# A REACTION-DIFFUSION MODEL FOR CONGESTION PROPAGATION IN URBAN NETWORKS

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Number of words: 4,998 + 2,000 (8 Figures) = 6,998
(Submitted for presentation only)

Transportation Research Board 96th Annual Meeting, January 8–12, 2017, Washington, D.C.
Special Call: Advances in Modeling and traffic management for large-scale urban networks.
Traffic Flow Theory and Characteristics
Committee: AHB45

## ABSTRACT

While cascade phenomena have been broadly studied by physicists, understanding and modeling of congestion propagation in large urban city networks still remains a challenge. Most efforts are mainly based on micro-simulations of link-level traffic dynamics without a proper treatment of physical laws. The main purpose of this paper is to reveal the process of congestion formation by exploring empirical and simulated data from large-scale urban networks. Specifically, the authors aim at studying the spatiotemporal relation of congested links, observing congestion propagation from an macroscopic perspective, and develop a dynamic model with a few number of parameters that can properly reproduce the spatiotemporal distribution of congestion and cascade phenomena of traffic. The model is based on two ingredients: a reaction and a diffusion term. The interaction of these two terms brings the model in a self-organized pattern that after appropriate calibration can reproduce realistic traffic scenarios. Vehicles spread through the urban network by diffusion as well as the values of average link speed according to a Fundamental Diagram that relies on density, flow and speed [(*9*), (*6*)]. The reaction term will be the responsible of any exogenous change of concentration of vehicles, e.g. exogenous demand. The combination of these two terms will reproduce many different traffic scenarios. The results presented show very good data matching with an available data set of more than 20k taxis GPS in Shenzhen during the morning peak hour.

**Keywords:** Traffic model; reaction-diffusion system; complex networks; traffic data analysis; congestion propagation.

## INTRODUCTION

In this work it is defined a reaction-diffusion model, inspired by a very general and well-know biological process (in [(*18*), (*13*), (*5*), (*29*)] are proposed just some of the various and wide-spreading applications in all scientific domains), to reproduce the congestion propagation throughout a network of urban streets. In the physical literature one can find many models that associate the traffic propagation to a diffusion gas-kinetic-like phenomenon or fluid dynamics process (several examples could be found in [(*23*), (*10*), (*11*), (*20*), (*22*), (*24*), (*16*), (*17*), (*12*)]). Others recent and promising traffic models come from the MFD (Macroscopic Fundamental Diagram) theory [(*6*), (*7*), (*19*)]. They suppose to divided the urban network into macro-regions which share the same MFD and calculate the inflow and out flow from one region to another [(*25*), (*16*), (*17*), (*27*), (*14*)]. This approach is very interesting because it does not need very accurate and local data, so often impossible to obtain, and look to the urban traffic from a macroscopic point of view that had also some advantages in terms of computational cost and complexity. Most of these models need to have a good and efficient clustering algorithm that would individuate the homogeneous regions with a clear non-ambiguous MFD and an aggregated OD matrix to take into account the different demand across regions.

Inspired by the advantages of a macroscopic model and with the aim to possible overcome to the lack of accurate data the model proposed is based mainly in two general principles. The minimal number of parameters that it needs to be calibrated whenever some traffic data are available is only two. Nevertheless, in order to reach more precision it is possible to calibrate more parameters per regions.

One of the fundamental step in these studies is to collect traffic data and try to replicate real traffic characteristics in simulation environments. Researchers in this domain have proposed many different models for traffic simulation that take into account the main features for congestion propagation and involves the structure of road network and the OD matrix [(*2*) , (*28*), (*21*), (*1*), (*26*)]. Models are important because they help us to calibrate some parameters to forecast an improvement or the reaction of traffic to some temporary accidents or events and, at the end, to have a better understanding of this phenomenon of congestion and its relationship with the characteristics of each particular urban framework.

Detailed models of traffic congestion with a reasonable spatio-temporal resolution requires entries of time dependent Origin-Destination (O-D) matrices that are sufficiently large. O-D matrices are difficult to be estimated with a high level of accuracy especially in the dynamic case. Many works have also highlighted the chaotic character of traffic dynamics especially under congested conditions [(*3*)]. Small perturbations to the O-D tables or small changes to drivers route choices can drastically change the aggregate outputs of detailed models [see for example (*4*)]. Developing a simple dynamic model for congestion, which will rely on a few number of parameters and would not require tedious calibration, it can be useful to reveal hidden information in the development and propagation of congestion and contribute to the development of efficient control strategies at a later stage.

In this work an elegant model is proposed. If one observes the congestion propagation from a macroscopic point of view it is quite evident to realize that it looks like to a biased diffusion phenomenon in a network. From this simply observation the model proposed tries to replicate this propagation by calibrating some parameters in order to match the real data come from different cities. In particular, some results from an available data set of more than 20k taxis of Shenzhen

(China) are shown in the following sections.

The model proposed for urban traffic involves the topological structure of the road network and local demand. Thanks to this model it is possible to reproduce, with a quite big accuracy, speed evolution during the peak hour but also simulate the traffic condition for a whole day, considering onset and offset of congestion with a very few number of parameters and without information about origin-destination matrices, detailed route choice and evolution of link speeds.. Based on the speed data, this model uses the diffusion process for Average Link Speed (ALS) throughout the network to simulate the congested component growing and propagating. During the process the diffusion term can not change the sum of all ALS values of the network but it changes only the distribution (see Methods for more details). We know that during a peak hour and in general during the day the global ALS changes (decreases in congested condition) that is the speed of each link evolves based on the speed o its neighbors and an overall congestion level. For this reason it needs also a reaction term that changes the ALS values by increasing or decreasing it. Here, the authors propose an unique function to simulate it and they associate to this a *reaction parameter* $\rho$ that can be depending on time and link or region and its reason is to weight the effect of function $f$.

