

# Constraint-Driven Design with Combinatorial Equilibrium Modelling

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## Abstract

This paper presents an extension of the Combinatorial Equilibrium Modelling (CEM) design framework. CEM gives the possibility to generate and explore multiple spatial equilibrium solutions in the early conceptual design phase. In addition to the form and force diagrams of graphic statics, CEM introduces a topological diagram, which allows to adjust the connectivity of a spatial strut-and-tie network in equilibrium, its combinatorial inner force state (tension-compression) as well as its metric values (such as element’s length or force magnitudes) in real time.

In order to solve more specific design problems within the CEM framework, hence to guide design explorations, constraints-driven adaptation procedures have been now developed. This paper describes how these interactive and automatic adaptation methods can be embedded in an overall structural design process containing alternating steps of lateral exploration and vertical adaptation. Moreover, the paper describes how design constraints can be transformed into objectives within non-linear optimization problems and which algorithms can be used to solve them.

**Keywords:** conceptual design, morphology, form finding, optimization, combinatorial equilibrium, topology

## 1. Introduction

The goal of structural design is primarily to reconcile the two disciplines of architecture and engineering. According to the Uruguayan architectural engineer Eladio Dieste, a “noble” and elegant approach for the creation of efficient structures is to build upon the principle of *resistance through form* (Anderson [2]). This implies that the form of the structure should be directly coupled with the load-bearing behaviour during the design process. Hence, if structural considerations are not taken into account from the beginning, technical and economical flaws are typically inevitable in a later stage of the design (Macdonald [16]). This paper presents an innovative approach towards the active generation and constraint-based control of forms in equilibrium in which intrinsic structural properties can be handled by the designer at every moment of the process. This approach relies directly on the Combinatorial Equilibrium Modelling (CEM) framework. Contrary to other techniques, such as force density method (Linkwitz and Schek [15]) and dynamic relaxation (Barnes [3]), in this case the search of equilibrium is not part of the problem, as it is always ensured by a-priori definition.

The paper is organized in six sections. Section 2 contains a brief overview of the mathematical concept behind the CEM framework and discusses the non-linear behaviour of the form and force diagrams, which represent the equilibrium state of the structure. Section 3 describes interactive and automatized design procedures to generate and manipulate the equilibrium state. A case study of a curved pedestrian bridge is used in Section 4 to illustrate the potential of the latest developments of the research. Sections 5 and 6 discuss the current results and describe the differences with other approaches.

## 2. Sequential equilibrium and its non-linear behaviour

Combinatorial equilibrium modelling (CEM) introduces a generative topological diagram  $T$ , which enables the possibility to easily control the load-bearing behaviour of a network in equilibrium by varying the connectivity of the network, the combinatorial state (tension or compression) as well as the metric values of its inner forces. The topological diagram  $T$  defines a design space with infinite different equilibrium states  $E$ . Each state can be represented by a form diagram  $F$  and a force diagram  $F^*$ .

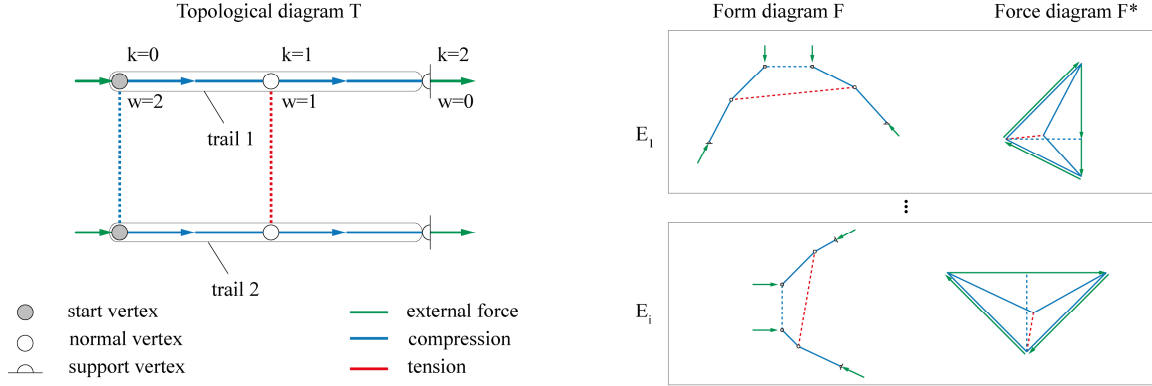


Fig. 1: Topological diagram  $T$  and two out of infinite possible equilibrium states

The topological diagram  $T$  is a directed graph. After fixing support vertices, the shortest path from each vertex to its closest support vertex can be obtained using shortest path algorithms Dijkstra [8]. The members along the shortest paths are called trail members (continuous lines) while the others are defined as deviation members (dashed lines). Start vertices (grey hatch) are those vertices of a trail with the maximum topological distance  $w$  from their corresponding support vertices. In the first step, the topological diagram is split into sequences  $k$  according to the topological distance  $w$  of the individual vertices to their corresponding support vertices. The overall equilibrium state of a network can then be developed sequence by sequence, from the start vertices ( $k=0$ ) to the support vertices ( $k=\max(w)$ ) using graphic statics (Ohlbrock and Schwartz [19]).

