Intelligent Traffic Control and Service in Big Data Environment, EPFL

Data-driven spatio-temporal discretization for pedestrian flow characterization

Marija Nikolić, Michel Bierlaire

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Outline

Introduction

Methodology

Application

Conclusion

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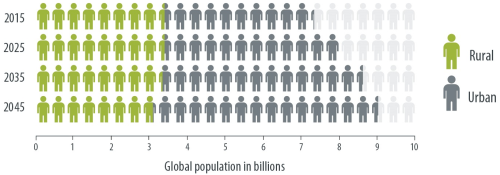
Urbanization

1950: **30%** of the population lives in cities
2014: **54%** of the population lives in cities

Challenges

Energy consumption, pollution, climate change
Increased traffic and congestion



Source: UN World Urbanization Prospects: 2011 Revision

Congestion: Pedestrian movements

Research challenges

Understand, describe and predict

Optimization of current infrastructure and operations

Efficient planning and management of future pedestrian facilities

Characterization

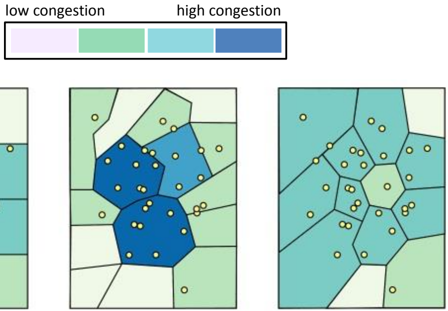
Quantities

- Density **k** (ped/m^2)
- Speed **v** (m/s)
- Flow **q** (ped/m·s)

Limitations

- Highly inspired by vehicular traffic
- Arbitrary spatial and temporal discretization

Discretization



Research challenges

- Results sensitive to minor changes
- Arbitrary discretization may introduce noise in data

How to define the discretization...

...independent of arbitrary chosen values?

How to define the discretization...

...independent of arbitrary chosen values?

Data-driven approach: Voronoi diagrams

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Context

Model

Space-time representation: $\Omega \subset \mathbb{R}^3$

Units: meters and seconds

$p = (x, y, t) \in \Omega$: physical position (x, y) in space at a specific time t

Assumption: Ω is convex (obstacle-free and bounded)

Data: trajectories

Continuous: $\Gamma_i : \{p_i(t) | p_i(t) = (x_i(t), y_i(t), t)\}$

Discrete (sample):
$\Gamma_i : \{p_{is} | p_{is} = (x_{is}, y_{is}, t_s)\}, t_s = [t_0, t_1, ..., t_f]$

3D Voronoi diagrams: 3DVoro

Definition

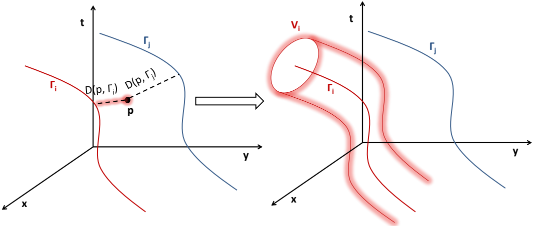
Associate $p \in \Omega$ with the closest Γ_i:

$$\delta_\Gamma(p, \Gamma_i) = \begin{cases} 1, & D(p,\Gamma_i) \leq D(p,\Gamma_j), \forall j \\ 0, & \text{otherwise} \end{cases}$$

$$D(p,\Gamma_i) = \min_{p_i}\{d(p,p_i)\}$$

Voronoi cell for Γ_i:

$$V_i = \{p \in |\delta_\Gamma(p,\Gamma_i) = 1\}$$



3DVoro: Distances

Spatial Euclidean distance

$$d_E(p, p_i) = \begin{cases} \sqrt{(x - x_i)^2 + (y - y_i)^2}, & t = t_i \\ \infty, & \text{otherwise} \end{cases}$$

Each point in time is independent

Motivated by the availability of snapshots of the floor area

All pedestrians must be observed at the exact same time

3DVoro: Distances

Time-Transform distances

$$d_{TT_1}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + v^2(t - t_i)^2}$$

$$d_{TT_2}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + \hat{v}_i(t_i)^2(t - t_i)^2}$$

$$d_{TT_3}(p, p_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2} + \hat{v}_i(t_i)|t - t_i|$$

Convert seconds into meters using speed

$d_{TT_1}(p, p_i)$, $d_{TT_2}(p, p_i)$: combine components based on the Euclidean norm

d_{TT_3}: weighted sum of two norms

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Predictive distance

$$d_P(p, p_i) = \begin{cases} \sqrt{(x_i^a - x)^2 + (y_i^a - y)^2}, & t - t_i \geq 0 \\ \infty, & \text{otherwise} \end{cases}$$

$$x_i^a = x_i^a(t) = x_i + (t - t_i)v_i^x(t_i)$$

$$y_i^a = y_i^a(t) = y_i + (t - t_i)v_i^y(t_i)$$

Accounts for the pedestrian dynamics

Anticipates future position when performing the assignment

Anticipation time: from zero to $t - t_i$

Points backward in time: infinitely distant

3DVoro: Distances

Mahalanobis distance

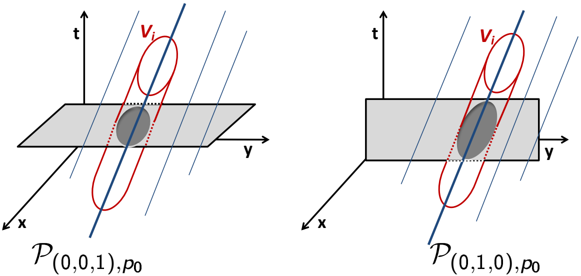
$$d_M(p, p_i) = \sqrt{(p - p_i)^T M_i (p - p_i)}$$

M_i: a change of variable matrix

Points in the movement direction of a pedestrian are “closer” than the points from other directions

Intersection with a plane

$\mathcal{P}_{(a,b,c),p_0}$: plane through p_0 with normal vector (a, b, c)



Voronoi-based traffic quantities

Consider $(x, y, t) \in \Omega$, and i such that $(x, y, t) \in V_i$

Density: $k(x, y, t) = \frac{1}{|V_i \cap \mathcal{P}_{(\mathbf{0},\mathbf{0},\mathbf{1}),(x,y,t)}|}$

Flow: $\vec{q}_{(a,b,0)}(x, y, t) = \frac{1}{|V_i \cap \mathcal{P}_{(a,b,\mathbf{0}),(x,y,t)}|}$

