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NON-LINEAR TRANSFER FUNCTION IDENTIFICATION OF PRESSURE PROBES USING SIREN DISKS

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Abstract:

This article presents a Siren Disk proof of concept for the dynamic excitation of pressure probes, and a method to reconstruct distorted signals due to pneumatic channels. Constraints in sensor installation require placing a pressure transducer distant from the measurement point. The transducer is usually connected through a pneumatic channel – creating a probe, which alter its dynamic response. The Siren Disk is used for the identification of transfer functions of different pressure probe geometries. The device is capable of producing pressure signals up to 10 kHz and 3.5 bara (peak-to-peak = 2.5 bars). The transfer function is obtained through the comparison of the probe signal to a flush mounted reference transducer that is subjected to the same pressure signal. The response of the probes was shown to be highly non-linear. Hence, a multi-dimensional transfer function is developed for the system identification of the probes. The function is based on the Fourier series, and consists of a set of sub transfer functions describing the average gain and phase lag for the offset and the harmonics. The approach is well suited to capture the non-linear frequency response of complex sensor installations. Experiments show that the flat response of transducers is jeopardized by the introduction of the low pass filter behavior from the pneumatic channels. The probe's signal was significantly distorted compared to the reference signal. The inverse transfer function is used to reconstruct the probe's signal in the time domain. Good agreement is found between the reconstructed and the reference signals even at excitation frequencies beyond the probe's resonant frequency. Hence, highlighting a wide range of validity for the proposed method.

Keywords: siren disk; dynamic response; unsteady pressure measurement; periodic excitation

Nomenclature

A	<i>Amplitude</i>	Φ_n	<i>Phase angle of n^{th} harmonic (deg.)</i>
$\bar{A}$	<i>Mean</i>	ω	<i>Frequency (Hz)</i>
A_n, B_n, Q_n	<i>Fourier series coefficients of n^{th} harmonic</i>		
D	<i>Diameter (mm)</i>		
f	<i>Excitation frequency (Hz)</i>		
f_{res}	<i>1st resonant frequency (Hz)</i>	FFT	<i>Fast Fourier transform</i>
G_n	<i>Gain in the n^{th} harmonic</i>	gof	<i>Goodness of fit</i>
L	<i>Length (mm)</i>	hpf	<i>Hole passing frequency (Hz)</i>
L_V	<i>Vertical Length (mm)</i>	Meas.	<i>Measured</i>
L_H	<i>Horizontal Length (mm)</i>	Recon.	<i>Reconstructed</i>
n	<i>Harmonic order</i>	Ref.	<i>Reference</i>
n_h	<i>Total number of harmonics</i>		
q	<i>Unsteady signal</i>		
q_0	<i>Time averaged offset</i>		
t, T	<i>Time (s)</i>		

1. Introduction

Time resolved accurate pressure measurements are of pronounced importance in the control, monitoring and understanding of a wide range of mechanical and aerospace systems. If the required measured pressure is constant over time, the transducer's sensitivity is the only factor that governs the measurement accuracy. However, many applications require the accurate measurement of unsteady pressures, which is crucial to observe and understand phenomena like turbulence, flow separation onset, or periodic flows in turbomachinery. Additionally, some of these applications are constraint in space (e.g. small-scale turbomachinery), or exposed to extreme temperatures (e.g. gas turbines), which prevent the flush mounting of fast response transducers. Hence, imposing the connection of the pressure transducers distant from the point of interest via pneumatic channels – creating a probe. In that case, further parameters are required to perform accurate time-resolved pressure measurements.

Bean [1] identified six parameters for the characterization of pressure transducers used in time-resolved measurement: (1) gain, (2) phase lag, (3) resonant frequency, (4) damping ratio, (5) rise time, (6) and overshoot. The identification of these parameters requires dynamic calibrators, which are capable of generating periodic pressure signals with controlled amplitude and frequency, or aperiodic step or impulse pressure signals with a short rise time and controlled amplitude. Aperiodic calibrators are generally based on shock tube or fast opening valve concepts [1–8], whereas periodic calibrators are either variable volume generators, rotating valves or sirens [1,2,5,9–14].