### 2.1. General equilibrium formulation

In the first step, for a given network with  $m$  trails  $i$  and  $w$  sequences  $k$ , equilibrium is imposed in each sequence  $k$  to obtain the matrix  $F_{k+1}^*$  of unknown force vectors  $f_{i,k+1}^*$  of the consecutive force trail members

$$F_{k+1}^* = -F_k^* - D_k^* - Q_k^* \quad (1)$$

with matrix  $F_k^*$  containing the preceding trail force vectors  $f_{i,k}^*$ , matrix  $Q_k^*$  containing the load vectors  $q_{i,k}^*$  at node  $i$  and matrix  $D_k^*$  containing the resultant deviation vectors  $d_{i,k}^*$ .

$$D_k^* = \begin{bmatrix} \vdots \\ \mathbf{d}_{i-1,k}^* \\ \mathbf{d}_{i,k}^* \\ \mathbf{d}_{i+1,k}^* \\ \vdots \end{bmatrix} = \mathbf{Ae} \quad \text{With } \mathbf{A} = \mathbf{C}_{i,k} \circ \mathbf{E}_{i,k} \circ \mathbf{M}_{i,k} \text{ and } \mathbf{e} = \begin{bmatrix} \vdots \\ 1 \\ \vdots \end{bmatrix} \quad (2)$$

The resultant deviation vector  $d_{i,k}^*$  acting at node  $i$ , is the sum of all entries in row  $i$  of the skew symmetric matrix  $\mathbf{A}$ . Matrix  $\mathbf{A}$  is the element-wise product ( $\circ$ ) of the connectivity matrix  $\mathbf{C}_k$  (compression = -1, tension = +1 or no connection = 0), the direction matrix  $\mathbf{E}_k$  (containing the unit vectors between the nodes) and the force magnitude matrix  $\mathbf{M}_k$  (containing the force magnitudes  $\mu$ , which are equivalent to the lengths of the deviation force vectors in the force diagram).

In the second step, the matrix  $\mathbf{P}_{k+1}$  of the unknown position vectors  $\mathbf{p}_{i,k+1}$  can be determined, considering the combinatorial state (plus or minus) and vector  $\mathbf{\Lambda}_{k+1}$  (containing the edge lengths  $\lambda_{i,k+1}$ , which are equivalent to the lengths of the trail edges in the form diagram).

$$\mathbf{P}_{k+1} = \mathbf{P}_k \pm \mathbf{\Lambda}_{k+1} * \left( \frac{\mathbf{F}_{k+1}^*}{|\mathbf{F}_{k+1}^*|} \right) \quad (3)$$

## 2.2. Equilibrium formulation for two trails

As an example, reducing the system to two trails (i and j) and combining equations(1), (2) and (3), the unknown position vector  $\mathbf{p}_{i,k+1}$  can be written as:

$$\mathbf{p}_{i,k+1} = \mathbf{p}_{i,k} \pm \lambda_{i,k+1} * \left( \frac{-\mathbf{f}_{i,k}^* - (c_{i,k,j,k} * (\mathbf{e}_{i,k,j,k}) * \mu_{i,k,j,k}) - \mathbf{q}_{i,k}^*}{|-\mathbf{f}_{i,k}^* - (c_{i,k,j,k} * (\mathbf{e}_{i,k,j,k}) * \mu_{i,k,j,k}) - \mathbf{q}_{i,k}^*|} \right) \quad (4)$$

Considering the combinatorial state of the topological diagram  $\mathbf{T}$  in Figure 1 and the start vertex of trail 1 ( $k=0$ ), where the preceding trail is null, equation (4) becomes:

$$\mathbf{p}_{1,1} = \mathbf{p}_{1,0} - \lambda_{1,1} * \left( \frac{-(-1 * (\mathbf{p}_{2,0} - \mathbf{p}_{1,0}) * \mu_{1,0,2,0}) - \mathbf{q}_{1,0}^*}{|(-1 * (\mathbf{p}_{2,0} - \mathbf{p}_{1,0}) * \mu_{1,0,2,0}) - \mathbf{q}_{1,0}^*|} \right) \quad (5)$$

Equation 5 depicts eleven independent parameters that could have an influence on the position vector in the next sequence ( $k=1$ ). Those eleven degrees of freedom include two start points  $\mathbf{p}_{1,0}$  and  $\mathbf{p}_{2,0}$  (one start point for each trail, with three coordinates each, resulting in six degrees of freedom) plus one trail element length  $\lambda_{1,1}$  plus one deviation force magnitude  $\mu_{1,0,2,0}$  plus three coordinates of the load vector  $\mathbf{q}_{1,0}^*$ .

