DIGITAL GRAPHIC STATICS –
SHARED DESIGN TOOLS FOR ARCHITECTS AND ENGINEERS

[Original English Version]
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Title: 数字化图解静力学设计方法 — 建筑师和结构工程师共享的设计工具
Author: Corentin Fivet 科朗坦 菲韦 (http://sxl.epfl.ch)
Translation: Meng XianChuan 孟宪川

Abstract

Aware of its heavy environmental impact, the construction industry is on the lookout for new tools that provide a closer control of the end performance of buildings, e.g. the energy and volume of materials that they demand and their ability to be repaired or recycled. Still, designers would favor tools that encourage unbiased creativity. So, what should the future design tools of architects and structural designers be like?

New directions are currently being explored to answer that fundamental question. This paper gives an account of experimental applications where forces and forms are designed simultaneously, within a process that fully integrates architectural and structural concerns.

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1. The shaping of structures

Thinking about the structure of an object involves seeking to understand the way form and material behave when subjected to forces. Structures can be found everywhere in nature, from the microscopic arrangement of atoms to gigantic underground caves. However, structures can also be shaped explicitly in order to meet specific requirements. Other than ensuring stability under expected loads, these requirements are generally not related to purely mechanical considerations.

Indeed, form and material affect architectural and engineering constructions in many other ways. These include their integration within the spatial context, their architectural quality, their spatial flexibility, their safety, their sustainability, their recyclability, their construction process and manufacturing, their aesthetics, their symbolism, and their cost. A quick illustration of the consequences of designs is shown on Fig. 1. Each stadium fulfill the same purpose, still the environmental impact of their design differ greatly.

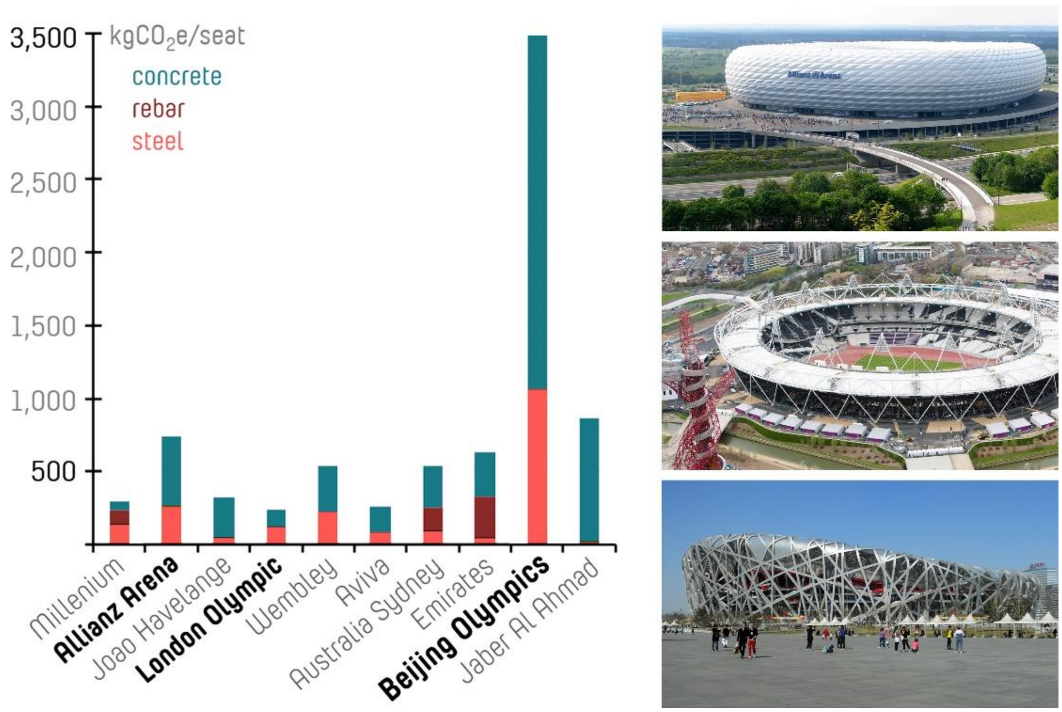


Fig. 1. Survey comparing the embodied carbon per seat of various stadiums worldwide. After [DeWolf·2014]

Shaping a structure means juggling with all these considerations. In addition, it happens every time a building is designed, even when its load-bearing behavior is hidden or implicit. In contemporary practice, the two main protagonists of this art are the architect and the structural engineer.

2. The architect and the structural engineer

In ancient times and up until the Renaissance, building expertise was primarily the concern of artisans — e.g. carpenters and stonecutters — who at times were directed by an individual — i.e. the master builder. As knowledge grew about the strength of materials, building techniques, stylistic forms and spatial qualities, practitioners became more specialized and a schism occurred in the eighteenth century Western world to produce two new practitioners: the architect and the engineer **[Picon·1988] [Addis·2007] [Saint·2007]**.

Although the architect and the engineer certainly develop different sensibilities and knowledge, it is a crude caricature to associate the former with the artist and the latter with the pragmatic **[Addis·1994] [Wells·2008] [Flury·2012]**. Another no less caricatured way of differentiating between them would be to say that the former has responsibility for the spatial form and the latter has responsibility for the structure's stability.

This comparison highlights the endless interference between the architect and the engineer: the former cannot do anything to the spatial form without challenging its stability and the latter cannot guarantee the stability of the structure without affecting its spatial form. Owing to this interference, they are forced to work together in one way or another.

Based on **[Baumberger·2012]**, four basic relationships between the architect and the structural engineer can be identified: monologues from the architect, monologues from the structural engineer, dialogues and soliloquies.

