

Adaptive sampling methodology for structural identification using radial-basis functions

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ABSTRACT

The aim of model-based structural identification is to identify suitable models as well as values for model parameters that determine structure behaviour through comparing measurements with predictions. Well known methodologies, such as traditional implementations of Bayesian model updating, have been shown to be inaccurate in cases characterised by systematic uncertainties and unknown spatial correlations. Error-domain model falsification (EDMF) is another approach to structural identification. This approach is easy to understand for practising engineers and can provide robust parameter identification without assumptions on spatial correlations. The performance of all approaches involving sampling is affected by the number of model evaluations that is generated based on prior knowledge of parameter-value distributions. This paper focuses on a new sampling technique, called radial-basis function sampling (RBFS), and its application to EDMF, to generate a set of candidate models that represent the behaviour of the structure with a certain confidence level. RBFS provides a good exploration of the parameter space even with a limited number of samples, which results in reduced computation times. A full-scale bridge in Singapore has been tested and a new index of sampling quality is proposed in order to compare this approach

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19 with other sampling techniques such as Latin hypercube sampling (LHS) and Markov chain Monte
20 Carlo (MCMC). Finally, a cross-validation method is employed to verify the robustness of the ap-
21 proach and the sensitivity of sampling on prediction reliability.

22 **Keywords:** adaptive sampling, radial-basis function, optimisation, structural model updating,
23 error-domain model falsification, surrogate models.

24 INTRODUCTION

25 Existing infrastructure elements, which are often designed and built for fixed lifetimes, need
26 to be maintained, retrofitted, adapted and replaced to meet new needs. The optimal planning of
27 maintenance requires an accurate knowledge of how existing structures behave. This helps avoid,
28 for example, replacement when structures have sufficient reserve. Also, expensive interventions
29 may be avoided through implementation of cheaper and more sustainable alternatives.

30 Structural identification methods are used to improve knowledge of structural behaviour. Mea-
31 surement data interpretation has been extensively employed for structural health monitoring in
32 the last decades, as reviewed in (Catbas et al. 2013). Many researchers have studied model-free,
33 sometimes called data driven, methods using data interpretation strategies (Posenato et al. 2010).
34 While these methodologies may be interesting for damage detection, infrastructure future-proofing
35 requires behaviour models to compare alternative scenarios and support decision-making. When
36 structures are modelled, for instance using finite elements (FE), and measurements are carried out,
37 model identification techniques are used to improve the accuracy of model predictions. Although
38 advanced simulations require long computation times and sensor equipment may be expensive,
39 quantifiable benefits arise when structural replacement and unnecessary interventions are avoided
40 (Smith 2016).

41 Despite the fact that measurements provide additional information for assessment of struc-
42 tures, this inverse problem involves many assumptions and sources of uncertainties. Raphael and
43 Smith (1998) proposed a multi-model approach, based on model falsification to overcome chal-
44 lenges associated with inverse problems. In this method, model-updating results consist of a set
45 of candidate models that explain the measurements taken from a structure. Robert-Nicoud et al.

46 (2005) applied the same methodology and determined threshold boundaries by combining model
47 and measurement errors.

48 Goulet et al. (2013a) proposed a probabilistic extension, called error-domain model falsifica-
49 tion (EDMF), for robust structural identification when there are systematic modelling uncertainties
50 and when correlations between measurement locations are unknown. This methodology helps
51 identify candidate models among an initial model population, generated according to prior knowl-
52 edge and engineering judgment, by using measurement values and probabilistically determined
53 thresholds to falsify incorrect model instances. First, uncertainties are combined and threshold
54 bounds are evaluated according to a reliability of identification. Then all the instances for which
55 residual values between predictions and measurements exceed these bounds, at one or more sensor
56 locations, are rejected. Pasquier et al. (2015a) compared traditional Bayesian model updating and
57 EDMF in terms of prediction accuracy and demonstrated that EDMF is more robust for both diag-
58 nosis and prognosis. Moreover, EDMF has been employed by Goulet et al. (2013b) to evaluate the
59 serviceability-limit-state reserve capacity of the Langesand Bridge and by Pasquier et al. (2014) to
60 evaluate the fatigue reserve capacity of the Aarwangen Bridge.

61 Structural identification methods work most efficiently when there is a prior identification of
62 the most *sensitive parameters* for model identification and prediction. Parameters are selected
63 using sensitivity analyses according to their relative importance on model predictions. Model
64 predictions are obtained directly using either a FE solver or a surrogate model (SM), which are
65 able to capture the essential behaviour of the real structure while being more efficient in terms
66 of computation speed. Several families of SMs have been employed in structural engineering,
67 such as response surfaces based on polynomial functions (Ren and Chen 2010), Kriging estimates
68 (Simpson et al. 2001), and radial-basis functions (RBF) (Buhmann 2000). Surrogate models based
69 on RBF often perform well for engineering applications (Holmström et al. 2008). Other methods
70 such as neural networks may also be adopted. Neural networks are usually employed for regression
71 and classification tasks and they often require large training sets to be effective. When using a
72 neural network, the objective function is hidden inside its layered structure and an analysis of the

73 relationship between parameters and objective function values becomes challenging.

74 In order to select representative sets of parameter values to be assigned to the initial model class,
75 sampling techniques are employed. These model samples, also referred to as model instances, are
76 usually evaluated using FE solvers. Uniform sampling techniques such as grid-based sampling
77 (GBS) and Latin hypercube sampling (LHS) have been employed with EDMF (Goulet et al. 2010;
78 Pasquier and Smith 2016). A direct stochastic algorithm, called PGSL (Raphael and Smith 2003),
79 has been applied to identify candidate models (Robert-Nicoud et al. 2005). In Bayesian model
80 updating, a well-established method for model updating is Markov chain Monte Carlo (MCMC),
81 which is a variant of Monte Carlo (MC) sampling. MCMC is used to sample from the target
82 distribution, which is proportional to the posterior distribution of parameter values.

83 Optimisation algorithms can be employed to increase sampling performance through focusing
84 the search around a particular area of the parameter domain. Adaptive sampling algorithms gener-
85 ate new samples by learning from the feedback of the previous samples. In this case, the feedback
86 is often based on the optimisation of an objective function.

87 The choice of the objective function affects algorithm performance. Gradient-based techniques
88 have been applied successfully in the absence of multiple optima and when objective functions are
89 smooth and continuous in order to identify parameters of statistical models. However, in struc-
90 tural identification, the objective function might have many local optima. In this case, stochastic
91 techniques such as genetic algorithms, physics-inspired algorithms and swarm algorithms have
92 been employed (Zhang et al. 2010a; Zhang et al. 2010b; Marwala 2010). These methods do not
93 require gradient information and are easily implemented. However, good solutions are often ob-
94 tained only by tailoring the method through parameter tuning and the convergence of this methods
95 is typically slow (Conn et al. 2009). Hybrid approaches which consist of both a stochastic and
96 a gradient-based optimisation have also been proposed (Christodoulou et al. 2008; Christodoulou
97 and Papadimitriou 2007). A comprehensive overview of these methods and their application in the
98 field of structural optimisation can be found in (Hare et al. 2013). Compared with non-gradient
99 optimisation methods, derivative-free optimisation techniques have shown to converge to globally

100 optimal solutions when enough evaluations are performed (Torn and Zilinskas 1989; Gutmann
101 2001). Some optimisation methods may converge towards a unique “optimal” solution. In struc-
102 tural identification, many models can explain the measurement data because of the inverse nature
103 of the problem. The presence of measurement and modelling uncertainties increases ambiguity.
104 Therefore, in order to increase identification accuracy, a range of good quality solutions is often
105 preferred to a single optimal solution. The advantage of derivative-free optimisation approaches
106 compared with most stochastic search algorithms is the computational efficiency, which is crucial
107 when evaluations have a non-negligible computational cost.

108 The need of efficient methodologies for sampling in EDMF has already been highlighted in pre-
109 vious studies (Goulet and Smith 2013a; Pasquier and Smith 2016). Traditional sampling methods,
110 such as GBS and LHS, are not able to exploit the knowledge acquired from samples that have been
111 already evaluated. Stochastic search methods provide efficient sampling only when model evalua-
112 tions can be computed fast - which is often not the case for complex models. Adaptive-sampling
113 techniques based on surrogate model optimization and compatible with EDMF can enhance sam-
114 pling accuracy while reducing computation times. Much work in this direction is still missing.
115 In addition, EDMF requires initial model sets that represent the entire population of plausible so-
116 lutions (i.e combination of parameter values). Inaccurate, biased or sparse sampling may lead to
117 identification shortcomings. Therefore, the sensitivity of sampling on prediction reliability needs
118 to be investigated.

119 Full-scale case studies are essential for validating model-updating methods since it is only at
120 this scale that uncertainties show realistic magnitudes. Brownjohn et al. (2001) highlighted that
121 field conditions affect the accuracy of measurements. Therefore, data collected during lab experi-
122 ments may not be representative of measurements collected under ambient conditions (Catbas et al.
123 2013). Lam et al. (2015) proposed an application of MCMC to update the model of a coupled-slab
124 system using field test data. They pointed out that, in real situations, the effects of both modelling
125 error and measurement noise are relatively large when compared to numerical examples or experi-
126 mental case studies under laboratory conditions. Therefore, posterior uncertainties when field data

127 are collected are higher than those obtained using laboratory experiments.

128 While many full-scale studies have been carried out, few researchers have systematically val-
129 idated data-interpretation proposals using full-scale structures (Simoen et al. 2013). Strong meth-
130 ods for result validation are required to assess the performance of model updating techniques. This
131 study implements an approach where cross-validation is carried out using models that have been
132 identified as suitable to predict at locations other than those used in model updating.

133 This paper proposes a new adaptive sampling methodology, to increase the performance of
134 structural identification. The impact of the employed sampling methodology on prediction reli-
135 ability is investigated. The first section provides background on error-domain model falsification
136 and sampling algorithms. The subsequent section introduces the new sampling methodology and
137 its application to EDMF. Finally, a full-scale case study is used to compare systematically the per-
138 formance of the proposed approach with those of traditional sampling algorithms and to validate
139 results of structural identification.

140 **BACKGROUND**

141 **Error-domain model falsification**

142 Initially proposed in (Goulet and Smith 2013a), EDMF helps identify plausible physics-based
143 models using information provided by measurement data. Plausible models are defined by n_θ pa-
144 rameter values and a model class. Each model class G_k has a unique parametrization that includes
145 characteristics such as material properties, geometry, boundary conditions and actions.

146 Let n_y be the number of measurement locations. For each location $i \in \{1, \dots, n_y\}$, R_i denotes
147 the real responses of a structure (unknown in practice) and \hat{y}_i corresponds to the measured value at
148 location i . Predictions $g_k(x_i, \Theta_k)$ of the model class G_k , which is usually based on a finite element
149 analysis, are evaluated at location x_i through assigning Θ_k , which corresponds to instances of the
150 parameter vector θ_k , to the model class. Since model-prediction uncertainty U_{i,g_k} and measure-
151 ment uncertainty $U_{i,\hat{y}}$ are unavoidable, model predictions and measurements are linked to the true

152 behaviour using Equation (1).

$$153 \quad g_k(x_i, \Theta_k) + U_{i,g_k} = R_i = \hat{y}_i + U_{i,\hat{y}} \quad \forall i \in \{1, \dots, n_y\} \quad (1)$$

154 Rearranging the terms:

$$155 \quad g_k(x_i, \Theta_k) - \hat{y}_i = U_{i,c} \quad (2)$$

156 where U_c is a vector representing the difference between uncertainties $U_{\hat{y}}$ and U_{g_k} . The left-hand
 157 side of Equation (2) is the difference between a model prediction and a measurement at location
 158 x_i , which is often called the residual $r_i = g(x_i, \Theta) - \hat{y}_i$.

159 The probability density function (PDF) describing the error in measurements $f_{U_{\hat{y}}}(\mathbf{u}_{\hat{y}})$ is usu-
 160 ally estimated by conducting multiple series of tests under site conditions. Manufacturer specifi-
 161 cations are often very optimistic lower bounds. The PDF describing the error in the model class
 162 $f_{U_{g_k}}(\mathbf{u}_{g_k})$ is estimated using values taken from the literature, stochastic methods, engineering
 163 judgment and local knowledge. In practical situations, uncertainties associated with the model
 164 class are usually much larger than measurement uncertainties. Thus, their quantification directly
 165 affects the performance of the method.

166 In the traditional implementation of EDMF, the identification process starts with the definition
 167 of an initial set of n_Ω model instances $\Omega_k = \{\Theta_{k,m}, m = 1, \dots, n_\Omega\}$, usually through employing
 168 uniform sampling techniques. Then the instances for which the residual values exceed defined
 169 threshold boundaries are falsified.

170 In EDMF, threshold bounds are defined through computing the shortest interval $\{u_{i,low}, u_{i,high}\}$
 171 that contains a probability equal to ϕ_d^{1/n_y} for the combined PDFs $f_{U_c}(\mathbf{u}_c)$ at each sensor location,
 172 as expressed in the following equation:

$$173 \quad \phi_d^{1/n_y} = \int_{u_{i,low}}^{u_{i,high}} f_{U_{c,i}}(u_{c,i}) du_{c,i} \quad \forall i \in \{1, \dots, n_y\} \quad (3)$$

174 In Equation (3) the confidence level ϕ_d is adjusted using the Sidák correction to take into account

175 that measurements at several locations are considered simultaneously to falsify model instances.
 176 The hyper-rectangular acceptance region adjusted with the Sidák correction (with dimensions cor-
 177 responding to the number of sensors) has been shown to be conservative regardless of the value of
 178 correlation between sensor locations (Goulet and Smith 2013a). In the field of structural engineer-
 179 ing, a value of 0.95 for the confidence level $\phi_d \in [0, 1]$ is commonly employed. Falsification is
 180 then performed according to the following equation:

$$181 \quad \Omega_k'' = \{\boldsymbol{\theta}_k \in \Omega_k \mid \forall i = 1, \dots, n_y \quad u_{i,low} \leq g_k(x_i, \boldsymbol{\theta}_k) - \hat{y}_i \leq u_{i,high}\} \quad (4)$$

182 where the candidate model set (CMS), Ω_k'' , is made up of all the initial model instances except
 183 those that have been falsified at one or more measurement locations. An instance Θ of a model
 184 class G is thus a candidate model if, for *each* sensor location $i \in \{1, \dots, n_y\}$, the residual value
 185 lies inside the interval defined by the threshold boundaries.