## 2.3. Non-linear optimization and CEM

Assuming that the load vector  $\mathbf{q}_{1,0}^*$  and the unit vector between the two start points  $\mathbf{p}_{1,0}$  and  $\mathbf{p}_{2,0}$  are perpendicular, the angle  $\alpha_{1,0}$  of the subsequent force trail can be analytically written as:

$$\alpha_{1,0} = \tan^{-1} \left( \frac{(\mathbf{p}_{2,0} - \mathbf{p}_{1,0}) * (\mu_{1,0,2,0})}{\mathbf{q}_{1,0}^*} \right) \quad (6)$$

Figure 2 displays the behaviour of the form and force diagrams as well as the function of the angle  $\alpha_{1,0}$  of the subsequent trail elements with respect to a parametric variation of the force magnitude  $\mu_{1,0,2,0}$ . As can be already seen in this simple example, the function is non-linear. Different optimization techniques can be used to solve non-linear problems. For example, in all those problems where the solution space can be represented by a continuous function (as the problem in Figure 2), gradient-based algorithms can be used to adapt the equilibrium state towards required constraints set by the design problem. In this case, to avoid to run into local optima, global optimization algorithms can be employed in advance. The aim of the present paper is to show how different optimization techniques can be used to explore the parametric variation towards a desired equilibrium state.

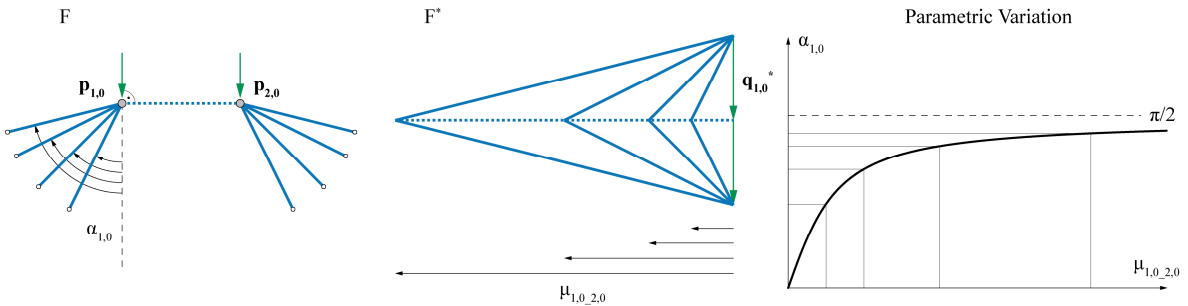


Fig. 2 Non-linear metric behaviour of the equilibrium state for the variation of  $\mu$

### 3. Lateral and vertical exploration in the equilibrium design process

In the design of equilibrium networks, one can differentiate between lateral and vertical design explorations. Given a network in equilibrium, **lateral (diverging) variations** can be defined as the variations obtained by changing the topology and the combinatorial state of the inner forces (compression-tension); each variation hence introduces an independent alternative design space. On the other hand, **vertical (converging) variations** can be considered as parametric variations within the same design space; vertical variations are, for example, the change in length of the network's elements, the adjustment of the magnitude of the inner forces or force densities, the modification of the start points' positions.

#### 3.1. Lateral explorations

In Figure 3 an overall equilibrium design process, which includes procedures for both lateral and vertical explorations, is represented schematically. The upper part of the scheme relates to lateral design exploration; here, divergent load-bearing mechanisms, each defined by a specific topological diagram  $\mathbf{T}_i$ , can be generated and visualized by the designer. A first procedure ( $L1$ ) is based on the construction of a topological diagram edge by edge using tensile or compressive elements. A second procedure ( $L2$ ) consists in the assembly of simple load-bearing modules into an overall topological diagram. A third procedure ( $L3$ ) relies on the initial definition of the topological diagram of a known structural typology used as a reference, which the designer can afterwards modify in terms of both connectivity and combinatorial state (compression-tension).