Velocity: $\vec{v}_{(a,b,0)}(x, y, t) = \frac{|V_i \cap \mathcal{P}_{(\mathbf{0},\mathbf{0},\mathbf{1}),(x,y,t)}|}{|V_i \cap \mathcal{P}_{(a,b,\mathbf{0}),(x,y,t)}|}$

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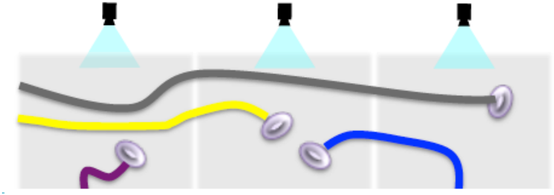
Lausanne train station

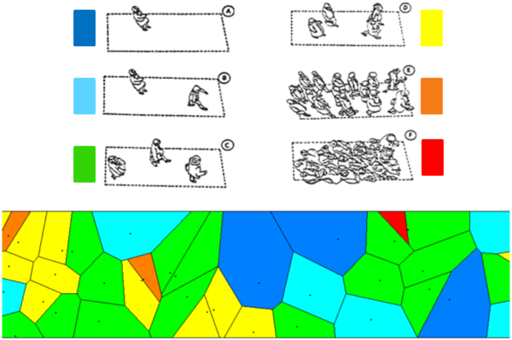
Lausanne train station: Data set

A large-scale network of smart sensors: a sparsity driven tracking (Alahi et al., 2014)

Dataset: 25,603 trajectories; February 2013



3DVoro illustration: Lausanne train station



- Data-driven discretization
- General framework
- Microscopic characterization
- Applicable to continuous and discrete data

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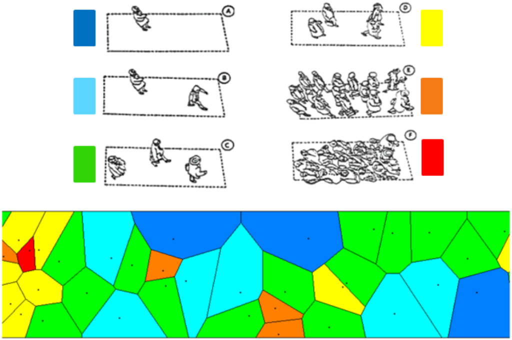

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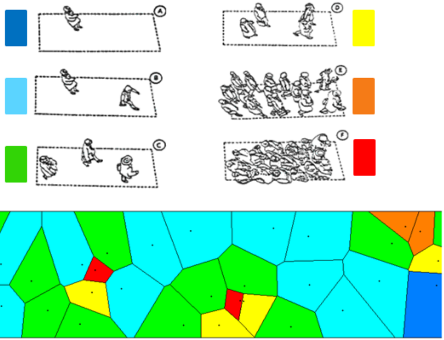

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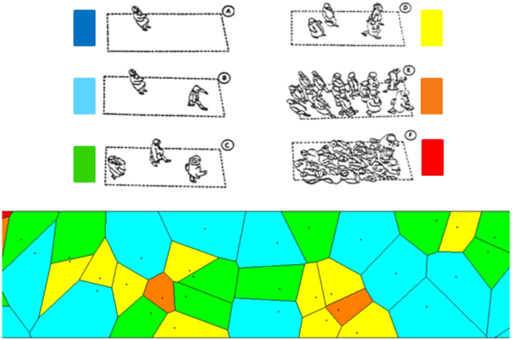

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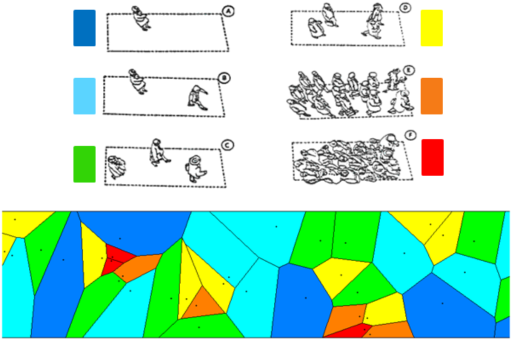

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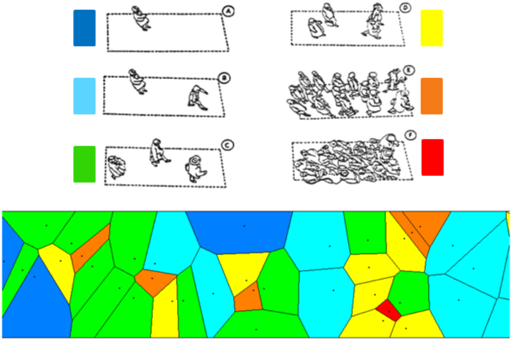

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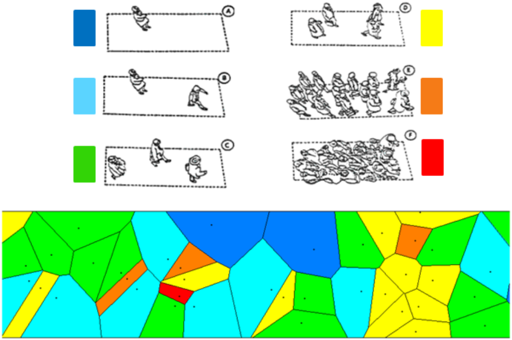

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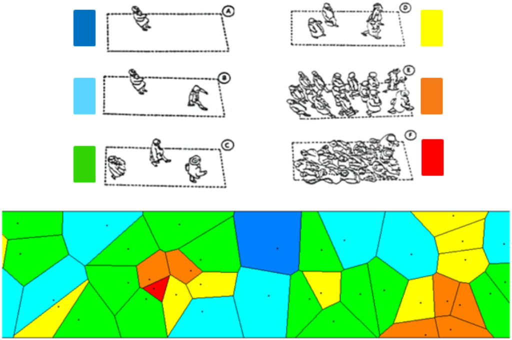

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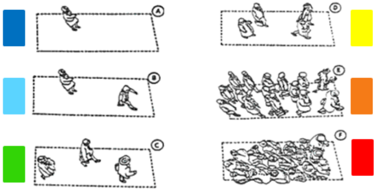

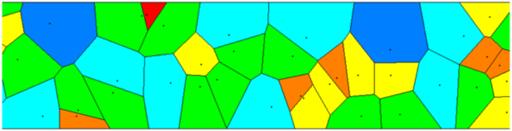

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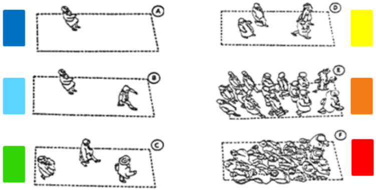

