A Unified Framework for Rich Routing Problems with Stochastic Demands

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Outline

1. Introduction
2. Key Modeling Elements
3. Capturing Demand Stochasticity
4. Optimization Model
5. Numerical Experiments
6. Conclusions and Future Research
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1. Introduction
2. Key Modeling Elements
3. Capturing Demand Stochasticity
4. Optimization Model
5. Numerical Experiments
6. Conclusions and Future Research
- Rich routing features (Lahyani et al., 2015)
Introduction

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- Distribution, collection, or other context
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- Real-world demand forecasting
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- Distribution, collection, or other context
- Stochastic demands that can be non-stationary
- Real-world demand forecasting
- Bridging the gap between theory and practice (Gendreau et al., 2016)
Contributions

- Complete or partial relaxation of iid normal assumption
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- Probabilities and costs of undesirable events, resp. recourse actions:
  - Cost of demand uncertainty
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- Computational tractability for a general inventory policy
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- Generality and practical relevance of the approach
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- Probabilities and costs of undesirable events, resp. recourse actions:
  - Cost of demand uncertainty
- Computational tractability for a general inventory policy
- Generality and practical relevance of the approach
- Real case study showing superiority wrt deterministic approaches
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1 Introduction

2 Key Modeling Elements

3 Capturing Demand Stochasticity

4 Optimization Model

5 Numerical Experiments

6 Conclusions and Future Research
Key Notations and Terms

- Planning horizon $\mathcal{T}$
Key Modeling Elements

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- Planning horizon $\mathcal{T}$
- Multiple depots, demand points $\mathcal{P}$, and supply points

- Heterogeneous fixed fleet $K$
- Demand stochasticity leads to stock-outs:
  - $\sigma_{it} = 1$ for stock-out of point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$, 0 otherwise
- And route failures:
  - Tours vs. trips: depot-delimited vs. supply point-delimited
  - $S \in S_k$: a trip in the set of trips performed by vehicle $k \in K$
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  - $\mathcal{S} \in \mathcal{G}_k$: a trip in the set of trips performed by vehicle $k \in \mathcal{K}$
Discretized Maximum Level (ML) Policy

- For tractable pre-processing of the stochastic information
- $l_{it}$: inventory of point $i \in \mathcal{P}$ at the start of period $t \in \mathcal{T}$
- $\Lambda_{it}$: inventory of point $i \in \mathcal{P}$ after delivery in period $t \in \mathcal{T}$
- $\omega_i$: inventory capacity of point $i \in \mathcal{P}$

Figure 1: Discretization example for $\Lambda_{it}$
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Demand Forecasting

- Stochastic non-stationary demand $\rho_{it}$ for point $i \in P$ in period $t \in T$:
  \[
  \rho_{it} = \mathbb{E}(\rho_{it}) + \varepsilon_{it}
  \]  

Demand Forecasting

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  \[
  \rho_{it} = \mathbb{E}(\rho_{it}) + \varepsilon_{it}
  \]  
  \(1\)

- Combine $\varepsilon_{it}$, $\forall t \in \mathcal{T}, i \in \mathcal{P}$ in a vector:
  \[
  \varepsilon = (\varepsilon_{11}, \ldots, \varepsilon_{1|\mathcal{T}|}, \varepsilon_{21}, \ldots, \varepsilon_{|\mathcal{P}||\mathcal{T}|})
  \]  
  \(2\)
Demand Forecasting

- Stochastic non-stationary demand $\rho_{it}$ for point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$:

$$\rho_{it} = \mathbb{E}(\rho_{it}) + \varepsilon_{it}$$  \hspace{1cm} (1)

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- Let $\varepsilon \sim \Phi$ satisfy $\text{var}(\varepsilon) = K$ for any covariance structure $K$


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  \tag{1}
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  \tag{2}
  \]

- Let $\varepsilon \sim \Phi$ satisfy $\text{var}(\varepsilon) = K$ for any covariance structure $K$

- Use any model that provides $\mathbb{E}(\rho_{it}), \forall t \in \mathcal{T}, i \in \mathcal{P}$ and $\Phi$
Capturing Demand Stochasticity