State of the art. Aperiodic calibrators provide time domain calibrations where gain and phase lag are identified by the ratio of the output to the input signals' Fourier (or Laplace) transforms [2,5]. Shock tubes are widely adopted as aperiodic calibrators; however, the incomplete burst of the diaphragm is a common issue, which results in low frequency pressure disturbances after the shock, making it challenging to obtain an accurate transfer function [3]. A similar device to a shock tube is the pressure pulse generator of Aronson and Waser [15] that relies on the rapid venting of a known static pressure on the sensor to be calibrated using a fast-acting poppet valve. The system is able to build up pressures up to 68 bara in a rise time less than 100 μ s. For broadband calibrations, fast opening devices are used in order to cover frequencies lower than 100 Hz in addition to shock tube tests [2,5].

Air periodic calibrators are generally used for low frequency (< 3 kHz) and low amplitude (<1 barg) calibration [5,10–13,16–18]. An ideal periodic calibrator would generate known pressures at a given frequency [2,4,5]. Lack of such accuracy is overcome by adding a reference pressure measurement in addition to the test sensor [2].

Kobata and Ooiwa [19] presented a square wave pressure generator using a rotating valve, which used air up to 1 barg and 1 kHz, reporting a well-shaped square wave at low frequencies. Hurst and Van De Weert [20] presented a spinning valve concept for the production of sinusoidal pressure signals. The valve had different holes interrupting a nozzle flow. The excitation range of the valve reached 2.8 kHz, for a pressure range between 0.068 barg and 0.689 barg, and a peak to peak pressure of 0.03 bar at maximum frequency. The calibrator was used to investigate the dynamic response of remote pressure transducers connected via flexible tubes to their measurement outlets. Whitmore et al. [17] used an oscillating piston and vacuum pump test rig, up to 2 kHz, for the same application.

The use of a liquid media allows the signal generators to reach higher amplitudes and frequencies, with the drawback of system contamination. Perls et al. [14] presented a sinusoidal pressure generator up to 10 kHz and with a peak to peak amplitude of 24 bars, using piezoelectric stacking with two working fluids, namely diphenyl metachloride and glycerol. Tsung and Han [21] introduced a square pressure wave generator using hydraulic oil as a working fluid with frequencies above 10 kHz and peak to peak amplitude up to 30 bara.

A dedicated study on the improvement of high frequency/amplitude periodic calibrators showed that siren type devices were the most promising solution to generate high amplitude pressure signal on a large range of frequencies [9]. It was also reported that sirens can produce periodic - not necessarily sinusoidal - low and medium pressure signals up to 1 kHz [22]. However, distortion of the generated signal into a saw-tooth like form was observed. Fridh et al. [23] reported pressure tap calibration up to 4 kHz using a reference pressure signal generated by a rotating hole-disk system. Unfortunately, the full description of such system is not available in the public domain.

Nature of the issue. The continuous development of technologies, creating increasingly complex and smaller systems - i.e. small scale turbomachinery [24,25] - makes it challenging to obtain accurate time resolved measurements. In some applications, even the smallest pressure transducers can violate the accuracy if flush mounted due to spatial averaging. Using small pressure taps with remotely mounted transducers will allow the increase of measurement points, yielding

detailed surface measurements. Remotely mounting the transducer will also prevent it from extreme temperature exposure, or any other aggressive flow conditions. Figure 1 shows different engineering applications requiring the remote mounting of the transducer: (a) pressure measurement in a small-scale compressor stage, (b) separation onset studies in external aerodynamics, and (c) pressure field measurement inside fluid film bearings and seals. Once the transducer is remotely mounted, both the pneumatic channel and the transducer are the components of a probe, exhibiting a different dynamic response that per se requires identification. The consequence is a distorted and shifted signal that does not reflect the real pressure signal. Also, for signals with high pressure amplitudes - relative to the mean, the dynamic response of the probe is no longer linear.

Given the previously stated challenges, it is found that the literature is lacking (1) a robust procedure for the identification of the non-linear transfer function of such probes, and (2) a practical method for the reconstruction of the distorted signal, especially at high frequencies and amplitudes.

Figure 1 – Typical pressure sensor applications in engineering systems, a) small scale turbomachinery, b) external aerodynamics, c) bearings and seals.

Goals and Objectives. The first goal of this paper is the definition of a procedure for the non-linear dynamic transfer function identification of pressure probes using the Siren Disk technology. The second goal is to identify a robust method for the reconstruction of the true pressure based on the distorted pressure signals. The objectives are to (1) design and build a Siren Disk excitation device, (2) prove its concept by exciting the probe on a wide frequency spectrum with varying amplitudes, (3) use the Siren Disk device to identify the response of various pneumatic probe geometries, (4) implement a Fourier series based transfer function to capture the non-linear behavior of the pneumatic channel, (5) reconstruct the measured signal using the identified transfer function and compare it to the reference signals.