The system of differential equations of the reaction-diffusion model is presented in the first section. Some discussion on the functional form of the reaction term and on the diffusion term is developed in the following two consecutive sections. The calibration topic of the model's parameters of three different solutions are proposed in the following section. Some results and figure are presented in the second to last section. In the very last section further extensions and applications are proposed.

## THE REACTION-DIFFUSION MODEL

The model is composed by two parts: a diffusion term and a reaction term. These will operate in a network, represented by a graph $\mathcal{G}(N, E)$ of $N$ nodes and $E \subset N \times N$ links. The diffusion part will be regulated by the combinatorial Laplacian $L = A - kI$ where $A$ is the corresponding adjacency matrix of graph $\mathcal{G}$ and $kI$ the diagonal matrix with the corresponding node degree $k_i, i \in N$ as entries. For the purpose of this work it will be more simple and useful to use the dual representation of the graph where component $i \in N$ will represent every link (road) $i$ and the elements of the adjacency matrix $A = \{a_{ij}\}$ will represent the intersections. In particular the element $a_{ij} = 1$ if and only if link $i$ and link $j$ are adjacent, 0 otherwise. In the matrix $L$ the elements on the diagonal $l_{ii} = k_i$ where $k_i$ is the number of the adjacent links of $i$.

Instead the reaction is regulated by a non linear function $f(\bar{u}, t)$ depending, in general, on the vector of ALS $\bar{u} = \{u_i\}_{i \in N}$ and the time $t$. The most general form of the differential equations for every component $i \in N$ of the vector $\bar{u}$ will be:

$$\frac{du_i(t)}{dt} = \rho(i,t) f(\bar{u}(t), t) + \sigma(i,t) \sum_{j=1}^{N} L_{ij} u_j(t). \tag{1}$$

The parameters $\rho$ and $\sigma$ will be the reaction and diffusion parameter respectively. In general, they could depend on space (link $i$) and/or time $t$.

The diffusion term $\sigma(i,t) \sum_{j=1}^{N} L_{ij} u_j(t)$ changes the distribution of ALS values among the links of the network while the reaction function will be the responsible for the change of the sum of ALS ($\sum_i u_i$). The weight for the two terms can be regulate by their respective parameters $\sigma$ and $\rho$.

It is easy to show that this system of differential equations with these two terms is possible to simulate almost any continuous transformation in urban traffic and replicate speed distribution in time. But the question is if it is also feasible in sense of computational cost and complexity. Estimating space- and time-dependent $\rho(i,t)$ terms could be very challenging as detailed local information might not always be available. It is also expected that traffic demand, turning movement ratios, route choice can influence the values of these parameters. Instead our objective is to properly identify a small number of these variables that can generate realistic aggregated congested patterns and their evolution across time and space, e.g. spatiotemporal distributions of link speeds. Thus, the final outcome of this work is not the accurate estimation of speeds for every link in a large network, but an elegant physical law that can generate realistic aggregated congestion patterns.

## THE REACTION TERM

As functional form for the reaction term it has been chosen the following one:

$$f(t,i,\bar{u}) = log(C(t,i) + du_i) \tag{2}$$

where $du_i = \sum_{j\in N(i)} \frac{(u_j - u_i)}{max\{u\} - min\{u\}}$ computes a normalized difference (not in absolute value) for speed of link $i$ to each neighbors $j \in N(i)$. The term $C(t,i)$ plays an important role for the increasing or decreasing general behavior. In most of the case it will be simply a constant, that is $C(t,i) = C$, to simulate a monotonically traffic behavior but it some other cases it can be depended on time and/or on space as a last example in this paper will show. This function has been selected among many others for various reasons. The first purpose was to match the simulation with the real data from Shenzhen during peak hour (6am - 8am), that is the onset of the daily morning congestion. The term inside the reaction function has been fixed to be constant, that is $C(t,i) = 1, \forall i \in N, t < T$. Here the principle is: for each link to look at its neighbors and based on the difference between the current link speed increase or decrease proportionally the speed $u_i$. For instance if link $i$ is surrounded by more congested links (and so $du_i < 0$) the function $f$ will return a negative value that decrease $u_i$. It simulates the fact that when a link surrounded by congested links tends to be congested as well and also it means that this part of the city likely has a high demand. In this way, from a free-flow regime (at 6am) one can automatically see the congestion rise faster in those regions where already there is a concentration of more congested links because of spatial correlations (see empirical evidences in (*7*)).

With this simple reaction function $f(i,\bar{u}) = log(C + du_i)$, it is possible to simulate onset and offset of congestion. In many cities the behavior of congestion follows roughly a sinusoidal function (see (*8*)) with two peak hours (morning and evening) in a normal day. Then the idea is to set the $C(t,i)$ in a function proportional to, for example, a cosinus depending on time. In particular, the authors chose $C(t,i) = 1 - 0.2 * cos((t/T) * \pi$, and so the reaction function becomes:

$$f(t,(\bar{u})) = log(1 - 0.2 * cos((t/T) * \pi) + du). \tag{3}$$

The results of a theoretical whole-day simulation are shown in Figure 7. This specific logarithmic functional shape for the reaction term has been chosen also for the follow mathematical characteristics:

$i)$ when $\alpha = 1$ the function $f(x) = log(\alpha + x)$ in zero results $f(0) = 0$. In this way if there is no difference between a link $i$ and all his neighbors the reaction term will have no effect on the speed of that link.

$ii)$ if $0 < \alpha + x < 1$, $|log(\alpha + x)| > |log(1 + [1 - (\alpha + x)]|$ that means for $\alpha = 1$, in average, a regular distribution of ALS of the whole network tends to decrease.