#### 3.2. Interactive vertical variations

The lower part of the scheme of Figure 3 shows two exemplary topological diagrams, generated through prior lateral exploration, each of them defining a specific load-bearing mechanism. These two examples are used to demonstrate different procedures for vertical design exploration, where the metric properties of the equilibrium state (form and force diagrams) can be directly modified by the designer to converge to a specific solution. On the lower left side of the scheme, an interactive design approach is shown. Two different procedures are here presented ( $VA$  and  $VB$ ). Procedure  $VA$  is based on the hypothesis that the support points are not a strong boundary constraint in the early phase of the design. Hence, following the conventional CEM approach (Ohlbrock and Schwartz [19]), specific metric properties of the structure can be freely changed in the form and force diagrams. These include the matrix  $\mathbf{P}_{k=0}$  of the start points  $\mathbf{p}_{i,k=0}$ , the matrix of trail lengths  $\mathbf{\Lambda}$  and/or the matrix  $\mathbf{M}$  of the deviation force magnitudes  $\mu$ . Following this procedure, the designer can interact in real-time with the resulting global form and force diagrams and explore various global equilibrium states. Procedure  $VB$  is meant for local parametric variations. Here, the designer can directly interact with the geometry of the form and/or force diagrams by transforming them in real time (D'Acunto et al[6]). Based on a numerical approach that makes use of a specific implementation of dynamic relaxation (Piker [20]), procedure  $VB$  enforces equilibrium of the network through the transformation, by keeping parallelism between the corresponding elements in the form and force diagrams.

#### 3.3. Vertical variations based on optimization

In order to explore equilibrium states that meet given design constraints, computationally driven procedures are necessary. In the lower right side of the scheme of Figure 3, two of these procedures ( $VC$  and  $VD$ ) are presented. Procedure  $VC$  can be used to meet given global constraints. The designer delegates the metric adaptation to a computational process in the background. Specifically, the designer defines a load-bearing mechanism, a load-case  $\mathbf{Q}$  (in the example two vertical point-loads), constraints for the equilibrium (in the example a boundary domain), fixes upper and lower bounds for each parameter and for each trail allocates an area for the support locations ( $\mathbf{p}_{1-4}$ ). Based on this setup, an optimization problem can be formulated. The objective can be defined as to find the set of parameters  $\mathbf{X}$ , which minimizes the sum of the distances  $\Sigma\delta$  between the actual locations of support

points ( $\mathbf{p}_{1-4}$ ) and the desired support area ( $\mathbf{p}_{d1-4}$ ). The boundary geometry can be taken into account by adding an extra penalty function to the objective.

