

Design Space of Modular Slab Systems with Discrete Stiffness Distribution and Irregular Column Layout

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Abstract

A known challenge of modular structural systems is to avoid the oversizing of its elements while allowing a wide variety of spatial layouts. Considering any given floor outline and the availability of square slab elements with three discrete bending and shear stiffness, this paper explores: 1) the minimal geometric and topological rules that ensure a positioning of columns satisfying serviceability requirements; and 2) for a given satisfying positioning of columns, the distribution of slab elements that minimizes the overall weight of the system. This study is part of a larger project aimed at developing a highly flexible floor system whose element dimensions do not constrain the positioning of the columns nor the shape of the floor plan. Hence, the structural system could be adapted over time to changing architectural needs.

Keywords: Modular slab systems, design space exploration, optimal support layout design

1. Introduction

The evolution of buildings cultivates a close relationship with the evolution of human needs and their constant trends towards new developments. From the beginning of the conceptual design, specific functional or spatial necessities are generally placed in the center with every design aspect revolving around them. In particular, structural systems, e.g. slabs, are designed to withstand loads that are specific to the activities being conducted on them. Moreover, the shape of the slab and the positioning of the columns are designed to satisfy specific architectural demands.

This common approach presents an environmental issue. Either the defined spaces or the structural capacity will not satisfy forthcoming functional needs, which may lead to premature destruction in all or in part. A first strategy to avoid such structural obsolescence is to design modular structural systems capable of being reconfigured into new shapes. An additional strategy is to ensure that reconfigurations will withstand future, sometimes unknown, structural demands while limiting the oversizing of the elements. Among other things, this last requirement calls for a perfect correlation between floor outline, column layout and stiffness distribution in the slab.

1.1. Related work

Chu et al. [3] introduced an evolutionary method to find the optimal design configuration for plates with discrete variable thicknesses subjected to different load cases, but maintaining constant overall weight. In the present paper, the total weight of the slab is regarded as an optimization goal while maximum relative deflection is put as a constraint.

The characterization of feasible column layouts produced in the next chapter is done by generating large sets of populations. The approach can be related to stochastic searches of column layouts. For instance, Scheurer [1] generated column layouts with agents in a closed environment while minimizing their number, considering axial capacity and column inclination. Vierlinger et al. [2] generated topological arrangements of V-shaped columns, continuously over multiple stories, before optimizing their orientation for limiting slab deflection.

1.2. Problem statement

This paper assumes that effective column layouts and floor outlines are currently unknown but will be given at the time of assembly. It also assumes a modular slab system whose square elements are available in three various thicknesses (Figure 1). This variability offers the possibility to optimize stiffness distribution in the slab system while avoiding superfluous oversizing of the elements. Normal, bending, and shear forces can be transferred within continuous elements through any of their lateral sides. The slab system is supported by axially loaded columns and its lateral stability is ensured by a stiff core not considered in this study. The floor area is constant whatever the outline of the floor plan, i.e. whatever the presence of openings or setbacks. Strength of the slab is not part of the study.

The first objective of this paper is to characterize the design space of column layouts that are capable of supporting a floor of given outline while satisfying serviceability (deflection criteria) everywhere. The end goal of this characterization is to outline practical design rules that allow the direct exploration of irregular, yet feasible, column layouts.

The second objective is to highlight the benefits in terms of weight minimization when a discrete stiffness distribution in the slab is available, considering a given, irregular floor outline and a given, irregular column layout.