186 Based on Equation (4), model instances that are falsified are assigned a null probability.

$$187 \quad \Pr(\Theta_k \notin \Omega_k'') = 0 \quad (5)$$

188 Since knowledge of uncertainty-distribution forms is typically poor, all the model instances that
 189 belong to the CMS are assigned a constant probability:

$$190 \quad \Pr(\Theta_k \in \Omega_k'') = \frac{1}{\int_{\theta_k \in \Omega_k''} d\theta_k} \quad (6)$$

191 It is very rare that a more sophisticated probability distribution for the CMS can be justified in
 192 practical situations.

193 When all initial model instances generated are falsified, the entire model class is falsified
 194 ($\Omega_k'' = \emptyset$). This means that no model is compatible with observations given the current estima-
 195 tion of model and measurement uncertainties. Thus it is usually a sign of incorrect assumptions in
 196 the model-class definition and uncertainty assumptions. Complete falsification helps avoid wrong

197 identification of parameter values and detect flaws in initial assumptions, highlighting one of the
 198 main advantages of EDMF compared with other methodologies. The choice of the parameters to
 199 be identified is carried out by: i) conducting a sensitivity analysis to determine the relative im-
 200 portance of each parameter in model predictions and ii) considering the final goal of the structural
 201 identification, such as loading capacity estimation. For example, when serviceability is the criti-
 202 cal limit state, relevant parameters include elastic material properties such as the Young's modulus
 203 and in-situ boundary conditions. In situations where the ultimate reserve capacity is investigated,
 204 relevant parameters include structural-element geometry and material strength values. The latter
 205 can be updated through non-destructive tests or laboratory tests. The choice of the initial interval
 206 and the distribution of parameter values to be adopted, usually uniform if no specific information
 207 is available, is based on engineering judgment. The parameters that most influence predictions are
 208 included in the vector of primary parameters θ_k and used to generate the initial model set Ω_k .

209 When a candidate model set is identified ($\Omega_k'' \neq \emptyset$), prediction tasks can be performed employ-
 210 ing the CMS to predict at unmeasured locations and to assess the reserve capacity of the structure.

211 Predictions Q_j at n_q locations are given by:

$$212 \quad Q_j = g_k(x_i, \Theta_k'') + U_{i,g_k}, \quad \forall i \in \{1, \dots, n_q\} \quad (7)$$

213 where $\Theta_k'' = \{\Theta_k | \theta_k \in \Omega_k''\}$ is a set of vectors of parameter values representing the CMS.

214 There is a trade-off between the simplicity of the FE model and the magnitude of model un-
 215 certainty U_{g_k} . For example, it is possible to employ a less precise model class G_k^* , which is
 216 characterised by higher model-uncertainty magnitudes $U_{g_k}^*$, to obtain more rapidly a good out-
 217 come compared with a detailed model. The “price to pay” in such cases is the loss of precision in
 218 prediction due to the higher variance of Q_j^* , according to Equation (7).

219 The performance of identification in reducing the initial parameter uncertainties depends on
 220 factors such as the initial choice of parameters, the sampling technique and the sensor configura-
 221 tion. Besides the selection of parameters to be considered as primary parameters, the generation

222 of the initial model set (IMS) affects structural identification. The IMS should cover the parameter
 223 domain that defines candidate models with sufficient density, in order to provide unbiased predic-
 224 tions. Many sampling techniques have been adopted in structural model updating. Each of them
 225 involves a trade-off between density and extension of space exploration.

226 **Sampling techniques**

227 The goal of EDMF is to falsify incorrect model instances, whose population should represent
 228 adequately, for each model class, the uncertainty connected to the parameter values after measure-
 229 ment.

230 In previous applications of EDMF mainly uniform sampling techniques, such as grid-based
 231 sampling (GBS) and Latin hypercube sampling (LHS) have been adopted to explore the model
 232 instance solution space (Goulet et al. 2010; Pasquier and Smith 2016). The vector of parameter
 233 range $I_j = [\theta_{j,low}, \theta_{j,high}] \quad \forall j \in \{1, \dots, n_\theta\}$ where $\theta_{low}, \theta_{high}$ represent respectively the lower and
 234 upper bounds for each parameter, is defined based on engineering judgment and the distributions
 235 of parameter values are usually uniform within the range. Conservative large bounds for parameter
 236 range are often used in order to ensure that the correct solution is within bounds.

237 *Grid-based sampling*

238 In GBS model instances are generated according to an n_θ -dimensional grid. Each parameter
 239 range is discretised into $\xi(I_j)$ intervals which define the density of the sampling. The IMS consists
 240 of a matrix having n_k rows and n_y columns, where the total number of combinations to be evaluated
 241 n_k , is calculated with the following equation:

$$242 \quad n_k = \prod_{j=1}^{n_\theta} \xi(I_j) \quad (8)$$

243 For example, in a model class defined by 5 primary parameters, each of which is discretized in 8
 244 uniform intervals, $8^5 = 32,768$ samples are evaluated.

245 *Latin hypercube sampling*

246 LHS generates model instances in a square n_θ -dimensional grid across the parameter space,
247 whereby each sample is the only one in each axis-aligned hyperplane containing it. This algorithm
248 represents a development of the Monte Carlo sampling methods and it is particularly adopted to
249 avoid clustering of samples. LHS requires that each parameter is divided into the same number
250 of intervals and the definition of the number of samples to be evaluated. The maximum number
251 of combinations for a LHS of M intervals and n_θ parameters can be computed with the following
252 equation:

$$253 \left(\prod_{n_\theta=0}^{M-1} (M - n_\theta) \right)^{n_\theta-1} = (M!)^{n_\theta-1} \quad (9)$$

254 The main drawbacks of LHS are that extreme points, such as corners of the design space, are not
255 necessarily covered, and that the selection of few samples can result in a poor exploration of the
256 domain.

257 *Optimal space filling*

258 Optimal space-filling sampling (SF) is a method whereby LHS is extended with post-processing.
259 SF is initialised as LHS and then optimised several times through maximising the distance between
260 samples. Samples are equally distributed throughout the design space with the objective of gaining
261 the maximum insight into the parameter domain with the fewest number of samples. SF shares
262 some of the same drawbacks as LHS, though to a lesser degree. Possible disadvantages are that
263 extreme points may not be covered and that a limited number of samples can result in a poor
264 exploration of the domain.

265 Figure 1 shows, for example, a two-dimensional problem that has two input parameters. Twenty
266 design intervals are considered and samples are generated using GBS, LHS and SF. Although GBS
267 provides an extensive exploration of the domain, it requires the highest number of samples (i.e.
268 400). Through avoiding samples with common rows and columns, LHS and SF require only 20
269 samples. However, LHS may generate clustered samples and skip parts of the parameter domain.
270 SF addresses extremes more effectively and provides a better coverage of the parameter domain.

271 GBS is not feasible when many parameters have to be considered simultaneously because its
 272 sampling complexity is exponential with respect to the number of parameters. LHS and SFS
 273 involve strategies to reduce the number of samples compared with GBS while providing a good
 274 exploration of the parameter domain. However as with GBS, poor sample density is likely when
 275 high-dimension domains are investigated.

276 *Markov chain Monte Carlo*

277 Adaptive sampling techniques can be applied to increase the performance of the sampling es-
 278 pecially when high-dimensional spaces are investigated. With non-adaptive sampling techniques,
 279 the IMS is first built based only on the parameter uncertainties. Adaptive sampling techniques re-
 280 quire an iterative process because the choice of the next sample depends on the parameter domain
 281 already explored.

282 Markov chain Monte Carlo (MCMC) is an algorithm that samples from a target distribution that
 283 is proportional to the posterior PDF by constructing a random walk that has the desired distribution
 284 at equilibrium. The most common method to construct the random walk between subsequent states
 285 is the Metropolis-Hastings algorithm, which is extensively used in Bayesian model updating. Let
 286 $p(\theta)$ denote the target PDF and $q(\theta)$ the proposal density, which depends on the current state
 287 $k \in \{1, \dots, n_k\}$. The algorithm proceeds as follows:

- 288 1. Sample from the proposal density: $q(\theta^c|\theta^k)$ to generate a candidate state θ^c from the previ-
 289 ous state θ^k ;
- 290 2. Evaluate the ratio $p_{accept} = \frac{p(\theta^c)q(\theta^k|\theta^c)}{p(\theta^k)q(\theta^c|\theta^k)}$;
- 291 3. If $p_{accept} \geq 1$, θ^c is accepted. Otherwise, the proposal state is accepted with probability
 292 $p_{accept} < 1$. If θ^c is accepted $\theta^{k+1} \leftarrow \theta^c$
- 293 4. A random number $r \in [0, 1]$ is generated. If $p_{accept} > r$ the proposal state is accepted and
 294 $\theta^{k+1} \leftarrow \theta^c$, otherwise $\theta^{k+1} \leftarrow \theta^k$.

295 By generating random numbers and then comparing them with p_{accept} , it is possible to visit regions
 296 having high posterior probability relatively more often than those associated with low posterior

297 probability. Additionally, even though the posterior had a maximum value and that point was
298 reached during the exploration, the algorithm would keep building the posterior distribution until
299 n_k samples are generated.

300 The efficiency of this approach is affected by the choice of the proposal density function form
301 and its optimal form is usually unknown beforehand. This issue is aggravated when the uncertain-
302 ties are correlated and when the posterior PDF is peaked. Moreover, traditional MCMC techniques
303 are inefficient to sample high-dimensional target PDF and cannot be applied if the PDF is mul-
304 timodal (Ching and Chen 2007). These issues have already been studied for Bayesian model
305 updating in (Beck and Au 2002), which proposed an adaptive Metropolis-Hastings (AMH) based
306 on intermediate simpler PDFs instead of the target one, and later developed in (Ching and Chen
307 2007) which proposed a transitional version of MCMC (TMCMC). The latter method is based on
308 the previous AMH but employs a re-sampling strategy, which is more robust against the increasing
309 number of parameters, to generate the intermediate PDFs and it has been shown to perform well
310 even when a peaked or multimodal posterior PDF has to be sampled. However, the main drawback
311 of this approach resides in the number of intermediate stages required to go through all the adjacent
312 PDFs, because the transition between one intermediate PDF and the next should be smooth, but
313 more stages mean more samples to be evaluated.

314 Goulet (2012) applied a combination of MCMC and GBS to obtain a CMS compatible with
315 EDMF using a likelihood function based on the k-order generalised Gaussian distribution. The
316 number of samples required by MCMC to get the same CMS as GBS was found to be approxi-
317 mately 20% lower. A performance was evaluated based on the size of the CMS compared to the
318 initial set and was evaluated only for a simple theoretical example, involving two parameters. Real
319 case studies are characterised by many primary parameters (usually 5 to 10) and large initial pa-
320 rameter uncertainties, which provide conservative wide initial intervals for parameter values. As
321 a consequence, MCMC performance is affected by high-dimensional target PDFs that have to be
322 sampled and iterative strategies should be adopted to tailor the proposal distribution to the problem
323 at hand.

324 According to Equation (6), all the model instances inside the CMS are assigned the same prob-
325 ability, which means that the target PDF will be uniform. If the CMS consists of only few model
326 instances, it is likely to observe a peaked posterior distribution, which can also be multimodal de-
327 pending on the combination of parameter values generated while sampling the parameter space.
328 In this circumstance, it is difficult to implement MCMC with Metropolis-Hastings because if the
329 proposal PDF is wide and it is likely that the peaked region will be reached only by chance. If the
330 proposal PDF is too narrow, the travel of the Markov chain will be slow and the peak could never be
331 reached in a reasonable number of samples. Though this issue can be faced by an adaptive evalua-
332 tion of the proposal distribution, sampling from a multimodal distribution may result in a Markov
333 chain which is trapped in one local peak. Moreover, to increase the performance of identification
334 and to provide redundant information, a large number of sensors, sometimes even more than the
335 over-instrumentation limit, are employed to obtain measurement values. Although the multivariate
336 Gaussian likelihood can be implemented to accommodate any number of measurement points, its
337 evaluation can be difficult when many sensors are considered.

338 In this paper a real case-study is employed to compare sampling techniques. An alternative
339 approach not explicitly related to Bayesian model updating that is appropriate for sampling high-
340 dimensional spaces through balancing domain exploration and result quality, and effective for low
341 number of samples, is proposed. The basics of this method are presented in the next section.

342 **Derivative-free optimization**

343 In many engineering applications, the goal is to optimise an objective function whose analytical
344 expression is unknown, and the function values are only available through a solver. In this study,
345 the objective function is expressed in terms of the residual values between FE predictions and
346 measurements. For complex structures, the solver evaluation is a time-expensive simulation and
347 therefore, estimating the partial derivatives of the response surface by finite-difference methods
348 within gradient-based methods is usually not convenient. Traditional derivative-free heuristics
349 such as simulated annealing, genetic algorithms, and particle swarm optimisation are also not
350 appropriate, since they often require many evaluations to find good quality solutions. Moreover,

351 they may not converge to the best solution if not enough samples are evaluated. These methods
 352 have been used in structural optimisation, as reported in (Hare et al. 2013).

