial x_2}{\partial \theta_H} = \frac{\partial x_2}{\partial \ell}\frac{\partial \ell}{\partial \lambda} = \left(\frac{\delta}{\ell\theta_L} + \frac{(1-\lambda)\delta\Delta\theta}{\lambda\ell^2\theta_H^2}\right)\hat{v} \geqslant 0.$$

Finally, differentiating x_2 with respect to δ yields

$$\frac{\partial x_2}{\partial \delta} = \left(-1 + \frac{\Delta\theta}{\ell\theta_L}\right)\hat{v},$$

which is greater than zero, provided that $\ell \leqslant \Delta\theta/\theta_L$. This completes the proof of part (ii). □

Proof of Lemma 5. Substituting Eqs. (25) and (26) into the incentive-compatibility constraint (27) and rearranging yields

$$2\delta\theta(1-x)\tau \geqslant \delta(1-x)\tau + r - (1+\delta)p - (1-2x)\tau,$$

which implies the result. □

Proof of Lemma 6. By Eq. (10), the inequality $\underline{x} > 0$ holds if

$$v \geqslant 2\tau. \tag{48}$$

If $x \leqslant \underline{x}$, then by La. 2, the rental price is $\phi = v - \tau x$. Comparing the optimal purchase premium in Eq. (14) with the one characterized in Eq. (31) and substituting the optimal rental price requires

$$\delta(v - 2(1-\theta_H)(1-x)\tau) \geqslant \delta\theta_H v.$$

Rearranging the terms yields

$$\delta(1-\theta_H)(v-2(1-x)\tau) \geqslant 0,$$

which holds by relation (48). If $x > \underline{x}$, then by La. 2, the sharing is deactivated by means of applying the induced choke-off strategy, and the rental price is $\phi = (2-3x)\tau$. The purchase premium in Eq. (31) is greater than the purchase premium in Eq. (31) if

$$\delta(v-2(1-\theta_H)(1-x)\tau) \geqslant 2\delta\theta_H(1-x)\tau,$$

Rearranging the terms yields

$$\delta(v-2(1-x)\tau) \geqslant 0,$$

which holds for all parameter values, given that $v > 2\tau$. This completes the proof. □

Proof of Lemma 7. By assumption, $\underline{x} = 1 - v/(2\tau) > 0$. By La. 2, if $x \leqslant \underline{x}$, the rental price is $\phi = v - \tau x$. The optimal $r(x)$ under the mass-selling strategy with sharing in Eq. (34) is greater than the purchase price $r(x) = \phi(x) + \pi(x)$, as determined by Prop. 1. That is,

$$(1+\delta)v - \tau x - 2\delta(1-\theta_L)(1-x)\tau \geqslant v - \tau x + \delta\theta_L v.$$

Rearranging the terms yields

$$\delta(1-\theta_L)(v-2(1-x)\tau) \geqslant 0,$$

which holds by assumption for all $x \in \mathscr{X}$. If $x > \underline{x}$, by La. 2 the rental price is $\phi = (2-3x)\tau$. The optimal purchase price under the mass-selling strategy with sharing is greater than without it if

$$(1+\delta)v - \tau x - 2(1-\theta_L)(1-x)\delta\tau \geqslant (2-3x)\tau + 2\delta(1-x)\theta_H.$$

Rearranging the terms yields

$$(1+\delta)v \geqslant 2(1+\delta)(1-x)\tau,$$

which is true by assumption for all $x \in \mathscr{X}$. This completes the proof. □

Proof of Proposition 2. By Eq. (36), sharing is feasible for all $x \in [\bar{x}, 1]$. In the presence of sharing, high-end selling is more profitable than mass selling if $\Pi_3(x) \geqslant \Pi_4(x)$. Using Eqs. (32) and (35)), and rearranging yields

$$(1-\lambda)\theta_L(-(1-\delta)v + \tau x - 2(1-\theta_L)(1-x)\delta\tau) \leqslant 2\delta\lambda(1-x)(\theta_H-\theta_L)\theta_H\tau.$$

Solving for $x \in [\bar{x}, 1]$ yields

$$x \geqslant \min\left\{1, \max\left\{\bar{x}, \frac{2\lambda\theta_H\delta(\theta_H-\theta_L)\tau + 2(1-\lambda)\theta_L\delta(1-\theta_L)\tau + (1-\lambda)\theta_L(1-\delta)v}{2\lambda\theta_H\delta(\theta_H-\theta_L)\tau + 2(1-\lambda)\theta_L\delta(1-\theta_L)\tau + (1-\lambda)\theta_L\tau}\right\}\right\}.$$

