Advanced Discrete Choice Model: What Do We Do With Them?

Michel Bierlaire

Transport and Mobility Laboratory
School of Architecture, Civil and Environmental Engineering
Ecole Polytechnique Fédérale de Lausanne

November 19, 2017
Outline

1. Demand and supply
2. Disaggregate demand models
3. Literature
4. A generic framework
5. A simple example
   - Example: one theater
   - Example: two theaters
6. Case study
7. Conclusion
Demand models

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch
Demand models

- Usually in OR:
  - optimization of the supply
  - for a given (fixed) demand
Aggregate demand

- Homogeneous population
- Identical behavior
- Price ($P$) and quantity ($Q$)
- Demand functions: $P = f(Q)$
- Inverse demand: $Q = f^{-1}(P)$
Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.
Demand and supply

Demand-supply interactions

**Operations Research**
- Given the demand...
- configure the system

**Behavioral models**
- Given the configuration of the system...
- predict the demand
Demand-supply interactions

Multi-objective optimization

Minimize costs

Maximize satisfaction
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Choice models

Disaggregate demand models

Behavioral models

- Demand = sequence of choices
- Choosing means trade-offs
- In practice: derive trade-offs from choice models
Choice models

Theoretical foundations

- Random utility theory
- Choice set: $C_n$
- $y_{in} = 1$ if $i \in C_n$, 0 if not
- Logit model:

$$P(i|C_n) = \frac{y_{in}e^{V_{in}}}{\sum_{j \in C} y_{jn}e^{V_{jn}}}$$
Logit model

Utility

\[ U_{in} = V_{in} + \varepsilon_{in} \]

- Decision-maker \( n \)
- Alternative \( i \in C_n \)

Choice probability

\[ P_n(i|C_n) = \frac{y_{in}e^{V_{in}}}{\sum_{j \in C} y_{jn}e^{V_{jn}}} . \]
Variables: $x_{in} = (p_{in}, z_{in}, s_n)$

Attributes of alternative $i$: $z_{in}$
- Cost / price ($p_{in}$)
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker $n$: $s_n$
- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.
Demand curve

Disaggregate model

\[ P_n(i|p_{in}, z_{in}, s_n) \]

Total demand

\[ D(i) = \sum_n P_n(i|p_{in}, z_{in}, s_n) \]

Difficulty

Non linear and non convex in \( p_{in} \) and \( z_{in} \)
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Stochastic traffic assignment

Features
- Nash equilibrium
- Flow problem
- Demand: path choice
- Supply: capacity
Selected literature

- [Dial, 1971]: logit
- [Daganzo and Sheffi, 1977]: probit
- [Fisk, 1980]: logit
- [Bekhor and Prashker, 2001]: cross-nested logit
- and many others...
Revenue management

Features

- Stackelberg game
- Bi-level optimization
- Demand: purchase
- Supply: price and capacity
Selected literature

- [Labbé et al., 1998]: bi-level programming
- [Andersson, 1998]: choice-based RM
- [Talluri and Van Ryzin, 2004]: choice-based RM
- [Gilbert et al., 2014a]: logit
- [Gilbert et al., 2014b]: mixed logit
- [Azadeh et al., 2015]: global optimization
- and many others...
Facility location problem

Features

- Competitive market
- Opening a facility impact the costs
- Opening a facility impact the demand
- Decision variables: availability of the alternatives

\[ P_n(i|C_n) = \frac{y_{in}e^{V_{in}}}{\sum_{j \in C} y_{jn}e^{V_{jn}}} \]
Selected literature

- [Hakimi, 1990]: competitive location (heuristics)
- [Benati, 1999]: competitive location (B & B, Lagrangian relaxation, submodularity)
- [Serra and Colomé, 2001]: competitive location (heuristics)
- [Marianov et al., 2008]: competitive location (heuristic)
- [Haase and Müller, 2013]: school location (simulation-based)
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A linear formulation

Utility function

\[ U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}. \]

Simulation

- Assume a distribution for \( \varepsilon_{in} \)
- E.g. logit: i.i.d. extreme value
- Draw \( R \) realizations \( \xi_{inr} \), \( r = 1, \ldots, R \)
- The choice problem becomes deterministic
Scenarios

**Draws**

- Draw $R$ realizations $\xi_{inr}, r = 1, \ldots, R$
- We obtain $R$ scenarios

$$U_{inr} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$  

- For each scenario $r$, we can identify the largest utility.
- It corresponds to the chosen alternative.
Capacities

- Demand may exceed supply
- Each alternative $i$ can be chosen by maximum $c_i$ individuals.
- An exogenous priority list is available.
- The numbering of individuals is consistent with their priority.
Priority list