353 In general, a derivative-free optimization problem can be cast in the following form:

$$\begin{aligned}
 & \max_{\boldsymbol{\theta}} f(\boldsymbol{\theta}) \\
 & \boldsymbol{\theta} \in [\boldsymbol{\theta}^L, \boldsymbol{\theta}^H],
 \end{aligned}
 \tag{10}$$

354 where $\boldsymbol{\theta} \in \mathbb{R}^{n_\theta}$ is the vector of the parameters, whose lower and upper bounds are defined by the
 355 vectors $\boldsymbol{\theta}^L \in \mathbb{R}^{n_\theta}$ and $\boldsymbol{\theta}^H \in \mathbb{R}^{n_\theta}$, respectively, and $f : \mathbb{R}^{n_\theta} \rightarrow \mathbb{R}$ is the objective function to
 356 optimise. The key feature of Equation (10) is that the analytic expression of f is unknown and
 357 the evaluation of the function value $f(\bar{\boldsymbol{\theta}})$, given a set of parameter values $\bar{\boldsymbol{\theta}}$, is provided only by a
 358 solver.
 359

360 An alternative approach is to adopt a surrogate model of the function f . The kriging-based
 361 EGO (Efficient Global Optimization) method (Jones et al. 1998), the radial-basis function (RBF)
 362 method (Gutmann 2001), and the stochastic RBF method (Regis and Shoemaker 2007) implement
 363 this idea. These approaches build global models of the function f , and not local models that are
 364 employed by trust-region methods (Conn et al. 2009). This ensures a convergence to the global
 365 optimal solution, if the number of simulations is large enough, and allows identification of good
 366 quality solutions within a limited number of simulations. Previous studies (Holmström et al. 2008)
 367 have shown that the RBF method performs well on engineering problems.

368 *The radial-basis function method*

369 The goal of the radial-basis function method is to approximate the unknown objective function
 370 f using predictions provided by a FE solver. The surrogate model of the objective function is
 371 an interpolant s that is built by means of radial basis functions. The RBFs are special functions
 372 $\omega(\|\boldsymbol{\theta} - \bar{\boldsymbol{\theta}}\|) : \mathbb{R}^+ \rightarrow \mathbb{R}$ that depend on the Euclidean distance r between a new set of parameter
 373 values $\boldsymbol{\theta}$ and parameter sets $\bar{\boldsymbol{\theta}}$ already employed in FE simulations.

374 There are several types of RBFs, and the most commonly employed in engineering are the
 375 linear RBF $\omega(r) = r$, the cubic RBF $\omega(r) = r^3$ and the thin plate spline RBF $\omega(r) = r^2 \log(r)$.

376 Given z parameter sets $\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_z \in [\boldsymbol{\theta}^L, \boldsymbol{\theta}^H]$ and the FEM predictions $f(\boldsymbol{\theta}_i)$, the RBF interpolant
 377 s_z to the points $(\boldsymbol{\theta}_i, f(\boldsymbol{\theta}_i)), \forall i \in \{1, \dots, z\}$ can be expressed as:

$$378 \quad s_z(\boldsymbol{\theta}) = \sum_{i=1}^z \lambda_i \omega(r) + p(\boldsymbol{\theta}), \quad (11)$$

379 where ω is the RBF employed, p is a polynomial and $\lambda_i \in \mathbb{R}$ are the coefficients of the interpolant
 380 that are found by solving a linear system. The polynomial p is needed to ensure the existence
 381 of the interpolant and depends on the RBF type. The polynomial guarantees that this system
 382 can be solved and that coefficients can be computed. Also, the minimum degree of the required
 383 polynomial depends on the RBF employed. For example, in the cubic and thin plate spline cases
 384 the polynomial must have a degree of at least 1, whereas for the linear case a 0-degree polynomial
 385 (i.e., a constant) is sufficient, as explained in (Costa and Nannicini 2015). Further details on the
 386 RBF method can be found in (Buhmann 2000).

387 According to the RBF method, after z simulations a target value f_z^* is set. The next set of
 388 parameter values $\boldsymbol{\theta}_{z+1}$ to be evaluated through the FE solver is the point in the domain which min-
 389 imizes the *bumpiness* (see Figure 2) of the RBF interpolant if this new set $(\boldsymbol{\theta}_{z+1}, f_z^*)$ is considered.
 390 The RBF method requires the function f to be smooth and an advantage of this methodology is
 391 that it is possible to obtain an analytical measure of the bumpiness (Costa and Nannicini 2015).
 392 Since smooth functions exhibit low bumpiness values, new parameter sets $\boldsymbol{\theta}_{z+1}$ are chosen through
 393 a bumpiness minimization procedure (further details are provided in the next section).

394 *RBF Optimization-RBFOpt*

395 RBFOpt is open-source software developed for radial-basis function optimization. The soft-
 396 ware and additional information are available in (Costa and Nannicini 2015). Using the notation
 397 introduced above, and by defining z as the counter for the number of simulations and `MAX_ITER`
 398 as the maximum number of allowed simulations, the RBF method implemented by RBFOpt is
 399 summarized by the following steps:

- 400 1. Select and evaluate a set of m starting points $S = \{(\boldsymbol{\theta}_1, f(\boldsymbol{\theta}_1)), \dots, (\boldsymbol{\theta}_m, f(\boldsymbol{\theta}_m))\}$;

- 401 2. Set $z = m$ to include the simulations already performed;
- 402 3. Compute the RBF interpolant for the points in S according to Equation (11);
- 403 4. Choose a target function value f_z^* ;
- 404 5. Find the point θ_{z+1} such that the interpolant to the points $S \cup (\theta_{z+1}, f_z^*)$ is the least bumpy;
- 405 6. Evaluate θ_{z+1} through the solver to obtain $f(\theta_{z+1})$;
- 406 7. Add $(\theta_{z+1}, f(\theta_{z+1}))$ to S and set $z = z + 1$;
- 407 8. If $z = \text{MAX_ITER}$ stop; otherwise return to step 3.

408 The first step involves the initialization of the RBF interpolant, for which $m > n_\theta$ starting
 409 points are needed. Although many strategies may be employed to define the starting points, RB-
 410 FOpt employs LHS to select $m = n_\theta + 1$ random points.

411 The target value f_z^* is chosen according to a cyclic strategy, which alternates between the ex-
 412 ploration of unknown zones of the domain and the identification of good approximations obtained
 413 by the surrogate model. The target value is a guess of the best value that the real objective func-
 414 tion may achieve. If this value is far from the current optima of the interpolant implies that the
 415 real objective function can achieve a much better value than those predicted by the interpolant.
 416 In this case, the points which can potentially yield this improvement are searched in unexplored
 417 part of the domain. When the target value is close to the optima of the current interpolant a good
 418 approximation of the real objective function is achieved, hence the next evaluation point will not
 419 be to far from points already sampled. Thus, cyclically changing the target value helps optimize
 420 the interpolant by avoiding local optimum traps. More details related to the target value definition
 421 can be found in (Costa and Nannicini 2015).

422 Figure 2 represents an illustrative example that describes the meaning of bumpiness. The blue
 423 circles are the points already evaluated θ_z . Considering a target value f_z^* , two predictions (inter-
 424 polants red and green) can be obtained according to the choice of the next point θ_{z+1} . The green
 425 prediction is less bumpy and thus, it will be chosen by RBFOpt.

426 In summary, RBFOpt is an iterative algorithm that optimises an objective function, whose
 427 analytical expression is unknown. To build the surrogate model of the function f (i.e. RBF in-

428 interpolant) predictions provided by a FE solver are employed. Moreover, this method is able to
429 provide good results rapidly, while other approaches such as neural networks, which often need
430 a large training set to be effective, would be computationally expensive. Also, RBF returns an
431 approximation of the objective function, thus allowing the option to explore other promising parts
432 of the candidate domain. Potentials exist for RBFOpt to be applied to EDMF, using an appropri-
433 ate objective function (the falsification function), for sampling purposes. The optimisation of this
434 function helps increase sampling density in the candidate domain. This new approach is explained
435 in the following sections.

436 SURROGATE MODELS FOR ADAPTIVE SAMPLING

437 FE models are often used to predict structural behaviour, given input data such as element ge-
438 ometry, material properties, boundary and interface conditions, load configurations, element type,
439 mesh size, etc. To reduce modelling uncertainty, the mesh can be refined and shell or solid elements
440 can be employed instead of linear elements. However, computation times increase. Surrogate mod-
441 els are substitutes for complex models, since they are able to capture the essential behaviour of a
442 structure using much less computation time than FE models. A common approach is to build
443 response surfaces based on polynomial functions, through minimising the least-square difference
444 between response surface and FE model predictions. When surrogate models are adopted to sim-
445 ulate the structural behaviour, an additional source of uncertainty associated with the accuracy of
446 the surrogate model has to be considered. Also, surrogate models may not represent adequately
447 certain types of non-linear structural response.

448 Figure 3 shows a surrogate model that approximates the structural behaviour of a cantilever
449 beam. The Young's modulus E is the only unknown parameter. The goal is to predict the displace-
450 ment δ at the free edge, under the distributed load q . The dashed hyperbolic line represents the true
451 behaviour of the beam. Although in this simple case the mathematical formulation of δ^{TRUE} is
452 well known, model predictions of real structures are usually provided by FE solvers. Since solver
453 computation can be time consuming, two displacements (δ_1 and δ_2) are evaluated using two values
454 of parameter E . Then a linear surrogate model δ^{SM} is built and used to calculate the displacement

455 δ for many samples of E .

456 The difference between the red line (δ^{SM}) and the dashed line (δ^{TRUE}) corresponds to the
457 uncertainty of the surrogate model. Since the analytical formulation of the SM is built considering
458 only few (in this example two) simulations, SM predictions may diverge from the true behaviour
459 for E-values far from the two training points. This error can be reduced by increasing the density
460 of training points and can be estimated by means of testing points, where both δ^{SM} and δ^{TRUE} are
461 evaluated and compared. Increasing the number of training points enhances the accuracy of the
462 SM; however, those points require time-consuming FE simulations.

463 Another implementation of surrogate models can be found in structural optimisation, as men-
464 tioned in the background section. The radial-basis function method - depicted in Figure 4 - is an
465 adaptive sampling strategy that helps focus the sampling of parameter values in particular regions
466 of the domain. Such a methodology can reduce the computation times required to reach a defined
467 level of sampling accuracy. Considering the same cantilever beam as in the previous example, let
468 assume that the goal is to identify E-values for which the residuals ($r = \delta - \delta^*$) between pre-
469 dicted and measured maximum deflections lie in a predefined interval $[r_{low}; r_{high}]$. This objective
470 is described by means of a uniform residual target distribution $f(r)$. A Surrogate Model (SM) is
471 used to generate samples of the parameter E , for which the residuals r follow the residual target
472 distribution. In Figure 4a $f(r)$ is defined in the prediction domain, while the analytic expression
473 the function in the parameter domain $f(E)$ is unknown. Therefore, a SM ($RBF(E)$) is used to
474 approximate the function f in the parameter domain (Figure 4b). Then, the SM helps find target
475 E-values - for which $f(E) = 1$. Hence, the SM is used as an objective function to guide the sam-
476 pling of E -values for which residuals follow the target distribution (Figure 4c). Compared with
477 random sampling, the SM generates more samples (i.e. E-values) in the parameter sub-domain for
478 which $f(E) = 1$. Moreover, no SM uncertainty has to be considered, since every model prediction
479 is computed using the FE solver. In this approach, the SM is used to approximate the unknown
480 function $f(E)$.

481 **RBF sampling for EDMF**

482 As mentioned above, surrogate models can be used for sampling purposes to help increase
483 sampling density in sub-domains of the parameter space. The idea is to use radial-basis functions
484 to create a surrogate model of a particular target distribution that characterises candidate models.
485 This approach is applied to a new framework for adaptive sampling in EDMF, which is introduced
486 in Figure 5.

487 This new methodology for sampling the parameter space is based on the optimisation of a
488 surrogate model that represents a particular target distribution called the falsification function (f_F).
489 The optimization, performed using RBFOpt, helps provide sets of parameter values associated
490 with residuals that lie within threshold bounds of falsification. The falsification function, which
491 can be seen as the projection of a uniform likelihood function in the parameter space, is used as an
492 objective function to guide the search of candidate models.

493 In order to compute the residuals r_i , model predictions are calculated using a FE solver and
494 measurement values y_i are collected from sensors. Furthermore, model and measurement uncer-
495 tainties are combined to calculate the threshold bounds, as stated in Equation (2).

496 The next section provides a detailed explanation of the new framework.

497 **Iterative optimization**

498 In order to initialise the RBF, a set of parameter vectors $\tilde{\theta}$ is defined by sampling in the initial
499 parameter ranges I_j using traditional techniques such as LHS. These vectors are assigned to the FE
500 model class. The static analysis is performed using a FE solver such as ANSYS and predictions
501 $g(x_i, \theta)$ at locations x_i are calculated. Then, residuals r_i between measurement values and struc-
502 tural predictions are computed. According to Equation (2), the combined uncertainties $U_{i,c}$ and
503 the residuals r_i are compared in order to perform falsification. The falsification function, which is
504 defined in the error domain, represents the target distribution. This distribution is approximated in
505 the parameter domain by means of a surrogate model (i.e. RBF interpolant). The RBF interpolant
506 is employed as an objective function for adaptive sampling. This improves the search for parameter
507 values that provide residuals which follow the target distribution.

508 RBFOpt generates the first RBF interpolant $rbf(\boldsymbol{\theta})$ which is a function $rbf : \mathbb{R}^{n_\theta} \rightarrow \mathbb{R}$ that
 509 provides the falsification value associated with the parameter values $\boldsymbol{\theta}$. The output of rbf -function
 510 are expressed in Equation (12)

$$511 \quad rbf(\boldsymbol{\Theta}) = \begin{cases} 1 & , \text{ if } \boldsymbol{\theta} \in \Omega'' \\ [0, 1[& , \text{ otherwise} \end{cases} \quad (12)$$

512 An instance $\boldsymbol{\Theta}$ of the parameter vector $\boldsymbol{\theta}$ that is in the candidate-model domain Ω'' , provides an
 513 interpolant value $rbf(\boldsymbol{\Theta}) = 1$.

514 The generation of the interpolant consists of two steps. First, the interpolant type (i.e. linear,
 515 cubic) is chosen by a leave-one-out cross-validation and the RBF type that provides the smallest
 516 error is selected. Second, the choice of the interpolant shape is performed according to a bumpiness
 517 minimization procedure.