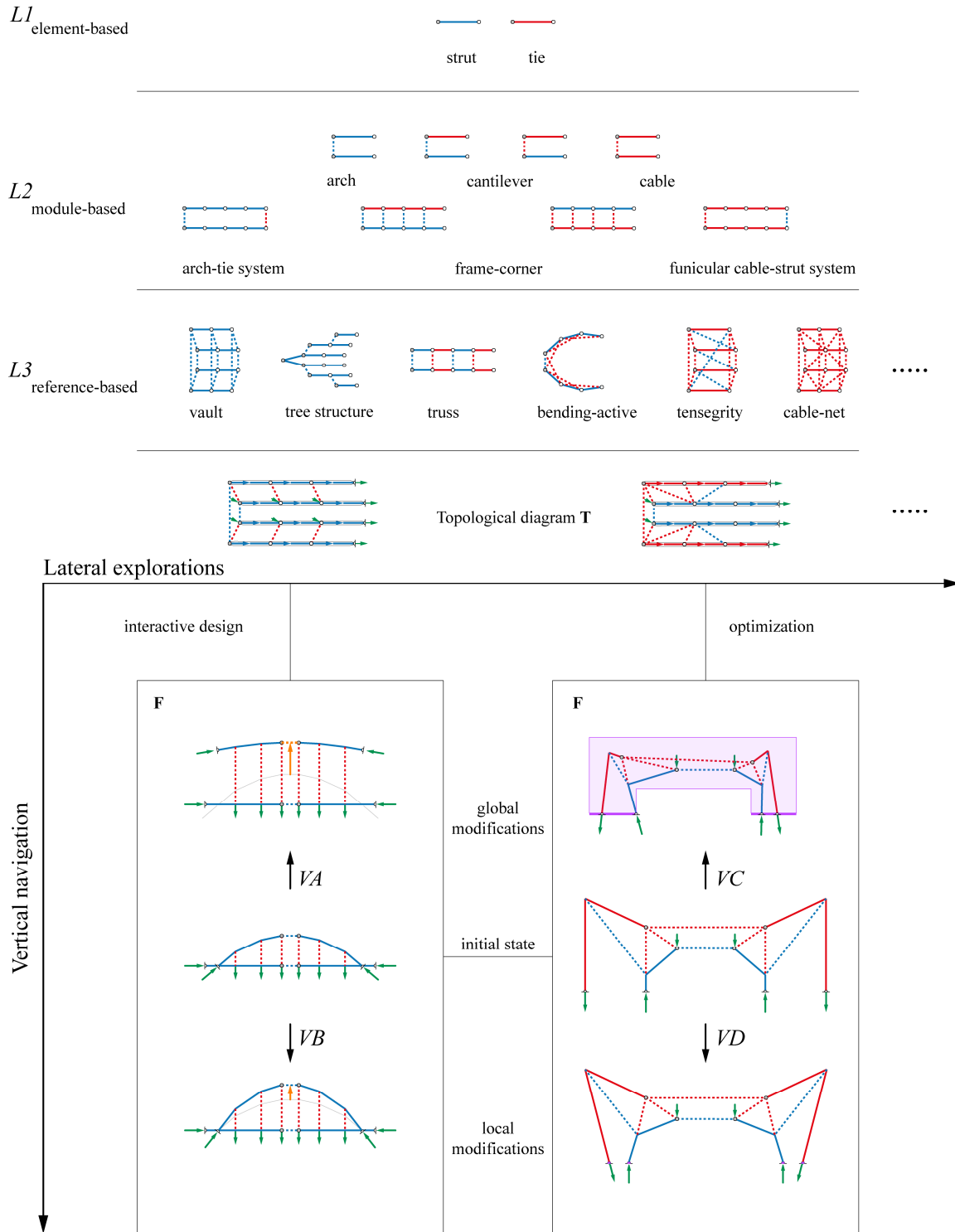


Fig. 3 Lateral ( $L1$ ,  $L2$  and  $L3$ ) and vertical procedures ( $VA$ ,  $VB$ ,  $VC$  and  $VD$ ) within the CEM design framework

The second computational-driven procedure, *VD*, can be seen as the local formulation of procedure *VC*. Based on an existing equilibrium state, local adjustments can be steered, by changing and/or adding more specific boundary constraints. In the example, the desired location of the supports is set as a local constraint. The adaptation of the equilibrium state of the network is then automatically found as the result of an optimization process. It is important to notice that the procedures *L1*, *L2*, *L3*, *VA*, *VB*, *VC* and *VD* should not be necessarily applied in a sequential way throughout the design process. On the contrary, it is always possible to move from lateral to vertical exploration and vice versa.

#### 4. Case studies

To test the different vertical procedures introduced in Section 3, the design of a pedestrian bridge spanning 20 meters has been taken into consideration as a main case study. The topological diagram **T** of Figure 4 has been taken as an initial load-bearing mechanism. This consists of two main compressive elements, a deck (width = 2 m) made of trails 2 and 3 and an arch made of trails 1 and 4, which are connected with each other by six deviation cables. Assuming that the dominant load case is a fully vertically loaded deck ( $q = 2.5 \text{ kN/m}^2$ ), six vertical point loads  $\mathbf{Q}_i$  can then be obtained:  $\mathbf{Q}_i \text{ [kN]} = 2.5 \text{ [kN/m}^2] * 2 \text{ [m]} * \lambda_{2,3} \text{ [m]}$ . Each equilibrium state for this load-bearing mechanism can be described by 32 independent parameters  $\mathbf{X}$  (4 start points each with 3 coordinates, 8 force magnitudes  $\mu$  and 12 trail lengths  $\lambda$ ). In order to keep the form as regular as possible, the length  $\lambda$  of each trail of the compressive deck and arch is set equal to the distance between the corresponding start points  $\mathbf{p}_{2,0}$  and  $\mathbf{p}_{3,0}$  (respectively  $\mathbf{p}_{1,0}$  and  $\mathbf{p}_{4,0}$ ). This reduces the amount of independent parameters in  $\mathbf{X}$  to 20. Specific constraints are then defined in the form of support point locations ( $\mathbf{p}_{i,3d}$ ). Moreover, the deck is constrained to lie within a horizontal plane  $\Omega$  located at height  $h$  from the origin.