$iii)$ It is possible to regulate $\alpha$ in basis of the demand in time. Usually, it follows a sinusoidal behavior and for this reason it has been chosen to set $\alpha = (C + 0.2 * \cos((2t/T) * \pi))$ where $t$ represents the time and $T$ the period of peak hour (morning p.h. and evening p.h.)

It is worth to notice that the term $C(i, t)$ can have many different forms that they will be reflected in the global average link speed. This is because increasing or decreasing $C(i, t)$ has as effect to increase or decrease the positive contribution of the function $f$ for speed change.

## THE DIFFUSION TERM

In the model the flow of vehicles among the links of the network is simulated by the diffusion term. In a link, inversely to the value of density of vehicles, the speed follows also diffusional behavior. In this sense if a link $i$ has a lower ALS then for some of its neighbors ( $u_i < u_j$ for $j \in N(i)$) the speed will increase proportionally to a parameter $\sigma$ in link $i$ and decrease in link $j$ because of diffusion. This term reflects the fact that drivers in front of a congested road tend to occupy a more empty link to be able to pass over the congestion and arrive to their destinations. In the results shown in this paper, $\sigma$ is a nonzero constant for all the network. It is clear that this diffusion parameter can be set for every couple of neighbor links in order to have a more detailed network that considers also flow direction. For instance it can be possible to set $\sigma(i, j) = 0$ (diffusion parameter between link $i$ and $j$) that would mean no correlation between link $i$ and $j$ and so no vehicles can choose link $j$ instead of link $i$. According to this strategy it is possible to calibrate even more specifically and in details the model framework. The diffusional parameter $\sigma$ can be set proportionally to the roads *intra-correlation* whenever it is possible to deduce from data (for instance from historical trips data and/or structural road characteristics). As detailed calibration is beyond the scope of this paper, we investigate if a model with a small number of parameters can replicate spatiotemporal features of complex networks.

## CALIBRATION OF REACTION AND DIFFUSION PARAMETERS

In the most general and simplest case it needs to calibrate only two parameters in order to obtain the closest results to the real data. All simulations start always from the same initial configuration that corresponds to the real ALS data in the Shenzhen network at 6am. It has been chosen an integration time step $dt$ appropriately little and a final time $T$ large enough such that the ALS reaches the value of the real data at 8am (as shown in Figure 3). This can be easily regulated tuning up and down the parameter $\rho$. Looking at the distribution of ALS during the simulation it can be possible also regulate the diffusion parameter $\sigma$. A higher value for $\sigma$ means that at the end the distribution is more homogeneous, in the sense that the difference among links will be eliminated faster than the reaction effect (that implements in fact dissimilarity). Then in one hand the reaction function tends to increase the highest values and decrease the lower values among links, and so create more heterogeneity in ALS values. On the other hand, $\sigma$ that is associate to the diffusion phenomenon, spreads the value and reduces the difference between neighbors links.

After some adjustments (see Methods) it has been found the best couple of $(\rho, \sigma)$ in order to get the minimum distance, in terms of Mean Square Error, between the results and the data from Shenzhen taxis dataset.

## RESULTS

### A toy example: the grid network

In order to show the mechanism and the main features of this model it will be presented in this section some examples about how the RD model behaves when the network is a regular grid and how the reaction and diffusion term influence the results. As already mentioned, the reaction term plays a fundamental role in *congestion generation* by regulating, in same sense, the heterogeneous demand on a urban network. In a first example shown in Figure 1 it is possible to appreciate the effect of two different reaction parameters $\rho_c$ and $\rho_p$, respectively for the city center $(10 \times 10)$ and in the periphery of what we can consider a squared city with a grid road network composed by $30 \times 30$ intersection and 1740 links in total. In this example it has been set $\rho_p = 1$ and $\rho_c = 2 * \rho_p$ and $\sigma = 0.01$ for all links. One can see that from a first initial condition mainly uncongested with a random distribution of speed ($u_i \in 16 \pm 10$) the network become more and more congested especially in the city center where at the end it appears a congested connected component that expands itself through the urban network. It can be noticed that the average speed decreases (Figure $1b$) and the distribution of ALV maintains a realistic behavior (Figure $1c$).

In Figure 2 are shown the results of a simulation of the RD model with an initial uncorrelated normal distribution of speed in the squared grid. In this case the reaction and diffusion parameters are the same for all links and in particular $\rho = 1.3$ and $\sigma = 0.01$. As one can see from some points of the grid some congested components grow and tend to aggregate themselves in less bigger ones. This phenomenon it has been observed in many real cases where at the beginning of a peak hour from some sparse little congested areas a city pass to have only few extended congested regions at the critical time of a peak hour.

From these two examples one can understand better the effect of the reaction term (Figure 1) and the diffusion term that simulate the propagation of congested areas (Figure 2).

### Simulation and real data: Shenzhen city

Shenzhen is a major city in the south of Southern China's Guangdong Province, situated immediately north of Hong Kong. The area become one of the most successful economic zones in China. The rapid foreign investment created one of the fastest growing cities in China, with an urban population close to 11 million. As expected Shenzhen has now large congestion problems both in the urban and freeway system of the city (see (*15*) for more details). All the results presented in this paper referred to Shenzhen networks and in particular to some available real data from 6am to 8am of Thursday 01/09/2011, a typical working day.