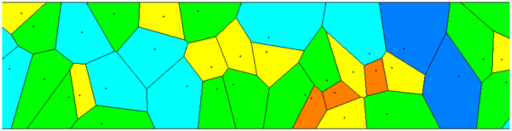

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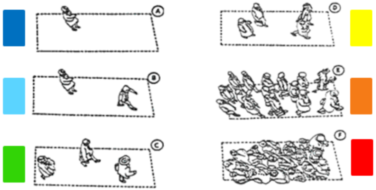

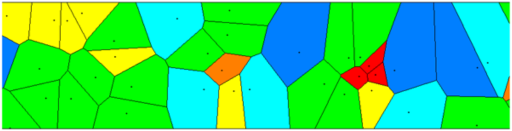

- Data-driven discretization
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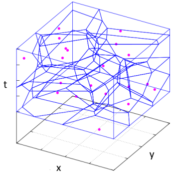
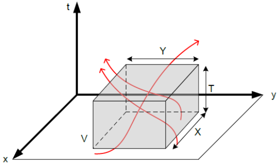
3DVoro performance: Synthetic data

NOMAD simulation tool (Campanella et al.; 2014)
Flow composition: uni-directional and bi-directional
Scenarios: low/high demand, homogenous/heterogeneous population

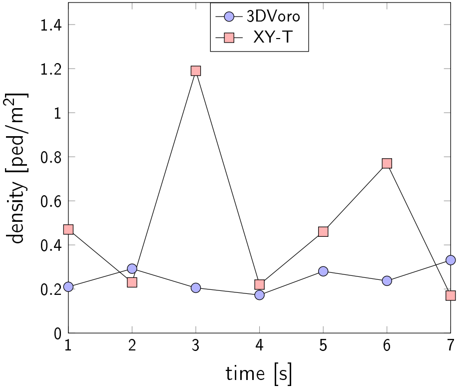
Analysis

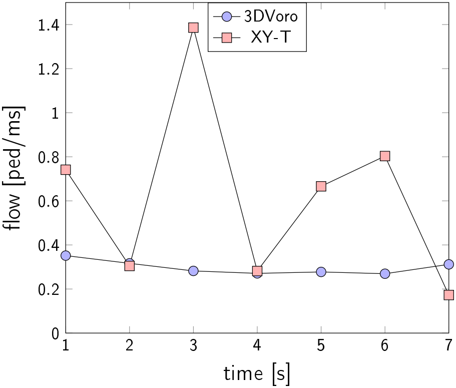
3DVoro and XY-T methods



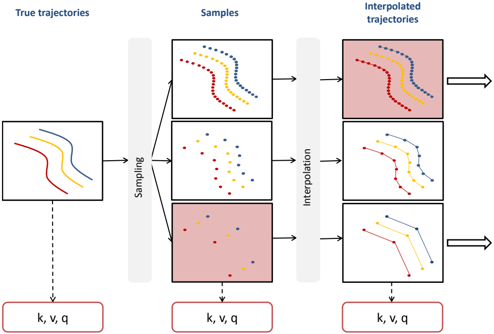


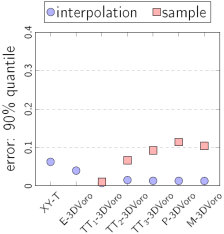
Nature of the results

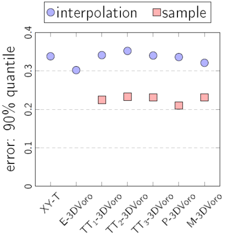




Robustness to sampling of trajectories







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Main findings

- Data-driven spatio-temporal discretization
- Well defined, flexible and general framework
- Smooth transitions in measured characteristics
- Robust to noise in the data
- Robust to sampling of trajectories

Future directions

- Anisotropy and presence of obstacles

Thank you

Intelligent Traffic Control and Service in Big Data Environment

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Marija Nikolić, Michel Bierlaire

- marija.nikolic@epfl.ch

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Characteristics of methods

Method	Scale	Spatial aggregation		Temporal aggregation		Data type
		Unit	Assumptions	Unit	Assumptions	
XY-T	Macroscopic	Area	Shape Size Location	Interval	Duration	Trajectories
Grid-based (GB)	Macroscopic	Cell	Size Location	Interval	Duration	Trajectories Sync. sample
Range-based (RB)	Macroscopic	Circle	Radius Location	Interval	Duration	Trajectories Sync. sample
Exponentially-weighted (EW)	Macroscopic	Range	Influence function Range of influence	Interval	Duration	Trajectories Sync. sample
Voronoi-based (VB)	Microscopic	Voronoi cell	Boundary conditions	Interval	Duration	Trajectories Sync. sample

3DVoro: Distances

Mahalanobis distance

$$S_1(t_i,\alpha) = p_i + \Delta t v_i(t_i) + \alpha d^1(t_i)$$

$$d^1(t_i) = \frac{v_i(t_i)}{||v_i(t_i)||}, ||d^1(t_i)|| = 1$$

$$S_2(t_i,\alpha) = p_i - \Delta t v_i(t_i) - \alpha d^1(t_i)$$

$$S_3(t_i,\alpha) = p_i + \alpha d^2(t_i)$$

$$d^2(t_i) = \begin{pmatrix} d^1_x(t_i) \\ d^2_y(t_i) \\ 0 \end{pmatrix}$$

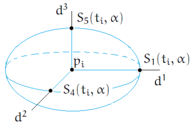
$$S_4(t_i,\alpha) = p_i - \alpha d^2(t_i)$$

$$S_5(t_i,\alpha) = p_i + \alpha d^3(t_i)$$

$$S_6(t_i,\alpha) = p_i - \alpha d^3(t_i)$$

$$d^3(t_i) = \begin{pmatrix} 0 \\ 0 \\ \Delta t \end{pmatrix}$$

$$d_M(S_j, p_i) = \alpha, j = 1, \dots, 6$$

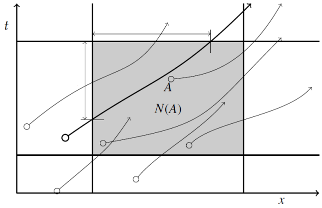


Edie (1963)

