

## Modal Response of Hydraulic Turbine Runners

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**Key words:** Dynamic Response, Hydraulic Turbines, Runner.

### Abstract

The mechanical design of hydraulic turbines is conditioned by the dynamic response of the runner that is usually estimated by a computational model. Nevertheless, the runner has complex boundary conditions that are difficult to include in the computational model. One of these boundary conditions is the water in which the runner is submerged. The effect of the added mass and damping of water can modify considerably the natural frequencies of the runner.

In order to analyze this effect on a Francis turbine runner, an experimental and a numerical investigation in a reduced scale model was carried out.

The experimental investigation was based on modal analysis. Several impact tests with the runner in air and in water were done. The response was measured with accelerometers located in different positions of the runner. Special attention was taken to determine the most suitable positions of measurements and impacts. From the modal analysis, the natural frequencies, damping ratios, and mode shapes were determined.

The simulation of the same runner was also carried out using a FEM method. First, some tests including a sensitivity analysis were done to check the accuracy of the numerical results. Second, the runner was simulated and the frequencies and mode shapes were calculated both in air and in water like in the experiment. The simulation was compared with the experimental results to determine its accuracy especially regarding the added mass effects. Similar mode shapes and frequency reduction ratios were obtained so the simulation gave rather good results.

In the paper, the frequencies, damping and mode shapes obtained in air and in water both from experiment and simulation are indicated. The same mode shapes obtained in air were

obtained in water but with lower natural frequencies and higher damping ratios. The difference in the natural frequencies is shown to be dependent basically on the added mass effect of the water and not on its added damping. This difference also depends on the geometry of the mode presenting different values for different mode shapes. Using non-dimensional values, the reduction in the natural frequencies can be extrapolated to other Francis runners presenting similar geometrical characteristics.

## **Introduction**

The power concentration in hydraulic turbines is constantly being increased when designing new machines or upgrading existing ones. As a consequence the hydraulic excitation forces on the turbine runners increase due to higher heads and fluid velocities. Moreover, the operation range is widened to satisfy the demand of larger regulation capacity. The operation at off-design conditions also results in larger forces that combined with a reduced thickness/weight ratio in runners can provoke even higher vibration levels. This dynamic excitation is the responsible of fatigue damage. A complete analysis of the modal behavior of the runner is necessary to prevent this type of damage. Since the runners are working submerged in water, the effects due to the presence of this heavy fluid must be taken into account. Therefore, this analysis has to consider the study of the excitation forces and of the runner response considering the water effect. Particularly, the current paper is concentrated only on the response of a hydraulic runner.

Although experimental and numerical modal analysis have been extensively used to study the dynamic response of structures, most of them have been limited to simple geometries (Ref 1, 2, 3, 4, 5). Furthermore, few works have considered the effects of surrounding fluid. In the case of hydraulic turbine runners, it is very difficult to test the real operation conditions from the experimental point of view. And from the numerical point of view, the main challenges are due to the complexity of the geometry, especially considering the mesh compatibility between the fluid and structure on the interfaces. In fact, the current research on turbine runners submerged in water has been limited to some simplified theoretical model only validated against few experimental results (Ref 6, 7, 8, 9).

In summary, the results of the modal response of a Francis turbine runner both in air and in water (Ref 10) are discussed in detail in this paper. The natural frequencies, mode shapes and frequency reduction ratios due to the water added mass effect obtained with an experimental modal analysis and with a numerical simulation with finite element method are compared showing a very good accuracy.

## **Theoretical background**

In theoretical considerations, in order to take into account the added mass effect due to the surrounding fluid the system has to be treated as a fluid-structure interaction problem. In that case, the structural dynamics equation is coupled with the equations of the fluid, which can be described as the discretized equation (1) as presented in (Ref 11):

$$\begin{bmatrix} M_s & 0 \\ M_{fs} & M_f \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_f \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{p} \end{Bmatrix} + \begin{bmatrix} K_s & K_{fs} \\ 0 & K_f \end{bmatrix} \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix} \quad (1)$$

where  $M_s$  = structure mass matrix;  $C_s$  = structure damping matrix;  $K_s$  = structure stiffness matrix;  $F_s$  = applied load vector;  $u$  = nodal displacement;  $M_f$  = fluid equivalent “mass” matrix;  $C_f$  = fluid equivalent “damping” matrix;  $K_f$  = fluid equivalent “stiffness” matrix;  $F_{sf}$  = fluid “load” produced by structure displacement at the interface;  $p$  = pressure on the nodes;  $M_{fs} = \rho_f R$ , defined as equivalent coupling “mass” matrix;  $K_{fs} = -R$ , defined as equivalent coupling “stiffness” matrix.