After having set the parameters $\rho$ and $\sigma$ the simulation returns with a good approximation the evolution of the average speed in the network like shown in Figure 3. The plot of the mean of all ALS is determined mostly by the reaction term and, in the case studied with fixed the function $f$, simply by the only reaction parameter $\rho$ that in fact gives a weight the reaction function $f$. To

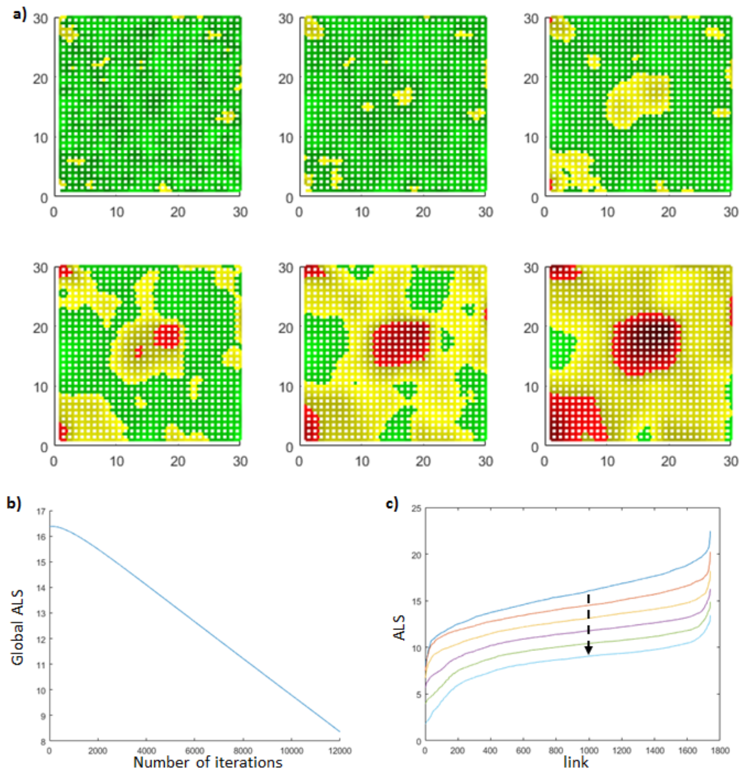


**FIGURE 1 RD model in a grid network $30 \times 30$ with differentiated reaction term. In panel $a)$ are reported 6 snapshot of the simulation of the RD model starting from a random distribution of speed and with different reaction parameters $\rho$ for the center $(10 \times 10)$ and the periphery. The green link are those considered as free flow $(ALS > 11)$. Yellow links quite congested $(7 < ALS < 11)$. Red links very congested $(ALS < 7)$. In panel $b)$ the plot of the average speed in the whole network during the simulation time. In panel $c)$ the sorted speed distribution corresponding to the 6 snapshots plotted in $a)$. The dark blue line corresponds to the beginning of the simulation light blue one to the most congested moment of the end of the simulated peak hour.**