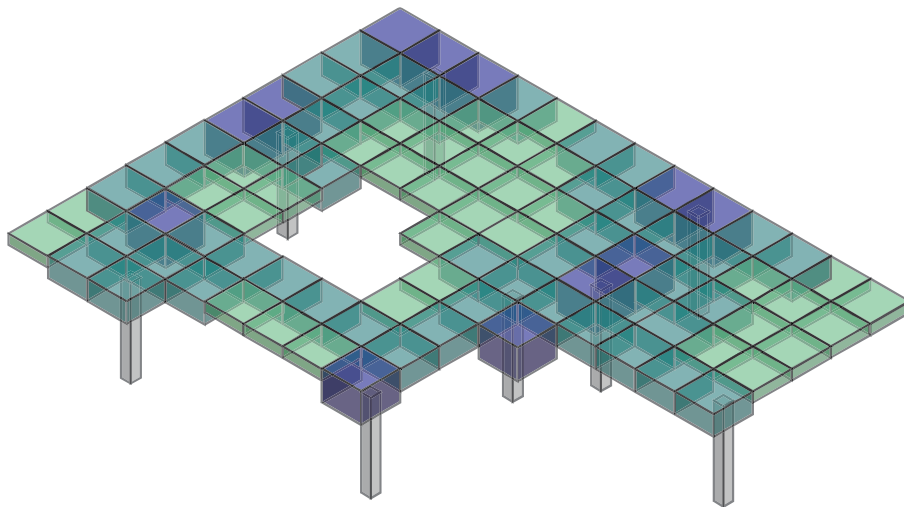


Figure 1: Structural system with slab elements of varying thicknesses whose arrangement adapts to the irregular floor outline and column positioning in order to fulfill deflection criteria while minimizing weight.

2. Design Space of Column Layouts Satisfying Serviceability Constraints

2.1. Methodology

This section deduces design rules to place columns on a given floor plan by characterizing a very large set of trial layouts. Although the characterization results from structural analyses, the attempt is to define practical design rules that are purely geometric and that can be applied without in-depth computation.

Although stiffness distribution and column positions are finite, the set of possible geometric features – in particular sizing features related to column spans and cantilevered areas –, is so large and heterogeneous that probabilistic studies following naive simulations are here preferred.

First, a set of floor plans is generated and random discrete column layouts are applied on them. A finite-element analysis then computes absolute deflections everywhere on the slab. Deflection points are subsequently mapped to geometric features of the floor plan and the column layout in order to separate configurations that satisfy serviceability from those that do not. The mappings are eventually plotted and used to draft design rules.

2.2. Generation of floor plans and column layouts

The floor plans are assemblies of 2×2 m² cells. Each column lies on a grid whose step is 50 cm. In a first step, a floor plan of constant geometry is considered. It is a plain 10×20 m² rectangle supported by 500.000 random distributions of columns, containing between 4 and 12 columns each.

In a second step, 75 floor plans are considered and 3000 random column layouts are applied on each of them. Twenty floor plans are generated manually in order to obtain extreme configurations such as narrow surface areas. The remaining are generated randomly. They all present openings and/or setbacks that are minimum 2 m wide. Dimensions of these floor plans are such that the floor area always lies between 180m² and 200 m² whatever the outline of the floor plan.

2.3. Computation of absolute deflections

For each generation, a finite element analysis computes the nodal deflections of the slab. Belytschko and Tsai [4] finite plate elements are implemented. Self-weight is considered and all elements are uniformly loaded with an area load of 10 kN/m², accounting for an approximate combination of dead and live loads. A Young's modulus of 30 GPa is considered everywhere. Since the aim is to check whether a given column layout will lead to a feasible solution, the thicker slab (36 cm) out of the three available ones is assumed everywhere. This thickness will eventually be reduced during the weight optimization performed in section 3.

2.4. Serviceability checks

Serviceability criteria are usually expressed as minimum ratios between absolute deflections and spanning or cantilevering lengths, which in the case of 2D irregular floors is subject to interpretation. Two types of lengths, Type I and Type II, are therefore identified and used for two serviceability limits, respectively $L/300$ and $L/150$. Lengths of type I are computed from a Delaunay triangulation of the columns (Figure 2 dark blue lines). Lengths of type II are measured from each column to (a) any vertex of the floor outline, to (b) any vertex of the corresponding Voronoi cell, and to (c) any closest point situated on a Voronoi edge (Figure 2 turquoise lines).