518 The RBF interpolant is iteratively optimised through selecting new instances $\boldsymbol{\Theta}_{z+1}$, evaluating
 519 their predictions and computing the residuals. The choice of the next parameter sample is based on
 520 two criteria, the improvement of the interpolant accuracy and the search of the interpolant global
 521 maximum.

522 Satisfying these two criteria forms the core of the RBF sampling algorithm (RBFS). The im-
 523 provement of the interpolant accuracy ensures a sufficient exploration of the parameter space while
 524 the search for the global maximum generates samples in the candidate domain. The framework in
 525 Figure 5 is performed iteratively until a stop or restart condition is reached. When the stop con-
 526 dition is reached, all model instances characterised by an RBF-value equal to 1 are automatically
 527 included in the CMS. Other instances are falsified.

528 The stop condition can be static or dynamic. In the former case, a maximum number of iter-
 529 ations is defined a priori, for instance, according to the available computation times. In the latter
 530 case, the variance of parameter values in the candidate domain is checked for each new candi-
 531 date model. New model instances are generated until the variation in the variance is lower than a

532 predefined limit.

533 A restart occurs when, after a given number of attempts, the current best solution (i.e. the
534 maximum objective-function value $rbf(\Theta)$ found so far) is not further improved. Consequently, a
535 new random set of initial points is evaluated and a new RBF interpolant has to be optimised. This
536 feature increases the exploration of the parameter domain. The entire process is summarised in the
537 flowchart depicted in Figure 6.

538 **Falsification Function**

539 The falsification function is defined using the threshold boundaries computed through Equation
540 (3). First, all the sources of uncertainties are combined using the Monte Carlo method and thresh-
541 old bounds corresponding to 95% and 99% confidence level are calculated and corrected using
542 the Sidák correction for a given number of measurements. The green area represents a rectangular
543 distribution defined by the thresholds bounds $(T_{95,low}; T_{95,high})$. More formally, the function is built
544 as follows:

$$545 \quad f_F(U_{i,c}) = \begin{cases} 1 & , \text{ if } u_{i,L}^{95} < U_{i,c} < u_{i,H}^{95}, \quad \forall i \\ 0 & , \text{ otherwise} \end{cases} \quad (13)$$

546 where $u_{i,L}^{95}$ and $u_{i,H}^{95}$ represent the lower (L) and upper (H) threshold boundaries at sensor location
547 i , calculated with a 95% confidence level. This confidence level is a standard engineering criterion
548 that is used in many engineering decision tasks. Due to the systematic uncertainties in model-class
549 definition, the f_F is not centered in zero.

550 Since values of the falsification function for previously generated model instances are em-
551 ployed to fit RBF interpolants, a perfectly rectangular objective function is not appropriate because
552 it cannot guide the search for optimal values. To guide the search, two triangular distributions are
553 attached to the rectangular distribution, as shown in Figure 7. These distributions have a 0 proba-
554 bility at the 99% confidence bounds. The triangular distributions do not affect the falsification of
555 model instances because all the samples outside the 95% interval (i.e. $f_F < 1$) are discarded.

556 **CASE STUDY**

557 The case study is a prestressed reinforced concrete bridge in Singapore. This structure, which
558 consists of four prestressed concrete beams, has a single span of 32 m and is supported at each
559 end by 4 bearing devices (Figure 8). The beams support and are connected to a reinforced con-
560 crete slab that is 22 cm thick. The structure has been modelled using ANSYS and the model
561 includes non-structural elements such as the asphalt pavement and precast concrete barriers. These
562 elements, which are usually included as permanent loads in design-stage models, are included in
563 order to reduce model-simplification uncertainties. A static load test involving 6 trucks, each with
564 a gross weight of 33 tons, was performed. A measurement system consisting of a laser tracker, 8
565 strain gauges (S) and 2 inclinometers (I) has been designed. In order to increase the accuracy of
566 deflection measurements, 4 prisms (P) were placed on the bottom faces of the main beams. Truck
567 configuration and sensor locations are depicted in Figure 9.

568 **Uncertainty definition**

569 The case study is used to estimate the performance of RBFS and to compare it with results from
570 LHS, SF and MCMC sampling. The parameters are defined according to a sensitivity analysis
571 of parameter impact on model predictions at measurement locations. Figure 10 shows the relative
572 importance of eight parameters that need to be defined in the FE model. Parameter selection is
573 often a tradeoff - in this study the five most sensitive parameters are considered for identification.
574 Although other parameters may provide additional insights, increasing the number of parameters
575 for identification would result in a sampling domain with higher dimensions that requires larger
576 sample sizes, in particular when non-adaptive sampling approaches are employed. In this study, the
577 initial model set (IMS) is generated through sampling the five-dimensional parameter space defined
578 by the Young's modulus of cast-in-place concrete, the Young's modulus of precast concrete, the
579 Young's modulus of barrier concrete, the rotational and the vertical stiffness of the bearing devices.

580 The initial interval for each parameter, defined using the prior knowledge available, is reported
581 in Table 1. This knowledge is based on a preliminary study and in-situ visual inspection. Intervals
582 of elastic material properties (i.e. Young's moduli) include mean values from structural codes.
583 Moreover, these intervals are conservatively wide in order to ensure that true parameter values

584 are included. Rotational stiffness intervals are able to describe boundary conditions in the range
585 between pinned and fixed constraint conditions. Vertical stiffness intervals are defined in order to
586 consider vertical displacements of beam supports up to $40mm$. Visual inspection did not provide
587 evidence of local change of material properties. Therefore, constant properties have been assigned
588 to every bridge element.

589 Table 2 describes modelling and measurement uncertainty sources. Uniform distributions are
590 adopted to describe model uncertainty sources - a range of plausible assumptions on uncertainty
591 forms are described in (Pasquier and Smith 2015b). The minimal and maximal bounds defined
592 in Table 2 are expressed as a percentage of the mean value of model predictions for modelling
593 uncertainties and as a percentage of the measured value or in absolute terms for measurement
594 uncertainties. The uncertainty associated with the FE model takes into account two aspects: i)
595 parameters that have not been considered for identification and ii) FE model simplifications. The
596 variation in predictions due to parameters not considered for identification are estimated and in-
597 cluded as uncertainties in model predictions. Model-simplification uncertainty is related to the
598 FE-model-class features. In this study, the bridge model is constructed in ANSYS using solid ele-
599 ments (SOLID 185) with perfectly connected interfaces between adjacent layers. This assumption
600 leads to a model class that is likely to overestimate the real stiffness of the bridge. Therefore, the
601 FE uncertainty distribution is not centered on zero. Bounds of uncertainty associated with mesh
602 refinement and bounds for additional uncertainty are taken from (Goulet et al. 2010). Finally, the
603 source of uncertainty associated with spatial variability originates from strain sensors measuring
604 behaviour that is affected by the spatial variability of material properties. This source compen-
605 sates for the choice of assigning constant material properties to every bridge element. Uncertainty
606 sources estimated in this paper are similar to those employed in previous studies that focus on
607 bridges (Goulet et al. 2010; Pasquier and Smith 2016).

608 Sensor accuracies are described by uniform distributions that are based on manufacturer speci-
609 fications. The measurement repeatability was assessed by taking multiple measurements under the
610 same load case. Since the noise was mainly due to the vehicle traffic across a lane that was kept

611 open during the test, a uniform distribution was used with bounds corresponding to the maximal
612 and minimal values recorded. For strain gauges, uncertainty also arises from the imperfect align-
613 ment of gauges with respect to the bridge longitudinal axis, which results in underestimation of
614 real stresses. Finally, additional noise associated with sensor installation has been considered for
615 inclinometers and strain gauges using field observations and conservative engineering judgment.
616 In order to perform model falsification, for each measurement location, a combined uncertainty
617 PDF is computed and threshold bounds are determined for a confidence level fixed at 95%. The
618 remainder of this paper investigates the impact of sampling methodologies employed to generate
619 the IMS on candidate-model identification.

620 **Candidate domain comparison (LHS, SF, MCMC and RBFS)**

621 Two non-adaptive sampling techniques (LHS and SF) and the traditional adaptive MCMC are
622 compared with the new approach introduced in this paper (RBFS). Using each sampling technique,
623 1000 initial model instances are generated to form the initial model set. In order to assure a fair
624 comparison, the same set of starting points is employed in RBFS and MCMC. Then, falsifica-
625 tion is performed using threshold bounds and a candidate model set (CMS) is obtained using each
626 sampling technique. Figure 11 shows the prediction values at each sensor location, which are ref-
627 erenced in accordance with Figure 9. Initial model instances are depicted in grey, while candidate
628 models are highlighted by dark lines. The asterisks represent measured values taken by each sen-
629 sor. LHS predictions are almost uniformly distributed throughout the parameter value range while
630 RBFS predictions are denser near to measured values at each location. Moreover, RBFS provides
631 more candidate models than LHS. RBFS provides larger prediction ranges than LHS at all sensor
632 locations. This aspect helps avoid wrong identification due to the incomplete exploration of the
633 prediction domain.

634 Figure 12 shows the parameter values that define model instances depicted in Figure 11. Both
635 techniques cover the parameter domain. However, RBFS focuses the sampling of parameter values
636 close to the candidate domain. For example, in Figure 12b sample density is denser for low values
637 of the longitudinal stiffness and high values of precast concrete Young's modulus than in Figure

638 12a. As shown in Figure 12c, RBFS increases the identified bounds for some parameters such as
639 Young's modulus of cast-in-place concrete and the rotational stiffness. Moreover, for the rotational
640 stiffness lower bound, RBFS helps identify a portion of the domain where LHS does not find can-
641 didate models. Similar observations can be made for the lower bound of cast-in-place concrete
642 Young's modulus. The reduction of parameter initial range after falsification is related to charac-
643 teristics of the measurement system. Some parameters such as the longitudinal stiffness and the
644 precast Young's modulus are well identified, while for other parameters such as the Young's mod-
645 ulus of cast-in-place concrete, falsification does not reduce the initial interval. However, RBFS
646 provides a thorough exploration of the candidate domain and helps avoid wrong falsification of
647 parameter values that are caused by poor sampling.

648 **Sensitivity of initial model set size**

649 The number of samples that needs to be evaluated for reaching a defined level of performance
650 is a crucial point for selecting the best sampling strategy. In the background section are mentioned
651 some challenges, related to the sample size, in order to ensure a sufficient coverage of the pa-
652 rameter domain. Increasing the initial sample size is an effective way to improve the exploration.
653 However, large sample sizes negatively affect evaluation times. For example, in this case study,
654 the computation time required by ANSYS to solve one model instance is about 180 seconds using
655 12 cores in parallel (i.e. 50 hours for 1000 simulations). In order to compare RBFS with LHS,
656 SF and MCMC, three initial sample sizes, which correspond to 500, 1000 and 2000 samples, are
657 defined. Sample sizes smaller than 500 samples are not reasonable considering the dimension of
658 the parameter domain and, thus, sampling performance would be largely affected by random gen-
659 eration of values. Sample sizes bigger than 2000 samples are discarded because of computation
660 time constraints. Figures 13 and 14 present values of rotational stiffness and cast-in-place concrete
661 Young's modulus that have been identified for three initial model sets that consist of 500, 1000 and
662 2000 samples.

663 Non-adaptive sampling techniques need many samples to provide a thorough exploration of
664 the candidate domain while adaptive sampling techniques are more effective in identifying large

665 parameter ranges using small numbers of model instances. For a large number of samples, the
666 parameter ranges provided by the four sampling techniques are similar.

667 RBFS provides the highest number of candidate models regardless of the initial sample size.
668 For the Young's modulus of cast-in-place concrete, RBFS is able to explore the entire candidate
669 domain with 500 samples and outperforms MCMC for both sampling density and identification
670 ranges. The employment of adaptive sampling techniques such as RBFS and MCMC increases
671 the exploration of the candidate domain. In terms of parameter-value range for a fixed number of
672 samples, RBFS often outperforms MCMC in exploring the candidate domain, especially when the
673 number of samples is low.

674 In the next section, a quantitative index is defined to compare sampling algorithms.

675 **Quality index**

676 The quality index is a metric to compare the performance of sampling techniques in terms of
677 parameter domain exploration. Let k be a sampling technique. For each parameter θ , the quality
678 index at iteration i (when i model instances are generated) is defined as the ratio between i) the
679 range of parameter values using sampling technique k at iteration i and ii) the range of parameter
680 values obtained using all sampling techniques at the final iteration. Then, the values obtained
681 for each parameter are averaged out. In this way, the quality index quantifies the increment of
682 candidate domain exploration at each evaluation and for each sampling technique. This index is
683 comparable to the performance profile used in derivative-free optimisation, where the performance
684 of several algorithms is compared with the performance of the best algorithm (Moré and Wild
685 2009).

686 More formally, let CMS_k be the matrix of candidate models for the sampling technique k .
687 $\text{CMS}_k(t, \theta)$ represents the value of the parameter θ for the t -th model in the candidate model set.
688 Let CMS_T be the matrix obtained by merging the CMS_k for each k and, if at iteration i a candidate
689 model is found, let $p(i)$ be its sequential position in the CMS. The quality index for the sampling
690 technique k at iteration i , indicated as $Q(k, i)$, is defined as follows:

$$Q(k, i) = \begin{cases} 0, & \text{if } i = 0, \\ Q(k, i - 1), & \text{if the } i\text{-th model does not belong to CMS}_k, \\ \frac{1}{n_\theta} \sum_{\theta=1}^{n_\theta} \left(\frac{\max_{t \leq p(i)} \text{CMS}_k(t, \theta) - \min_{t \leq p(i)} \text{CMS}_k(t, \theta)}{\max_t \text{CMS}_T(t, \theta) - \min_t \text{CMS}_T(t, \theta)} \right), & \text{otherwise.} \end{cases} \quad (14)$$

Figure 15 shows the comparison of the candidate domain generated using RBFS and four LHS. LHS₀ denotes the default LHS-setting implemented in ANSYS. Three more populations (LHS₁, LHS₂ and LHS₃) have been generated through a random selection of the sampling seed. The quality of candidate domain exploration provided by LHS is affected by the random choice of the seed value. However, there is no direct relationship between the choice of the seed value and the LHS performance. For example, although LHS₃ is among the best of the LHS when the number of samples is less than 500, it provides the lowest quality of exploration when more than 1000 samples are generated. RBFS outperforms all sample seeds of LHS and most successful LHS seed at high-sample number (LHS₁) provides lower quality sampling compared with RBFS up to 1000 model samples. Moreover, RBFS is more likely to provide higher quality results than those provided by LHS, especially for low sample numbers (< 900).