2. Siren Disk Description and Design

The Siren Disk test rig is comprised of (1) a Siren Disk assembly, (2) a driving electrical motor, (3) a pressure source, (4) a reference sensor, and (5) a test probe to be calibrated in the vicinity of the reference sensor. The target pulsation frequency is 10 kHz with pressure amplitudes starting from 0.5 barg up to 3.5 barg.

Figure 2 – The Siren disk calibration apparatus

The Siren Disk is designed to generate interruptions at the exit of a nozzle - Figure 2. The nozzle is placed downstream of a pneumatic line with a maximum gauge pressure of 8 bar - pressure regulated upstream. The nozzle is convergent, with an inlet and exit diameters of 20 mm and 10 mm respectively, and a length of 40.7 mm.

The main constraint in the disk design is the tip speed that is limited to 110 m/s due to mechanical stress. Another constraint is ensuring equal and synchronized opening and closing times for the reference transducer and the test probe. Finally, the distance between the reference transducer and the test probe needs to be kept minimal - Figure 3. These design constraints yield a 280 mm diameter disk, with 80 holes, a pitch of 10 mm between holes, and a maximum rotational speed of 7500 rpm. Such design is able to produce a hole passing frequency (hpf) of 10 kHz. The disk holes are of a semi-oval shape having straight side walls, hence, guaranteeing synchronized opening and closing over both the reference transducer and the test probe. A micrometric x-y table allows the accurate alignment of the reference transducer and the test probe with the center of the nozzle and the siren disk holes. An optical proximity probe is placed on top of the Siren Disk holes in order to measure the hpf.

Figure 3 - Holes of the siren disk

Structurally, the Siren Disk is designed for minimum inertia and maximum stiffness. A thin stainless steel disk plate with the 80 holes is bolted between two rigid holders with web supports. The thin disk minimizes the distance between the nozzle exit and the measurement point. The Siren Disk is assembled on a spindle, which is connected to an electric motor via flexible coupling - Figure 4. Dynamically, there are two excitation sources to the rotor, (1) the synchronous

rotor imbalance (rotational speed), and (2) flow induced excitation resulting from the interaction between the air jet and the Siren Disk (hpf). Therefore, the rotor assembly was designed such that its eigenfrequencies do not coincide with the rotational speed or the hole passing frequency.

Figure 4 - Schematic of the siren disk set-up

3. Siren Disk Qualification

Four L-Shaped probes with different lengths are tested to qualify the Siren Disk - Figure 5. The pressure transducer is placed at the end of the horizontal side, while the supply nozzle is facing the vertical end of the pneumatic channel. Tube lengths and aspect ratios are summarized in Table 1.

Table 1 L-Shaped probes under study

<i>Probe</i>	<i>Diameter, D [mm]</i>	<i>Vertical Length, L_V [mm]</i>	<i>Horizontal Length, L_H [mm]</i>	<i>Total Channel Length, $L=L_V+L_H$ [mm]</i>	<i>Total Channel L/D</i>
<i>1</i>	1.8	16.5	14	30.5	16.9
<i>2</i>	1.8	23.5	19	42.5	23.6
<i>3</i>	1.8	21	29	50	27.8
<i>4</i>	1.8	21	4	25	13.8

Figure 5 - Schematic for the L-Shaped Pressure Probe

Two Kulite xcq-062 pressure transducers are used in the test rig. The first serves as a reference transducer with a range of 7 bara. The second is implemented inside the test probe with a range of 35 bara. Both transducers are equipped with identical protective B screens that limit their flat response to 20 kHz [26]. The screens attenuate the high frequency components (>20 kHz) of the measured pressure signal. The sensitivities are 1.4 mV/V/bars and 0.286 mV/V/bars for the reference transducer and the test probe respectively.

The data is acquired at a sampling rate of 200 kHz in order to ensure at least 20 samples per cycle at the highest rotor speed of the siren disk. The pressure excitation is introduced as a ramp in the rotor speed of the Siren Disk from 0 Hz up to 10 kHz (0-7500 rpm) in 15 seconds followed by a similar deceleration ramp. The test ramps provide time intervals of 50 milliseconds with a constant hpf (± 17 Hz). The pressure level was randomly changed during the hpf ramp (0 – 4.5 barg), yielding signals at different amplitudes for a given frequency. The different pressure levels help in identifying the system's dependency on the peak-to-peak amplitude. This procedure is applied for the four L-shaped probes under investigation.