Dividing the nominator and denominator by $\lambda\theta_H(\theta_H-\theta_L)\tau$ and using the facts that $\ell = ((1-\lambda)\theta_L)/(\lambda\theta_H)$, $\Delta\theta = \theta_H - \theta_L$, and $\hat{v} = v/\tau$ produces our claim. □

Proof of Lemma 8. By Eq. (37), $x_3 < 1$ provided that $(1-\delta)\hat{v} < 1$. If this is the case, then differentiating x_3 with respect to ζ yields

$$\frac{\partial x_3}{\partial \zeta} = \frac{2\delta((1-\delta)\hat{v}-1)}{(2\delta(1+\zeta(1-\theta_L)) + \zeta^2)^2} < 0. \tag{49}$$

By definition,

$$\zeta = \frac{\ell}{\Delta\theta} = \frac{(1-\lambda)\theta_L}{\lambda\theta_H\Delta\theta},$$

is decreasing in λ, θ_H, and $\Delta\theta$ and increasing in θ_L. Hence by Eq. (49), x_3 is increasing in λ, θ_H, and $\Delta\theta$. Differentiating x_3 with respect to θ_L yields

$$\frac{dx_3}{d\theta_L} = \frac{\partial x_3}{\partial \zeta}\frac{\partial \zeta}{\partial \theta_L} + \frac{\partial x_3}{\partial \theta_L} = \frac{2\delta((1-\delta)\hat{v}-1)}{(2\delta(1+\zeta(1-\theta_L)) + \zeta)^2}\left(\frac{1-\lambda}{\lambda(\Delta\theta^2)} + \zeta^2\right) < 0.$$

Differentiating x_3 with respect to $\hat{v}$ yields

$$\frac{\partial x_3}{\partial \delta} = \frac{\zeta(1-\delta)}{(2\delta(1+\zeta(1-\theta_L)) + \zeta^2)^2} > 0.$$

Differentiating x_3 with respect to δ yields

$$\frac{\partial x_3}{\partial \delta} = \frac{2\zeta(1+\zeta(1-\theta_L))(1-\hat{v}) - \zeta^2\hat{v}}{(2\delta(1+\zeta(1-\theta_L)) + \zeta^2)^2} < 0,$$

given that $\hat{v} = v/\tau > 1$. This completes the proof. □

Proof of Lemma 9.

(i) Let $D_r \geqslant 0$ and $D_\phi \geqslant 0$ be the demand for the purchase and rental service without sharing, respectively. Then

$$\Pi(x) = D_r\left(r(x) - c\right) + D_\phi\left(\phi(x) - \frac{c}{2}\right),$$

as described by Eqs. (13), (15), and (18). Differentiating the profit function with respect to x yields

$$\frac{\partial \Pi(x)}{\partial x} = D_r \frac{\partial r(x)}{\partial x} + D_\phi \frac{\partial \phi(x)}{\partial x} \leqslant 0,$$

since by Eqs. (16) and (19), the rental price and the purchase premium are such that

$$\frac{\partial \phi(x)}{\partial x} \in \left\{-\tau, -3\tau\right\} \leqslant 0,$$

and

$$\frac{\partial \pi(x)}{\partial x} \in \left\{0, -2\delta\theta_L\tau, -2\delta\theta_H\tau\right\} \leqslant 0.$$

Note that

$$\frac{\partial r(x)}{\partial x} = \frac{\partial \pi(x)}{\partial x} + \frac{\partial \phi(x)}{\partial x} < 0.$$

Since by the maximum theorem (Berge, 1959), there is no jump in the profit function, $\Pi(x)$ is continuous and decreasing in x.
For parts (ii)–(iii) of this proof, we assume that the model parameters are nondegenerate, i.e., the parameters τ, $(1-\lambda)$, λ, θ_H, and θ_L are strictly positive, and $\theta_L \leqslant \theta_H < 1$.

(ii) For all $x \in [\bar{x}, x_3]$, the profit is given by Eq. (32). The profit function is increasing in x if $\Pi_4'(x) \geqslant 0$. This requires

$$\Pi_3'(x) = \lambda\theta_H(2\delta(1-\theta_H)-1)\tau - 2(1-\lambda)\theta_L\tau \geqslant 0.$$

Rearranging the terms and using the definition of $\ell = (1-\lambda)\theta_L/(\lambda\theta_H)$ yields

$$\delta \geqslant \frac{\ell}{1-\theta_H} + \frac{(1/2)}{1-\theta_H},$$

as claimed.