As reported in the background section, MCMC requires the definition of a proposal distribution, which is usually Gaussian. A narrow proposal leads to inefficient sampling and many iterations to converge. A wide proposal may never find the candidate-model domain. A common approach is to keep adjusting the proposal width during iterations and to check the acceptance ratio of new samples. This strategy requires sample sizes large enough to ensure the convergence towards good width values. In this study, four proposal widths are employed to sample four IMSs, which consists of 2000 samples each. In this way, a fair comparison of sampling quality using 2000 samples can be carried out.

Figure 16 shows the quality index of RBFS and four MCMC characterised by different proposal widths. MCMC₁ denotes the best parameter setting found after four attempts. The quality of candidate domain exploration provided by MCMC is affected by the proposal width employed.

714 This is a drawback of this methodology since a good MCMC performance requires the tailoring of
715 the proposal width to the problem at hand. However, the proposal width is not the only aspect to
716 consider. The poor performance of the tested MCMC compared with RBFS is due to the limited
717 number of samples. Indeed, after 2000 samples, MCMC has not converged. Additionally, the
718 multi-dimensional posterior and the uniform likelihood adopted by EDMF reduce the ability of
719 MCMC to focus the sampling in the candidate domain and to identify candidate models. In this
720 case study, RBFS provides higher quality results than those provided by MCMC for all sample
721 numbers. Moreover, since RBFS does not require initial tuning, there is less risk of low-quality
722 results.

723 Figure 17 reports the final comparison between the four sampling techniques, in which the
724 default value of the LHS seed is used (LHS_0). For MCMC, the best proposal width is chosen.
725 Once again RBFS provides the best performance. After 2000 evaluations it performs the most
726 accurate exploration of the candidate domain. Moreover, RBFS provides a quality of 75% in just
727 180 evaluations. For comparison, MCMC requires 630 iterations to reach the same quality level.
728 For a limited number of FE evaluations (i.e. 500), RBFS provides the highest number of candidate
729 models and the best exploration of the parameter domain. In this case, the quality index of RBFS
730 is twice the one of LHS.

731 Moreover, RBFS requires the least number of samples to reach high levels of quality (one-fifth
732 of the evaluations needed with MCMC for a quality index of 75%). A 75% quality is reached only
733 after 1380 samples using SF and after more than 1800 samples using LHS.

734 **Cross-validation of structural identification**

735 Full-scale case studies are essential for validating model updating methods since it is only at
736 this scale that uncertainties show realistic magnitudes. Unfortunately, many studies employ only
737 simulated measurements. Likewise, data collected during lab experiments may not be representa-
738 tive of measurements collected under ambient conditions. Strong methods for result validation are
739 required to assess the performance of model updating techniques. In this study, a cross-validation
740 is carried out using candidate models that have been identified to predict at locations that were not

741 used for model updating.

742 Figure 18 shows an example of cross-validation. First, 2000 initial samples are generated
743 through RBFS. Then, falsification is performed using the threshold bounds. Predictions are ob-
744 tained by combining model uncertainties with predictions of the candidate model set according
745 to Equation (7). The CMS prediction distributions are shown (Figure 18). Because of the Sidák
746 correction, the width of threshold bounds depends on the number of measurements that are used
747 for falsification. Therefore, in order to perform the cross-validation, threshold bounds have to be
748 calculated considering only the measurements that are used for falsification. Since use of EDMF
749 involves the hypothesis that all candidate models are equivalently likely due to a lack of knowledge
750 of real distribution forms, uniform prediction distributions (PDs) within 95% threshold bounds are
751 assigned to the CMS (Figure 18). The PDs at three locations (P1, S1 and I1), which have not been
752 used for falsification, are plotted. For each sensor, the PD includes the measured value. Therefore,
753 the cross-validation is verified and EDMF prognoses are robust.

754 Further investigation is carried out to assess the sensitivity of sampling on prediction reliability.
755 Five sensors (three deflection prisms P2, P3, P4 and two inclinometers I1, I2) have not been used
756 for falsification and one of the two inclinometers (I2) is used for cross-validation. Four sampling
757 techniques (MCMC, LHS, RBFS, and SF) are employed to generate four IMSs of 500 and 1000
758 samples. The uniform PD at location I2 and the measured value are plotted in Figure 19. In this
759 picture, the y-axis value of each uniform distribution is scaled to improve the visual interpretation
760 of PDs. Table 3 summarizes cross-validation results.

761 The CMS provided by RBFS is validated when 500 initial samples are considered, while CMSs
762 provided by MCMC and SF are validated only when the initial model set is large enough. There-
763 fore, EDMF reliability can be affected by the exploration of the candidate domain provided by
764 the adopted sampling algorithm. Interestingly, RBFS provides effective sampling even when the
765 number of samples is low.

766 **SUMMARY AND CONCLUSIONS**

767 In this paper a new sampling methodology - referred to as RBFS - based on radial-basis func-

768 tions (RBF) is employed in combination with error-domain model falsification (EDMF) to perform
769 structural identification. First, the finite element model of a bridge is built, the most sensitive pa-
770 rameters are selected for sampling and uncertainties are quantified. Then, several sampling tech-
771 niques are compared according to their performance in exploring the parameter domain while pro-
772 viding predictions that are compatible with the measured behaviour of the structure. RBFS over-
773 performs traditional approaches (LHS and SF) and adaptive algorithms such as MCMC. Therefore,
774 RBFS can increase the performance of EDMF by reducing computation times. Moreover, RBFS
775 sampling accuracy helps avoid identification shortcoming and biased predictions.

776 Specific conclusions are as follows:

- 777 • Grid-based sampling is not feasible when many parameters are considered simultaneously.
778 LHS and SFS involve a reduced number of samples; however, poor sample density is likely
779 when high-dimensional domains are investigated.
- 780 • RBFS outperforms traditional uniform sampling techniques such as LHS and SF even for
781 low numbers (500) of samples (FE evaluations). For the case study that was examined,
782 RBFS provides the most effective exploration of the parameter domain and helps avoid the
783 wrong falsification of parameter values connected with sampling shortcomings.
- 784 • The employment of adaptive sampling techniques such as RBFS and MCMC increases
785 the exploration of the candidate domain. In terms of parameter-value range for a fixed
786 number of samples, RBFS usually outperforms MCMC in exploring the candidate domain,
787 especially when the number of samples is low.
- 788 • The quality index proposed in this paper is useful for comparing sampling techniques.
789 When a quality level is established as a target (i.e. 75%), RBFS requires the least number
790 of samples. For example, MCMC may require more than four times the number of samples
791 than RBFS to obtain the same quality level.
- 792 • EDMF prediction reliability is affected by the exploration of the candidate domain provided
793 by the adopted sampling algorithm. RBFS provides effective sampling even when the
794 number of initial samples is low.

795 In order to generalise the conclusions above, other case studies involving real-scale measure-
796 ments are ongoing. Future work will focus on assessing the performance of RBFS when more
797 parameters are selected and when alternative definitions of model class and uncertainty magni-
798 tudes are considered. Also, further adaptive sampling algorithms have to be compared. Finally, the
799 implication of sampling methodologies in the assessment of reserve capacity for existing bridges
800 will be investigated.

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Parameters	Lower bound	Upper bound
Young's modulus of cast-in-place concrete	20 GPa	35 GPa
Young's modulus of precast concrete	25 GPa	50 GPa
Young's modulus of barrier concrete	3 GPa	40 GPa
Rotational stiffness of bearing devices	9 log(Nmm/rad)	13 log(Nmm/rad)
Vertical stiffness of bearing devices	8 log(N/mm)	11 log(N/mm)

TABLE 1. Parameter initial intervals.

Uncertainty source	Displacements - (P)		Rotations - (I)		Strains - (S)	
	Min	Max	Min	Max	Min	Max
FE model (%)	-5	13	-5	13	-5	13
Mesh refinement (%)	-1	1	-1	1	-1	1
Spatial variability (%)	-	-	-	-	-5	5
Additional uncertainty (%)	-1	1	-1	1	-1	1
Sensor accuracy	-0.05 mm	0.05 mm	-1 μ rad	1 μ rad	-2 $\mu\epsilon$	2 $\mu\epsilon$
Repeatability	-0.15 mm	0.15 mm	-4 μ rad	4 μ rad	-4 $\mu\epsilon$	4 $\mu\epsilon$
Sensor orientation (%)	-	-	-	-	0	6
Sensor installation (%)	-	-	-5	5	0	5

TABLE 2. Modelling and measurement uncertainty sources.

Sampling technique	Number of initial samples	
	500	1000
Optimal space filling - SF	×	✓
Latin hypercube sampling - LHS	×	×
Markov chain Monte Carlo - MCMC	×	✓
Radial-basis function sampling - RBFS	✓	✓

TABLE 3. Outcome of prediction cross-validation at location I2 (✓: successful cross-validation, ×: not successful cross-validation).

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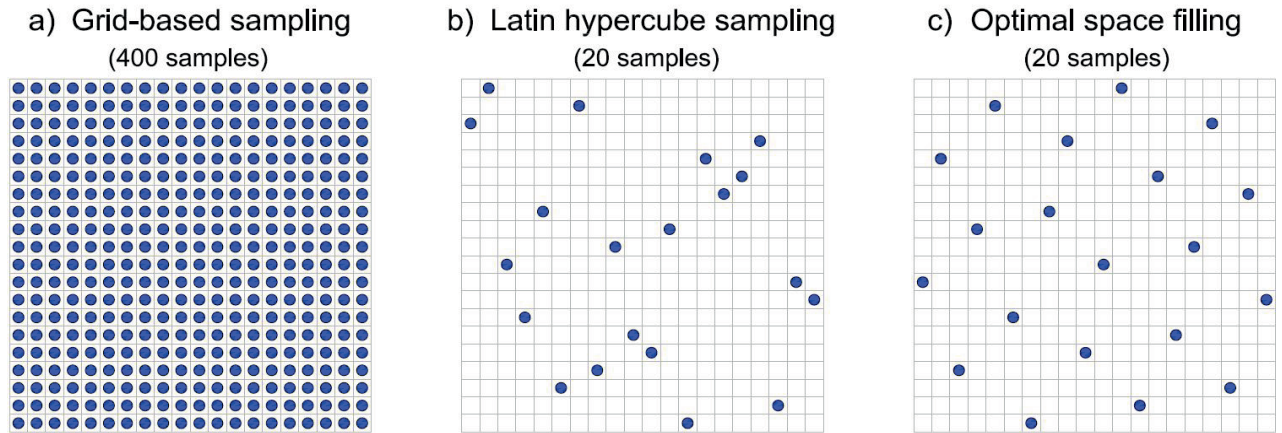


FIG. 1. Examples of sampling in a 2D domain using: a) grid-based sampling, b) Latin hypercube sampling (LHS) and c) optimal space-filling sampling (SF).

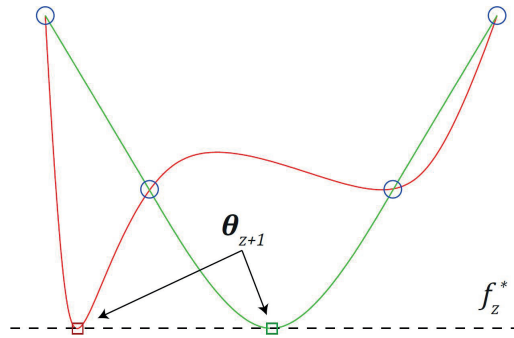


FIG. 2. Interpolant selection according to the bumpiness minimization criterion. The blue circles are the points already evaluated θ_z . Considering a target value f_z^* (dashed line), two interpolants (red and green) can be obtained according to the choice of the next point θ_{z+1} (square). The green interpolant is less bumpy.