$$k(A) = \frac{\sum_{i=1}^{N} t_i}{dxdt}$$

$$q(A) = \frac{\sum_{i=1}^{N} x_i}{dxdt}$$

$$v(A) = \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} t_i}$$



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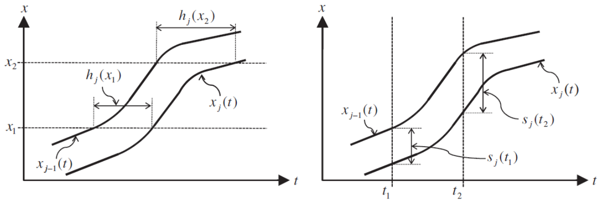
Jabari et al. (2014)

$$k(x,t) = \frac{1}{s_i(t)}, \text{ for } x \in [x_i(t), x_{i-1}(t))$$

$$q(x,t) = \frac{1}{h_i(x)}, \text{ for } t \in (t_{i-1}(x), t_i(x)]$$

$$v(x,t) = \frac{s_i(t)}{h_i(x)}, \text{ for } x \in [x_i(t), x_{i-1}(t)), t \in (t_{i-1}(x), t_i(x)]$$



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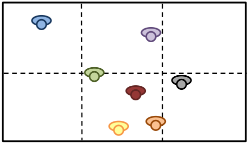
Fruin (1971)

$$k(x,y,t) = \frac{N_A(t)}{|A|}, \text{ for } (x,y) \in A$$

$$\vec{q}(x,y,t) = k(x,y,t)\vec{v}(x,y,t)$$



$$\vec{v}_i(t) = \frac{\begin{pmatrix} x_i(t_2) \\ y_i(t_2) \end{pmatrix} - \begin{pmatrix} x_i(t_1) \\ y_i(t_1) \end{pmatrix}}{t_2 - t_1}$$

$$\vec{v}(x,y,t) = \frac{\sum_{i=1}^{N_A} \vec{v}_i(t)}{N_A}, \text{ for } (x,y) \in A$$

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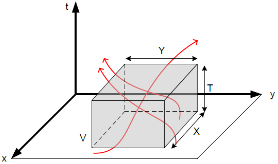
van Wageningen-Kessels et al. (2014)
Saberi and Mahmassani (2014)

$$k(A) = \frac{\sum_{i=1}^{N} t_i}{dxdydt}$$

$$\vec{q}(A) = \begin{pmatrix} q_x(A) \\ q_y(A) \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{N} x_i}{dxdydt} \\ \frac{\sum_{i=1}^{N} y_i}{dxdydt} \end{pmatrix}$$



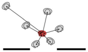
$$\vec{v}(A) = \begin{pmatrix} v_x(A) \\ v_y(A) \end{pmatrix} = \begin{pmatrix} \frac{\sum_{i=1}^{N} x_i}{\sum_{i=1}^{N} t_i} \\ \frac{\sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} t_i} \end{pmatrix}$$

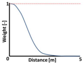
Helbing et al. (2007)

$$f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right) = \frac{1}{\pi R^2} \exp\left(-\frac{\left\|\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right\|^2}{R^2}\right)$$

$$k(x,y,t) = \sum_i f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)$$

$$\vec{q}(x,y,t) = k(x,y,t)\vec{v}(x,y,t)$$

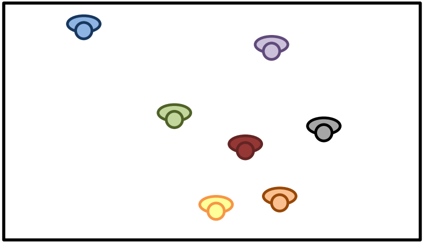




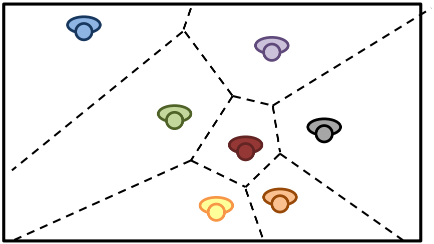
$$\vec{v}_i(t) = \frac{\begin{pmatrix} x_i(t_2) \\ y_i(t_2) \end{pmatrix} - \begin{pmatrix} x_i(t_1) \\ y_i(t_1) \end{pmatrix}}{t_2 - t_1}$$

$$\vec{v}(x,y,t) = \frac{\sum_i \vec{v}_i(t) f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)}{\sum_i f\left(\begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right)}$$

Voronoi diagrams: 2D

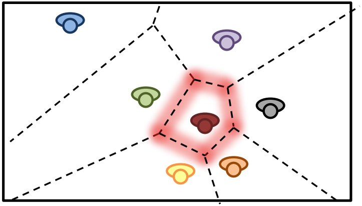


Voronoi diagrams: 2D



Steffen and Seyfried (2010)

$$k(x,y,t) = \frac{1}{|A_i|}, \text{ for } (x,y) \in A_i$$



$$\vec{v}(x,y,t) = \frac{\begin{pmatrix} x_i(t_2) \\ y_i(t_2) \end{pmatrix} - \begin{pmatrix} x_i(t_1) \\ y_i(t_1) \end{pmatrix}}{t_2 - t_1}$$

q: half a person has passed a segment if half of the Voronoi cell has passed it

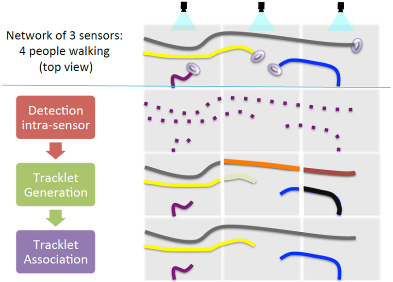
Lausanne data

Tracklet generation

- A graph-based tracking algorithm is implemented to link the detected points
- A directed graph: vertices representing the 3D coordinates of detected pedestrians, edges defining the connectivity between vertices
- The connectivity prevents too long or unrealistic connections

Tracklet association

- Task: find the set of trajectories Θ that best explains the extracted tracklets
- Formally:maximizing the a-posterior probability of Θ given the set of tracklets



3DVoro: Robustness to noise in the data

100 sets of pedestrian trajectories synthesized per scenario

$\theta_r^M(p) = (k_r^M(p), v_r^M(p), q_r^M(p))$: a vector of indicators at point p obtained by applying the method M to the r^{th} set of trajectories

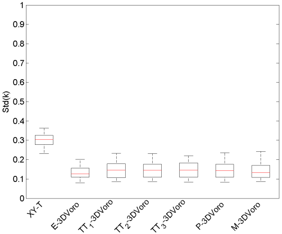
The standard deviation of the indicators at p as