Monologues result from the total control of the project by the architect or the engineer, with the other at his service. Monologues from architects generally lead to formal, demonstrative buildings. Monologues from engineers generally lead to pragmatic, cost-effective buildings. In contrast, dialogues occur when architect and engineer are equal partners in the discussion. This kind of cooperation is generally hard to maintain because of the opposition between protecting personal egos and being challenged by the other's work. However, it usually tends to produce the most successful outcomes. Finally, soliloquies occur in rare cases where the architect and the engineer are the same person. Because of both domains being mastered, the work produced occasionally has a tendency to be demonstrative.

These four different relationships are particularly noticeable at the beginning of the project when the initial ideas and design decisions are established, i.e. during the conceptual design stage.

3. The conceptual design stage

The conceptual design stage is here referred to as the process evolving from the brief towards the soft definition of the dominant features of the future design. Also called 'design exploration', it may involve shaping a new structural concept, transforming a preexisting one or obtaining a fast feedback of its performance (Fig. 2).

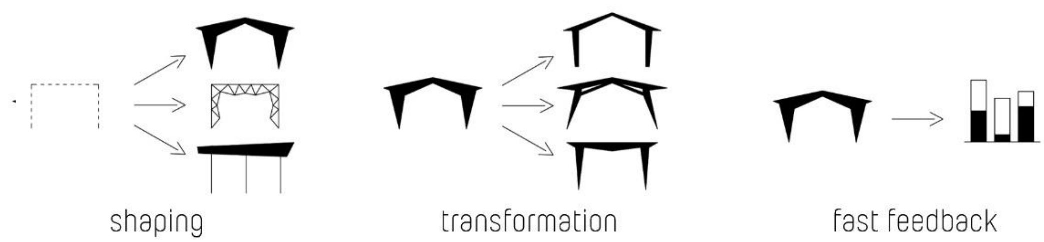


Fig. 2. The three purposes of the conceptual design stage.

This process may evolve linearly, back-and-forth or in loops. In every case, the decisions made during the conceptual design stage are crucial because most of them will never be challenged thereafter mostly for economic reasons (Fig. 3). Speaking of architectural projects, it is at this moment that, consciously or not, the designer fixes the mechanical behavior of the future load-bearing structure. As such, the early explorations already frame the scope and the efficiency of the subsequent refinements happening during the next stages of the design, e.g. during structural analysis and structural detailing. The design exploration will be more or less versatile and more or less efficient depending on the tools that are used.

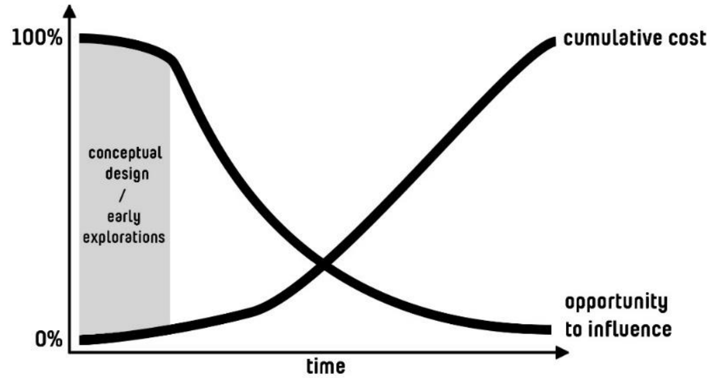


Fig. 3.The reason why choices made at the conceptual design stage are crucial. After [Paulson·1976].

4. The role of design tools

The current importance attached to the academic development of appropriate tools and methods for architects and structural engineers is directly linked to the essence of structural design: the shaping of structures is not a science. It is not a process undertaken to find the unique solution to a given problem through rational calculations. It is a project. It is a process at the beginning of which the definition of the problem is as unknown as the result.

> *"[Structural] art is solving problems which cannot be formulated before they have been solved. The search goes on, until a solution is found, which is deemed to be satisfactory. There are always many possible solutions, the search is for the best — but there is no best — just more or less good."*
> *Ove Arup, in* **[Addis·1994, page 7]**

> *"All the great masters of structural design have reminded us repeatedly that structural design is not a science;*
> *it is a craft that relies on judgment rather than absolute certainty"*
> **[Allen·2010, page xiv]**

Tools are meant to support designers and not to replace them. Still, tools, together with experience and intuition, greatly influence the final product in a variety of ways: they condition the conduct of the design process; they determine the set of parameters that can be acted upon and the set of values that will result; and they impose the speed of execution, the level of interactivity, the level of accuracy etc.

The choice of a tool or method over another one is dependent not only on the initial data available — e.g. the material that will be used or the quality of the bearing soil — but also on the desired results — e.g. aesthetics or solid-to-void ratios. This feature sometimes leads engineers and architects to develop their own customized tools and methods in order to achieve particular results within particular frameworks **[Krasny·2008]**.

5. Drawbacks of contemporary structural design tools

Architects and engineers have many tools at hand but none of them is currently well suited for the early exploration of structural systems. Indeed, tools mostly used by the architects to sketch their conceptual project do not take gravity or the strength of material into account. Tools mostly used by the engineers are of two kinds: analysis tools and form-finding tools. While the former compute the inner stresses of a given geometry in order to size or assess its structural parts, the latter produce the optimal geometry according to a given set of contextual constraints (e.g. mechanical properties and boundary conditions). As a result, none of these tools is properly suited to assist the design from its inception, i.e. when neither the geometry nor the contextual constraints are known.

Moreover, these tools impose a specific chronology of design. Users have to resolve the design process in the way that is directed by the software's own algorithmic reasoning, with specific inputs and outputs. As a quick illustration, the two following issues cannot be dealt with any current tool: « What geometric changes would make a particular rod in a truss become in tension? » « What additional weight would enable the reaction stresses on a given footing to be sufficiently vertical so as to be supported by the ground? ». These two questions are not unsolvable; they just require the use of static equilibrium laws in a different way from that proposed by customary structural analysis tools.