(iii) For all $x \in [x_3, 1]$, the profit is increasing if the derivative of the profit function in Eq. (35) is positive. That is,

$$\Pi_4'(x) = (\lambda\theta_H + (1-\lambda)\theta_L)(2\delta(1-\theta_L)-1)\tau \geqslant 0.$$

Hence, the profit function is increasing if

$$\delta \geqslant \frac{(1/2)}{1-\theta_L},$$

concluding the proof. □

Proof of Lemma 10.

(i) By Props. 1 and 2, the sharing market is inactive when $x \leqslant \bar{x}$. Hence, the gains from trade are

$$\mathrm{GT}(x) = D_\phi\left(v - \frac{c}{2}\right) + D_r(v-c) + \left(D_{rH}\theta_H + D_{rL}\theta_L\right)v.$$

For all $x \in [0, x_1]$, by Prop. 1 the rental demand is $D_\phi^0 = D_\phi^1 = \lambda\theta_H + (1-\lambda)\theta_L$. Thus, the gains from trade become

$$\mathrm{GT}_0 = \lambda\theta_H\left(2 + 2\ell\right)\left(v - \frac{c}{2}\right). \tag{50}$$

For all $x \in (x_1, x_2]$, the rental demand is $D_\phi^0 = (1-\lambda)\theta_L$ and $D_\phi^1 = \lambda\theta_H(1-\theta_H) + (1-\lambda)\theta_L$. The demand for purchase comes exclusively from young high-type agents: $D_r = D_{rH} = \lambda\theta_H$. Hence, the gains from trade are

$$\mathrm{GT}_1 = \lambda\theta_H\left(\left(2\ell + 1 - \theta_H\right)\left(v - \frac{c}{2}\right) + \left(v - c\right) + \theta_H\right). \tag{51}$$

For all $x \in (x_2, \bar{x}]$, the rental demand in the second period is $D_\phi^1 = \lambda(1-\theta_H)\theta_H + (1-\lambda)(1-\theta_L)\theta_L$. The demand for ownership comes from both high- and low-type agents in $\mathscr{C}^0$: $D_r = D_{\rho H} = \lambda\theta_H + (1-\lambda)\theta_L$. Hence, the gains from trade are

$$\mathrm{GT}_2 = \lambda\theta_H\left(\left(\ell\left(1-\theta_L\right) + 1 - \theta_H\right)\left(v - \frac{c}{2}\right) + \left(\ell + 1\right)\left(v - c\right) + \left(\theta_H + \ell\theta_L\right)v\right). \tag{52}$$

(ii) By Props. 1 and 2, the sharing market is inactive when $x > \bar{x}$. Hence, the gains from trade become

$$\mathrm{GT}(x) = D_\phi\left(v - \frac{c}{2}\right) + D_r(v-c) + \left(D_{rH} + D_{rL}\right)v.$$

For all $x \in (\bar{x}, x_3]$, by Prop. 2 high-end selling & rental is the optimal strategy. The rental demand is $D_\phi = D_\phi^0 + D_\phi^1 = 2(1-\lambda)\theta_L$. The demand for purchase comes from young high-type consumers, and it is equal to $D_r = \lambda\theta_H$. The gains from trade are

$$\mathrm{GT}_3 = 2\left(1-\lambda\right)\theta_L\left(v-\frac{c}{2}\right) + \lambda\theta_H(2v-c).$$

By Prop. 2, mass selling is optimal for all $x \in (x_3, 1]$. The rental demand vanishes, as $D_\phi = D_\phi^0 + D_\phi^1 = 0$. The demand for purchase is $D_r = \lambda\theta_H + (1-\lambda)\theta_L$. The gains from trade amount to

$$\mathrm{GT}_4 = (\lambda\theta_H + (1-\lambda)\theta_L)(2v-c).$$

Clearly, $\mathrm{GT}_3 = \mathrm{GT}_4$, as claimed.

(iii) It is enough to show that the gains from trade with sharing are larger than the maximum gains from trade without sharing, i.e.,

$$\mathrm{GT}_3 - \max\{\mathrm{GT}_0, \mathrm{GT}_1, \mathrm{GT}_2\} = D_{rH}(1-\theta_H)\theta_H + D_{rL}(1-\theta_L)\theta_L > 0,$$

concluding the proof. □

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