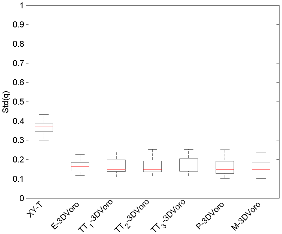
$$\sigma_R^M(p) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}(\theta_r^M(p) - \mu_R^M(p))^2}$$

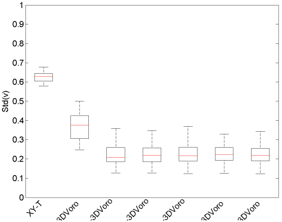
$\mu_R^M(p) = \frac{1}{R}\sum_{r=1}^{R}\theta_r^M(p)$, $R = 100$

The procedure is repeated for 1000 randomly selected points p

3DVoro: Robustness to noise in the data - Sc.I







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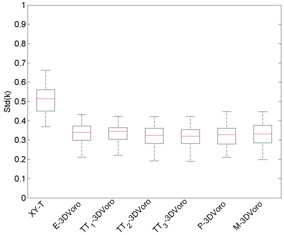
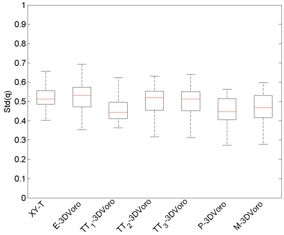
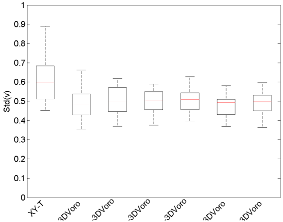
3DVoro: Robustness to noise in the data - Sc.II







3DVoro: Robustness to sampling frequency

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$1.47e^{-02}$	/	$1.25e^{-02}$	/	$1.25e^{-02}$	/	$6.25e^{-02}$	/
E-3DVoro	$1.17e^{-02}$	/	0	/	$4.48e^{-04}$	/	$3.96e^{-02}$	/
TT_1-3DVoro	$2.70e^{-03}$	$6.70e^{-03}$	0	0	$3.00e^{-04}$	$2.30e^{-03}$	$7.30e^{-03}$	$1.02e^{-02}$
TT_2-3DVoro	$5.80e^{-03}$	$3.50e^{-02}$	0	$2.80e^{-03}$	$6.00e^{-04}$	$2.08e^{-02}$	$1.50e^{-02}$	$6.69e^{-02}$
TT_3-3DVoro	$5.40e^{-03}$	$4.34e^{-02}$	0	$8.00e^{-03}$	$6.00e^{-04}$	$2.83e^{-02}$	$1.32e^{-02}$	$9.22e^{-02}$
P-3DVoro	$8.20e^{-03}$	$5.36e^{-02}$	0	$6.10e^{-03}$	$2.40e^{-03}$	$3.03e^{-02}$	$1.30e^{-02}$	$1.14e^{-01}$
M-3DVoro	$4.50e^{-03}$	$5.65e^{-02}$	0	$6.80e^{-03}$	$1.10e^{-03}$	$4.55e^{-02}$	$1.28e^{-02}$	$1.04e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$1.90e^{-01}$	/	$1.00e^{-01}$	/	$1.50e^{-01}$	/	$3.38e^{-01}$	/
E-3DVoro	$1.64e^{-01}$	/	$1.12e^{-02}$	/	$1.46e^{-01}$	/	$3.02e^{-01}$	/
TT_1-3DVoro	$2.54e^{-01}$	$1.27e^{-01}$	$1.35e^{-02}$	$9.00e^{-03}$	$1.16e^{-01}$	$8.97e^{-02}$	$3.41e^{-01}$	$2.25e^{-01}$
TT_2-3DVoro	$1.64e^{-01}$	$1.22e^{-01}$	$1.44e^{-02}$	$1.06e^{-02}$	$1.21e^{-01}$	$7.30e^{-02}$	$3.52e^{-01}$	$2.33e^{-01}$
TT_3-3DVoro	$1.89e^{-01}$	$1.24e^{-01}$	$1.84e^{-02}$	$1.09e^{-02}$	$1.24e^{-01}$	$7.88e^{-02}$	$3.40e^{-01}$	$2.31e^{-01}$
P-3DVoro	$3.19e^{-01}$	$1.21e^{-01}$	$3.26e^{-02}$	$6.20e^{-03}$	$1.43e^{-01}$	$7.43e^{-02}$	$3.36e^{-01}$	$2.10e^{-01}$
M-3DVoro	$1.97e^{-01}$	$1.24e^{-01}$	$3.48e^{-02}$	$9.90e^{-03}$	$1.41e^{-01}$	$7.72e^{-02}$	$3.21e^{-01}$	$2.31e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Uni_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$2.05e^{-02}$	/	0	/	$1.25e^{-02}$	/	$5.00e^{-02}$	/
E-3DVoro	$1.43e^{-02}$	/	0	/	$2.67e^{-02}$	/	$2.64e^{-02}$	/
TT_1-3DVoro	$8.00e^{-03}$	$4.55e^{-02}$	0	0	$8.00e^{-04}$	$1.75e^{-02}$	$2.36e^{-02}$	$8.52e^{-02}$
TT_2-3DVoro	$1.49e^{-02}$	$1.07e^{-01}$	0	0	$3.20e^{-03}$	$5.72e^{-02}$	$3.33e^{-02}$	$2.21e^{-01}$
TT_3-3DVoro	$1.24e^{-02}$	$1.60e^{-01}$	0	0	$3.50e^{-03}$	$9.62e^{-02}$	$2.98e^{-02}$	$3.41e^{-01}$
P-3DVoro	$2.10e^{-02}$	$1.66e^{-01}$	0	0	$4.20e^{-03}$	$1.16e^{-01}$	$5.27e^{-02}$	$3.64e^{-01}$
M-3DVoro	$1.31e^{-02}$	$2.40e^{-01}$	0	0	$2.50e^{-03}$	$1.75e^{-01}$	$2.91e^{-02}$	$5.58e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$5.29e^{-01}$	/	$1.63e^{-01}$	/	$4.75e^{-01}$	/	$1.01e^{00}$	/
E-3DVoro	$4.02e^{-01}$	/	0	/	$2.49e^{-01}$	/	1.03E+00	/
TT_1-3DVoro	$4.06e^{-01}$	$2.90e^{-01}$	$3.10e^{-01}$	$2.48e^{-02}$	$2.64e^{-01}$	$1.65e^{-01}$	$9.21e^{-01}$	$7.12e^{-01}$
TT_2-3DVoro	$3.92e^{-01}$	$4.58e^{-01}$	$2.85e^{-01}$	$2.34e^{-01}$	$2.48e^{-01}$	$2.34e^{-01}$	$9.30e^{-01}$	1.11E+00
TT_3-3DVoro	$4.41e^{-01}$	$5.07e^{-01}$	$2.89e^{-01}$	$5.89e^{-02}$	$2.37e^{-01}$	$3.06e^{-01}$	$9.81e^{-01}$	1.17E+00
P-3DVoro	$4.31e^{-01}$	$3.71e^{-01}$	$1.40e^{-03}$	0	$2.58e^{-01}$	$1.80e^{-01}$	$9.43e^{-01}$	$7.29e^{-01}$
M-3DVoro	$4.34e^{-01}$	$5.01e^{-01}$	$3.16e^{-01}$	$1.36e^{-01}$	$2.75e^{-01}$	$3.52e^{-01}$	$9.96e^{-01}$	$9.80e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Uni_{HD-HeteroPop}$