In most cases, these tools are hermetic black boxes. As such, they produce results that are difficult to interpret or even to understand. They are not meant to help improving the structural behavior of the model. This does not encourage a direct return to initial choices in order to fix or improve them. In addition, users are required to have a precise idea of how they will conduct the modelling beforehand; otherwise, they would be forced to start again due to unexpected discoveries

Owing to this, results are rarely communicated to the architect who subsequently becomes distanced from the structural issue:

> *"For the architect too, the separation of disciplines has not solely brought benefits. The growing difficulty of understanding how structures function definitely represents an impoverishment."*
> **[Muttoni·2005, page v]**

For most common basic projects developed in contemporary offices, these drawbacks will have little detrimental impact on the structure's quality. However, structures that require closer attention might suffer from weaknesses, e.g. safety, architectural quality, construction costs, environmental damages.

6. Learning from great designers

This observation does not mean in any sense that contemporary computerized analysis, optimization and form-finding tools are unsuitable for structural design. Rather, it means that they should be supplemented – not replaced – by new kinds of tools such as tools dedicated to early structural design explorations.

Stemming from that, the question is "What should be a tool for early structural design explorations like?" The strategy we followed to answer that question is to study the work of recognized structural designers who have worked wonders in a context in which computers were not necessarily available. Those designers are usually either trained or considered as both architect and engineer.

From a general point of view, structural designers who care about quality projects first seek to employ tools and methods that maximize the opportunity for their creativity and intuition to percolate through. As such, they look for clarity, speed and interactivity.

> *"Creativity is necessary not just for issues around form, but also for purely technical aspects: processes, materials and static systems. This creativity is the difference between people who are happy to calculate and real engineers."*
> *Jürg Conzett, in* **[Conzett/Solt·2008, page 29]**

The techniques that favor these qualities are many and varied. We selected three of them, which, according to us, are of primary importance:

- (1) design-oriented use of simplifying assumptions;
- (2) problem reduction guaranteeing permanent control;
- (3) extensive use of graphical methods and geometry.

(1) The real behavior of every structure will always remain unknown to its designer since it is still nowadays impossible to model in its full complexity. Engineers work therefore with idealized representations that are understood and controllable. Consequently, there is always a gap between the real material behavior and its theoretical behavior, whichever approach is chosen. These gaps are qualified by so-called simplifying assumptions.

For instance, a simplifying assumption may be one that allow the application of the "lower-bound theorem of plasticity" **[Heyman·1995] [Heyman·1996] [Heyman·1999] [Ochsendorf·2005]**. It has recently been used to perform early design explorations of a compression-only monument at MIT, Cambridge. The complexity of shaping a system of blocks that obtains its stability by simple contact has been reduced to the single handling of lines of thrust thanks to the lower-bound theorem of plasticity (Fig. 4, left). Coupled with a parametric design tool, this theoretical simplification allowed the designers to check the stability of dozens of proposals in real-time (Fig. 4, right), at a moment where very little was known about the actual detailing of the monument.

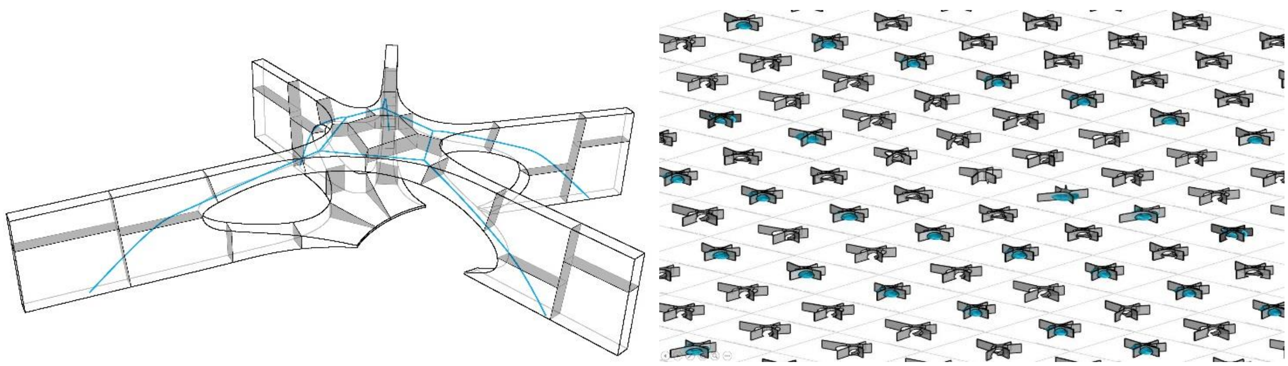


Fig. 4. Design exploration of the Collier Memorial by Höweler+Yoon Architects, after [Ochsendorf/…·2016].
Left: the thrust line (blue) is a function of the shape of each block. If it stays everywhere within the geometry of the monument, it means that the monument is stable under self-weight.
Right: a sample of all the various geometries that could have been tested rapidly thanks to the thrust line.

(2) A second technique is to reduce the design issue to a small number of critical parameters (or equations, variables or relationships) that alone control all the major questions of the design issue. Applications of this technique have been accomplished in many different forms.

Antoni Gaudi (1852·1926) used hanging models to control both the shape of the buildings and their stability under dead loads **[Krasny·2008 page 58] [Huerta·2006]**; in the design of vaults all that was required was to add and remove weights and ropes. Robert Maillart (1872·1940) drew parabolas in order to correct repeatedly the apparently free-form bottom and upper chords of the Salginatobel Bridge (Fig. 6) **[Fivet/Zastavni·2012] –** the entire curvature of a parabola is controllable with only three points. Felix Candela (1910·1997) synthesized the entire structural problem (and, to some extent, the architectural issue) into a single equation describing hyperbolic paraboloids **[Faber/Candela·1963] [Garlock/…·2008]**. When designing the bowstring arch of the Nijmegen City Bridge (Fig. 5) worked the topology of the arch to reduce the number of geometric parameters describing it, which then greatly simplified the control of the variation of these parameters during the optimization **[Ney/…·2010, page 166]**.