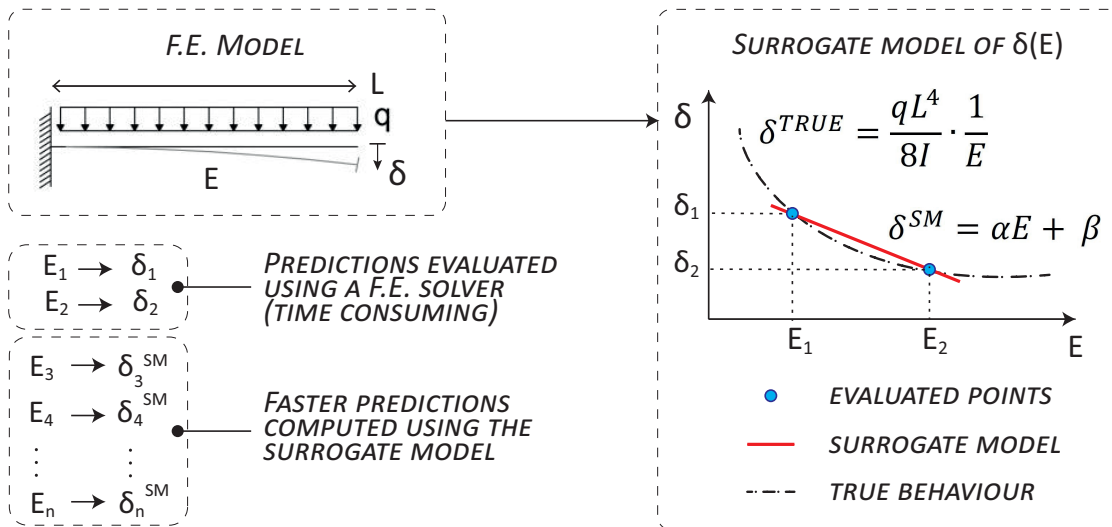


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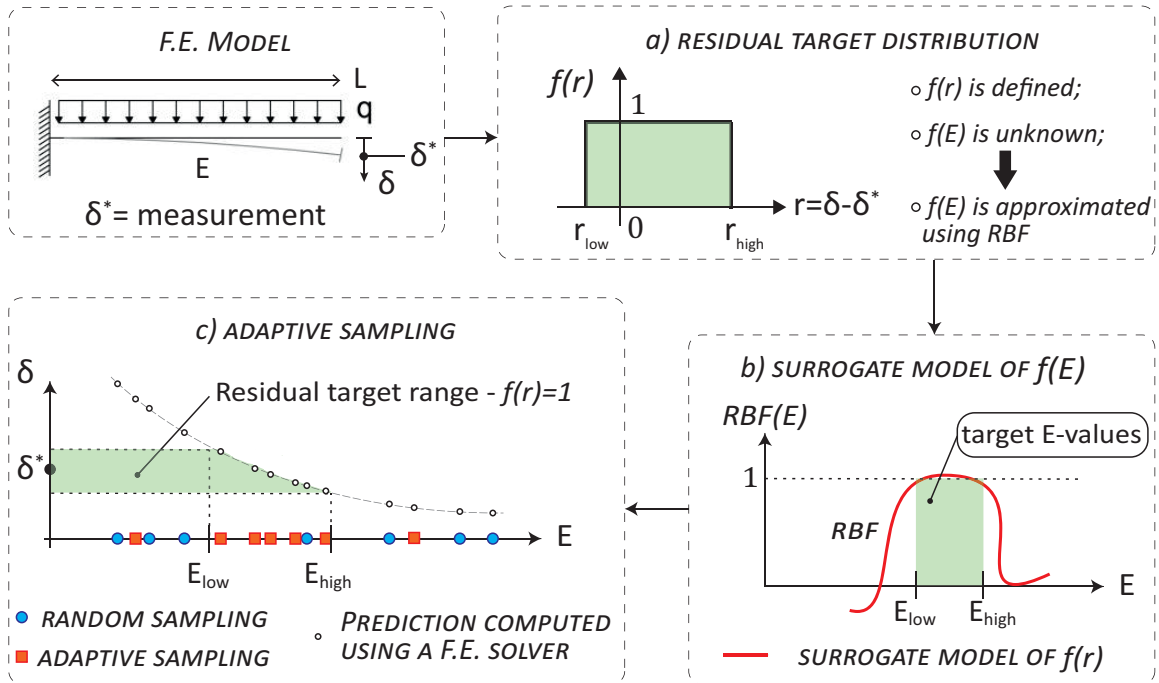


FIG. 4. Surrogate-model of the target distribution. A Surrogate Model (SM) is used to generate samples of the parameter E , for which the residuals ($r = \delta - \delta^*$) between predicted and measured maximum deflections follow the uniform target distribution. The residual target distribution $f(r)$ is defined in the prediction domain a), while the analytic expression the function in the parameter domain $f(E)$ is unknown. Therefore, a SM ($RBF(E)$) is used to approximate the function f in the parameter domain b). Then, the SM helps find target E -values for which $f(E) = 1$. Hence, the SM is used as an objective function to guide the sampling of E -values for which residuals follow the target distribution c).

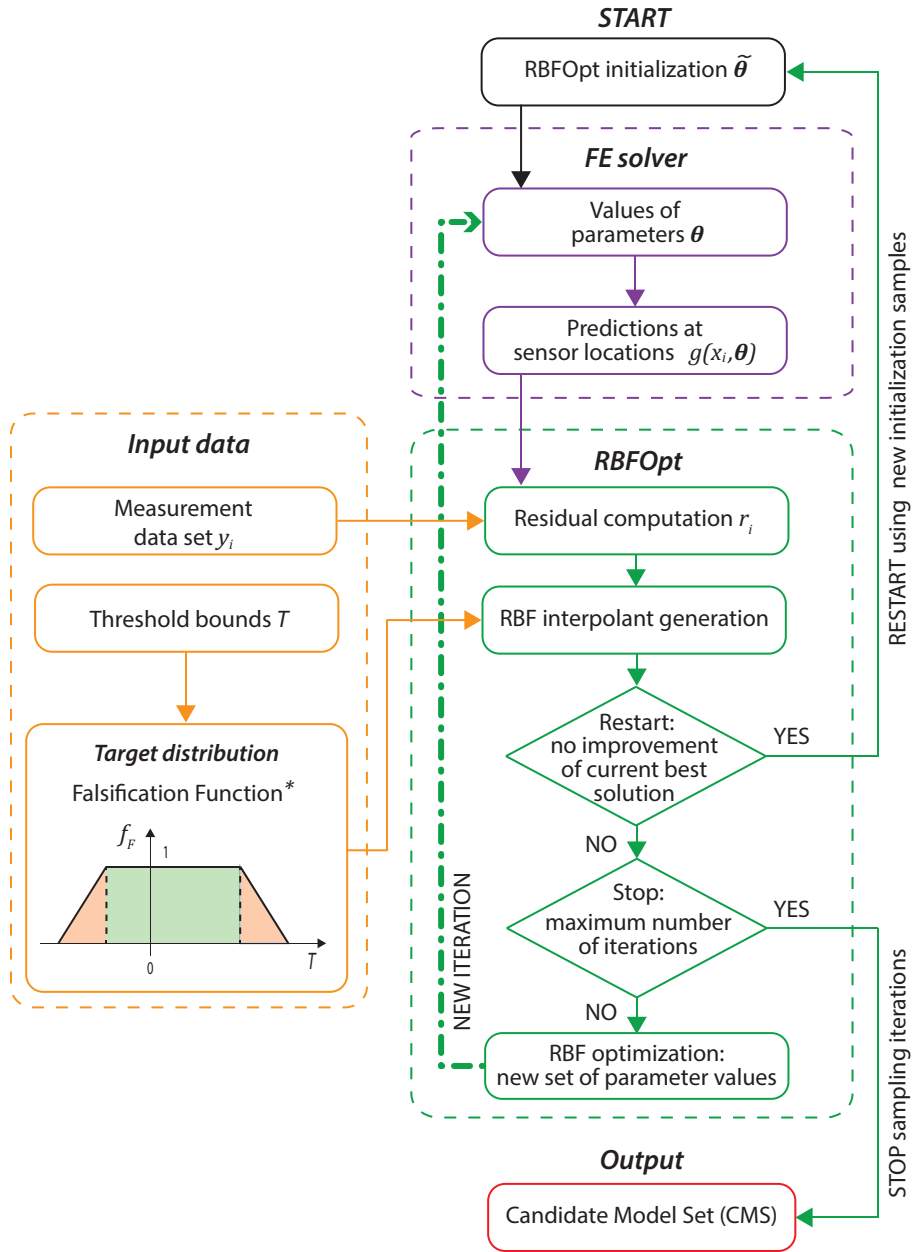


FIG. 5. General framework of the radial-basis function sampling (RBFS). (*) For the Falsification Function refer to Figure 7.

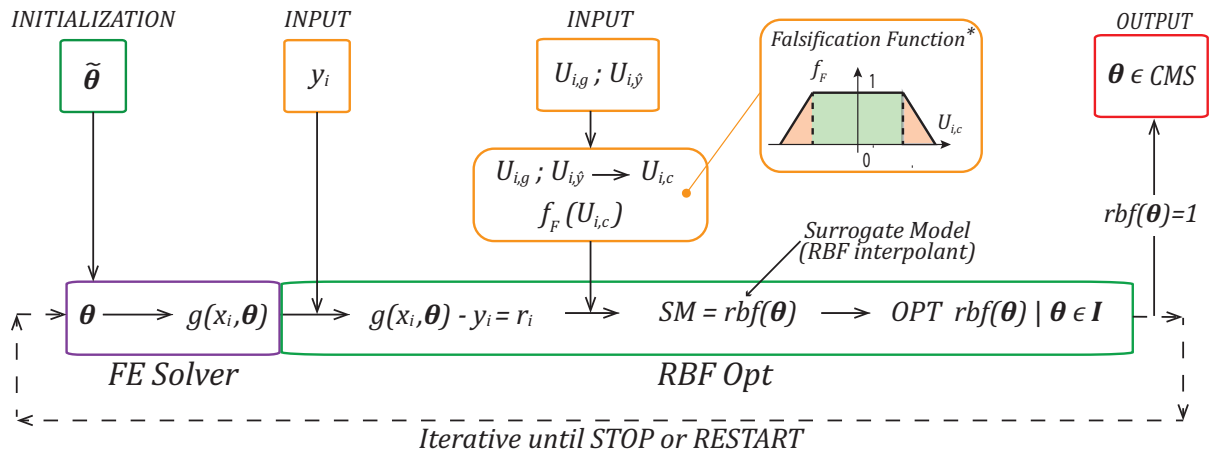


FIG. 6. Flow chart of the radial-basis function sampling (RBFS). (*) For the Falsification Function refer to Figure 7.

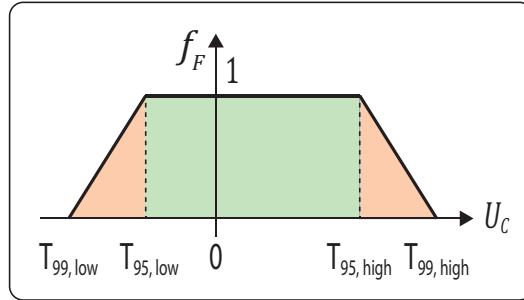


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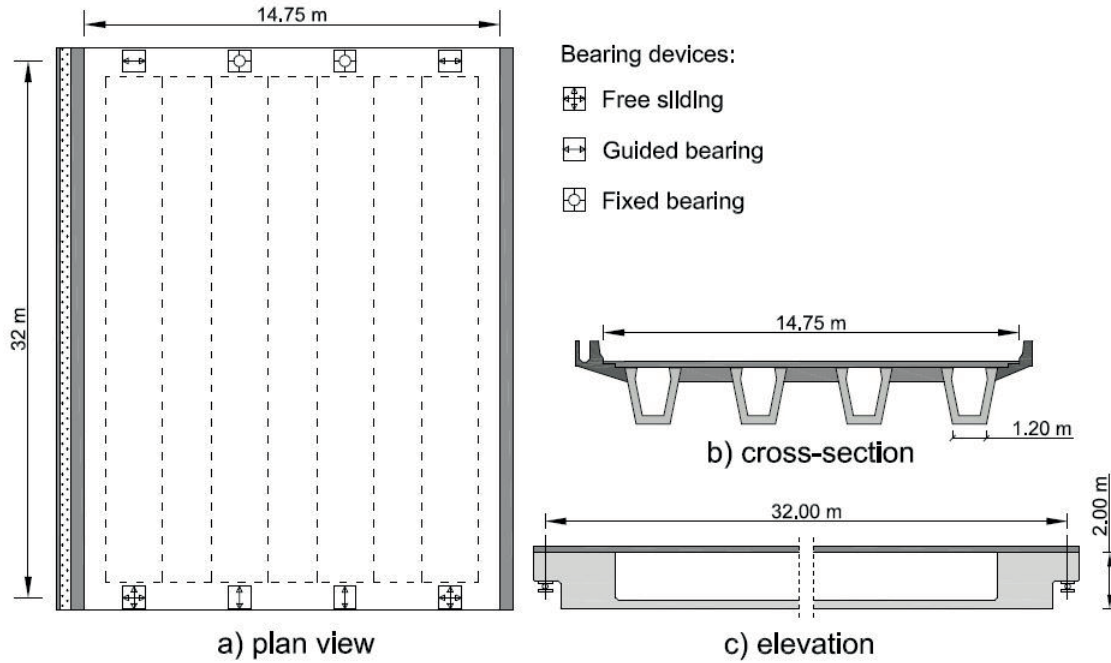


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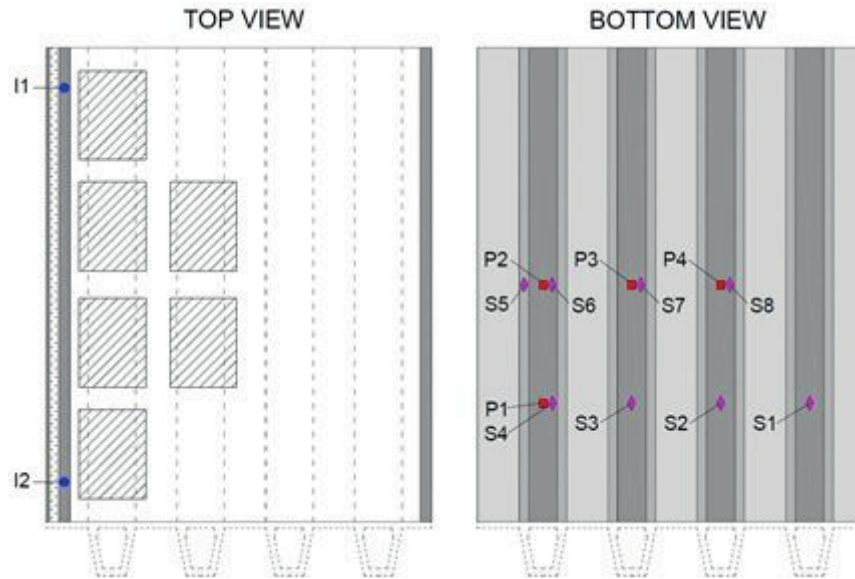


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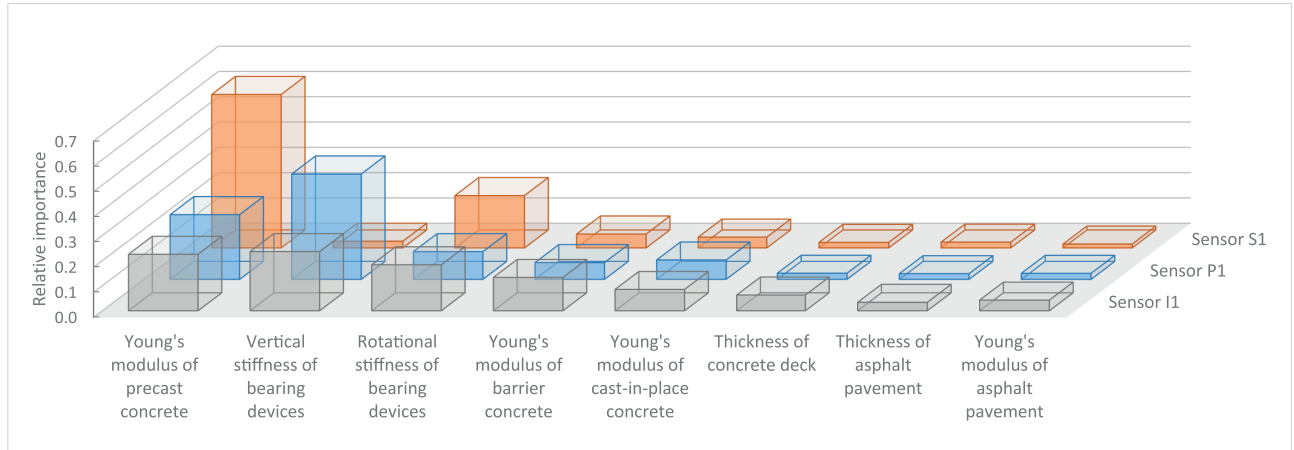


FIG. 10. Relative importance of parameters θ on model predictions at three sensor locations (P1, S1 and I1).