Figure 6 shows the measured pressures across the siren disk opening compared with the nozzle pressure at zero disk speed. It is observed that the nozzle set pressure drops by 35% ($\pm 1\%$) in gauge value. The figure also demonstrates that both the reference sensor and the probe are exposed to the same steady pressure.

Figure 6 - The measured steady pressure relative to the set nozzle pressure

Figure 7a shows the pressure signal spectrum of the uninterrupted impinging jet - i.e. disk at zero rotational speed. The observed response is attributed to turbulent pressure fluctuations. However, it is considered negligible when compared to the spectrum excited by the rotation of the Siren Disk - Figure 7b. The frequency spectrum of both the reference transducer and probe-3 clearly shows the existence of the fundamental frequency as well as its harmonics, which is a consequence of the step change in pressure levels due to the opening/closing cycles produced by the disk. Comparing the probe to the reference transducer, it is observed that while the signal components are at the same frequencies, yet the amplitudes are varying for each harmonic. Figure 7b also shows that signal components beyond the 5th harmonic can be neglected as they are at least two orders of magnitude smaller than the fundamental component.

Figure 7 - Frequency spectrum for (a) uninterrupted flow and (b) periodic flow at 3.5 kHz, measured by the reference transducer and Probe-3.

A final qualification is the study of the wind effect due to the Siren Disk rotation. The static pressure is measured during the rotation of the Siren Disk while the supply nozzle is closed. Figure 8 represents the relative pressure drop as a function of the hpf (i.e. rotor speed), showing a maximum static pressure drop due to rotation of approximately 1% at maximum hpf. Hence, the wind effect is negligible compared to the target pressure levels in this study.

Figure 8 - Pressure drop due to disk rotation

4. Proof of Concept

The four probes under investigation were tested up to 10 kHz. Figure 9 compares the time domain pressure signal for the reference transducer and test probe-3 at four different excitation frequencies. It is observed that the signal of the test probe is amplified - Figure 9d - or attenuated - Figure 9a, b, and c, and also delayed, as a function of the excitation frequency. Such behavior is the typical signature of the pneumatic channel in the probe. Another interesting observation is the distortion of the probe's signal creating a saw tooth, even at relatively low excitation frequencies.

At low hpf, the reference signal has a shape similar to a square. However, at higher hpf, the harmonics of the reference signal are less pronounced, yielding a signal closer to a sine wave - Figure 9a and b. This can be attributed either to the cutoff frequency of the transducer screens (20 kHz), or to the aerodynamic nature of the jet. Nonetheless, the results confirm the Siren Disk's capability to generate periodic pressure signals up to 10 kHz, while achieving amplitudes at the same order of magnitude of the mean pressure.

Figure 9 - Reference and test probe time domain signals for excitations of 10, 7, 3 and 1.2 kHz for Probe-3

Six data sets are obtained at several pressure levels (1.5 bara to 4.5 bara) - Figure 10. **Error! Reference source not found.** Five sets are used in the transfer function development (data sets 1-5), and one (data set 6) is used as a test case for signal reconstruction.

Figure 10 - Reference data pressure levels for Probe-3

5. Non-linear Transfer Function Identification

In order to reconstruct the original signal based on the test probe signal, the latter's dynamic response needs to be identified. This is done by building a transfer function between the reference and test probe. A typical method is to create a non-parametric transfer function in the frequency domain by taking the ratio between the Fourier (or Laplace) Transforms of the probe and the reference transducer.

A linear system identification approach was first attempted, however, the signal reconstruction based on the inverse transfer function was unsuccessful. These results signaled the potential non-linear behavior of the pneumatic channels. Hence, a different methodology is developed for that purpose.

Methods based on the Fast Fourier Transform (FFT) are presented by several authors in prior works [7,27,28]. The signal decomposition based on such a method is an excellent mean to identify the frequency spectrum. However, the FFT was proven inadequate in the calculation of the phase angle. This is due to the increasing uncertainty in handling harmonics of low amplitude [29–32]. Since the phase shifts play an essential role in the accurate reconstruction of the measured data - Figure 9, the FFT methodology should not be adopted for applications where the details of the pressure signal are important.