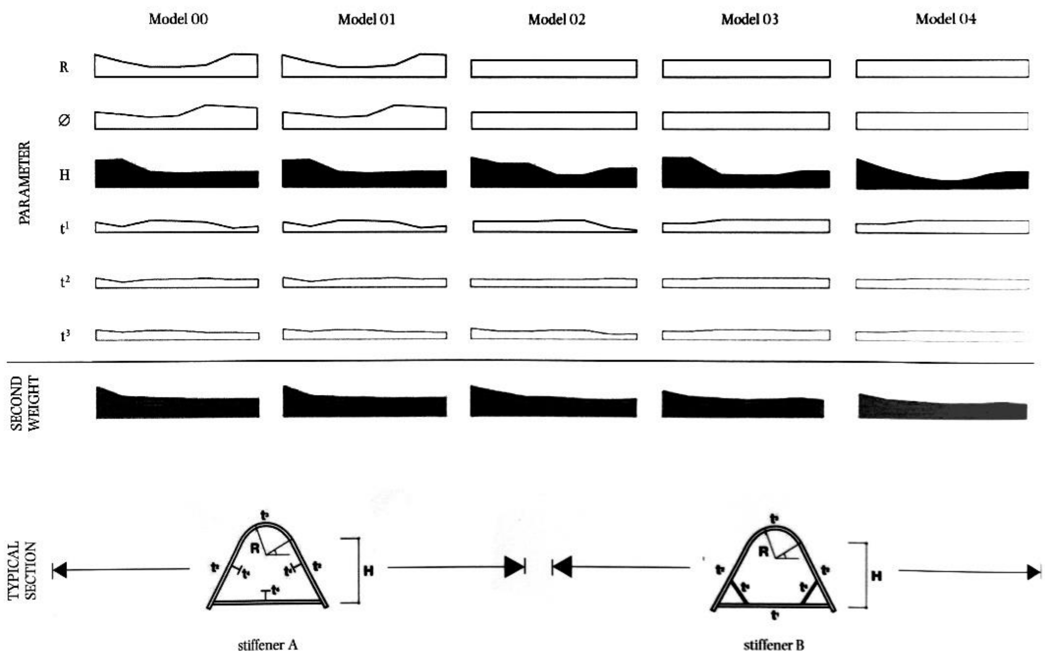


Fig. 5. **Laurent Ney's 'De oversteek' Bridge in Nijmegen, The Netherlands, 2013. Below, optimization of the arch with a minimum of controlled parameters. [Ney/...·2010]**

(3) The third technique concerns the extensive use of sketches, drawings, graphical methods and geometry. Although it appears that these are being used less and less in contemporary practices, they are still an important technique in the designer's toolbox **[Fivet·2013]**. Moreover, graphics are one of the very few remaining common grounds between the architect's work and the engineer's work.

> *"The qualitative evaluation of the forces using an inductive process — for example, graphic statics — does not require exact calculation, just practice and experience. This method is understandable to architects too, and offers a good basis for working together. [...] A common language needs to be learned — an indispensable prerequisite for a close dialogue between the architect and the engineer. [...] This would be a culture in which the dialogue between architects and structural engineers can begin to grow — a culture that would enable the development of designs in which structural and formal needs merge."*
> **[Joseph Schwartz in Flury·2012]**

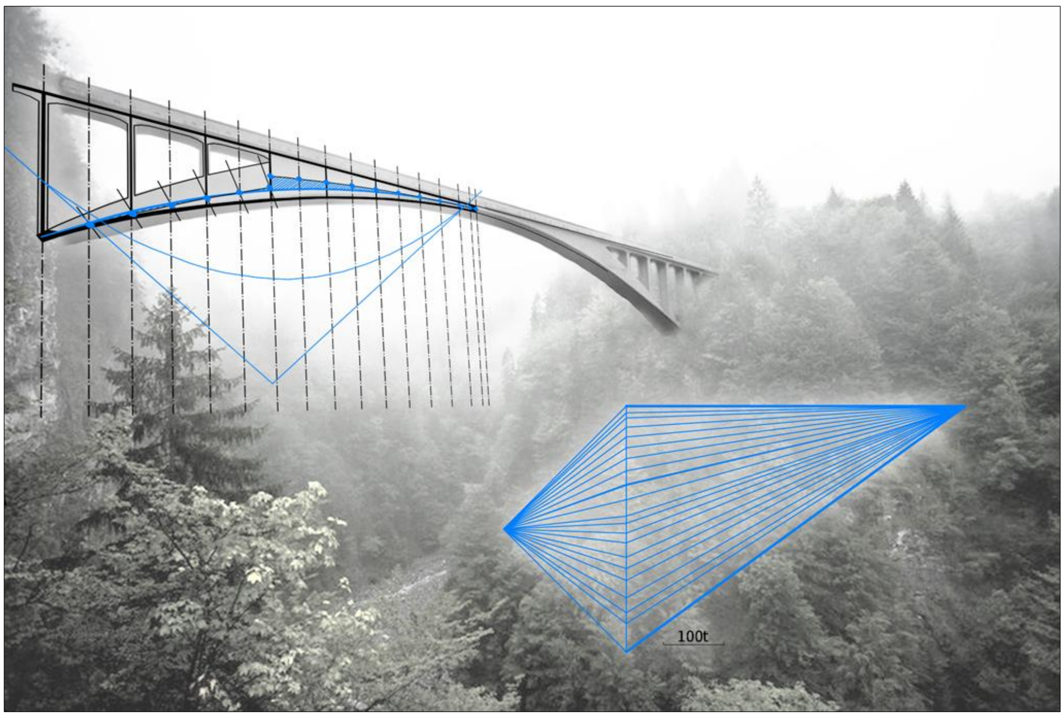
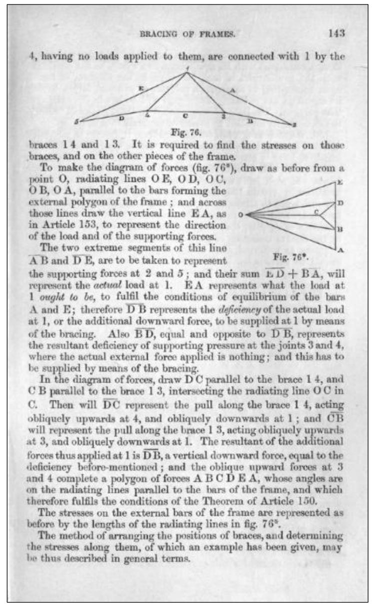