Stock-out Probabilities: Branching

Figure 2: State Probabilities

\[
\begin{align*}
\sigma_{ig} &= 0 \\
\sigma_{ig} &= 1
\end{align*}
\]

\[
\begin{align*}
\sigma_{i(g+1)} &= 0 \\
\sigma_{i(g+1)} &= 1
\end{align*}
\]

\[
\begin{align*}
\sigma_{i(g+2)} &= 0 \\
\sigma_{i(g+2)} &= 1
\end{align*}
\]

\[
\begin{align*}
\sigma_{i(g+3)} &= 0 \\
\sigma_{i(g+3)} &= 1
\end{align*}
\]

\[
\begin{align*}
P(\Lambda_{ig} - \rho_{ig} - \rho_{i(g+1)} > 0 | \Lambda_{ig} - \rho_{ig} > 0) &= \sigma_{i(g+2)} = 0 \\
P(\Lambda_{ig} - \rho_{ig} - \rho_{i(g+1)} \leq 0 | \Lambda_{ig} - \rho_{ig} > 0) &= \sigma_{i(g+2)} = 1
\end{align*}
\]

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\]

\[
\begin{align*}
P(\omega_i - \rho_{i(g+1)} > 0) &= \sigma_{i(g+2)} = 0 \\
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Stock-out Probabilities: Formulation and pre-computing

- DVar: $y_{ikt} = 1$ if vehicle $k \in \mathcal{K}$ visits point $i \in \mathcal{P}$ in period $t \in \mathcal{T}$
Stock-out Probabilities: Formulation and pre-computing

- **DVar:** \( y_{ikt} = 1 \) if vehicle \( k \in \mathcal{K} \) visits point \( i \in \mathcal{P} \) in period \( t \in \mathcal{T} \)

- Stock-out probability at point \( i \in \mathcal{P} \) in period \( t \in \mathcal{T} \):

  \[
  p_{it}^{\text{DP}} = \mathbb{P} (\sigma_{it} = 1 | \bigwedge_{im} : m = \max (0, g < t: \exists k \in \mathcal{K} : y_{ikg} = 1)) \tag{3}
  \]
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  \]

- For a discretized ML policy, we can pre-compute expression (3), $\forall i \in \mathcal{P}, t \in \mathcal{T}$, with $\varepsilon \sim \Phi$ and $\text{var}(\varepsilon) = K$ using simulation
Stock-out Probabilities: Formulation and pre-computing

- DVar: \( y_{ikt} = 1 \) if vehicle \( k \in K \) visits point \( i \in P \) in period \( t \in T \)

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\[
\hat{p}_{it}^{DP} = \mathbb{P}(\sigma_{it} = 1 \mid \Lambda_{im} : m = \max(0, g < t : \exists k \in K : y_{ikg} = 1)) \quad (3)
\]

- For a discretized ML policy, we can pre-compute expression (3), \( \forall i \in P, t \in T \), with \( \varepsilon \sim \Phi \) and \( \text{var}(\varepsilon) = K \) using simulation

- The complexity is linear in the number of discrete levels
Capturing Demand Stochasticity

Rte Failure Probabilities: Formulation

- $S_t \in \mathcal{I}$: demand points in $\mathcal{I}$ visited in period $t \in \mathcal{T}$
Rte Failure Probabilities: Formulation

- \( S_t \in \mathcal{I} \): demand points in \( \mathcal{I} \) visited in period \( t \in \mathcal{T} \)

- Quantity delivered by vehicle \( k \in \mathcal{K} \) in trip \( \mathcal{I} \in \mathcal{G}_k \):

\[
\Gamma_{\mathcal{I}} = \sum_{S_0 \in \mathcal{I}} \sum_{s \in S_0} (\Lambda_{s0} - l_{s0}) + \sum_{t \in \mathcal{T} \setminus 0} \sum_{S_t \in \mathcal{I}} \sum_{s \in S_t} \left( \Lambda_{st} - \Lambda_{sm} + \sum_{h=m}^{t-1} \rho_{sh} \right),
\]

where \( m = \max(0, g \in \mathcal{T} : g < t : \exists k' \in \mathcal{K} : y_{sk'g} = 1) \)
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  \]

  where \( m = \max(0, g \in \mathcal{T}: g < t: \exists k' \in \mathcal{K}: y_{sk'g} = 1) \)