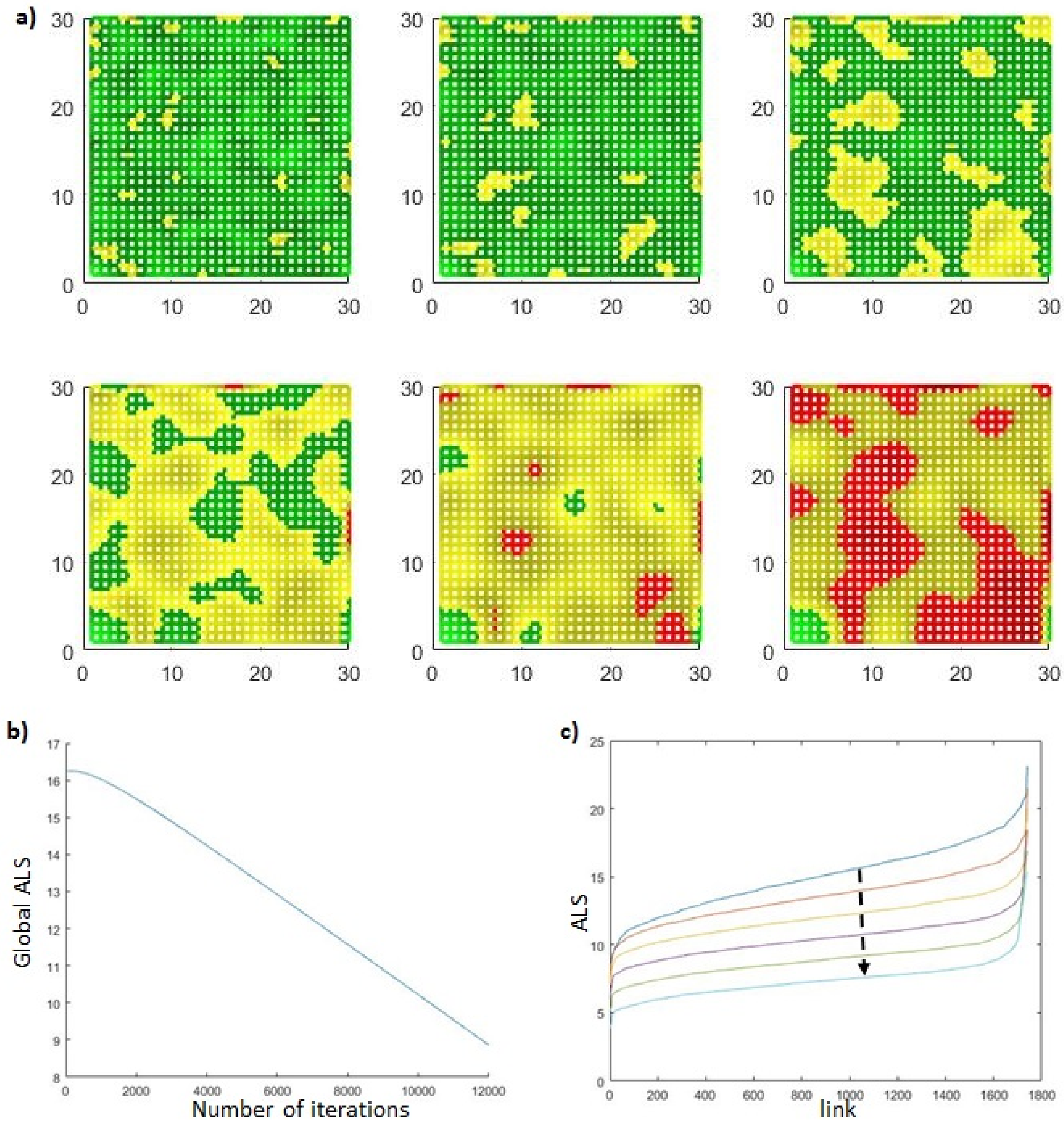


**FIGURE 2 RD model in a grid network starting with a normal distribution of speed. In panel $a)$ 6 snapshots of one simulation starting from a normal distribution of speed. the reaction and diffusion parameters $\rho$ and $\sigma$ are the same for the whole network. The green link are those considered as free flow ($ALS > 11$). Yellow links quite congested ($7 < ALS < 11$). Red links very congested ($ALS < 7$). In panel $b)$ the plot of the average speed in the whole network during the simulation time. In panel $c)$ the sorted speed distribution corresponding to the 6 snapshots plotted in $a)$. It is possible that many sparse congested area tends to decrease further their speed as time pass and they propagate through the network.**

an higher value of $\rho$ corresponds a steeper decrement in time of the mean of ALS. This is due also to the concavity of the function $f$ (for $ii$) as indicated earlier).

In Figure 4 it is reported the ALS distribution for the 2013 links of the Shenzhen network every 5 minutes. The comparison with the real data is very eloquent. These results have been reached thanks to the calibration of diffusion parameter $\sigma$. In fact high value of $\sigma$ tends to homogenize the speed values in the whole network very fast, at least faster than the differential effect of the reaction term. In this case the results would be all the same speed in all links. In the other hand with a low value of $\sigma$ the final distribution will be mostly divided between the minimum and the maximum speed always because of the effect of reaction term without an homogenizing contradictory.

Figure 5 shows the ALS colored map of Shenzhen and the comparison of the results from RD model with real data (show in Figure 6). It is possible to notice two congested components grow and propagate in the network, in particular a bigger one in the top part of the network and a smaller one on the left-bottom part. In the simulation plot one can see the same two congested parts that follow the behavior expressed by the real data in Figure 6.

As already introduced this model is able not only to replicate the onset of congestion during a peak hour but also it can easily be set to simulate the urban traffic for one or more days, with onset and offset of congestion. This can be done, for example changing the reaction function 4 in 3. The effect of the sinusoidal term $cos(t/T) * \pi$ is to move the term $C(t, i)$ back and forwards around the value $1$. This fact gives to the reaction term the tendency to increase or decrease in average the global ALV because more values $du_i$ will return positive (or negative) increment, i.e. $f(du_i) > 0$ ( or $f(du_i) < 0$). A cyclic behavior has been tested in Shenzhen network and the results are shown in Figure 7. As one can see a congestion grows and propagates during the first peak hour that we can suppose to be the morning peak hour and then, after a period of better traffic condition, another congestion rises in coincidence with the next peak hour. The interesting thing is that here has been found an equilibrium between the reaction and the diffusion terms that enable the model to do not homogenize all the ALS on the network with the diffusion factor but it keeps information of the historical data simulating congestion in those located and the most crowed parts of the city.

This encouraging results seems to suggest that the intuition of the dependence of the reaction term by the neighbors values it is reasonable and quite accurate. But if in one hand this reaction term reproduces good results remaining simple and *global* in the other hand it strongly depends on the initial link speed values. A way to avoid rough analysis and predictions is to use historical data to analyze regional demand and then test the model changing the network structure and see how fast the congestion propagates.

As one might expect, if Shenzhen network is divided in *homogeneous clusters* (see (*27*), (*14*)) and the parameters are calibrated to match the regional real data with the simulation one will obtain a better result. Indeed the comparison of the two approaches (global and regional) has been done and the results are shown in Figure 8.

## Methods

The code for the simulations of the RD have been run in Matlab using the network information provide by Shenzhen dataset. The results come from the integration of the differential system

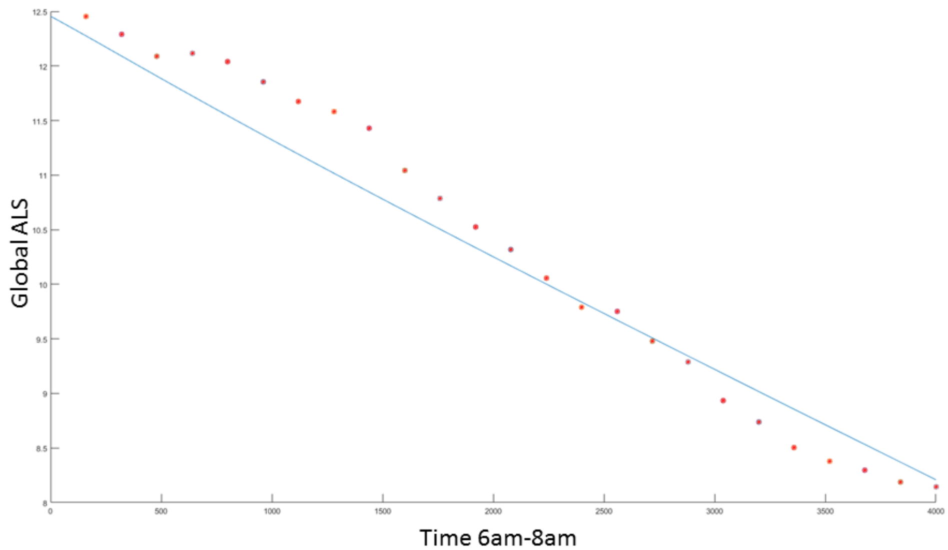


**FIGURE 3** **Mean of ALS for Shenzhen downtown ($\sum_{i \in N} u_i$). In red points are reported the values calculated from the real data during the peak hour from 6am to 8am every 5 minutes. In blue line the same measure computed during the simulation of the RD model.**