Alternatively, a Fourier series decomposition is used, where the periodic signals of the reference and the test probe are represented as the sum of the offset (mean) and the periodic components at the fundamental frequency and its harmonics:

$$q(t) = q_0 + \sum_{n=1}^{n_h} A_n \cos(n\omega t) + \sum_{n=1}^{n_h} B_n \sin(n\omega t) = q_0 + \sum_{n=1}^{n_h} Q_n \sin(n\omega t + \Phi_n) \quad \text{Eq. 1}$$

$$Q_n = \sqrt{A_n^2 + B_n^2} \quad \text{Eq. 2}$$

$$\Phi_n = \arctan\left(\frac{B_n}{A_n}\right) \quad \text{Eq. 3}$$

where

$$q_0 = \frac{1}{T} \int_0^T q(t) dt \quad \text{Eq. 4}$$

$$A_n = \frac{2}{T} \int_0^T q(t) \cos(n\omega t) dt \quad \text{Eq. 5}$$

$$B_n = \frac{2}{T} \int_0^T q(t) \sin(n\omega t) dt \quad \text{Eq. 6}$$

This decomposition procedure is performed on signal segments obtained at a given hpf with the objective of characterizing the nature of the transfer function. As a first step, the optical signal of the hole passing is used to identify the fundamental frequency of the segment through an FFT. Consequently, one pressure cycle of the segment is considered for the Fourier series. Only the five first harmonics are included, as they were shown previously to be the most predominant of the spectrum - Figure 7. A linear transfer function for each harmonic - gain and phase lag - is calculated as follows:

$$G_n(\omega) = \frac{Q_{n_{meas.}}(\omega)}{Q_{n_{ref.}}(\omega)} \quad \text{Eq. 7}$$

$$\Phi_{n_{correction}}(\omega) = \Phi_{n_{meas.}}(\omega) - \Phi_{n_{ref.}}(\omega) \quad \text{Eq. 8}$$

where n is the order of the harmonic, starting with n = 1 for the fundamental frequency. The gains of the initial 5 harmonics were calculated and averaged (Eq. 7) for the different pressure signals (1.5 bara to 4.5 bara - data sets 1 to 5) - Figure 11. In a linear system, the gain at a given frequency would be the same whether or not it is a fundamental or a harmonic (i.e. $G_2(\omega) = G_1(2\omega)$). It is observed though, that the gain functions are different for each harmonic in terms of amplitude and shape, thus confirming the non-linearity of the pneumatic channel.

Figure 11 - Harmonics gains for Probe-3 - plotted with respect to absolute frequency values.

In addition, the offset gains (Eq. 9) are calculated and averaged at the five different mean pressure levels:

$$G_0(\omega) = \frac{q_{0_{meas.}}(\omega)}{q_{0_{ref.}}(\omega)} \quad \text{Eq. 9}$$

Figure 12 represents the averaged offset gain and the fundamental phase lag as a function of the fundamental hpf for the 4 probes under investigation. It should be noted that the offset and the amplitude are inter-dependent due to the nature of the Siren Disk. The error bars indicate the deviation in gain and phase lag due to the nozzle

pressure level variation. The non-constant nature of the offset gain is emphasizing the signature of a non-linear system.

The phase shifts of the fundamental frequency are highly dependent on the probe geometry, suggesting diverse natural frequencies for the different probes - Figure 12 b, d, f, h. The resonant frequencies of the four probes - identified through the 90° and 270° phase shift - vary between 1.5 and 2.1 kHz. The highest resonant frequencies are achieved by the shortest channels, which is in agreement with basic principles of organ pipe or Helmholtz resonator models.

Figure 12 - L-shaped probes offset gain and phase lag

The mathematical implications of the identified non-linearity are:

1. The gain at a given frequency is dependent on the order of the harmonic ($G_2(\omega) \neq G_1(2\omega)$).

2. The offset gain G_0 is a function of the hpf.
3. The gains are dependent on the amplitude/mean of the input pressure signal.

The non-linear gain function is then described as follows:

$$G_n = G_n(\omega_1, A, \bar{A}) \quad \text{Eq. 10}$$

where ω_1 is the fundamental frequency, A is the amplitude, and $\bar{A}$ is the signal offset. In order to assess these implications, the averaged harmonic gain functions of probe-3 are plotted with respect to the fundamental frequency and analyzed - Figure 13. The error bars due to the varying nozzle pressure are shown similar to Figure 12 and are contained within a 10% band. It is observed that the errors increase around frequencies where the gradient of the functions varies significantly. However, it will be shown later -section 7- that the error propagation is insignificant due to the low amplitudes of the harmonics relative to the fundamental. As a consequence, the effect on the pressure mean and amplitude can be neglected for these test probes operated at mean pressure ranging from 1.25 to 3.25 bara with amplitudes from 0.25 to 2.25 bar and the gain functions can be represented as a function of the hpf.