Fig. 6. **Robert Maillart's Salginatobel Bridge in Switzerland, 1928, and a summary of its graphic statics diagrams: (left) line of thrust and bending moments in the arch and (right) force magnitudes.**

7. Graphic statics

The roots of graphic statics go back to the 1858's works of J.W.M. Rankine **[Rankine·1858, pages 139-144]** (Fig. 7) and J.C. Maxwell **[Maxwell·1864]** and, thereafter, to those of S. Earnshaw, F. Jenkin, C. Culmann, L. Cremona, R.H. Bow and M. Lévy. By the end of the nineteenth century, graphic statics became one of the most popular tool for structural analysis (Fig. 8).


BRACING OF FRAMES. 143

4, having no loads applied to them, are connected with 1 by the

Fig. 76.

braces 1 4 and 1 3. It is required to find the stresses on those braces, and on the other pieces of the frame.

To make the diagram of forces (fig. 76*), draw as before from a point O, radiating lines O E, O D, O C, O B, O A, parallel to the bars forming the external polygon of the frame; and across those lines draw the vertical line E A, as in Article 153, to represent the direction of the load and of the supporting forces.

Fig. 76*.

The two extreme segments of this line $\overline{AB}$ and $\overline{DE}$, are to be taken to represent the supporting forces at 2 and 5; and their sum $\overline{ED} + \overline{BA}$, will represent the *actual* load at 1. E A represents what the load at 1 *ought to be*, to fulfil the conditions of equilibrium of the bars A and E; therefore $\overline{DB}$ represents the *deficiency* of the actual load at 1, or the additional downward force, to be supplied at 1 by means of the bracing. Also $\overline{BD}$, equal and opposite to $\overline{DB}$, represents the resultant deficiency of supporting pressure at the joints 3 and 4, where the actual external force applied is nothing; and this has to be supplied by means of the bracing.

In the diagram of forces, draw D C parallel to the brace 1 4, and C B parallel to the brace 1 3, intersecting the radiating line O C in C. Then will $\overline{DC}$ represent the pull along the brace 1 4, acting obliquely upwards at 4, and obliquely downwards at 1; and $\overline{CB}$ will represent the pull along the brace 1 3, acting obliquely upwards at 3, and obliquely downwards at 1. The resultant of the additional forces thus applied at 1 is $\overline{DB}$, a vertical downward force, equal to the deficiency before-mentioned; and the oblique upward forces at 3 and 4 complete a polygon of forces A B C D E A, whose angles are on the radiating lines parallel to the bars of the frame, and which therefore fulfils the conditions of the Theorem of Article 150.

The stresses on the external bars of the frame are represented as before by the lengths of the radiating lines in fig. 76*.

The method of arranging the positions of braces, and determining the stresses along them, of which an example has been given, may be thus described in general terms.

Fig. 7. First printed diagrams of graphic statics in 1858, [Rankine·1858].

In the late twentieth century, the development of computers capable of complex numerical calculus (rather than drawing calculus) ended the use of graphic statics in most structural design practices and in engineering schools amid numerical analysis tools. For a couple of years now there is a comeback of graphic statics both in education **[Zalewski·1997]** and in practice **[Conzett·2003, pages 100-109] [Beghini·2013]**. Today, graphic statics is meant to assist the architect and the structural engineer during early structural explorations, the structural problem involved is reduced to its essentials: the shaping of static equilibria.

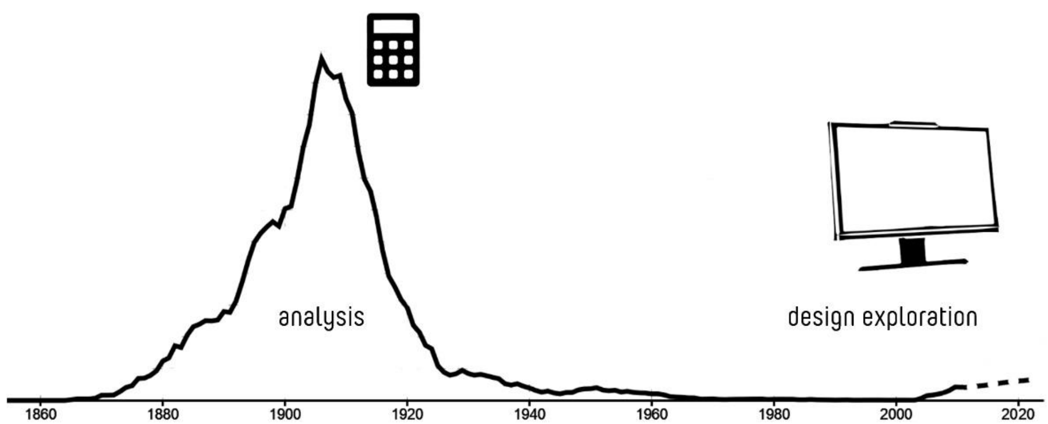


Fig. 8. All publications with "graphic statics" in the title, in English, French, German, Italian & Spanish, graph generated with the help of Google Ngram view.

