**INTRODUCTION**

Fluid injection at a pressure below the local minimum principal total stress in a fault may (re)activate shear crack propagation (hydroseisming). Because of the presence of asperities along the fault’s surfaces, the fault hydraulic width increase with the slip (up to a constant value).

The question we want to address in this contribution is the following: does the increase of hydraulic width (dilatancy) affect the shear crack propagation along the fault? does it play a role in the shear crack propagation of unstable faults?

Garagash & Germanovich [1] showed that a fault subjected to locally elevated pore pressure associated with fluid injection hosts different limiting regimes depending on how far the initial stress state is from its strength level. Notably when a fault is stressed almost to its static strength level (critically loaded fault), a large slip zone is expected. Hence at the nucleation time, the pressurized region is within the slipping patch. On the other hand, for a sufficiently pressurized fault (i.e., when the pore pressure is just enough to activate the slip), the slipping patch is much slower than the diffusive growth of the pressurized zone.

In addition to this, they showed that the regime of propagation of such pressurized faults can be ultimately stable or unstable depending on whether the initial shear stress state is greater or lower than the fault residual strength. In the former case the shear crack propagates with a moderate velocity (quasi-static) as it is induced by fluid pressure diffusion (but it might turn into a dynamic instability followed by an arrest). In the latter case, the shear crack instantly propagates quasi-statically, then, as slip accumulate along the fault, the quasi-static crack growth become unstable and the shear crack runs away.

The effect of dilatancy leads to a local reduction of pore-pressure at the shear crack tip depending on the ability (viscosity-related) of the fluid to flow as the newly created void space, leading to a stabilising effect [2].

**NUMERICAL SCHEME DESCRIPTION**

The elasticity equation (1) is solved numerically using Displacement Discontinuity Method (with piecewise linear shear displacement discontinuities), whereas the fluid flow is discretised using Finite Volume Method (3).

The algorithm solves the fully coupled problem with an implicit time integration that enforce the M-C criterion (2). As a result, the slippage length is computed (or equal) to the fault shear strength:

\[
\tau(x, t) = \frac{\sigma_{f}}{\mu} f(x, t) \leq \tau_{f},
\]

where \(\tau(x, t)\) is the background shear stress, \(\sigma_{f}\) is the shear modulus, \(\nu\) is the Poisson’s ratio, \(\mu\) is the shear dilatancy density, \(\Delta \sigma = \sigma_{f} / \mu\) is the simple Cauchy kernel.

Within the fault region, we assume the shear weakening Mohr-Coulomb failure criterion. The shear stress on the fault must be less (or equal) to the fault shear strength:

\[
\tau(x, t) \leq f(\delta) (\sigma_{n} - \mu \tau) - p(x, t)) \leq \tau_{f},
\]

where \(p\) is the ambient pore-pressure distribution in the fault. \(\sigma_{n} = p(x, t)\), also denoted as \(\sigma_{r}\), is the effective stress normal to the fault and the friction coefficient \(\alpha\) is assumed to weaken linearly with slip (Figure 1 - center).

Under the lubrication approximation and under the assumption of slightly compressible liquid of compressibility \(c_{f}\), the average mass conservation in the fault reduces to the following continuity equation:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho \mathbf{v}_{f}}{\partial x} = 0,
\]

where \(\rho\) is the fluid density and \(\mathbf{v}_{f}\) is the fluid velocity. Under the assumption that the flow is incompressible, then \(\mathbf{v}_{f} = \mathbf{v} = \nabla \tau / \mu \nabla p\).

\[
\frac{\partial \Delta \sigma}{\partial t} + \frac{\partial \Delta \sigma \mathbf{v}_{f}}{\partial x} = 0,
\]

where \(\Delta \sigma\) is the increment of dilatancy with respect to its initial value and \(\alpha_{w}\) is the patch length scale, \(\epsilon_{f} = \Delta \sigma / \alpha_{w}\) is the increment of dilatancy with respect to its initial value and \(\alpha_{w}\) is the patch length scale.

**REFERENCES**


**QUASISTATIC SHEAR CRACK PROPAGATION UNDER FLUID INJECTION**

Let us consider an impermeable linear elastic medium with a long fault under a uniform background stress field characterised by the normal \(\sigma_{n}\) and shear \(\tau_{f}\) components (Figure 1 - left). In addition to this, let us suppose the presence of a shear crack of a finite length 2\(x\) acting on the fault.

Figure 1: Model of shear weakening dilatant fault and loading conditions (left); Friction weakening law (center); Dilatant hardening law (right).

By applying the distributed shear dilatation theory, under a Quasi-Static approximation [7], the shear stress depends on the slip by means of the following singular integral equation of the first kind:

\[
\sigma_{f}(x, t) = \frac{G}{2\pi} \left( \int_{-a}^{a} \frac{\sigma_{f}(x')}{\sqrt{(a-x')^{2} + y^{2}}} \, dx' \right) + \frac{1}{2\pi} \left( \int_{-a}^{a} \frac{\sigma_{f}(x')}{\sqrt{(a-x')^{2} + y^{2}}} \, dx' \right)
\]

where \(\sigma_{f}(x, t)\) is the background shear stress, \(G\) is the shear modulus, \(\alpha_{w}\) is the Poisson’s ratio, \(\Delta \sigma = \sigma_{f} / \mu\) is the shear dilatancy density, \(\Delta \sigma = \sigma_{f} / \mu\) is the simple Cauchy kernel.

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where \(\Delta \sigma\) is the increment of dilatancy with respect to its initial value and \(\alpha_{w}\) is the patch length scale, \(\epsilon_{f} = \Delta \sigma / \alpha_{w}\) is the increment of dilatancy with respect to its initial value and \(\alpha_{w}\) is the patch length scale.

**RESULTS**

Solutions of the governing equations for an ultimately stable & unstable fault (in terms of normalised crack half-length \(a / \alpha_{w}\) and normalised peak slip \(\delta / \alpha_{w}\)) as a function of normalised dimensionless time \(\alpha_{w} / \mu \Delta \sigma / \alpha_{w}\) are hereunder reported.

![Graph 1](GRaph.png)

Figure 2: Qualitative development of the normalised crack half-length for an ultimately stable fault \((a / \alpha_{w} = 0.55 - 0.75)\) and for an unstable fault \((a / \alpha_{w} = 0.75 - 0.95)\), for various values of dimensionless crack half-length \(a / \alpha_{w}\), and a value of constant overpressure \(\Delta \sigma / \mu\). Force effects correspond to the GoG’s results for quasi-static crack propagation without dilatant hardening [3].

The corresponding evolution of pore pressure profile for an ultimately stable fault characterised by \(a / \alpha_{w} = 0.55\) is

![Graph 2](GRaph.png)

Figure 3: Pore pressure evolution for an ultimately stable fault characterised by \(a / \alpha_{w} = 0.55\) and \(\epsilon_{f} = \Delta \sigma / \alpha_{w} = 0.01\) (left). Snapshot of pore pressure profile at \(\alpha_{w} / \mu \Delta \sigma / \alpha_{w} = 2.9\) for an ultimately stable fault characterised different values of dimensionless dilatancy \(\epsilon_{f} = \Delta \sigma / \alpha_{w}\) (right).