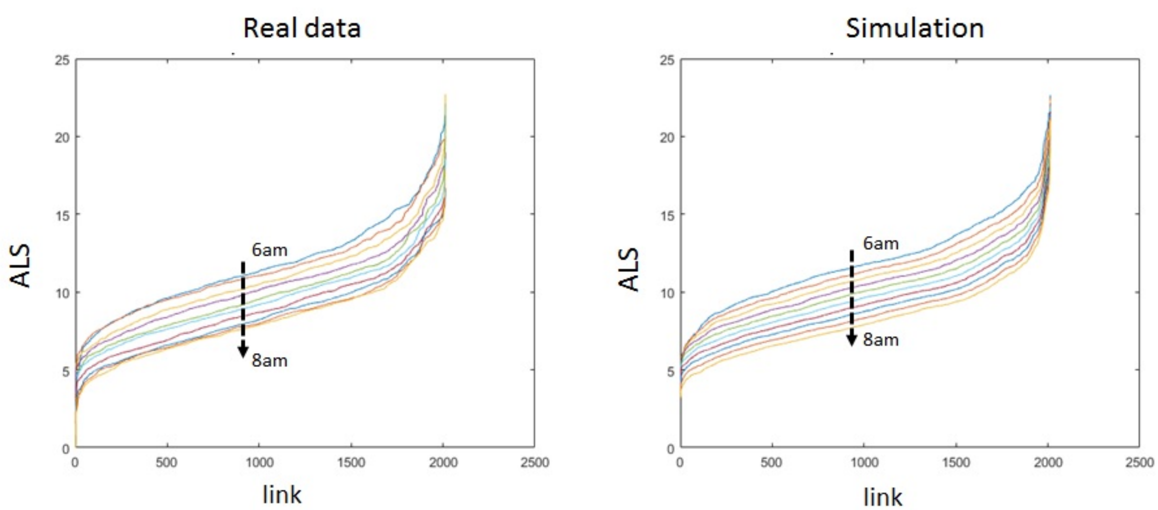


**FIGURE 4** **The comparison between the real data sorted speed distribution (on the left panel) and the link speed distribution obtained by simulation of RD model (on the right panel). According to the average speed that decreases in both case with time from 6am to 8am the different line represent the distribution every 5 minute.**

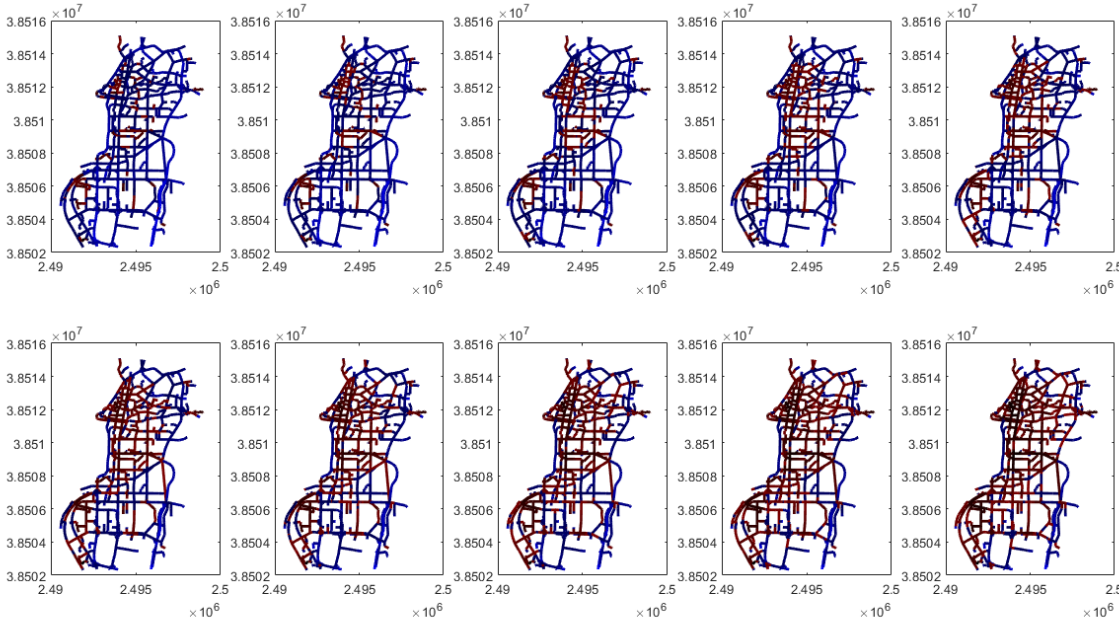


**FIGURE 5 Simulation results for ALS for Shenzhen downtown network shown through a Contour Map. Blue color means high speed link close to the free-flow condition, red link lower link speed that means congestion. The simulation replicates the congestion propagation from 6am to 8am of a typical working day in Shenzhen downtown.**