Application dependent
- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework
The list of customers must be sorted
References

- Technical report: [Bierlaire and Azadeh, 2016]
- TRISTAN presentation: [Pacheco et al., 2016]
- STRC proceeding: [Pacheco et al., 2017]
Demand model

- Population of $N$ customers ($n$)
- Choice set $C$ ($i$)
- $C_n \subseteq C$: alternatives considered by customer $n$

Behavioral assumption

- $U_{in} = V_{in} + \varepsilon_{in}$
- $V_{in} = \sum_k \beta_{ink} x_{ink}^e + q^d(x^d)$
- $P_n(i|C_n) = \Pr(U_{in} \geq U_{jn}, \forall j \in C_n)$

Simulation

- Distribution $\varepsilon_{in}$
- $R$ draws $\xi_{in1}, \ldots, \xi_{inR}$
- $U_{inr} = V_{in} + \xi_{inr}$
Supply model

- Operator selling services to a market
  - Price $p_{in}$ (to be decided)
  - Capacity $c_i$
- Benefit (revenue \textminus cost) to be maximized
- Opt-out option ($i = 0$)

**Price characterization**
- Continuous: lower and upper bound
- Discrete: price levels

**Capacity allocation**
- Exogenous priority list of customers
- Assumed given
- Capacity as decision variable
MILP (in words)

\[
\text{MILP} \\
\text{max } \text{benefit} \\
\text{subject to } \text{utility definition} \\
\text{availability} \\
\text{discounted utility} \\
\text{choice} \\
\text{capacity allocation} \\
\text{price selection}
\]
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A simple example

Context
- $C$: set of movies
- Population of $N$ individuals
- Competition: staying home watching TV
One theater – homogenous population

Alternatives
- Staying home: $U_{cn} = 0 + \varepsilon_{cn}$
- My theater: $U_{mn} = -10.0p_{m} + 3 + \varepsilon_{mn}$

Logit model
$\varepsilon_{m}$ i.i.d. EV(0,1)
Demand and revenues

![Graph showing demand and revenues as functions of price. The graph displays two curves: one for demand and one for revenues. The demand curve peaks at a certain price and then decreases, while the revenues curve shows a peak before declining as well.](image-url)
Optimization

Solver
GLPK v4.61 under PyMathProg

Data
- \( N = 1 \)
- \( R = 1000 \)

Results
- Optimum price: 0.276
- Demand: 57.4%
- Revenues: 0.159
Demand and revenues

![Graph showing demand and revenues over price range from 0 to 1.]

- Demand curve peaks at a certain price and decreases as price increases.
- Revenues curve reflects the demand curve and starts at zero price, increasing with price until a peak, then declining.

Michel Bierlaire (EPFL)
Heterogeneous population

Two groups in the population

\[ U_{mn} = -\beta_n p_m + c_n \]

Young fans: 2/3
\[ \beta_1 = -10, \ c_1 = 3 \]

Others: 1/3
\[ \beta_2 = -0.9, \ c_2 = 0 \]
Demand and revenues

A simple example

Example: one theater

Demand and revenues

Price

Demand

Revenues

Young fans

Others
Optimization

Data
- $N = 3$
- $R = 500$

Results
- Optimum price: 0.297
- Customer 1 (fan): 52.4% [theory: 50.8%]
- Customer 2 (fan): 49% [theory: 50.8%]
- Customer 3 (other): 45.8% [theory: 43.4%]
- Demand: 1.472 (49%)
- Revenues: 0.437
Demand and revenues

![Graph showing demand and revenues as functions of price.](image-url)
Two theaters, different types of films
Two theaters, different types of films

**Theater \( m \)**
- Attractive for young people
- Star Wars Episode VII

**Theater \( k \)**
- Not particularly attractive for young people
- Tinker Tailor Soldier Spy

Heterogeneous demand
- Two third of the population is young (price sensitive)
- One third of the population is not (less price sensitive)
Two theaters, different types of films

Data

- Theaters $m$ and $k$
- $N = 9$
- $R = 50$
- $U_{mn} = -10p_m + 4$, $n =$ young
- $U_{mn} = -0.9p_m$, $n =$ others
- $U_{kn} = -10p_k + 0$, $n =$ young
- $U_{kn} = -0.9p_k$, $n =$ others

Theater $m$

- Optimum price $m$: 0.390
- Young customers: 3.48 / 6
- Other customers: 1.08 / 3
- Demand: 4.56 (50.7%)
- Revenues: 1.779

Theater $k$

- Optimum price $k$: 1.728
- Young customers: 0.0 / 6
- Other customers: 0.38 / 3
- Demand: 0.38 (4.2%)
- Revenues: 0.581
Two theaters, same type of films