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3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$6.50e^{-02}$	/	0	/	0	/	$8.65e^{-03}$	/
E-3DVoro	$1.20e^{-02}$	/	0	/	0	/	$4.66e^{-03}$	/
TT_1-3DVoro	$3.58e^{-03}$	$1.08e^{-02}$	0	0	0	$1.02e^{-03}$	$4.16e^{-03}$	$6.15e^{-03}$
$TT_2$3DVoro	$8.13e^{-03}$	$1.18e^{-02}$	0	0	0	$2.35e^{-03}$	$8.09e^{-03}$	$1.29e^{-02}$
TT_3-3DVoro	$1.49e^{-02}$	$2.06e^{-02}$	0	$3.91e^{-03}$	0	$8.43e^{-03}$	$7.46e^{-03}$	$3.10e^{-02}$
P-3DVoro	$2.29e^{-02}$	$5.42e^{-02}$	0	$1.94e^{-03}$	0	$2.72e^{-02}$	$9.25e^{-03}$	$1.06e^{-01}$
M-3DVoro	$2.15e^{-02}$	$4.82e^{-02}$	0	$4.31e^{-02}$	0	$2.42e^{-02}$	$7.69e^{-03}$	$1.29e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.66e^{-01}$	/	0	/	$6.84e^{-02}$	/	$7.00e^{-01}$	/
E-3DVoro	$1.65e^{-01}$	/	0	/	$1.19e^{-01}$	/	$3.40e^{-01}$	/
TT_1-3DVoro	$1.68e^{-01}$	$1.29e^{-01}$	$3.50e^{-02}$	$5.02e^{-02}$	$8.50e^{-02}$	$5.70e^{-02}$	$3.85e^{-01}$	$2.62e^{-01}$
TT_2-3DVoro	$1.70e^{-01}$	$1.02e^{-01}$	$4.52e^{-02}$	$5.63e^{-02}$	$8.49e^{-02}$	$6.15e^{-02}$	$3.82e^{-01}$	$5.57e^{-01}$
TT_3-3DVoro	$1.80e^{-01}$	$1.18e^{-01}$	$4.82e^{-02}$	$6.06e^{-02}$	$8.80e^{-02}$	$6.55e^{-02}$	$3.83e^{-01}$	$2.65e^{-01}$
P-3DVoro	$2.02e^{-01}$	$1.60e^{-01}$	$3.69e^{-02}$	$4.84e^{-02}$	$9.36e^{-02}$	$6.73e^{-02}$	$4.14e^{-01}$	$3.01e^{-01}$
M-3DVoro	$1.80e^{-01}$	$1.55e^{-01}$	$4.80e^{-02}$	$3.36e^{-02}$	$1.01e^{-01}$	$9.27e^{-02}$	$4.38e^{-01}$	$3.08e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Bi_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$2.85e^{-02}$	/	0	/	$3.28e^{-03}$	/	$1.00e^{-01}$	/
E-3DVoro	$3.00e^{-02}$	/	0	/	$9.64e^{-03}$	/	$6.50e^{-02}$	/
TT_1-3DVoro	$1.15e^{-01}$	$2.78e^{-02}$	0	0	$7.90e^{-04}$	$8.78e^{-03}$	$2.32e^{-02}$	$4.94e^{-02}$
TT_2-3DVoro	$9.72e^{-02}$	$9.34e^{-02}$	0	0	$3.21e^{-03}$	$5.16e^{-02}$	$3.50e^{-02}$	$2.15e^{-01}$
TT_3-3DVoro	$4.89e^{-02}$	$1.05e^{-01}$	0	0	$2.83e^{-03}$	$5.91e^{-02}$	$3.56e^{-02}$	$2.62e^{-01}$
P-3DVoro	$1.15e^{-01}$	$1.70e^{-01}$	0	$3.33e^{-02}$	$4.79e^{-03}$	$6.28e^{-02}$	$4.65e^{-02}$	$2.61e^{-01}$
M-3DVoro	$1.15e^{-01}$	$1.52e^{-01}$	0	$8.33e^{-02}$	$4.55e^{-03}$	$7.20e^{-02}$	$5.35e^{-02}$	$3.51e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T								
E-3DVoro	$2.79e^{-01}$	/	0	/	$1.29e^{-01}$	/	$7.14e^{-01}$	/
TT_1-3DVoro	$4.49e^{-01}$	$2.58e^{-01}$	$5.70e^{-03}$	$1.99e^{-03}$	$1.54e^{-01}$	$1.34e^{-01}$	$8.43e^{-01}$	$6.64e^{-01}$
TT_2-3DVoro	$3.71e^{-01}$	$2.98e^{-01}$	$4.28e^{-02}$	$9.34e^{-02}$	$1.61e^{-01}$	$1.40e^{-01}$	$8.07e^{-01}$	$7.90e^{-01}$
TT_3-3DVoro	$9.82e^{-01}$	$3.56e^{-01}$	$4.34e^{-02}$	$6.70e^{-03}$	$1.64e^{-01}$	$1.38e^{-01}$	$7.76e^{-01}$	$7.74e^{-01}$
P-3DVoro	$3.82e^{-01}$	$3.15e^{-01}$	$2.32e^{-03}$	$6.74e^{-03}$	$1.53e^{-01}$	$1.61e^{-01}$	$9.09e^{-01}$	$7.22e^{-01}$
M-3DVoro	$4.08e^{-01}$	$3.77e^{-01}$	$1.89e^{-02}$	$1.47e^{-02}$	$1.90e^{-01}$	$1.74e^{-01}$	$7.91e^{-01}$	$8.18e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of density indicator - $Bi_{HD-HeteroPop}$

