

Sinobeam: Focused Beamforming for PET Scanners

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Abstract—Focused beamformers have been extensively used in phased-array signal processing, leading to simple and efficient imaging procedures, with high sensitivity and resolution. The beamshape acts as a spatial filter, scanning the intensity of the incoming signal for particular locations. We introduce beamforming in the context of Positron Emission Tomography (PET), and propose a new beamformer called Sinobeam. Inspired by the Flexibeam framework [1], we sample the beamforming weights from an analytically-specified beamforming function. Since the weights are data-independent, the resulting imaging algorithm is extremely efficient, while presenting better resolution and contrast than state of the art methods as demonstrated by simulation.

In classical phased-array signal processing, *beamforming* combines coherently networks of antennas, so as to achieve desired radiation patterns. Mathematically speaking, this translates into a linear combination of the data, spatially filtering the observed signal with the array *beamshape*. By analogy, we hence define *beamformed data* in the context of positron emission tomography as:

$$\boldsymbol{\mu} = \mathbf{w}^T \mathbf{n} = \sum_{d=1}^D w_d n(\theta_d, p_d), \quad (1)$$

where $n(\theta_d, p_d)$ is the number of gamma ray coincidences recorded by the detector pair d with coordinates $(\theta_d, p_d) \in [0, \pi] \times \mathbb{R}$. It follows a Poisson distribution with mean:

$$\mathbb{E}[n(\theta_d, p_d)] = \int_{\mathbb{R}^2} \lambda(\mathbf{x}) \delta(p_d - \langle \mathbf{x}, \boldsymbol{\xi}_{\theta_d} \rangle) d\mathbf{x}, \quad (2)$$

where $\boldsymbol{\xi}_{\theta_d} = [\cos(\theta_d), \sin(\theta_d)] \in \mathbb{R}^2$ and $\lambda : \mathbb{R}^2 \rightarrow \mathbb{R}_+$ is the *intensity function*, proportional to the metabolic activity that we wish to recover. Plugging (2) into (1) yields on expectation

$$\mathbb{E}[\boldsymbol{\mu}] = \int_{\mathbb{R}^2} \lambda(\mathbf{x}) \left[\sum_{d=1}^D w_d \delta(p_d - \langle \mathbf{x}, \boldsymbol{\xi}_{\theta_d} \rangle) \right] d\mathbf{x} = \langle \lambda, \mathbf{b} \rangle.$$

Hence, beamforming the data equivalently filters the intensity function λ with the *beamshape* $\mathbf{b}(\mathbf{x}) := \sum_{d=1}^D w_d \delta(p_d - \langle \mathbf{x}, \boldsymbol{\xi}_{\theta_d} \rangle)$. Assuming one could choose *beamforming weights* $\{w_d(\mathbf{y})\}_d$ such that $\mathbf{b}(\mathbf{x}) \simeq \delta(\mathbf{x} - \mathbf{y})$ for some $\mathbf{y} \in \mathbb{R}^2$, then the reproducing property of the Dirac delta function would allow us to form the following estimate of λ :

$$\hat{\lambda}(\mathbf{y}) = \hat{\boldsymbol{\mu}}(\mathbf{y}) = \mathbf{w}(\mathbf{y})^T \mathbf{n}, \quad \forall \mathbf{y} \in \mathbb{R}^2.$$

This imaging procedure is called *imaging by beamforming* or *B-scan* in classical phased-array signal processing. In this paper, we design a Dirac-like beamshape using the Flexibeam framework [1]. First, we work with a notional *continuous detector field* on $[0, \pi] \times \mathbb{R}$, and derive the *beamforming function* associated to the Dirac beamshape. Then, we sample this beamforming function for the specific detectors composing the scanner in use, and form the *empirical beamshape*. For a continuous detector ring, the beamshape is given by

$$\mathbf{b}(\mathbf{x}) = \int_0^\pi \int_{\mathbb{R}} \omega(\theta, p) \delta(p - \langle \mathbf{x}, \boldsymbol{\xi}_\theta \rangle) dp d\theta = \int_0^\pi \omega(\theta, \langle \mathbf{x}, \boldsymbol{\xi}_\theta \rangle) d\theta, \quad (3)$$