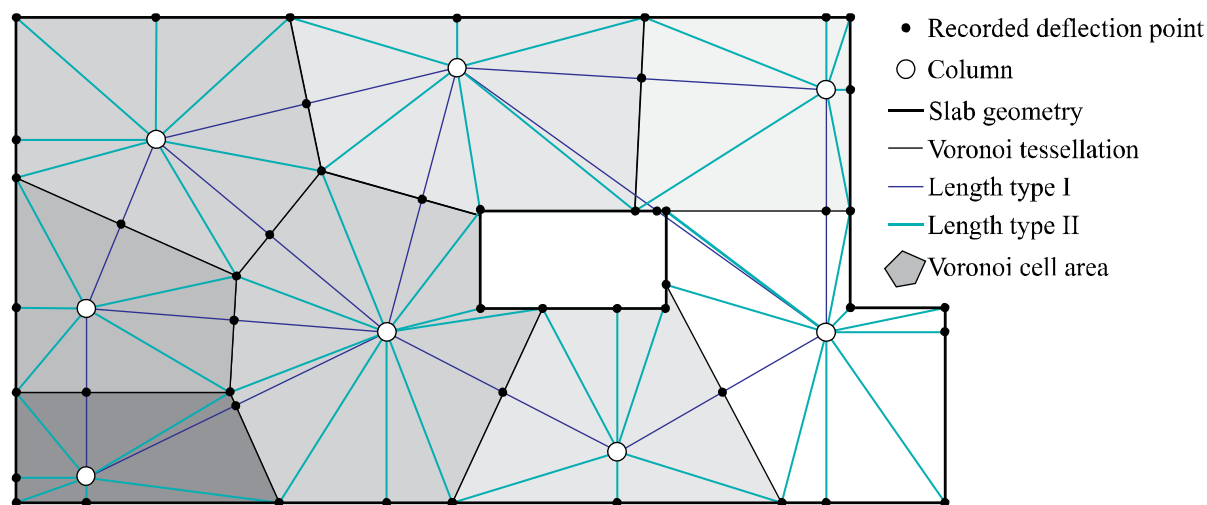


Figure 2: Computed parameters

2.5. Result: Probabilistic distribution of successful layouts

The first analysis determines the success rate of layouts for different average tributary areas (Figure 3). As it could be expected, the number of configurations satisfying serviceability increases proportionally to the inverse of the average tributary area, i.e. proportionally to the number of columns. Also, for the same number of columns, the chances of obtaining valid column layouts for an irregular floor plan (right) are lower than for a rectangular floor plain (left). It can also be seen that the chances of obtaining valid results when the average tributary area is sufficiently lower (16.6 in the case of the rectangular floor) are already high (95%) and that an increase of columns, whatever their actual position in the plan, is not necessary.

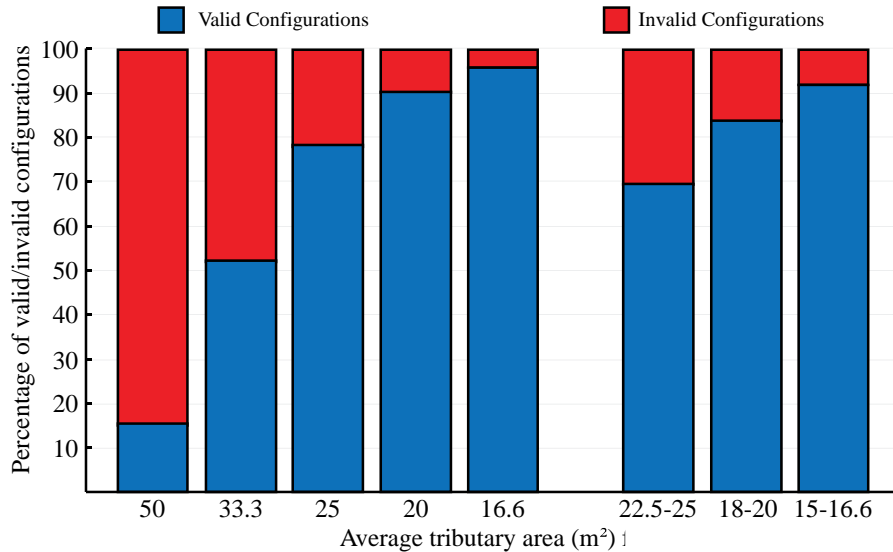


Figure 3: Success rate by tributary area for the rectangular floor plan (left) or for irregular floor plans (right).

2.6. Result: Distribution of successful spans and cantilevers

In Figure 4, random layouts of 10 columns applied on the rectangular floor plan are compared according to the maximum relative deflection they produce and to the corresponding decisive length, either of type I or type II. The blue points on the left hand side of the graph correspond to very short spans that deflect in upward direction, with a high value of the relative deflection due to the proximity of longer spans with high deflections in absolute terms. The figure highlights the proeminence of layouts where the lengths of type II are associated with the highest relative deflection.