**Theater $m$**
- Expensive
- Star Wars Episode VII

**Theater $k$**
- Cheap (half price)
- Star Wars Episode VIII

**Heterogeneous demand**
- Two third of the population is young (price sensitive)
- One third of the population is not (less price sensitive)
Two theaters, same type of films

Data
- Theaters $m$ and $k$
- $N = 9$
- $R = 50$
- $U_{mn} = -10p + 4$, $n =$young
- $U_{mn} = -0.9p$, $n =$others
- $U_{kn} = -10p/2 + 4$, $n =$young
- $U_{kn} = -0.9p/2$, $n =$others

Theater $m$
- Optimum price $m$: 3.582
- Young customers: 0
- Other customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater $k$
- Closed
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Challenge

- Select a real choice model from the literature
- Integrate it in an optimization problem.
Parking choices

- $N = 50$ customers
- $C = \{\text{PSP, PUP, FSP}\}$
- $C_n = C \quad \forall n$

- PSP: 0.50, 0.51, \ldots, 0.65 (16 price levels)
- PUP: 0.70, 0.71, \ldots, 0.85 (16 price levels)
- Capacity of 20 spots
Choice model: mixtures of logit model [Ibeas et al., 2014]

\[ V_{FSP} = \beta_{AT} AT_{FSP} + \beta_{TD} TD_{FSP} + \beta_{\text{Origin INT } FSP} \text{Origin INT } FSP \]

\[ V_{PSP} = \text{ASC}_{PSP} + \beta_{AT} AT_{PSP} + \beta_{TD} TD_{PSP} + \beta_{\text{FEE}} \text{FEE}_{PSP} \]

\[ + \beta_{\text{FEE}_{PSP \text{(LowInc)}}} \text{FEE}_{PSP \text{LowInc}} + \beta_{\text{FEE}_{PSP \text{(Res)}}} \text{FEE}_{PSP \text{Res}} \]

\[ V_{PUP} = \text{ASC}_{PUP} + \beta_{AT} AT_{PUP} + \beta_{TD} TD_{PUP} + \beta_{\text{FEE}} \text{FEE}_{PUP} \]

\[ + \beta_{\text{FEE}_{PUP \text{(LowInc)}}} \text{FEE}_{PUP \text{LowInc}} + \beta_{\text{FEE}_{PUP \text{(Res)}}} \text{FEE}_{PUP \text{Res}} \]

\[ + \beta_{\text{AgeVeh \leq 3}} \text{AgeVeh \leq 3} \]

- **Parameters**
  - Circle: distributed parameters
  - Rectangle: constant parameters

- **Variables:** all given but FEE (in bold)
Experiment 1: uncapacitated vs capacitated case (1)

- Capacity constraints are ignored
- Unlimited capacity is assumed

- 20 spots for PSP and PUP
- Free street parking (FSP) has unlimited capacity
Case study

Experiment 1: uncapacitated vs capacitated case (2)

Uncapacitated

Log Solution time (s)  Revenue

Log Solution time (s)  Revenue

Capacitated

Log Solution time (s)  Revenue

Log Solution time (s)  Revenue
Experiment 1: uncapacitated vs capacitated case (3)

Uncapacitated

![Graph showing Price and Demand for the uncapacitated case.]

Capacitated

![Graph showing Price and Demand for the capacitated case.]

Experiment 2: price differentiation by segmentation (1)

- Discount offered to residents
- Two scenarios (municipality)
  - Subsidy offered by the municipality
  - Operator obliged to offer reduced fees
- We expect the price to increase
  - PSP: \{0.60, 0.64, \ldots, 1.20\}
  - PUP: \{0.80, 0.84, \ldots, 1.40\}
Scenario 1

Scenario 2
Experiment 2: price differentiation by segmentation (3)

Scenario 1

Scenario 2
Other experiments

**Impact of the priority list**
- Priority list = order of the individuals in the data (i.e., random arrival)
- 100 different priority lists
- Aggregate indicators remain stable across random priority lists

**Benefit maximization through capacity allocation**
- 4 different capacity levels for both PSP and PUP: 5, 10, 15 and 20
- Optimal solution: PSP with 20 spots and PUP is not offered
- Both services have to be offered: PSP with 15 and PUP with 5
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Summary

Demand and supply
- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models
- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models
Optimization

Discrete choice models
- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation
- Linear in the decision variables
- Large scale
- Fairly general
Ongoing research

- Decomposition methods
- Scenarios are (almost) independent from each other (except objective function)
- Individuals are also loosely coupled (except for capacity constraints)


Bibliography III


Bibliography V


Bibliography VI