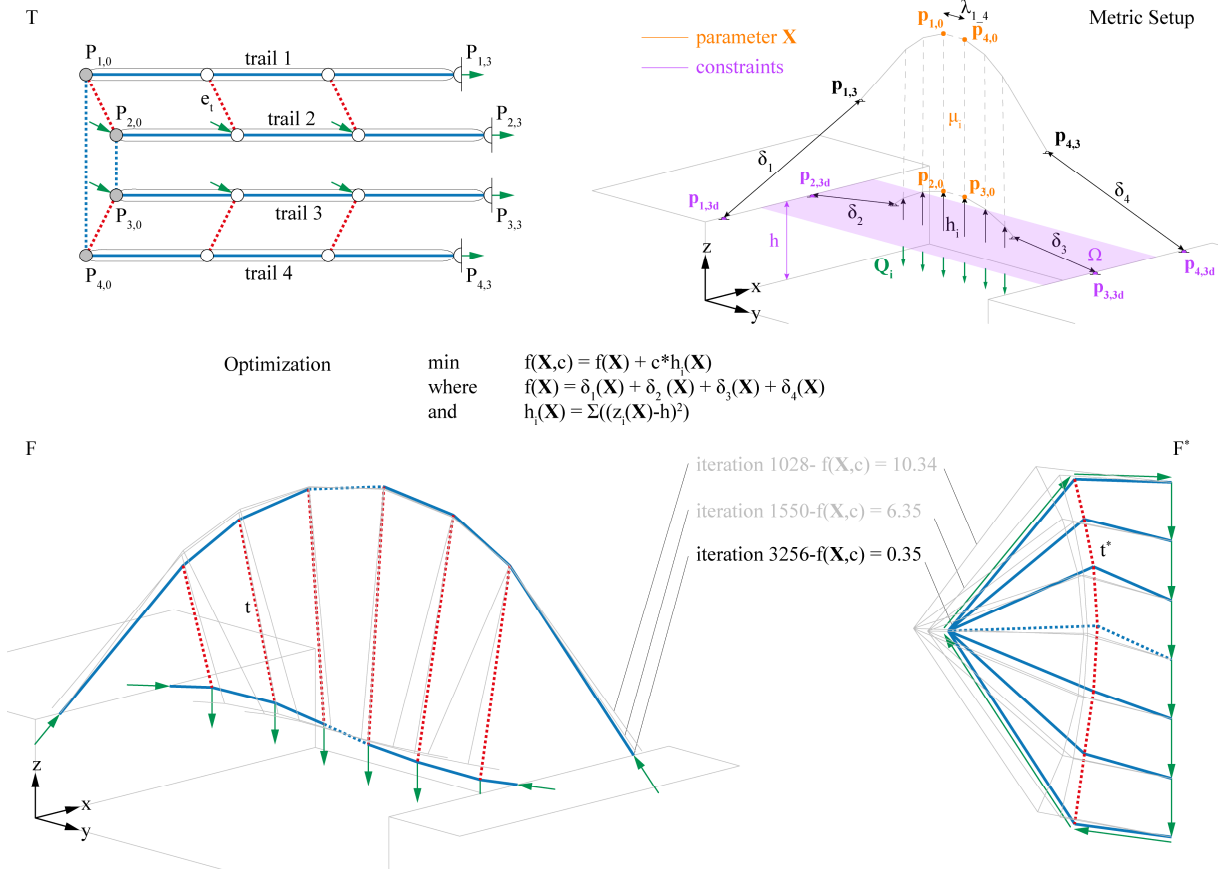


Fig. 4: Procedure *VC* – Topological diagram, metric setup (top), converging form diagrams

Based on this setup, a non-linear objective function  $f(\mathbf{X})$  (Bletzinger [4]) is defined, which here is described by the sum of the distances ( $\sum \delta_i(\mathbf{X})$ ) between the four initial support points and the four desired ones. The constraint related to the horizontality of the deck has been translated into a quadratic exterior penalty function  $p(\mathbf{X},c) = c \cdot \sum ((z_i(\mathbf{X})-h)^2)$  and added to  $f(\mathbf{X})$ . The lower part of Figure 4 shows two intermediate iterations of the 3D form  $\mathbf{F}$  and the force  $\mathbf{F}^*$  diagrams together with their final state after the objective function has been minimized within a defined threshold. The optimization problem has been solved with the BOBYQA algorithm (Powell [21]), which is embedded within the Rhino Grasshopper plugin *Goat* (Flöry et al. [9]). This algorithm which is part of the NLOpt library (Johnson [11]) approximates the function  $f(\mathbf{X})$  in each iteration locally with quadratic terms.

Based on the converged equilibrium state found with procedure *VC*, procedure *VB* is then applied to explore further vertical variations. The support locations are regarded as strong boundary constraints and only local transformations, such as the force redistributions within the network, are possible. Figure 5 shows the resulting 3D form diagram  $\mathbf{F}$  after an interactive transformation of the initial 3D force diagram  $\mathbf{F}^*$ . Shifting the pole (orange vector) away from the tensile hanger forces (red) leads, after the numerical simulation has converged, to an increment of the inner forces in the arch and ultimately results in an overall shallower geometry.