This equation considers the following assumptions (Ref 12):

The fluid is slightly compressible (density changes due to pressure variations);

The fluid is non-viscous (no viscous dissipation);

The flow is irrotational;

There is no mean fluid flow;

Changes of mean density and pressure in different areas of the fluid domain remain small.

Therefore, for a problem involving fluid-structure interaction the fluid element generates all the matrices with subscript  $f$  in addition to the coupling matrices  $\rho_f R$  and  $R$ . The matrices with subscript  $s$  are generated by the compatible structural element used in the model.

## Experimental setup and numerical model

### Experimental setup

The tested model runner was constructed following the IEC (International Electrotechnical Commission) Standards for international test acceptance. It is a replica at a reduced scale of 1:10 of a Francis turbine runner with a specific speed of 0.56. The model has 17 blades and a diameter of 409 mm. Its shape is shown in **Figure 1**. The material used is a bronze alloy with density of 8300 kg/m<sup>3</sup> and Young’s Modulus of 110 Gpa.

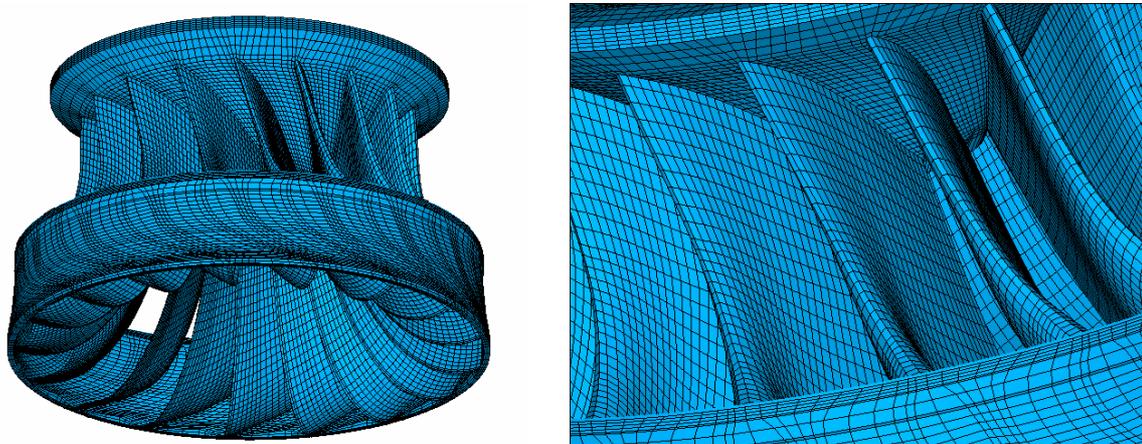
To get the runner system parameters as a free body with no constrains, the runner was suspended with a flexible string. For the tests in water, the runner was submerged in water contained in a squared tank.

A series of impact tests were applied on 16 positions in the band and on 6 positions in each blade, giving a total of 118 impact positions. Six impact were done on each position. The detailed results obtained with these tests have been published by Rodriguez et al. (Ref 13).

### Numerical model

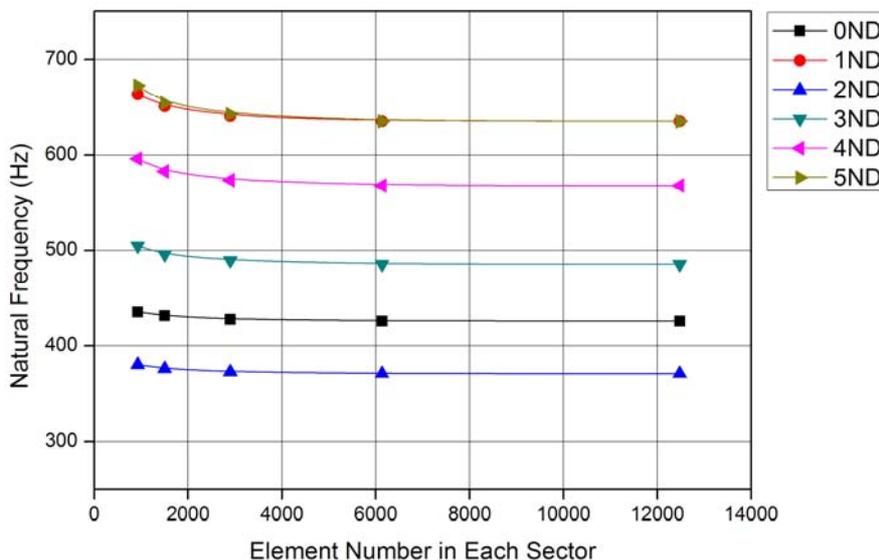
Based on the cyclic symmetrical characteristic of the structure, one sector including one blade

and covering an angle of  $360/17$  degrees was used to do the simulation. Then the results were expanded to the whole runner (Ref 14, 15). The mesh, as shown in **Figure 1**, was built up by discretizing the CAD model using hexahedral elements, which provided more accuracy.