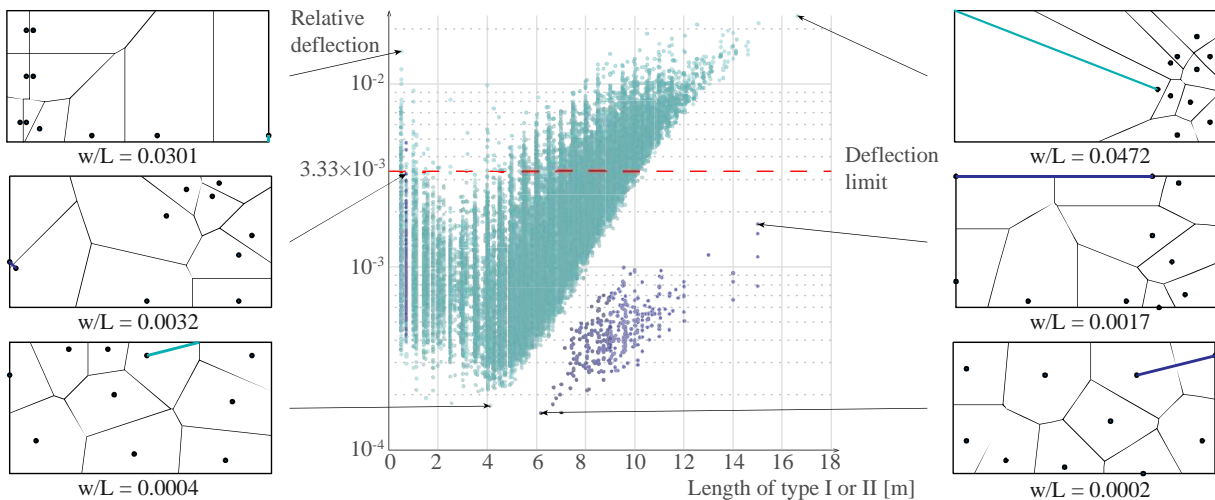


Figure 4: Relative deflection against critical deflected length for rectangular geometry and 10-column layouts. Layouts where lengths of type I (dark blue) or type II (turquoise) are decisive.

Figure 5 provides another analysis of the same data set and Figure 6 extends it to irregular floor plans. Maximum lengths of type II are plotted against maximum lengths of type I. Whereas the full range of maximum lengths of type I provides invalid layouts, both figures show that short maximum lengths of type II (below 6m for this case study) ensure valid layouts. This property can be directly implemented as design criteria.