Figure 13 - Harmonic gains for Probe-3 - Error bars cover 6 different pressure levels

Given the presented non-linearity, a multi-dimensional transfer function approach is developed for the system identification of the test probes. It is consisting of a set of sub transfer functions describing the pressure averaged gain and phase lag for the offset and the harmonics up to the 5th order. The transfer functions are mathematically represented by piecewise polynomial fits. The gain functions for the harmonics are deduced relative to each fundamental frequency (hpf).

6. Signal Reconstruction.

The probe signal of data set 6 (not used in the system identification) is reconstructed using the multi-dimensional transfer function developed a priori. The signal reconstruction on a cycle at a hpf = ω_1 is computed as follows:

$$q_{recon.}(t) = \frac{q_{0_{meas.}}}{G_0(\omega_1)} + \sum_{n=1}^5 \frac{Q_{n_{meas.}}}{G_n(\omega_1)} \sin(n \cdot \omega_1 \cdot t + \Phi_{n_{meas.}} - \Phi_{n_{correction}}(\omega_1)) \quad \text{Eq. 11}$$

Figure 14 compares samples of reconstructed signals with reference and measured signals of data set 6 suggesting excellent agreement between the reference and reconstructed signals. The presented examples in Figure 14 are for Probe-3 and at four excitations frequencies (1200 Hz, 3000 Hz, 7000 Hz and 9520 Hz). The figure shows that the transfer function identification and the reconstruction method are successful even at fundamental excitation frequencies far beyond the probe's resonant frequency (1.325 kHz for Probe-3).

Figure 14 - Measured and reconstructed signals for excitations of 1.2, 3, 7 and 9.5 kHz for Probe-3

7. Accuracy of Signal Reconstruction

The signal reconstruction accuracy is estimated using a goodness of fit (gof) variable, which compares the reconstructed signal to the reference signal as follows:

$$gof = \left(1 - \frac{\sqrt{\sum (q_{ref} - q)^2}}{\sqrt{\sum (q_{ref} - \bar{q}_{ref})^2}} \right) \quad \text{Eq. 12}$$

The resulting goodness values (data set 6) are plotted as a function of the hpf for each probe - Figure 15. For the purpose of comparison, the probe data before reconstruction are phase corrected, and used to calculate a baseline goodness relative to the reference signal. The reconstructed signal has goodness values less than 80% for low (<1 kHz) and high (>8 kHz) frequencies. In the range of 1 kHz to 8 kHz, goodness values higher than 80% are achieved. However, with recurring drops at frequencies where the second harmonic gradients change sign - Figure 13. Note that the goodness of the reconstructed data samples shown in Figure 14 vary between 70% and 85%, suggesting that a goodness of 70% represents a good match between the two signals. These results confirm the adequacy of the averaged multi-dimensional transfer function approach in the system identification, and signal reconstruction of pneumatic channels.

Figure 15 - Goodness of fit values of reconstructed signals for L-Shaped probes (data set 6)

8. Conclusions

The paper presents a Siren Disk proof of concept capable of generating pressure signals up to 10 kHz. The signals are up to 4.5 bara, with peak-peak values up to 3.5 bar. The produced pressure signals are used in the system identification of four pressure probes comprised of a pressure transducer and a pneumatic channel.

It is found that the probe's response is nonlinear in amplitude/mean, as well as frequency, hence, linear system identification toolboxes are inadequate. The transfer function of the pressure probe is deduced relative to a reference pressure signal using a Fourier series. The average gain is used for the different amplitude/mean variations, as it deviated only $\pm 10\%$ between 1.25 bara and 3.25 bara mean pressure. Independent gain functions are deduced for the offset and the first five harmonics as a function of the fundamental frequency, thus creating a multi-dimensional transfer function.

The flat response range of the probes tested are limited to very low frequencies (< 200 Hz). Beside the fact that the pneumatic channels will significantly violate the defined cutoff frequency of the transducers, they will also cause misleading time averaged measurements – amplification or attenuation function of the fluctuation frequency.

The inverse transfer function is then used to reconstruct the probe signal back to the reference signal of a new dataset. Hence, eliminating the effect of the pneumatic channel on the measurement. The averaged transfer functions perform well, suggesting a minor effect of pressure within the range under investigation. The signal reconstruction is shown to be successful for a wide range of frequencies, even beyond the natural frequency of the probes.

The observed non-linearity in the response of the pneumatic probes highlights the need for generic nonlinear models capable of describing their dynamic behavior, including the acoustic waves interactions, and the geometrical features of the probe.

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