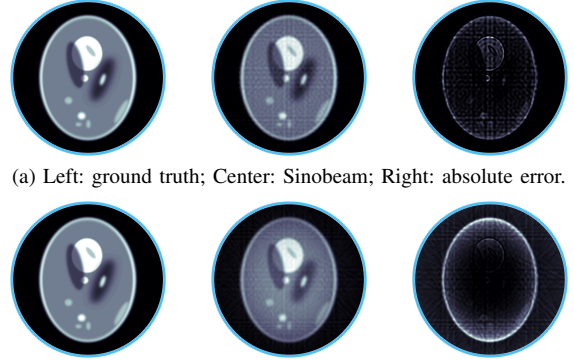


Fig. 1: Imaging with Sinobeam and comparison to state of the art.

where $\omega : [0, \pi] \times \mathbb{R} \rightarrow \mathbb{R}$ is called the *beamforming function*. From the inversion formula of the 2D Radon transform we know that

$$\delta(\mathbf{x} - \mathbf{y}) = \int_0^\pi [h * \mathcal{R}\{\delta(\mathbf{x} - \mathbf{y})\}(\theta, \cdot)](\langle \mathbf{x}, \boldsymbol{\xi}_\theta \rangle) d\theta, \quad (4)$$

with $h : \mathbb{R} \rightarrow \mathbb{R}$ the *Ramp filter* defined in the Fourier domain as $\hat{h}(f) = |f|$. Hence, choosing the beamforming function ω in (3) as

$$\omega(\theta, p) = [h * \mathcal{R}\{\delta(\mathbf{x} - \mathbf{y})\}(\theta, \cdot)](p) \quad (5)$$

yields the desired beamshape $\mathbf{b}(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{y})$. In practice, since the data is noisy and to avoid numerical instabilities, a truncated Ramp filter must be used: $\hat{h}(f) = |f| \chi([-f_d, f_d])$, for some $f_d > 0$. Then, the relationship (4) only holds approximately but (5) admits a convenient closed form:

$$\omega(\theta, p; \mathbf{y}) = \begin{cases} f_d^2 & \text{if } p = \langle \mathbf{y}, \boldsymbol{\xi}_\theta \rangle, \\ \frac{f_d \sin[2\pi f_d(p - \langle \mathbf{y}, \boldsymbol{\xi}_\theta \rangle)]}{\pi(p - \langle \mathbf{y}, \boldsymbol{\xi}_\theta \rangle)} + \frac{\cos[2\pi f_d(p - \langle \mathbf{y}, \boldsymbol{\xi}_\theta \rangle)] - 1}{2\pi^2(p - \langle \mathbf{y}, \boldsymbol{\xi}_\theta \rangle)^2} & \text{else.} \end{cases}$$

For a given scanner, choosing $w_d(\mathbf{y}) = \omega(\theta_d, p_d; \mathbf{y})$ for a finite number D of detector pairs with coordinates (θ_d, p_d) yields an empirical beamshape approximating (4), which we call *Sinobeam*. The main lobe of the achieved beamshape gets narrower as the number of detectors D increases while the sidelobes intensities decrease as f_d becomes larger. Fig. 1 compares a Sinobeam-obtained image with an image generated using the filtered back projection algorithm, for simulated Poisson data and $D = 31125$ detector pairs. The Sinobeam image in Fig. 1a appears sharper, with higher contrast and an improved dynamical range (dark ellipsoids appear darker). Moreover, the filtered back projection image in Fig. 1b has non uniform errors, with a higher sensitivity in the centre than at the edges. Finally, the Sinobeam image is also much cheaper to compute and heavily parallelisable.

REFERENCES

- [1] P. Hurley and M. Simeoni, “Flexibeam: analytic spatial filtering by beamforming,” in *International Conference on Acoustics, Speech and Signal Processing (ICASSP), IEEE*, March 2016.