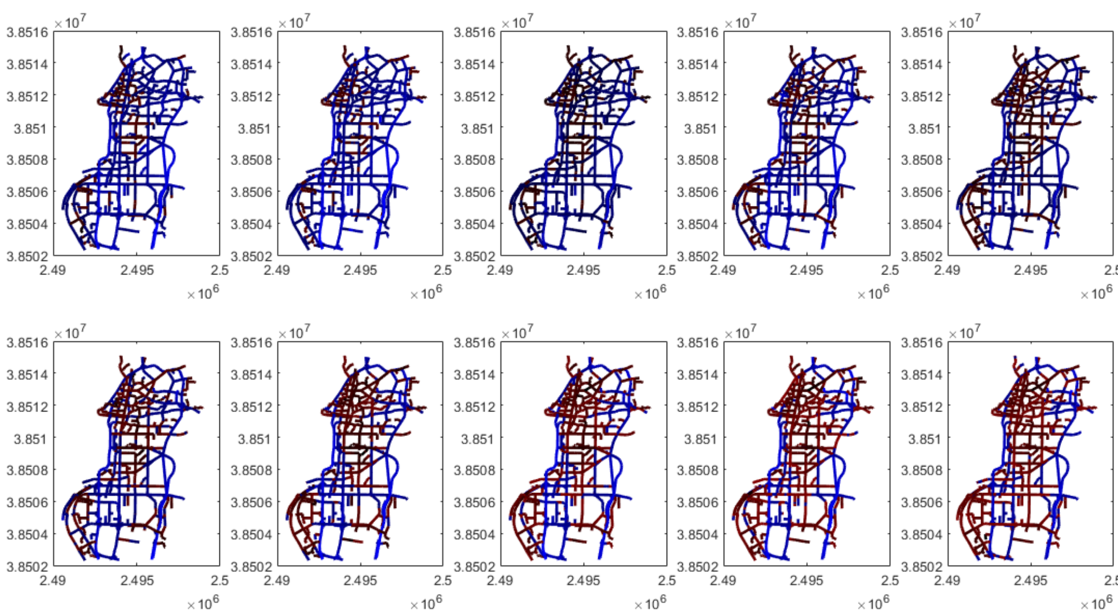


**FIGURE 6 Countour Map of Shenzhen downtown with the colors representing the real data: blue link = high speed link (free-flow), red link = low speed link (congestion) from 6am to 8am. One can see the two congestion parts growing with the time and spread overall the network.**

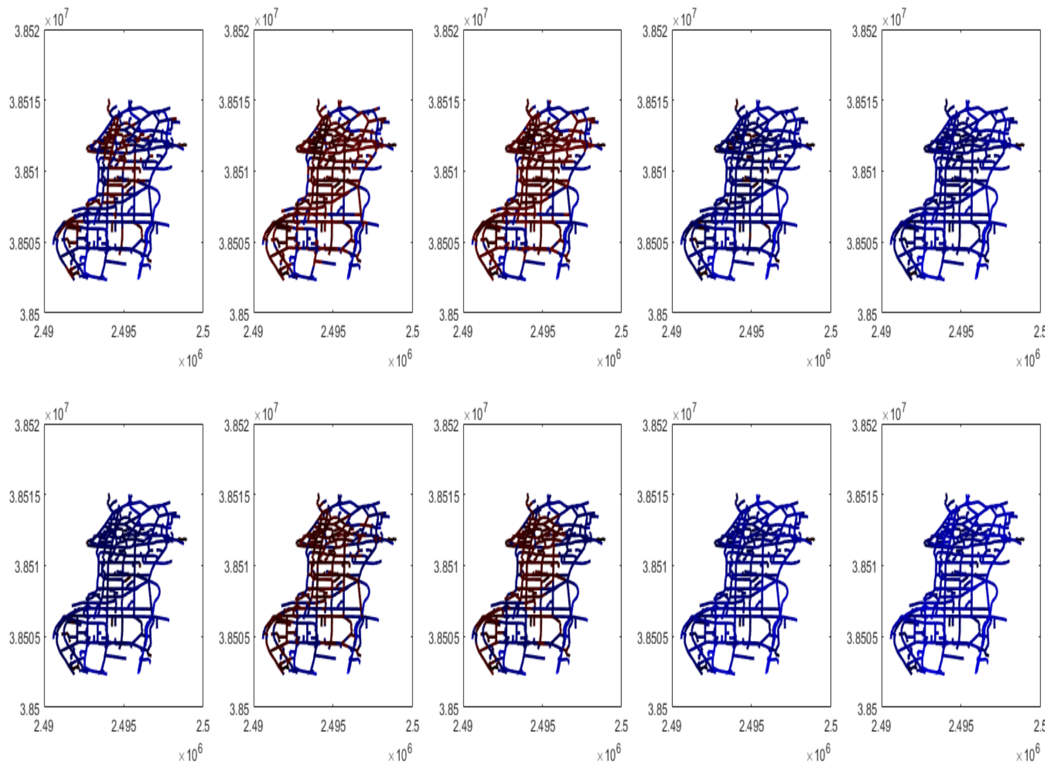

**FIGURE 7 The contour map of Shenzhen downtown for a simulation that replicates a typical working day in the Chinese city. The function $f$ has defined as in eq. 3. One can see the congestion of the morning peak hour appears and disappears and then a new congestion, corresponding to the afternoon peak hour, increases and decreases again. Ideally the panel have to be read from the top left to the bottom right and the follow the city condition from 6am to 8pm every $\approx 1.5$ hours**