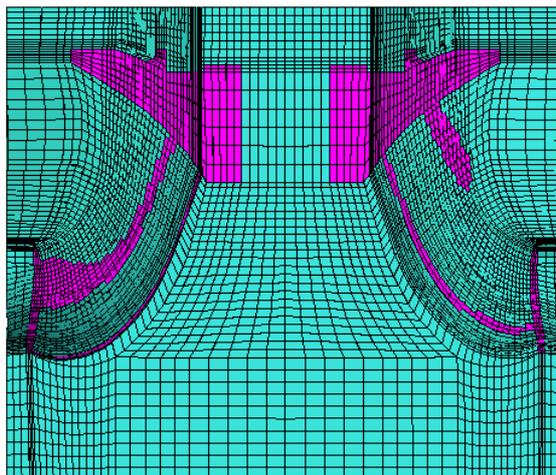
**Figure 1** Discretized finite element mesh with hexahedral elements

A sensitivity analysis was carried out to determine the influence of the mesh density. Five meshes with different element densities were simulated, each one including approximately a double number of elements than the previous one. In **Figure 2**, the natural frequency values are plotted for all of them. It can be seen that the results converge when the element density is increased. Therefore, the mesh with 6133 elements per sector, previously shown in **Figure 1**, was selected for the simulation.



**Figure 2** Results of the mesh sensitivity analysis

To do the simulation in water, the model of the runner was surrounded by a cylindrical fluid domain considering the cyclic symmetric characteristic of the runner. The boundary conditions were carefully treated according to the experimental test. In **Figure 3**, the draft of the model and FEM mesh are shown.



**Figure 3 Fluid domain (blue) around the runner (pink)**

## Results

The modal parameters of the runner, both in air and submerged in water, have been well determined with the experiments and the simulation. These results have been classified based on the mode shapes because they have been found to be similar between the structure in air and in water. Due to the cyclic symmetric characteristics of Francis turbine runners, the vibration modes can be classified according to the numbers of nodal diameters (ND) appearing on the band part. Defined by the condition  $ND=0$ , the modes are singlet and natural frequencies are distinct. The modes with  $ND \neq 0$  are doublet; they have a pair of mode shapes with the same natural frequency (Ref 16).

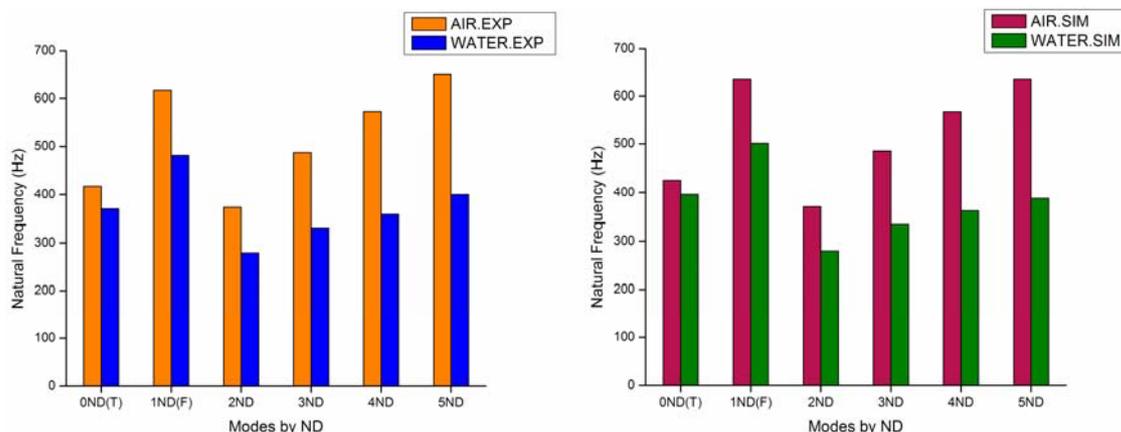
Since in our case it has been observed that the crown generally has small displacements compared to the band and the blades, the mode shapes have been described employing the number of nodal diameter (ND) on the band as the criterion. As shown in **Figure 5** and **Figure 6**, the 0ND mode corresponds to a torsion mode without band deformation and the 1ND mode corresponds to a flexion mode with the band swinging like a pendulum. For the modes with 2 or more NDs, the band behaves bending with deformation in the radial direction.