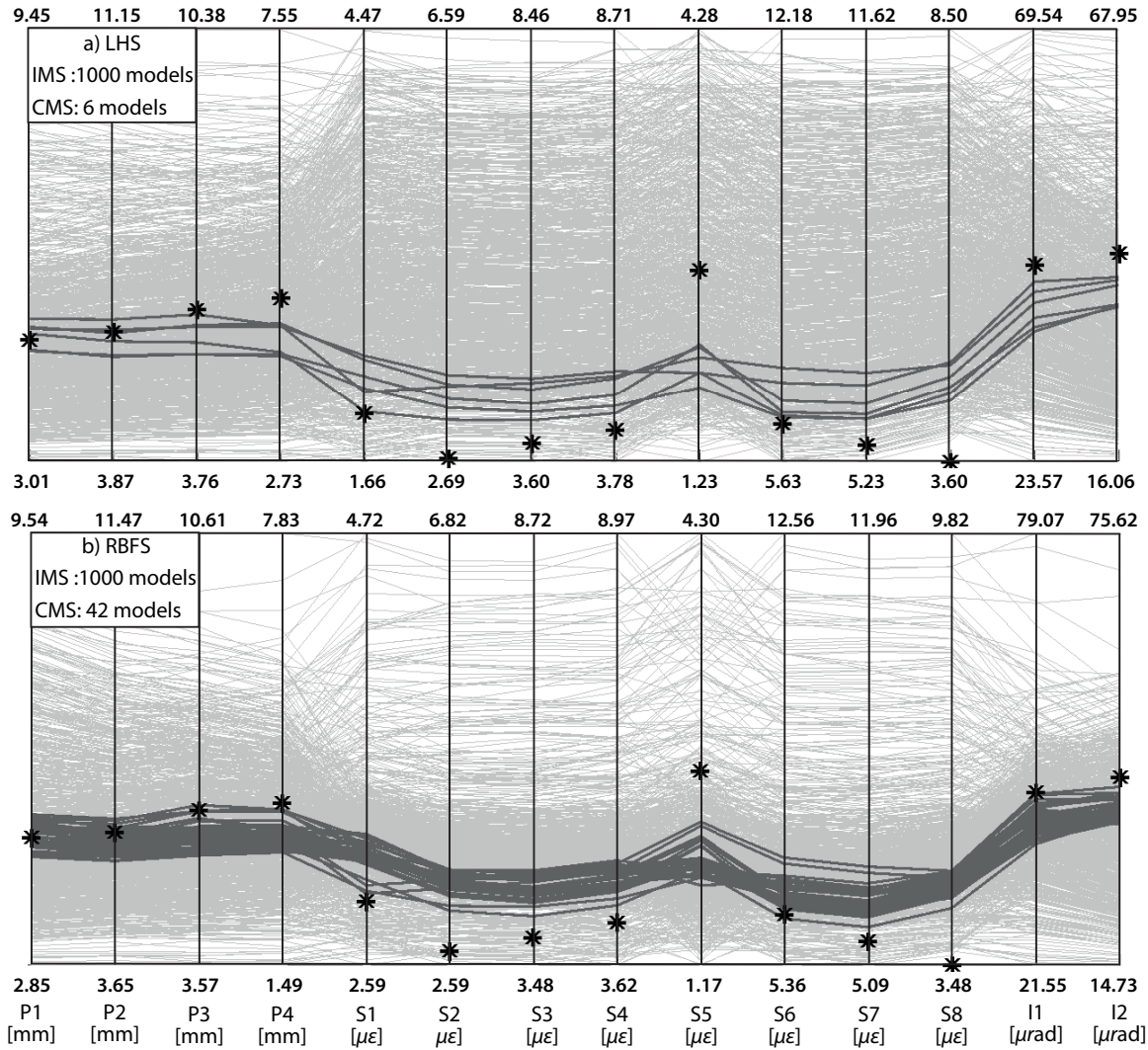


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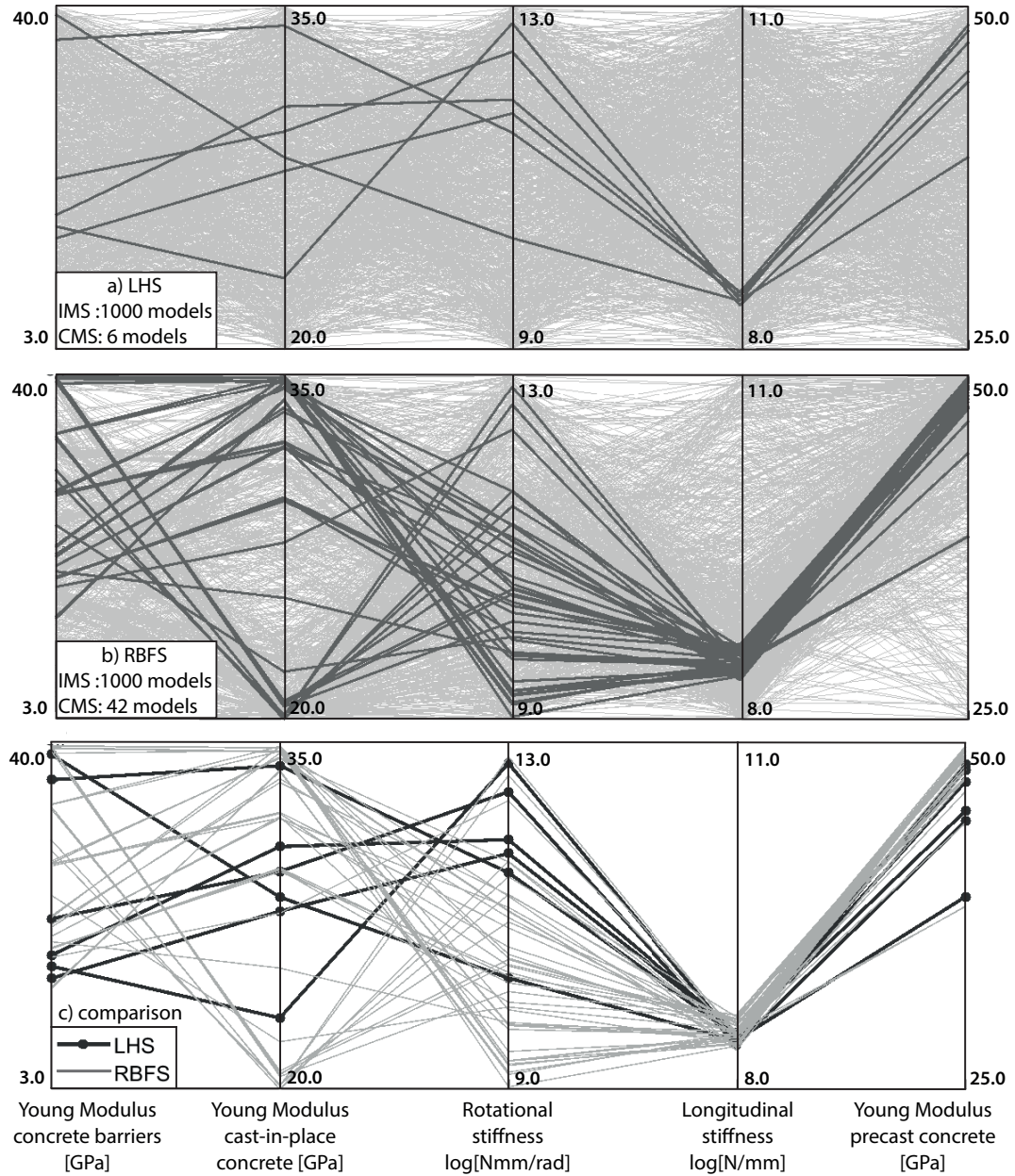


FIG. 12. Parameter values describing 1000 model instances generated using LHS and RBFS. Each vertical axis represents a parameter. Initial model instances (grey lines) and candidate models (dark lines) are plotted and initial values of each parameter are given for each axis. The comparison of candidate models is shown in c).

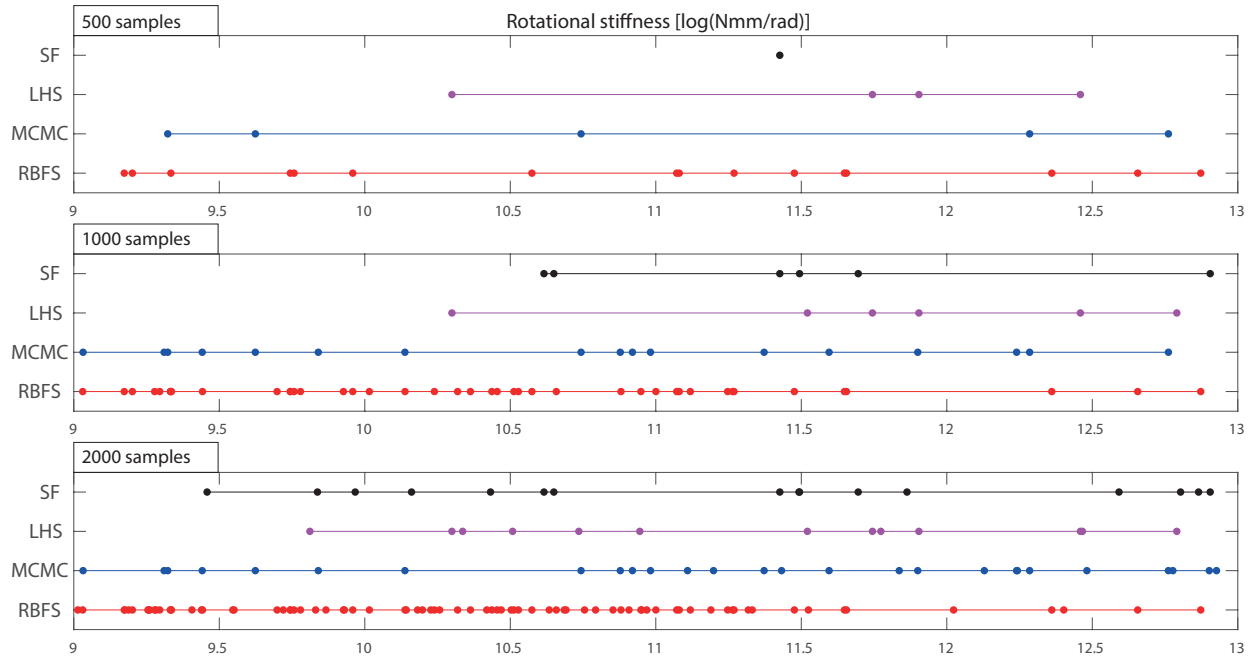


FIG. 13. Comparison of rotational stiffness values identified using RBFS, MCMC, LHS and SF. Falsification has been conducted considering three initial model sets that consist of 500, 1000 and 2000 samples. RBFS provides the largest CMS and the most effective exploration of the candidate domain, even with 500 samples.