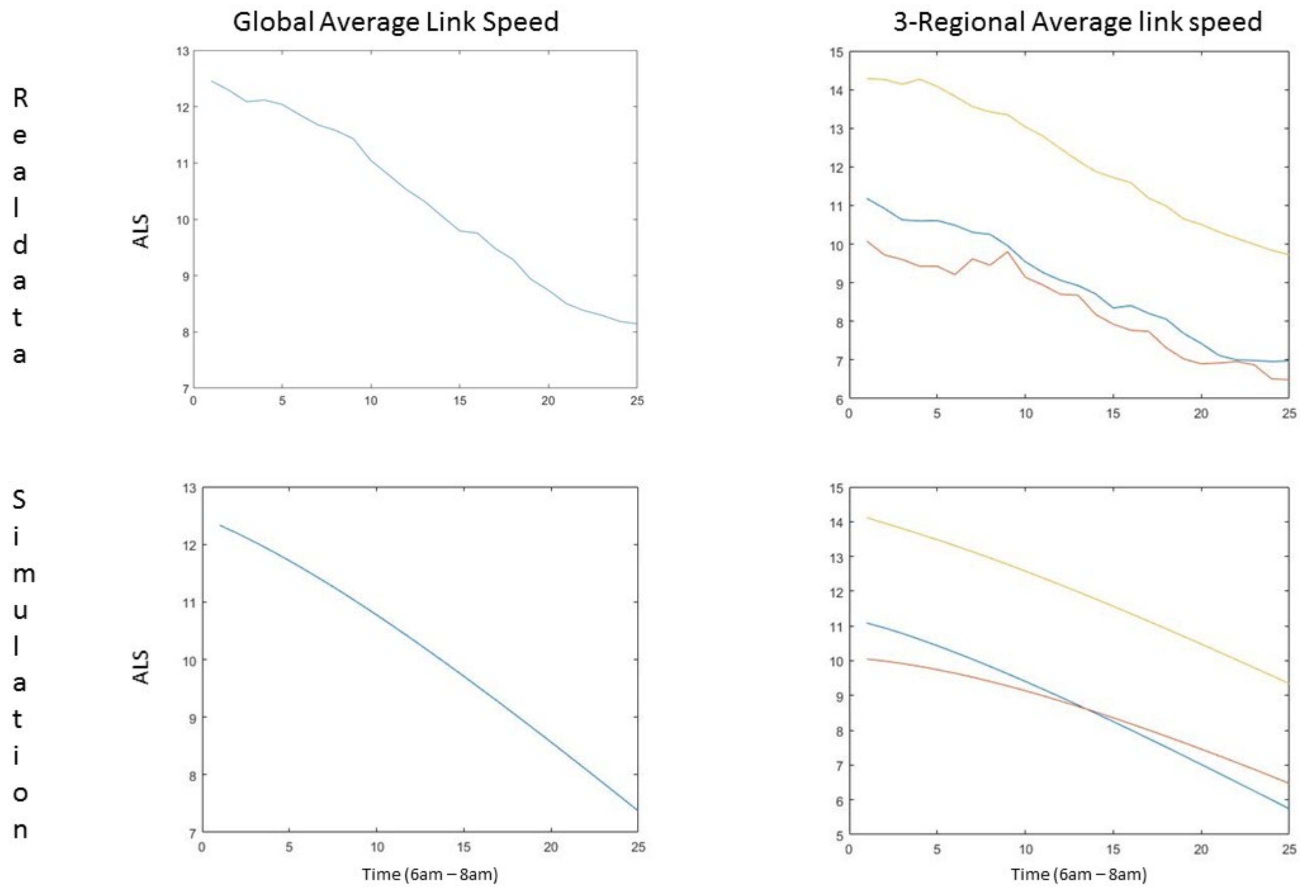


**FIGURE 8 Link speed average for the whole network of Shenzhen on the left and for three homogeneous regions on the right side. The comparison between the available real data and the results of the simulation of the RD model with 3 different couples of $(\rho_k, \sigma_k)$ for $k = 1, 2, 3$.**

4, and in particular the authors chose the following values:

$$u_i(t+1) = u(t) + 0.001 * (0.2 * log(1 + \sum_{k \in N(i)} u_i(t) - u_k(t)) + 0.01 * \sum_{j=1}^{N} L_{ij} u_j(t) \quad (4)$$

with $u_i(t)$ the ALS of link $i$ at time step $t$, $N(i)$ the set of neighbors of link $i$, $L$ the combinatorial Laplacian of the graph $\mathcal{G}(N, E)$ associated to the network. Moreover the network has been considered as undirected graph. The stopping time has been set at $T = 3000$.

## CONCLUSION AND FURTHER WORKS

In this paper it has been exposed the main pillars for a family of new models for urban traffic based on finding accordance and equilibrium between two contrasting terms: reaction and diffusion. Reaction and diffusion goes towards two opposite directions in terms of similarity. The first one increases the differences between ALS and traffic condition in the different part of the city while the second one spreads the ALS values over the network decreasing the differences among neighbors links. Due to the network topological structure and the existence of heterogeneous spatial distribution of demand it could exist different scenarios for different cities that this model proposes to replicate once set the best reaction principle (function $f$ ) and parameters calibration ($\rho$ and $\sigma$). The main advantage of this model if certainly the simplicity and the easiness to make the global mean of ALS increase or decrease just changing the terms $C(i,t)$ and/or the reaction weight $\rho$. This fact enables us to simulate the whole day traffic and so the onset and off set of congestion during the day. This easiness to manage the global ALS it can be used also to calibrate the parameters according to some clusters of real data. As shown in the Figure 8, the division in three regions allowed us to calibrate the respective regional means of ALS (on the panels on the right of the Figure 8) keeping the accordance with the global mean (panels on the left).

The RD model although with simple assumptions has already given very interesting and good results. But this is just one solution over many others of the principle of this very general model in the contest of urban traffic. For instance many other reaction functions $f$ could be chosen to simulate the increasing or decreasing in demand of the different part of the urban network and this change can be set dependent as in the studied case or independent by the ALS of the links in the previous time step. It is possible also to consider that the reaction term for each link is dependent by an exogenous demand but at the same time by the condition of the neighborhood. This last one seems the most interesting and general case by it needs to have *a priori* an OD matrix for the network and this is not always the case for researchers.

Future work will focus on evaluation of the ratio $\frac{\sigma}{\rho}$ between the two parameters for different cities and see the similarity and the difference among them. This can be a useful tool to classify the cities in term of mobility and congestion reaction and to study the structural causes that generate a better or worse behavior in traffic condition and in particular in congestion propagation.

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