- Route failure probability:

  \[
  p^{RF}_{\mathcal{I}, k} = \mathbb{P}(\Gamma_{\mathcal{I}} > \Omega_k)
  \]
The route failure probabilities cannot be pre-computed
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Impose iid error terms $\varepsilon$ by setting:

$$
\Phi(\varepsilon) = \prod_{t \in T} \prod_{i \in P} \Phi'(\varepsilon_{it}),
$$

where $\Phi'$ is the marginal distribution of $\varepsilon_{it}$.
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Impose iid error terms $\varepsilon$ by setting:

$$\Phi (\varepsilon) = \prod_{t \in T} \prod_{i \in P} \Phi' (\varepsilon_{it}),$$  \hspace{1cm} (6)

where $\Phi'$ is the marginal distribution of $\varepsilon_{it}$

Use simulation to pre-process empirical distribution functions to be used at runtime (limited number)
Objective

- Expected Stock-Out and Emergency Delivery Cost (ESOEDC), using stock-out cost $\chi$ and emergency delivery cost $\zeta$:

$$\sum_{t \in T} \sum_{i \in P} \left( \chi + \zeta - \zeta \sum_{k \in K} y_{ikt} \right) p_{it}^{DP} \quad (7)$$

- Expected Route Failure Cost (ERFC), using supply point detour cost $C_S$ and weight multiplier $\psi$:

$$\sum_{k \in K} \sum_{S \in S_k} \psi C_S p_{RF,S,k} \quad (8)$$

- Deterministic cost components (routing, work balancing, visits, etc.)

Overestimates the real cost due to modeling simplifications

- Do-nothing vs. optimal reaction policy
Objective

- **Expected Stock-Out and Emergency Delivery Cost (ESOEDC),** using stock-out cost $\chi$ and emergency delivery cost $\zeta$:
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  \]

- Deterministic cost components (routing, work balancing, visits, etc.)
- Overestimates the real cost due to modeling simplifications
  - Do-nothing vs. optimal reaction policy
Deterministic Constraints

- Open and multi-period tours
- Periodicities, service choice
- Accessibility restrictions
- Time windows, max tour duration, equity
- Inventory management (inventory policy)
- Vehicle capacity management
- etc...
Probabilistic Constraints

- Instead of capturing stochasticity in the objective, control it in the constraints
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- Maximum stock-out probability, for a constant $\gamma^{DP} \in (0, 1]$:
  \[ p_{it}^{DP} \leq \gamma^{DP} \quad \forall t \in T, i \in P \]  (9)

- Maximum route failure probability, for a constant $\gamma^{RF} \in (0, 1]$:
  \[ p^{RF}_{\mathcal{S}, k} \leq \gamma^{RF} \quad \forall k \in K, \mathcal{S} \in \mathcal{S}_k \]  (10)
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Numerical Experiments

Solution Methodology

- Adaptive large neighborhood search
- Developed by Markov et al. (2016)
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- Adaptive large neighborhood search
- Developed by Markov et al. (2016)
- Excellent performance on classical VRP and IRP benchmarks
- Performance on real-world stochastic waste collection IRP instances:
  - Stability: on average 1-2% between best and worst over 10 runs
  - Speed: 10-15 min per problem, suitable for operational purposes
Waste Collection IRP: Instances

- 63 instances from Geneva, Switzerland
- Rich routing features
Waste Collection IRP: Instances

- 63 instances from Geneva, Switzerland
- Rich routing features
- Test stochastic policies varying the:
  - Emergency Collection Cost (ECC) $\zeta$
  - Route Failure Cost Multiplier (RFCM) $\psi$
Waste Collection IRP: Instances

- 63 instances from Geneva, Switzerland
- Rich routing features
- Test stochastic policies varying the:
  - Emergency Collection Cost (ECC) $\zeta$
  - Route Failure Cost Multiplier (RFCM) $\psi$
- Against deterministic policies varying the:
  - Container Effective Capacity (CEC)
  - Truck Effective Capacity (TEC)
Waste Collection IRP: Instances

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- Test stochastic policies varying the:
  - Emergency Collection Cost (ECC) $\zeta$
  - Route Failure Cost Multiplier (RFCM) $\psi$
- Against deterministic policies varying the:
  - Container Effective Capacity (CEC)
  - Truck Effective Capacity (TEC)
- Simulate undesirable events on final solution for original capacities
Figure 3: Geneva Service Area
Waste Collection IRP: Stochastic vs. Deterministic

Figure 4: Routing Cost and Number of Overflows
## Waste Collection IRP: Calculating Route Failures