The natural frequencies found for the same mode shapes are listed in **Table 1** and plotted in **Figure 4**.

**Table 1 Natural frequencies of the runner in air and in water(Hz)**

	0ND (Torsion)	1ND (Flexion)	2ND	3ND	4ND	5ND
Exp. Air	417.50	616.75	373.50	487.50	573.75	649.75
Exp. Water	370.50	481.50	279.50	331.25	359.00	400.00
Sim. Air	425.87	635.53	370.80	485.61	568.18	635.46
Sim. Water	383.28	498.90	280.17	335.49	362.34	387.89

**Note: 1. Exp.=experiment, Sim.=simulation; 2. For all doublet modes, average value of 2 frequencies are listed in table.**



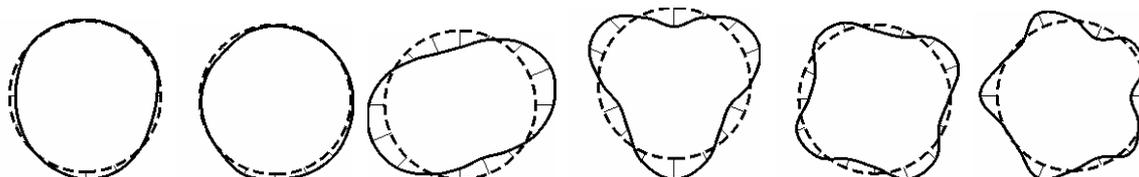
**Figure 4** Natural frequencies in air and in water (T: torsion, F: flexion)

The damping values obtained with the experiment for the runner in air and submerged in water are listed in Table 2.

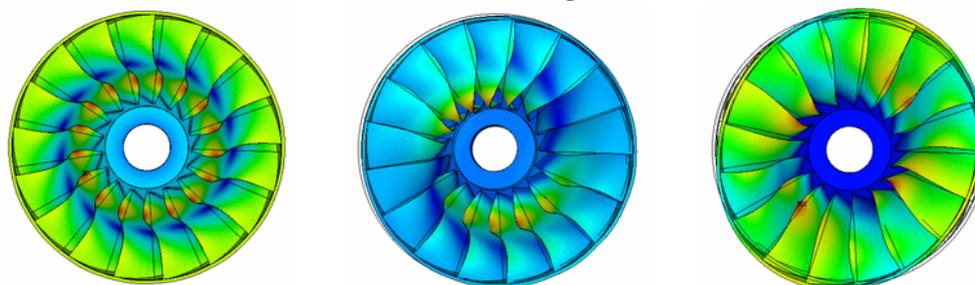
**Table 2** Dampings obtained by experiment

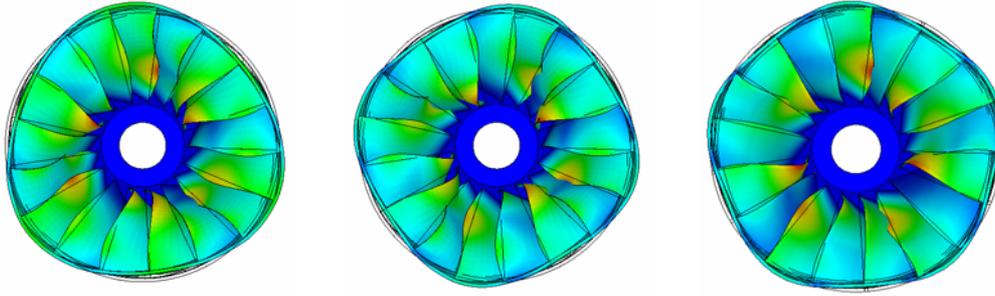
	0ND (T)	1ND (F)	2ND	3ND	4ND	5ND
In air	0.0047	0.0033	0.0056	0.0068	0.0040	0.0039
In water	0.0068	0.0120	0.0070	0.0082	0.0070	0.0069

The mode shapes of the band for the runner in air are shown in **Figure 5** and **Figure 6** from a bottom view. It must be noted that the same modes have been found with the experiment and the simulation. Moreover, the mode shapes obtained in water are just avoided to be repeated since they are similar to those in air.