Graphic Statics consists of a set of methods acting on two distinct diagrams: the geometry of the strut-and-tie network is drawn in a so-called form diagram in which a distance between two points measures a length, while the corresponding inner forces are drawn in a force diagram in which a distance between two points measures a magnitude of force (Fig. 9).

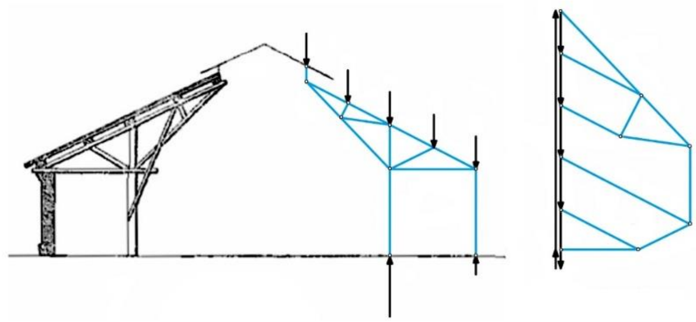


Fig. 9.Graphic statics diagrams for a roof truss in static equilibrium: (left) the form diagram and (right) the force diagram.

Since each node of the form diagram is supposed to be in static equilibrium, each vectorial addition of the force acting upon it must be zero. Hence, the graphic representation of the magnitudes of these forces in the force diagram must form a closed polygon. If this force polygon exists for every single node of the truss, then both the translational and the rotational equilibrium of the entire structure is guaranteed.

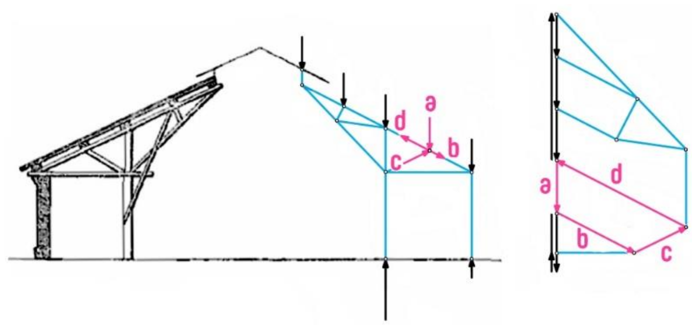


Fig. 10. Example of a force polygon embedded in the force diagram. The node in the form diagram is ensured to be in static equilibrium because the vectorial sum of the forces applied to it is zero.

Methods of graphic statics enable the structural designer to shape an initial structure while controlling at the same time both the geometry of the structure and its inner forces. One can change the desired geometry and then understand how the inner forces must adapt. Or, more interestingly, one can change the inner force and then understand how the geometry must adapt. Because the simultaneous understanding of both diagrams reveals the commonly hidden properties that govern the inner distribution of forces, graphic statics greatly favors the design of efficient and expressive structures.

The nature of the diagrams allows the model to be the abstraction of a variety of building systems **[Fivet/Zastavni·2013]**: reticular structures (be they determinate or not, pre-stressed, self-stressed or mechanisms); thrust lines within masonry structures; load paths or discontinuous stress-fields inside reinforced concrete elements or other materials showing a plastic behavior; bending moments and deflections of regular and freeform beams; equilibrium of stabilizing slopes, retaining walls and foundations. Because they use particular theories (e.g. Euler-Bernoulli beam theory or the lower-bound theorem of plasticity), specific assumptions must apply for these various applications to be valid. These conditions may either be observed by the designer himself or imposed by specific geometric constraints. Moreover, the designer can start the design exploration without having to define the effective building system beforehand and he can switch from one to another in the middle of his exploration.

The inherent high abstraction of the graphic statics diagrams gives room for many interpretations by the designer. However, it also implies that the presented tool cannot be used alone and that a comprehensive structural analysis may still need to be performed afterwards with other tools, e.g. to check deflections and dynamic responses. However, it is expected that this subsequent check may often be redundant and does not require a substantial change of the initial sketch, as it is often the case when the structural behavior is defined from the beginning **[Ochsendorf/...·2016]**.

8. Computer-aided graphic statics

The recent enhancement of graphical user interfaces have led to the development of new computerized frameworks. If combined with such computer power, graphic statics can gain new benefits: the generation of diagrams can be faster and is always correct. In addition, additional features can be added to provide a more comprehensive experience.

The first application consists in coupling graphic statics with a computerized parametric tool, such as Geogebra or Grasshopper. Generic examples of such parameterization in Geogebra can be found in eQuilibrium, a platform developed by the Block Research Group for education **[VanMele/...·2012]**. Grasshopper allows the geometric objects of the CAD environment Rhino3D to be controlled parametrically by applying successive geometric operations. The common operations of graphic statics (e.g. to link two points with a line, to copy a parallel line, to move a point) are straightforward to program within Grasshopper (Fig. 11). Once the model is created with such operations, any initial input (e.g. the position of a point in the force diagram or in the form diagram) can be varied and new geometries and new load paths are updated in real-time. Optimization routines can also be implemented right away.