### Table 1: Impact of ECDFs on Tractability

<table>
<thead>
<tr>
<th>ALNS version</th>
<th>Bins</th>
<th>ECC</th>
<th>RFCM</th>
<th>Cost (CHF)</th>
<th>Runtime (s.)</th>
<th>ECDF calls (millions)</th>
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<td></td>
<td></td>
<td></td>
<td>Best</td>
<td>Avg</td>
<td>Worst</td>
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<td>Original</td>
<td>–</td>
<td>100.00</td>
<td>1.00</td>
<td>662.65</td>
<td>666.64</td>
<td>672.87</td>
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<tr>
<td>ECDFs</td>
<td>1000</td>
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<td>869.52</td>
<td>903.81</td>
</tr>
</tbody>
</table>

*Note.* ECDF: Empirical Cumulative Distribution Function

*Note.* Bins: Number of bins in the ECDF binning implementation
Waste Collection IRP: Overestimation

Figure 5: Do-nothing vs. Optimal Reaction Policy
Facility Maintenance Problem: Instances

- 94 instances derived from the same data
- Probability of breakdown depends on last visit
Facility Maintenance Problem: Instances

- 94 instances derived from the same data
- Probability of breakdown depends on last visit
- Compare stochastic policies varying the:
  - Emergency Repair Cost (ERC) $\zeta$
  - Maximum allowed probability of breakdown $\gamma^{DP}$
Facility Maintenance Problem: Instances

- 94 instances derived from the same data
- Probability of breakdown depends on last visit

Compare stochastic policies varying the:
- Emergency Repair Cost (ERC) \( \zeta \)
- Maximum allowed probability of breakdown \( \gamma^{DP} \)

Against deterministic policies varying the:
- Minimum number \( \nu \) of required visits over \( T \)
Facility Maintenance Problem: Instances

- 94 instances derived from the same data
- Probability of breakdown depends on last visit
- Compare stochastic policies varying the:
  - Emergency Repair Cost (ERC) $\zeta$
  - Maximum allowed probability of breakdown $\gamma^{DP}$
- Against deterministic policies varying the:
  - Minimum number $\nu$ of required visits over $T$
- Simulate undesirable events on final solution
Figure 6: Routing Cost and Breakdowns for Stochastic Approach

(a) Routing Cost

(b) Breakdowns
Facility Maintenance Problem: Stochastic vs. Deterministic

Table 2: Performance Indicators for Stochastic Approach

<table>
<thead>
<tr>
<th>Model</th>
<th>ERC</th>
<th>$\gamma_{DP}$</th>
<th>Avg RC (CHF)</th>
<th>Avg EERC (CHF)</th>
<th>Avg Num Breakdowns</th>
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</thead>
<tbody>
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<td></td>
<td></td>
<td>75th Perc.</td>
</tr>
<tr>
<td>Prob. obj</td>
<td>250.00</td>
<td>–</td>
<td>1108.69</td>
<td>312.94</td>
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<tr>
<td>Prob. const</td>
<td>–</td>
<td>0.08</td>
<td>1010.44</td>
<td>0.00</td>
<td>3.91</td>
</tr>
</tbody>
</table>

Table 3: Performance Indicators for Deterministic Approach

<table>
<thead>
<tr>
<th>Model</th>
<th>ERC</th>
<th>$\nu$</th>
<th>Avg RC (CHF)</th>
<th>Avg EERC (CHF)</th>
<th>Avg Num Breakdowns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>75th Perc.</td>
</tr>
<tr>
<td>Deterministic</td>
<td>–</td>
<td>2</td>
<td>1945.96</td>
<td>0.00</td>
<td>3.16</td>
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<tr>
<td>Deterministic</td>
<td>–</td>
<td>1</td>
<td>1140.10</td>
<td>0.00</td>
<td>4.28</td>
</tr>
</tbody>
</table>

Note. Avg RC: Average routing cost

Note. Avg EERC: Average Expected Emergency Repair Cost
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Conclusions and Future Research

Conclusions

- Stochastic, non-stationary demands with few distributional assumptions
- Rich routing features
- Cost of demand uncertainty
- Tractability through pre-processing
- Negligible deviation of modeled from real cost

Future Research

- More tests on real-world benchmarks
- Lower bounds: column generation
Conclusions and Future Research

Conclusions

- Stochastic, non-stationary demands with few distributional assumptions
- Rich routing features
- Cost of demand uncertainty
- Tractability through pre-processing
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Conclusions and Future Research

Thank you

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A Unified Framework for Rich Routing Problems with Stochastic Demands
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References