3DVoro: Robustness to sampling frequency

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$4.30e^{-03}$	/	0	/	$3.40e^{-03}$	/	$1.16e^{-02}$	/
E-3DVoro	$1.55e^{-01}$	/	0	/	$3.56e^{-02}$	/	$4.99e^{-01}$	/
TT_1-3DVoro	$9.60e^{-03}$	$2.31e^{-02}$	0	0	$2.20e^{-03}$	$9.38e^{-03}$	$2.79e^{-02}$	$4.85e^{-02}$
TT_2-3DVoro	$2.04e^{-02}$	$7.66e^{-02}$	0	$4.10e^{-03}$	$5.80e^{-03}$	$4.48e^{-02}$	$6.48e^{-02}$	$1.68e^{-01}$
TT_3-3DVoro	$1.81e^{-02}$	$9.15e^{-02}$	0	$8.00e^{-04}$	$5.70e^{-03}$	$4.51e^{-02}$	$5.42e^{-02}$	$2.15e^{-01}$
P-3DVoro	$2.98e^{-02}$	$1.38e^{-01}$	0	$5.90e^{-03}$	$1.41e^{-02}$	$7.90e^{-02}$	$5.75e^{-02}$	$2.92e^{-01}$
M-3DVoro	$1.88e^{-02}$	$1.46e^{-01}$	0	$2.00e^{-04}$	$5.90e^{-03}$	$1.04e^{-01}$	$5.95e^{-02}$	$3.22e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean IT	Mean SoP	Mode IT	Mode SoP	Median IT	Median SoP	90% quantile IT	90% quantile SoP
XY-T	$5.80e^{-01}$	/	1.02	/	$3.26e^{-01}$	/	1.42	/
E-3DVoro	1.77	/	$4.36e^{-02}$	/	$7.11e^{-01}$	/	1.27	/
TT_1-3DVoro	$5.42e^{-01}$	$5.40e^{-01}$	$2.28e^{-02}$	$2.10e^{-03}$	$3.43e^{-01}$	$3.02e^{-01}$	1.04	$9.66e^{-01}$
TT_2-3DVoro	$5.11e^{-01}$	$5.56e^{-01}$	$1.39e^{-01}$	$8.20e^{-03}$	$3.15e^{-01}$	$3.17e^{-01}$	1.07	1.04
TT_3-3DVoro	$6.08e^{-01}$	$5.52e^{-01}$	$3.72e^{-02}$	$7.50e^{-03}$	$3.29e^{-01}$	$3.18e^{-01}$	1.05	1.05
P-3DVoro	$5.60e^{-01}$	$5.41e^{-01}$	$8.75e^{-02}$	$1.30e^{-03}$	$3.32e^{-01}$	$3.04e^{-01}$	$9.76e^{-01}$	$9.82e^{-01}$
M-3DVoro	$5.03e^{-01}$	$5.43e^{-01}$	$3.93e^{-02}$	$6.91e^{-02}$	$3.76e^{-01}$	$3.15e^{-01}$	1.08	$9.52e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of velocity indicator - $Uni_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.92e^{-02}$	/	$9.60e^{-03}$	/	$6.20e^{-03}$	/	$3.42e^{-02}$	/
E-3DVoro	$3.17e^{-02}$	/	0	/	$6.30e^{-03}$	/	$3.86e^{-02}$	/
TT_1-3DVoro	$1.57e^{-02}$	$6.18e^{-02}$	0	0	$6.10e^{-03}$	$1.87e^{-02}$	$3.23e^{-02}$	$1.30e^{-01}$
TT_2-3DVoro	$1.83e^{-02}$	$1.38e^{-01}$	0	$1.73e^{-02}$	$7.90e^{-03}$	$4.27e^{-02}$	$3.82e^{-02}$	$3.88e^{-01}$
TT_3-3DVoro	$1.85e^{-02}$	$1.88e^{-01}$	0	$1.00e^{-01}$	$8.00e^{-03}$	$6.46e^{-02}$	$4.08e^{-02}$	$4.87e^{-01}$
P-3DVoro	$2.93e^{-02}$	$2.05e^{-01}$	0	$7.96e^{-02}$	$9.00e^{-03}$	$9.82e^{-02}$	$6.49e^{-02}$	$5.29e^{-01}$
M-3DVoro	$2.14e^{-02}$	$3.16e^{-01}$	0	$5.10e^{-03}$	$8.00e^{-03}$	$1.47e^{-01}$	$4.37e^{-02}$	$8.21e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$5.73e^{-01}$	/	1.15	/	$3.51e^{-01}$	/	1.58	/
E-3DVoro	1.01	/	$8.57e^{-01}$	/	$3.85e^{-01}$	/	1.67	/
TT_1-3DVoro	$5.82e^{-01}$	$5.80e^{-01}$	$8.69e^{-01}$	$5.85e^{-02}$	$4.51e^{-01}$	$3.13e^{-01}$	1.40	1.28
TT_2-3DVoro	$5.76e^{-01}$	$5.67e^{-01}$	$9.40e^{-01}$	$1.02e^{-01}$	$3.75e^{-01}$	$2.64e^{-01}$	1.54	1.16
TT_3-3DVoro	$5.79e^{-01}$	$5.94e^{-01}$	$8.50e^{-01}$	$5.73e^{-02}$	$3.70e^{-01}$	$2.77e^{-01}$	1.46	1.29
P-3DVoro	$5.66e^{-01}$	$5.62e^{-01}$	$8.92e^{-01}$	$4.61e^{-02}$	$3.83e^{-01}$	$2.95e^{-01}$	1.38	1.26
M-3DVoro	$6.27e^{-01}$	$7.11e^{-01}$	$9.13e^{-01}$	$1.43e^{-02}$	$5.05e^{-01}$	$2.86e^{-01}$	1.55	1.49

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of velocity indicator - $Uni_{HD-HeteroPop}$