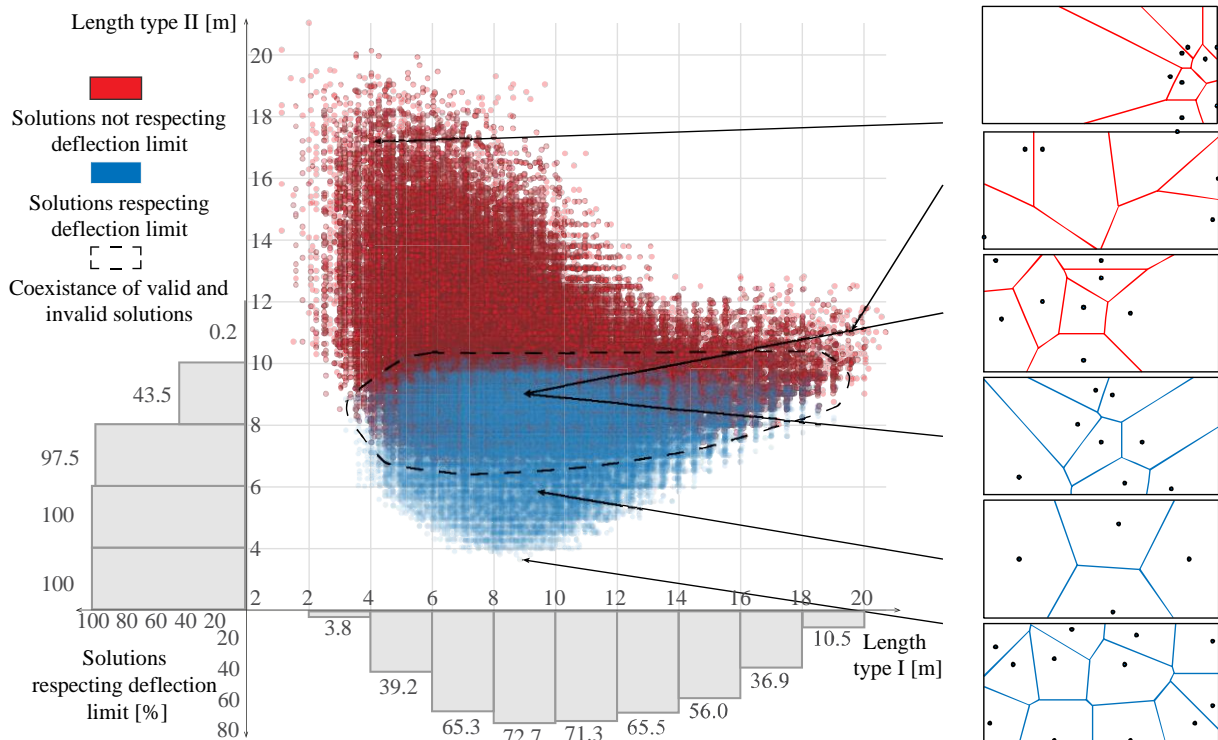


Figure 5: Distribution of column layouts for a rectangular slab with an area of 200 m².

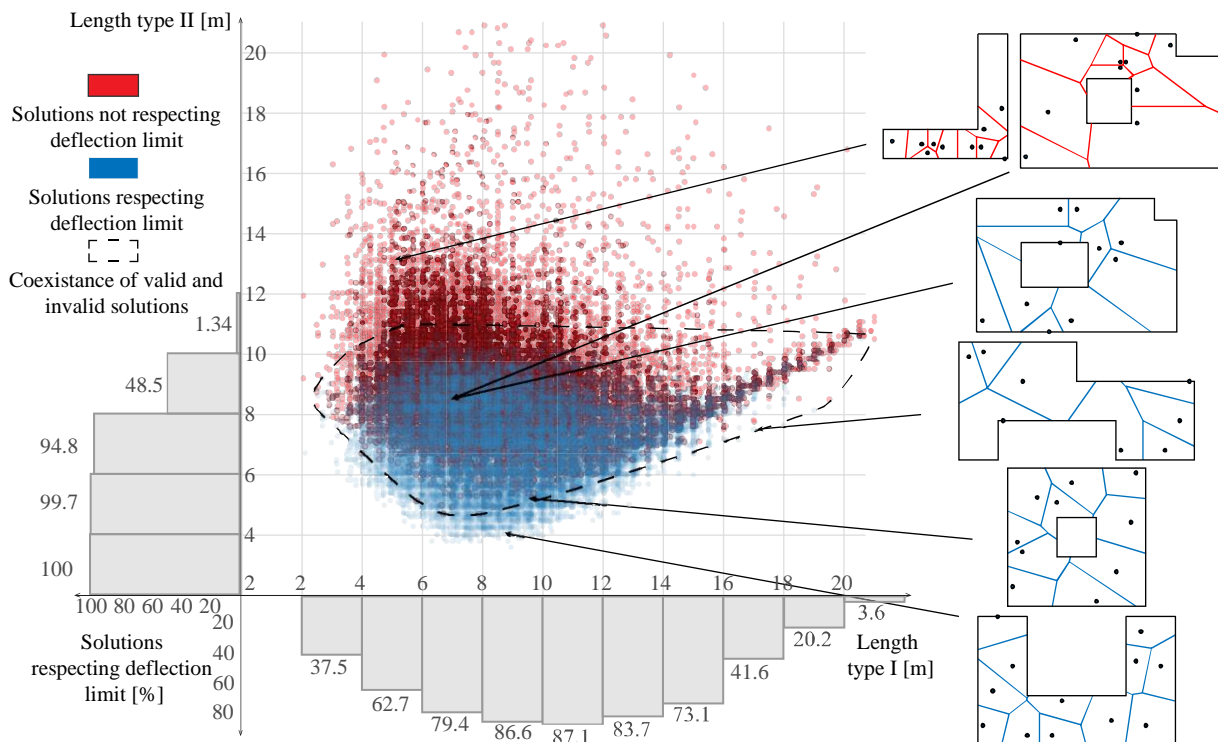


Figure 6: Distribution of column layouts for various floor outlines of area between 180 m² and 200 m².