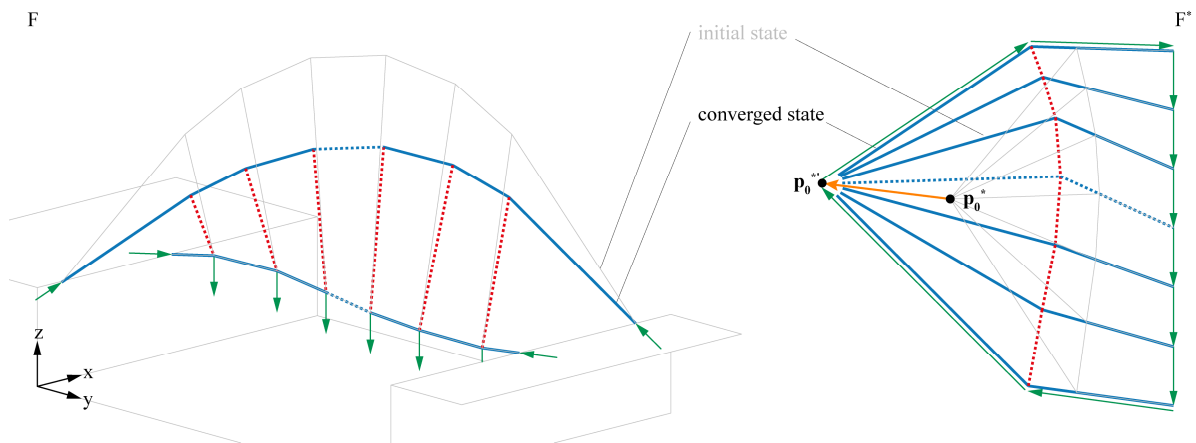


Fig. 5: Procedure *VB* - Transformations of vector-based force diagrams and corresponding change in the form.

The next exemplary design step is characterized by an intermediate lateral exploration, while the metric values are mostly kept unchanged. The initial load bearing mechanism (represented by  $\mathbf{T}$ ) with two arches is transformed into a cantilever-like load-bearing mechanism ( $\mathbf{T}'$ ) through procedure *L2*. Then following procedure *L1* an additional strut is added in order to redirect the tensile element to the ground ( $\mathbf{T}''$ ). Figure 6 shows the transformation of the topological diagram and the corresponding 3D form diagram. Starting from the modified topological diagram  $\mathbf{T}''$ , procedure *VA* is carried out. Within this procedure, two parameters (force magnitude  $\mu_{1,0\_4,0}$  and the z-coordinate of  $\mathbf{p}_{4,0}$ ) are interactively adjusted in order to explore further design solutions. Both diagrams at the bottom of Figure 6 show five possible equilibrium states each corresponding to an independent parametric variation of  $\mu_{1,0\_4,0}$  and  $\mathbf{p}_{4,0}$ .