**Figure 5** Mode shapes obtained from the experiment in air (ordered by number of ND from left to right)





**Figure 6 Mode shapes obtained by simulation in air (ordered by number of ND from left to right)**

In order to check the level of accuracy of the simulation compared to the experiment, the differences in natural frequency values have been calculated with equation (2). They are listed in **Table 3**.

$$\Delta(\%) = \left[ \frac{(U_{sim.} - U_{exp.})}{U_{exp.}} \right] \times 100 \quad (2)$$

where  $\Delta(\%)$  is the difference in percent, and  $U_{sim.}$  and  $U_{exp.}$  are the natural frequency values obtained by simulation and by experiment respectively.

**Table 3 Deviation of simulation results compared with experiment ones (%)**

	0ND (T)	1ND (F)	2ND	3ND	4ND	5ND
Natural frequency (in air)	2.00	3.04	-0.72	-0.39	-0.97	-2.20
Natural frequency (in water)	3.45	3.61	0.24	1.28	0.93	-3.03

It can be clearly noticed that the maximum difference is of about  $\pm 3.6\%$ . Moreover the mode shapes, both in air and in water, also represent good consistency between simulation and experiment. Therefore, the simulation has been found to be accurate enough to calculate the modal characteristics of such a turbine runner structure and, in turn, to correctly determine the added mass effect.

## Discussion

### Added mass effect

Comparing the results in air and in water, a significant decrease of the natural frequencies can be observed. This reduction is due to the effect of the added mass induced by the surrounding water. The added mass effect can be quantified by calculating the frequency reduction ratio  $\delta$  of each mode shape, defined with equation (3):

$$\delta = (f_a - f_w) / f_a \quad (3)$$

where  $f_a$  and  $f_w$  are the natural frequencies in air and in water respectively. In **Table 4**, the frequency reduction ratios obtained are listed.

**Table 4 Frequency reduction ratios of the runner modes of vibration**

	0ND (T)	1ND (F)	2ND	3ND	4ND	5ND
Sim.	0.10	0.21	0.24	0.31	0.36	0.39
Exp.	0.11	0.22	0.25	0.32	0.37	0.38

It can be clearly noticed that the natural frequencies are considerably reduced by the presence of fluid. The frequency reduction ratio shows a significant variation from 0.10 to 0.39 depending on the corresponding mode shape.

In order to extrapolate the results about added mass effect, a non-dimensional added mass factor for each mode can be defined with equation (4): (Ref 10)

$$\gamma_i = \left[ \left( \frac{f_{a,i}}{f_{w,i}} \right)^2 - 1 \right] \frac{\rho_s}{\rho_f} \quad (4)$$

where  $\gamma_i$  is the non-dimensional added mass factor, which remains constant for a specified mode (i), classified by the features of the mode-shape (ND, etc.).

The non-dimensional added mass depends only on the geometrical characteristics of the structure, which is determined by the design of the runner. In other words, the relationships between natural frequencies in air and in water, obtained by the method employed in this investigation, can also be valid for any other Francis turbine runners with geometrical similarity. This can be practically used to estimate the natural frequencies of other geometrically similar turbine runners constructed with different materials and dimensions.

### **Damping**

The damping found has low values in all the modes. This is the typical behavior for a structure vibrating with small amplitudes and high frequencies in stagnant fluid in absence of wave radiation. It can be seen that the dampings are increased by the presence of water, varying depending on the mode shapes, but they are not high enough to affect significantly the values of the natural frequencies.

### **Conclusion**

The modal characteristics of a reduced scale model of a Francis turbine, in air and in water, have been investigated experimentally and numerically. Comparison of results indicates that the simulation results show good agreement with the experimental ones.

Meanwhile the mode shapes in water are similar to those in air, the presence of water reduces considerably the natural frequencies while the dampings are increased.

The added mass effect of the surrounding still water has been quantified by calculating the frequency reduction ratio, which varies in a range of 0.10~0.39 depending on the geometry of the mode shape.

A non-dimensional added mass factor has been derived which can be used to extrapolate the natural frequencies of geometrically similar runners of different materials.

Therefore, the methodology used in this investigation has been proved to be valid so that it can be extended to study the dynamic behavior of any other hydraulic turbine runner.

### **Acknowledgements**

This research has been carried out in the HYDRODYNA project. The authors would like to acknowledge the LMH-EPFL and Voith Siemens Hydro Power Generation for their collaboration.

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