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3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$1.93e^{-02}$	/	0	/	$1.77e^{-02}$	/	$7.73e^{-02}$	/
E-3DVoro	$1.65e^{-02}$	/	0	/	$5.60e^{-03}$	/	$3.75e^{-02}$	/
TT_1-3DVoro	$3.00e^{-04}$	$7.60e^{-03}$	0	0	0	$2.60e^{-03}$	$8.00e^{-04}$	$1.74e^{-02}$
TT_2-3DVoro	$1.40e^{-03}$	$4.16e^{-02}$	0	0	0	$3.17e^{-02}$	$3.60e^{-03}$	$8.99e^{-02}$
TT_3-3DVoro	$1.30e^{-03}$	$4.65e^{-02}$	0	$4.32e^{-02}$	0	$3.48e^{-02}$	$3.90e^{-03}$	$1.14e^{-01}$
P-3DVoro	$2.70e^{-03}$	$4.69e^{-02}$	0	$1.41e^{-02}$	$8.00e^{-04}$	$2.27e^{-02}$	$5.50e^{-03}$	$1.29e^{-01}$
M-3DVoro	$1.20e^{-03}$	$5.09e^{-02}$	0	$4.75e^{-02}$	0	$3.54e^{-02}$	$2.50e^{-03}$	$1.23e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$2.55e^{-01}$	/	$1.45e^{-01}$	/	$2.45e^{-01}$	/	$5.06e^{-01}$	/
E-3DVoro	$4.17e^{-01}$	/	$6.50e^{-02}$	/	$1.27e^{-01}$	/	$3.83e^{-01}$	/
3DVoro-δ_{TT_1}	$1.74e^{-01}$	$1.50e^{-01}$	$1.79e^{-01}$	$8.00e^{-04}$	$1.13e^{-01}$	$8.77e^{-02}$	$3.21e^{-01}$	$2.98e^{-01}$
TT_1-3DVoro	$2.07e^{-01}$	$1.53e^{-01}$	$1.92e^{-01}$	$1.00e^{-04}$	$1.39e^{-01}$	$8.52e^{-02}$	$3.71e^{-01}$	$3.29e^{-01}$
TT_2-3DVoro	$2.33e^{-01}$	$1.52e^{-01}$	$2.05e^{-01}$	$3.00e^{-04}$	$1.48e^{-01}$	$8.46e^{-02}$	$3.63e^{-01}$	$3.27e^{-01}$
TT_2-3DVoro	$2.17e^{-01}$	$1.43e^{-01}$	$1.53e^{-01}$	$1.40e^{-03}$	$1.34e^{-01}$	$8.49e^{-02}$	$3.01e^{-01}$	$2.98e^{-01}$
M-3DVoro	$1.75e^{-01}$	$1.48e^{-01}$	$1.83e^{-01}$	$1.00e^{-04}$	$1.36e^{-01}$	$9.11e^{-02}$	$3.43e^{-01}$	$3.22e^{-01}$

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of flow indicator - $Uni_{LD-HomoPop}$

3DVoro: Robustness to sampling frequency

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$2.75e^{-02}$	/	$2.30e^{-03}$	/	$1.75e^{-02}$	/	$7.21e^{-02}$	/
E-3DVoro	$1.09e^{-02}$	/	0	/	$8.70e^{-04}$	/	$2.83e^{-02}$	/
TT_1-3DVoro	$7.80e^{-03}$	$6.06e^{-02}$	0	0	$7.00e^{-04}$	$1.21e^{-02}$	$2.22e^{-02}$	$1.58e^{-01}$
TT_2-3DVoro	$1.05e^{-02}$	$1.45e^{-01}$	0	0	$1.10e^{-03}$	$6.08e^{-02}$	$2.78e^{-02}$	$3.11e^{-01}$
TT_3-3DVoro	$1.06e^{-02}$	$2.03e^{-01}$	0	0	$1.00e^{-03}$	$8.27e^{-02}$	$2.19e^{-02}$	$4.64e^{-01}$
P-3DVoro	$1.62e^{-02}$	$1.95e^{-01}$	0	$4.86e^{-02}$	$1.80e^{-03}$	$8.54e^{-02}$	$3.70e^{-02}$	$4.90e^{-01}$
M-3DVoro	$1.29e^{-02}$	$3.06e^{-01}$	0	0	$1.60e^{-03}$	$1.48e^{-01}$	$2.92e^{-02}$	$8.95e^{-01}$

(a) Sampling frequency: 3 s^{-1}

Method	Mean		Mode		Median		90% quantile	
	IT	SoP	IT	SoP	IT	SoP	IT	SoP
XY-T	$5.18e^{-01}$	/	$3.50e^{-01}$	/	$4.48e^{-01}$	/	1.09	/
E-3DVoro	$6.54e^{-01}$	/	$3.69e^{-01}$	/	$2.03e^{-01}$	/	1.54	/
TT_1-3DVoro	$4.99e^{-01}$	$4.02e^{-01}$	$1.06e^{-01}$	$6.49e^{-02}$	$3.24e^{-01}$	$1.81e^{-01}$	1.35	$9.43e^{-01}$
TT_2-3DVoro	$5.66e^{-01}$	$4.16e^{-01}$	$1.47e^{-01}$	$5.55e^{-02}$	$2.73e^{-01}$	$1.73e^{-01}$	1.57	1.21
TT_3-3DVoro	$5.91e^{-01}$	$4.45e^{-01}$	$1.53e^{-01}$	$1.57e^{-01}$	$2.94e^{-01}$	$1.71e^{-01}$	1.68	1.31
P-3DVoro	$4.81e^{-01}$	$4.28e^{-01}$	$5.53e^{-02}$	$3.98e^{-02}$	$2.22e^{-01}$	$1.89e^{-01}$	1.34	1.12
M-3DVoro	$6.41e^{-01}$	$4.47e^{-01}$	$9.07e^{-02}$	$4.55e^{-02}$	$3.97e^{-01}$	$1.73e^{-01}$	1.66	1.24

(b) Sampling frequency: 0.5 s^{-1}

Robustness to the sampling frequency of flow indicator - $Uni_{HD-HeteroPop}$

3DVoro: Robustness to sampling frequency

Interpolation

- Higher sampling frequency
- Time-Transform distances lead to the best performance (TT1-3DVoro)

Samples

- Lower sampling frequency
- $Uni_{LD-HomoPop}$: the distances that take into account the speed and/or direction of pedestrians (TT_2-3DVoro, P-3DVoro and M-3DVoro)
- $Uni_{HD-HeteroPop}$: Time-Transform distances (TT_1-3DVoro)

General

- Time-Transform: more data available (the sampling frequency equal to 3 s^{-1} or the demand equal to 3.6 pedestrians per second)
- Distances accounting for the dynamics: less data available (the sampling frequency equal to 0.5 s^{-1} and the demand equal to 1.2 pedestrians per second)