2.7. Result: Distributions of tributary area

In Figure 7 and Figure 8, the scattered results of the analysis are plotted in terms of relative deflection against various parameters chosen as attempts to quantify the homogeneity of column distribution. Tributary areas (Figure 7) are equivalent to the areas of Voronoi cells (Figure 2). The distribution of tributary areas is more precisely grasped with coefficients of variations (Figure 8 left). However, there are particular cases where all cell areas are equal although the spans between columns might differ. Homogeneity is therefore further described as the standard deviation of lengths type II, i.e. distances between column and each cell vertex (Figure 8 right).

Figure 7 right shows that 90% of the configurations are valid when the maximum tributary area is less than 2 times the average tributary area. Similarly, a coefficient of variation of cell areas lower than 0.4 or an average of maximum standard deviation of lengths type II lower than 1.0 are required in order to ensure a success rate higher than 90%. Due to the simplicity of its computation, the ratio of maximum tributary area over average tributary area is probably preferable as a design rule.

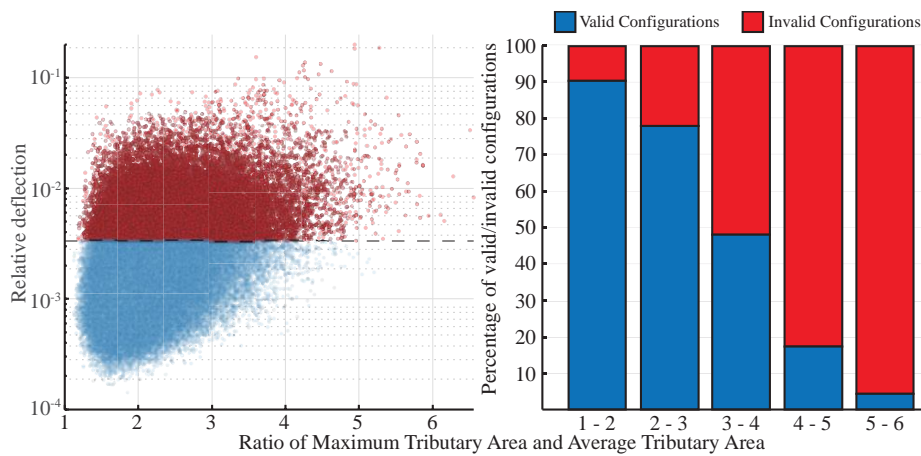


Figure 7: Maximum tributary area over average tributary area for irregular floor plans.

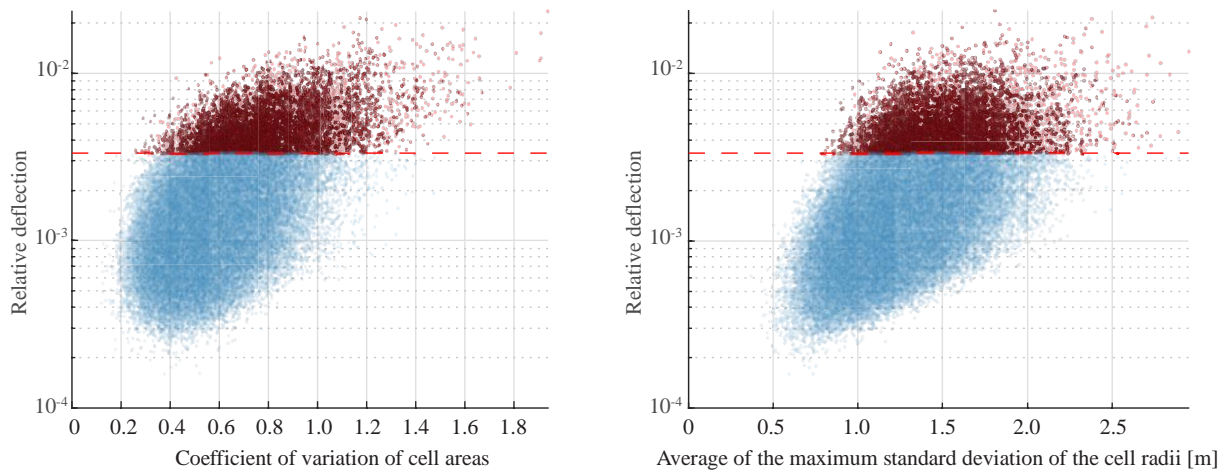


Figure 8: Coefficient of variation of cell area (left) and average of the maximum standard deviation of the cell length type II (right) for the rectangular floor plan and 10 columns.