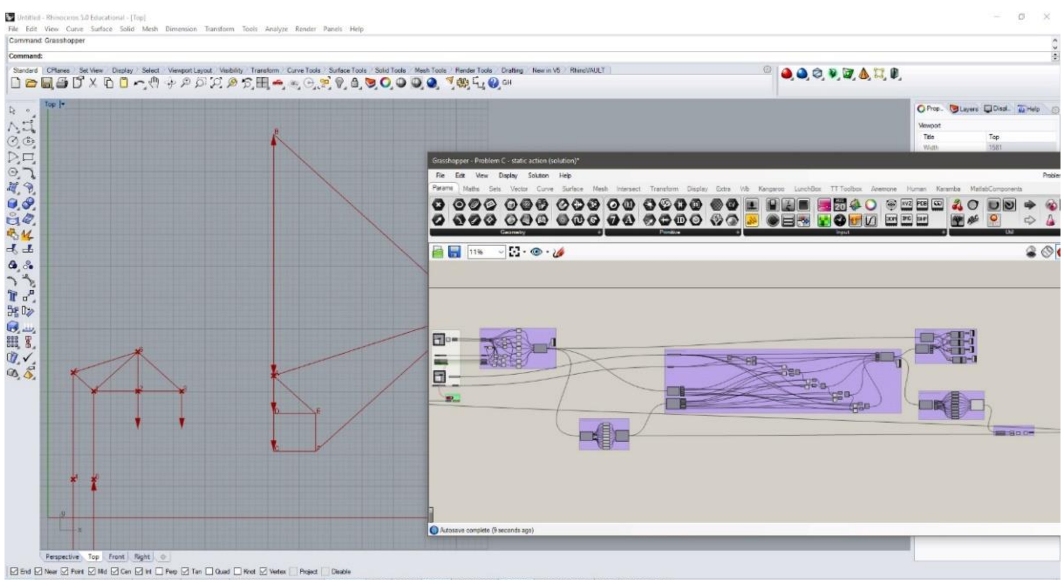


Fig. 11. Graphic statics diagrams (left) that are parametrically controlled in Grasshopper (right).

Such methods have been taught and applied recently during a one-week workshop in Nanjing University. Fig. 12 shows the final designs produced by the students. None of them knew graphic statics before starting the workshop but by the end of the week, all of them were able to produce an original architectural structure where the form and the forces are in line.

Fig. 12. Student's designs from Nanjing University produced after a one-week introduction to graphic statics and Grasshopper.

Still, this type of use of graphic statics is fairly similar to the old classical way of building the diagram on a piece of paper: (1) the diagrams are only in equilibrium once they are completely constructed and (2) the designers must have a clear idea of the final result before starting the construction. In order to counter these two drawbacks, the Structural Xploration Lab (http://sxl.epfl.ch) is currently developing Constraint-Based Graphic Statics **[Fivet/Zastavni·2013] [Fivet/Zastavni·2014] [Fivet/Zastavni·2015]**.

The first idea behind constraint-based graphic statics is to let the user create or transform existing strut-and-tie networks by means of successive operations that never jeopardize their static equilibrium. The very firstly drawn structure is already in equilibrium — meaning that the form diagram and the force diagram are already completed — and it remains in static equilibrium after each transformation. For instance, it is impossible to add a new force without adding an equal but opposite reaction. As a result, the designer can branch out potential systems in equilibrium at every step of the construction process. Design decisions happens in the designer's mind at every step.