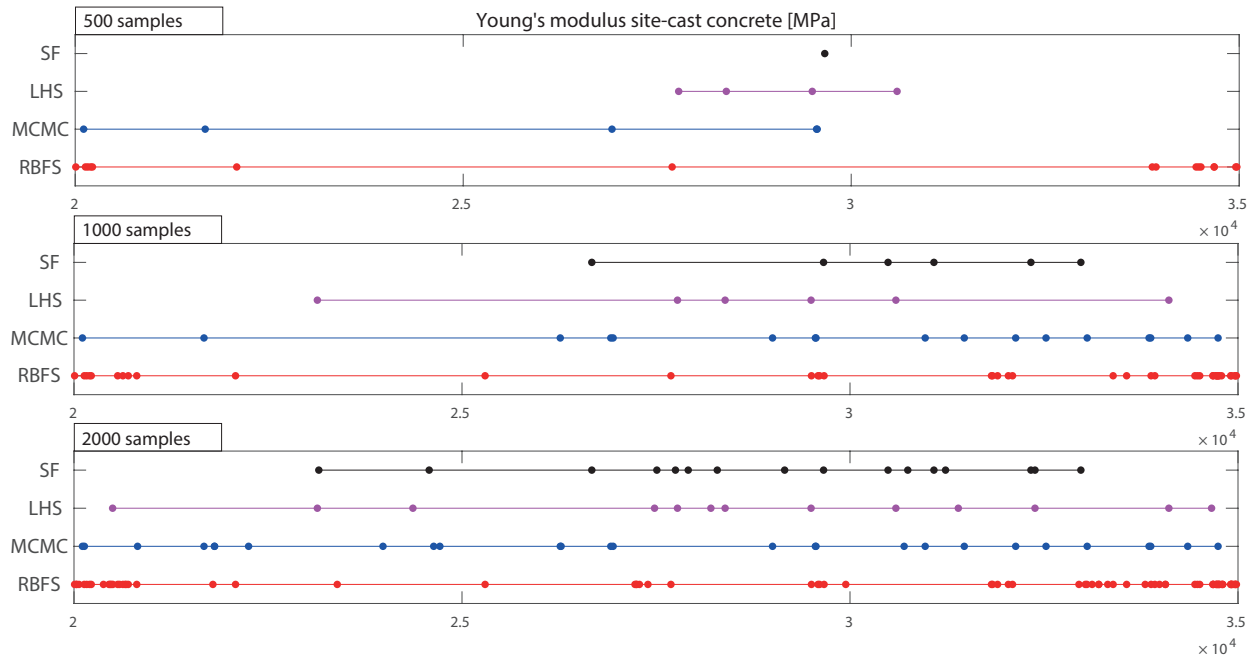


FIG. 14. Comparison of Young's modulus values for cast-in-place concrete, identified using RBFS, MCMC, LHS and SF. Falsification has been conducted considering three initial model sets that consist of 500, 1000 and 2000 samples. RBFS covers the entire candidate domain within 500 samples, performing better than MCMC.

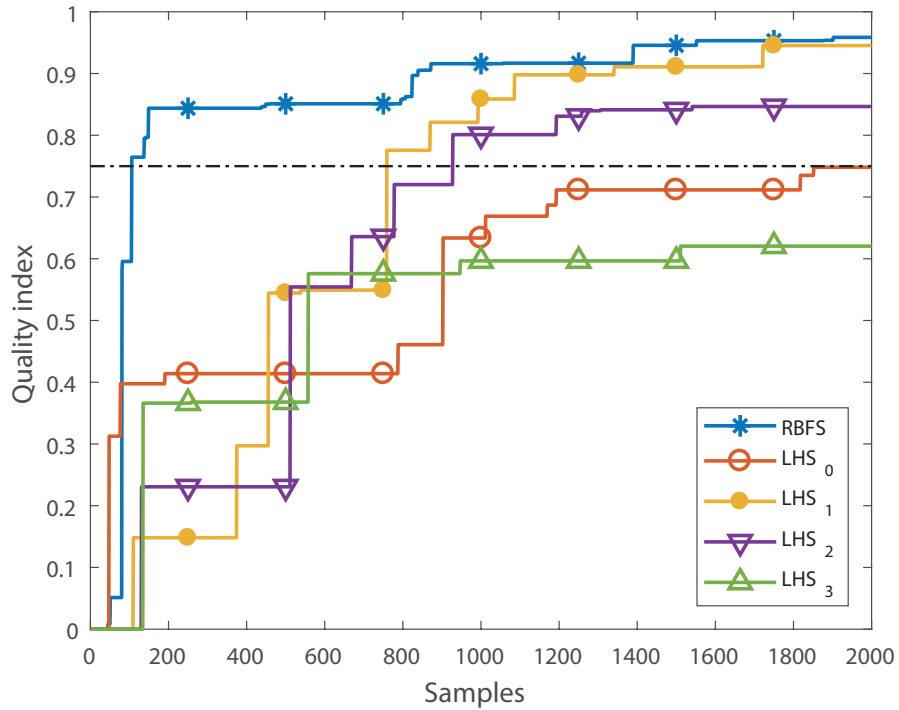


FIG. 15. Comparison of the candidate domain provided by 2000 samples generated using RBFS and four LHS. LHS₀ denotes the default LHS-setting implemented in ANSYS. Three more populations (LHS₁, LHS₂ and LHS₃) have been generated through a random selection of the sampling seed. The quality of candidate domain exploration provided by LHS is affected by the random choice of the seed value.

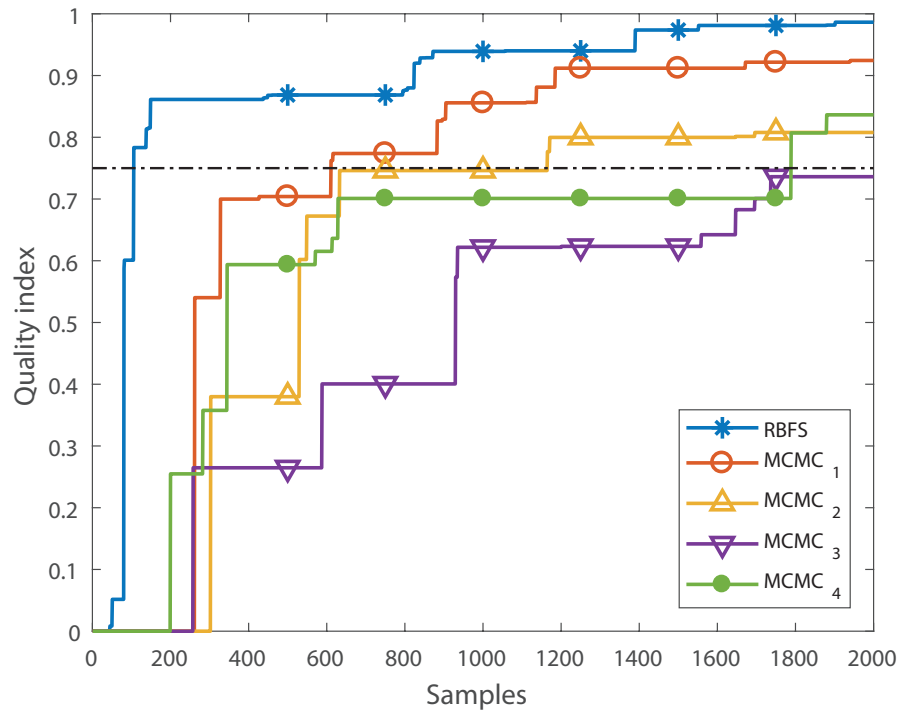


FIG. 16. Comparison of the candidate domain provided by 2000 samples generated using RBFS and four MCMC characterised by different proposal widths. MCMC₁ denotes the best parameter setting found after four attempts. Since RBFS does not require initial tuning, there is less risk of low-quality results.

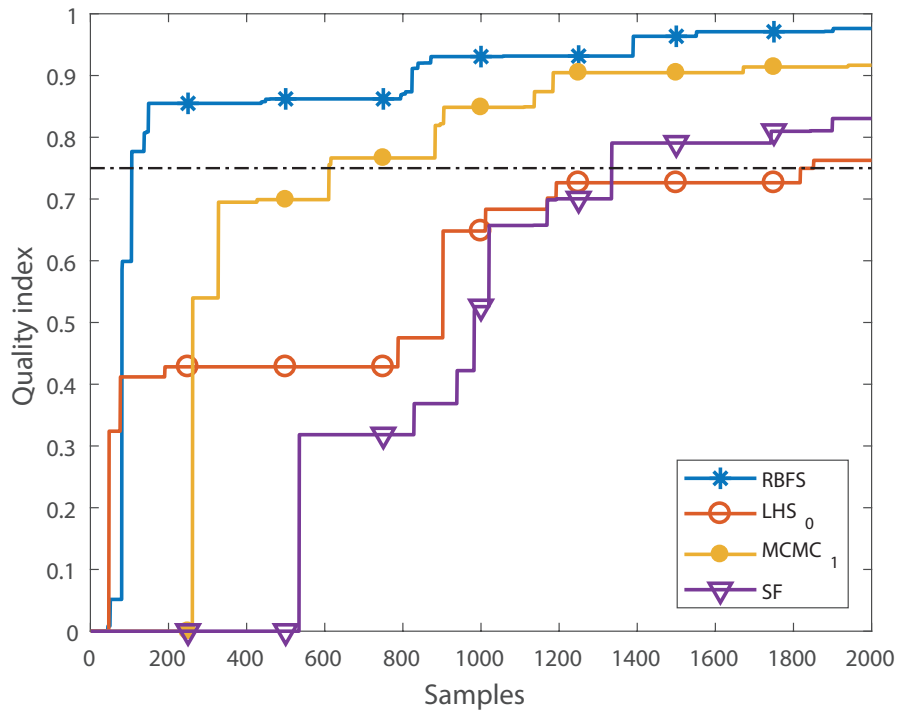


FIG. 17. Comparison of the candidate domain provided by 2000 samples generated using RBFS, LHS₀, MCMC₁ and SF. The main drawback of non-adaptive sampling is the slow exploration of the domain with increasing number of samples.

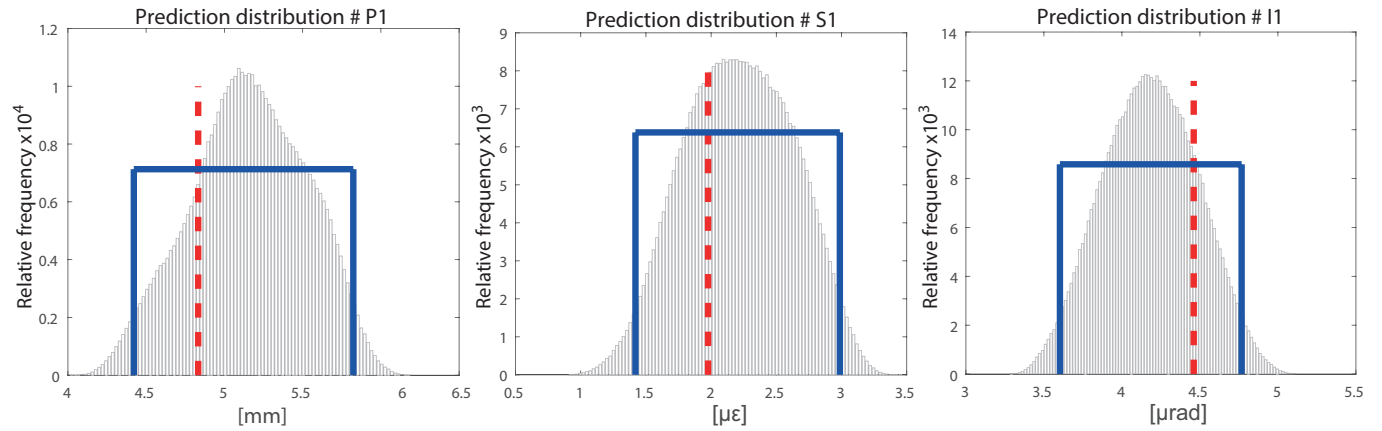


FIG. 18. Distribution of candidate-model-set predictions (PD) at three sensor locations (P1, S1 and I1) that have not been used for falsification. The uniform PDs for a 95% confidence and the CMS prediction distributions are plotted. For each sensor, the PD includes the measured value. The PDs have been identified using 2000 initial samples generated through RBFS.

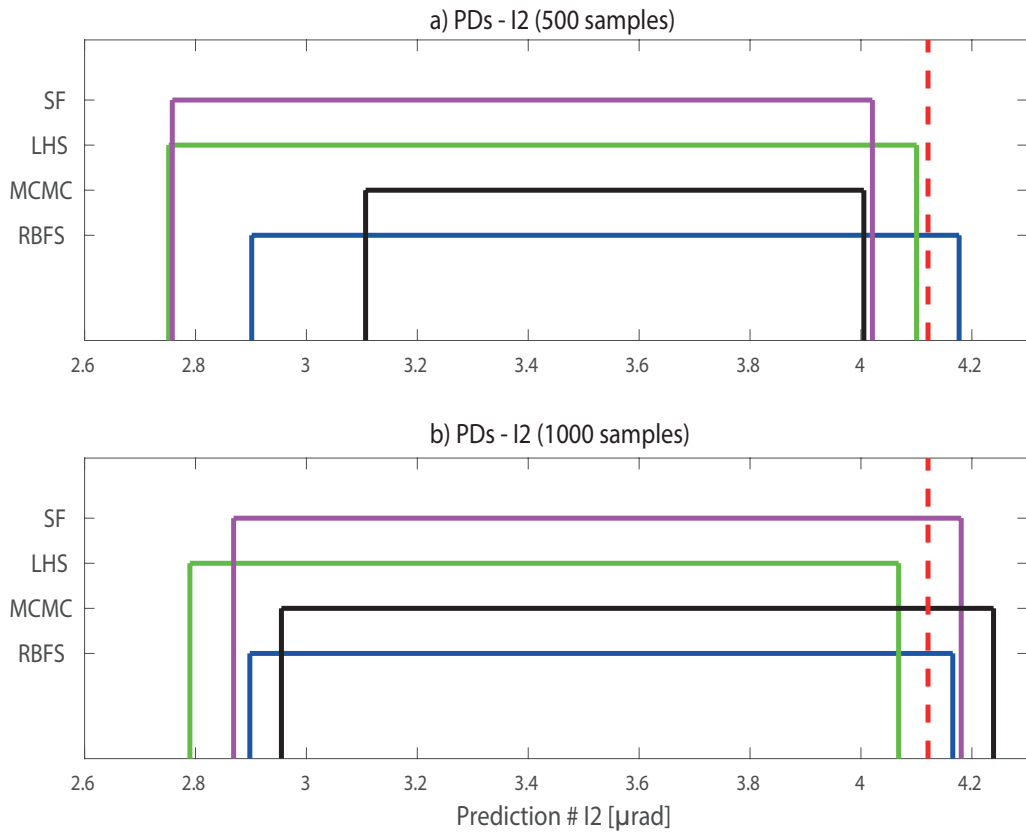


FIG. 19. Uniform distributions of candidate-model-set predictions (PD), for a 95% confidence, at sensor location I2. Sensors P2, P3, P4, I1 and I2 have not been used for falsification and two PDs have been identified using respectively 500 (a) and 1000 (b) initial samples. The measurement value at location I2 is shown (dashed line).