2.8. Discussion

A number of parameters are kept constant during this study, e.g. the floor area and the thickness of the slab. Further design space characterization studying the variation of these parameters are needed before finalizing design rules.

Due to the scattered distribution present in most plots, one strategy to identify additional design rules would be to extract subsets of column layouts that satisfies specific geometric constraints, e.g. minimum distance between two columns, or maximum rectangular area between any floor edge and the closest column.

3. Weight optimization of the slab system with discrete stiffness distribution

The third part of this paper considers a discrete distribution of slab thicknesses over the floor plan. Starting from a floor outline and a valid configuration of column layout, a genetic algorithm is implemented to minimize the weight of the slab system while varying the thickness of each slab element and ensuring serviceability requirements (as defined in section 2.4). Three thicknesses are available: 12, 24, and 36cm. The optimization has been implemented for the initial rectangular slab and a randomly generated slab, as well as for regular and irregular column layouts (Figure 9).

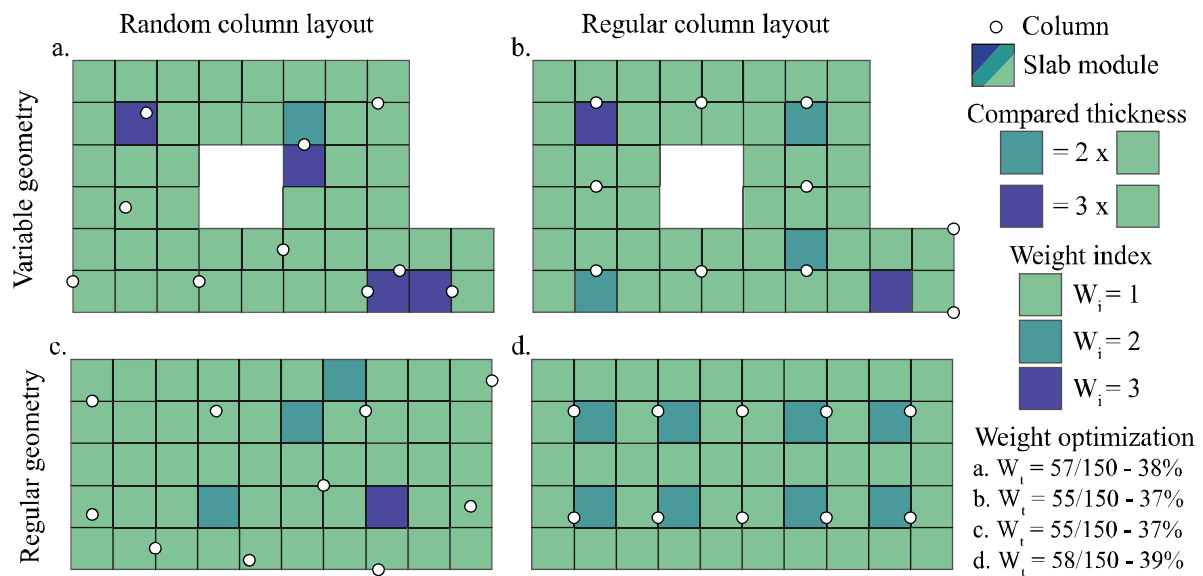


Figure 9: Optimized slab configurations

Three remarks can be drawn from this study. First, irregular layouts of columns lead to slab systems that are as efficient as regular layouts. Second, the mainly ‘flat’ thickness distribution on Figure 9 can be explained by the short spans and the high density of columns in that example. Third, the non-trivial positioning of stiff elements may be due to the fact that the shown result does not correspond to global optimality yet.

4. Conclusion

This paper studied the design space of a modular slab system over a large set of floor outlines and irregular column layouts. The distribution of solutions satisfying serviceability criteria has been plotted against various geometric and structural features. However, further studies would be needed to check their validity in larger sets including other floor plans of other sizes, other slab thicknesses, and additional load cases. A better geometric characterization of the positioning of each column should also lead to interpretations that are more precise.

The modular nature of the system permits adapting the stiffness distribution of the slab to any column layout, minimizing the mass of the slab and thus the resource expenses. Discrete optimization of the modules’ thickness provides efficient solutions for irregular distributions of columns that are comparable to classical regular column grids in terms of use of materials.

5. References

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