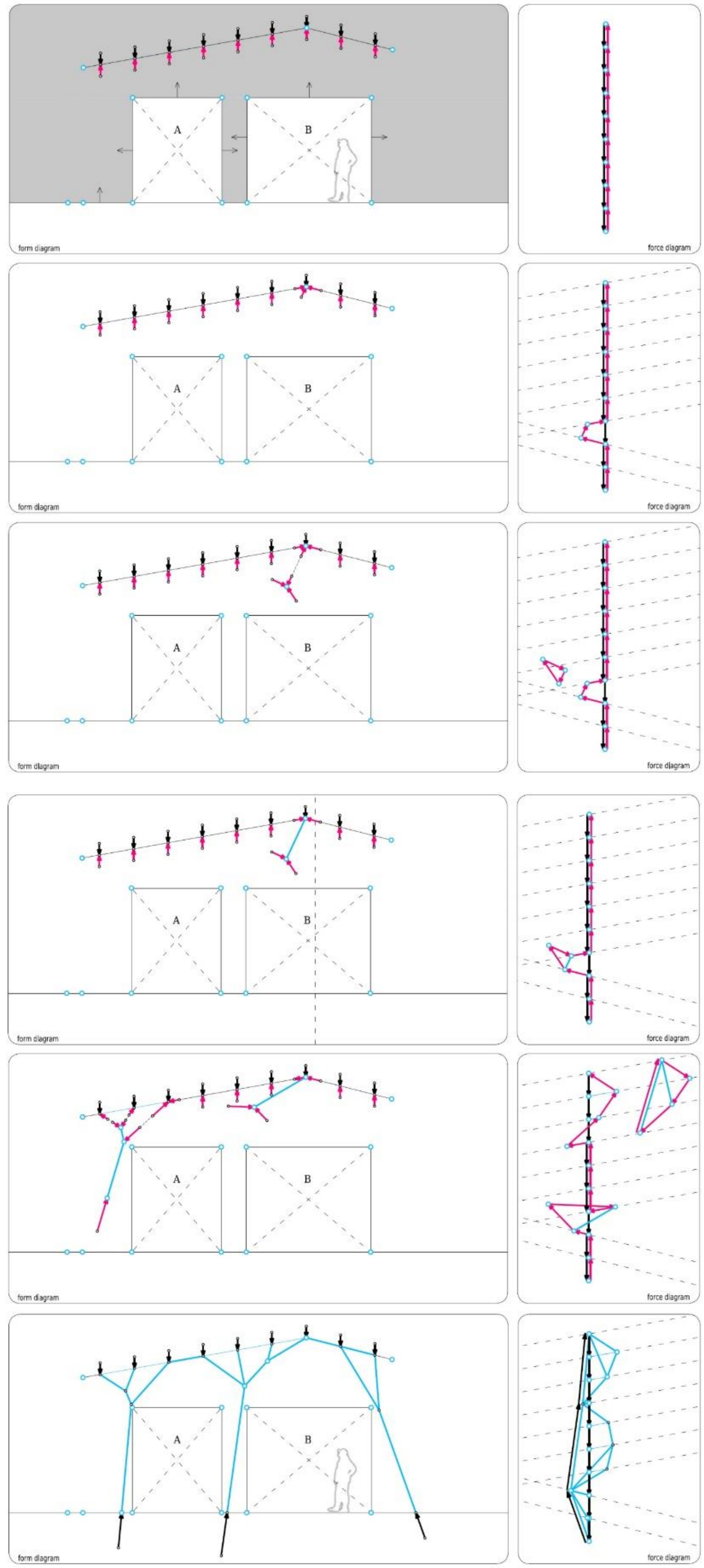


Fig. 13. Successive manual operations leading to the design of a reticulated roof truss. The system remains in static equilibrium at every step of the process.

Since the only parameters of these networks are points in two diagrams, it is enough to constrain the position of these points inside a graphical region in order to constrain all the geometrical and mechanical properties of the strut-and-tie network. These geometric constraints can be computationally propagated to any other node defining the parameterization. As a result, every node in the form diagram and in the force diagram will be restricted inside a graphical region that is equal to its solution space, i.e. the set of all positions for which no constraint applied on the diagram is violated (Fig. 14). This added feature is useful because the field of all solutions to a particular design issue can be evaluated before even exploring them.

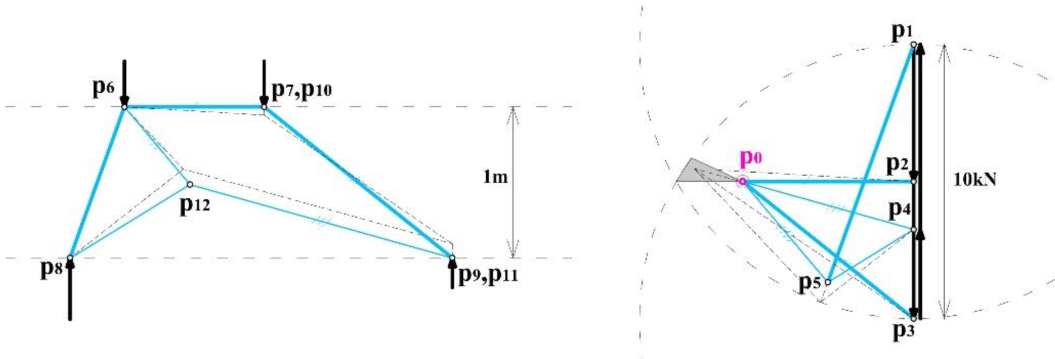


Fig. 14. The shaded area in the force diagram (right) is the solution space of the node p_0 such that the strut-and-tie network (left) is not higher than 1 m, and the magnitudes in the bars are below 10kN. The designer consequently knows that moving p_0 to the extreme right of the shaded area is equivalent to reducing the magnitudes of forces inside the system.

An additional diagram, has been recently suggested in order to unlock the chronological nature of design furthermore **[Ohlbrock/…·2016]**. It consists in a diagram that controls the topology of the structure and from which both form and force diagrams can be deduced.

Combined with computers, graphic statics can also be used in order to simplify the inner working of computer algorithms. For instance, **[Todisco/…·2015]** uses it to define post-tensioned free-formed trusses and **[Lee·2016]** generated random trusses with grammar rules inspired by graphic statics.

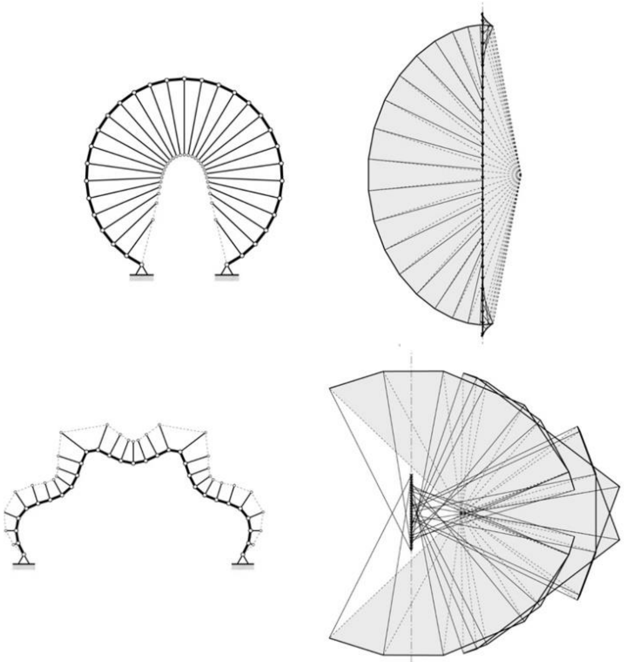


Fig. 15. Two post-tensioned networks in static equilibrium (left), constructed from a force diagram (right), after [Todisco/…·2015].

Last but not least, the computerization of graphic statics has led to the development of 3-dimensional graphic statics, a field that may have a huge impact on the design of free-form spatial structural systems in the next decade **[Akbarzadeh·2015] [D'Acunto/... 2016] [Konstantatou·2016]**.

9. Conclusion

New conceptual design tools are needed for both architects and structural designers in order to improve rationality and creativity. Still, the definition of such tools is a very young field of research. Graphic statics is one of the few means that can be used to create this new generation of design tools. Other directions can be found in the work of **[Mueller/...·2013]** or **[VonBuelow/...·2012]** for instance. One day, maybe, every architect's design tool will embed knowledge about structural stability. One day, maybe, every structural engineer's job will be solely to create custom design tools for the architects.

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