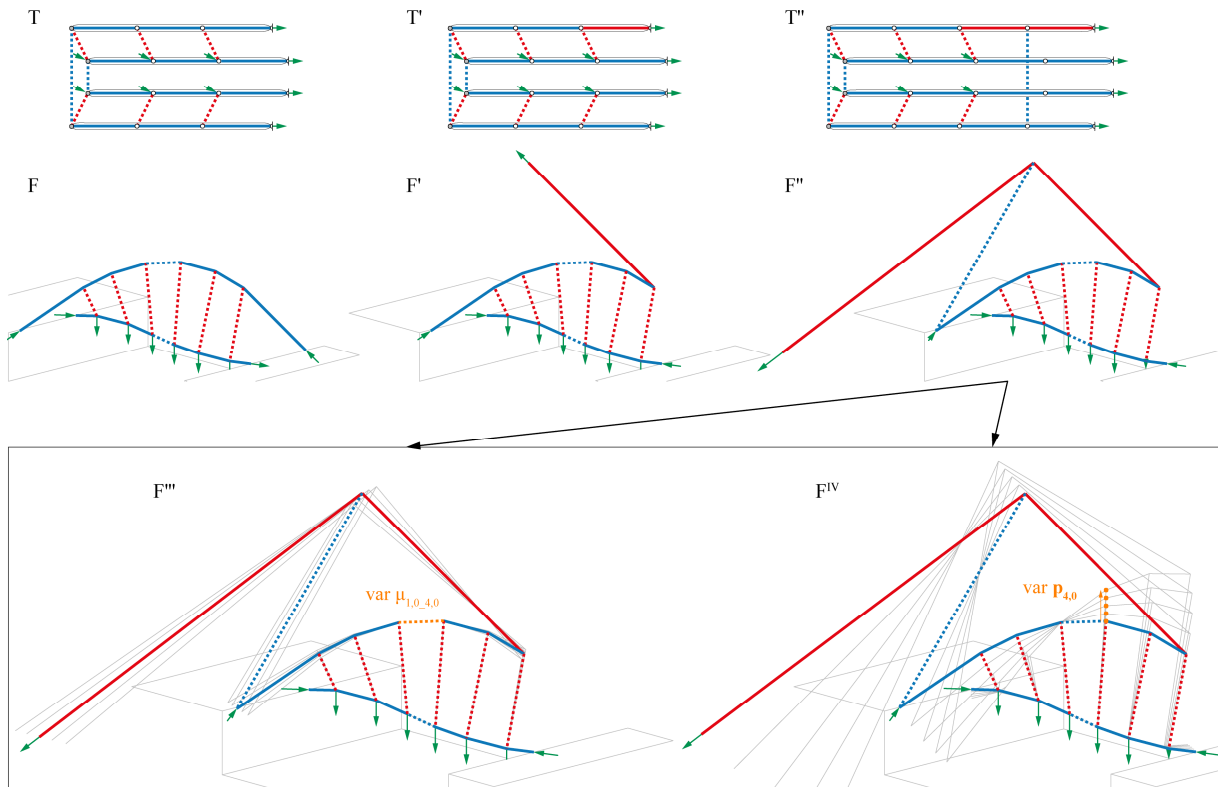


Fig. 6: Procedure *L1*, *L2* and *VA*: Lateral & Vertical exploration by varying *T* and parameter *X*

In order to demonstrate a more complex design application, a second case study related to an arena roof is taken into consideration. The initial equilibrium state of Figure 7 (grey lines) is generated using the CEM by means of procedure *L3*. A design objective is then formulated to constrain the position of the supports and the geometry of the projection of the roof on the ground (purple points and curves). Through the application of procedure *VD*, the converged equilibrium network is then found with the help of an optimization process.

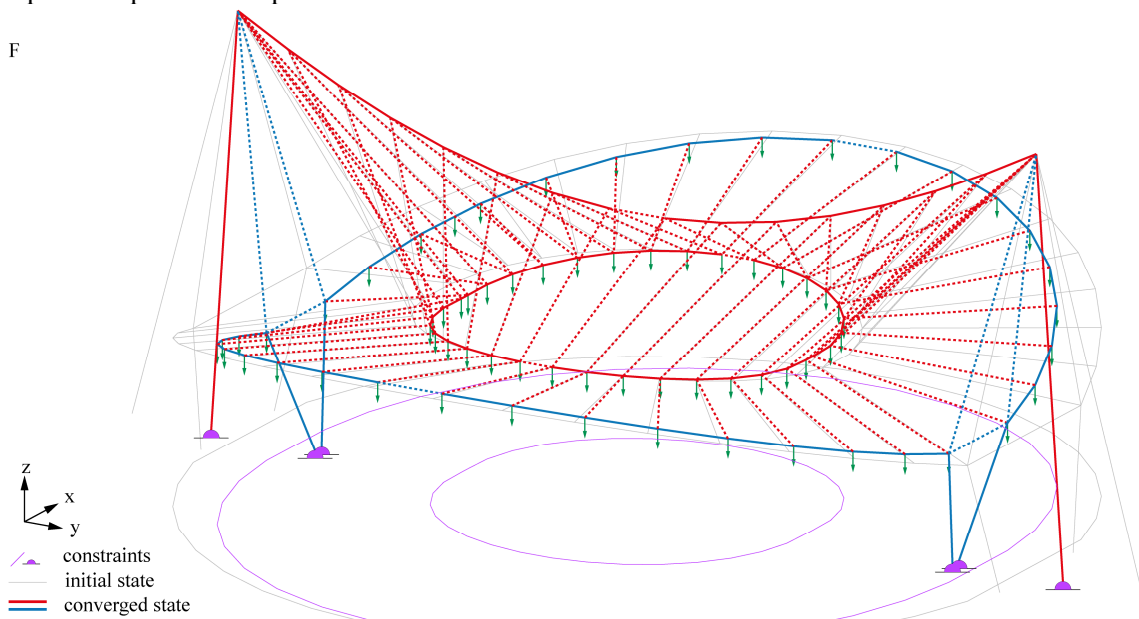


Fig. 7: Procedure *VD*: Local adaptation of equilibrium state (grey: initial state, red-blue: final state)



## 5. Discussion

The present paper introduces a series of procedures allowing the designer to interact with digital structural models in equilibrium, while opening up to the possibility to automate part of the design process. Pierluigi Nervi [18] pointed out that the overall aim of a structural design process is to *find an agreement* between the hard, objective static necessities and formal, rather soft and subjective properties. In other words, the goal of structural design is not to find an optimal design solution, but rather a satisfactory design within the given context (Ove Arup in Addis [1], p.7). Hence, studying lateral alternatives with radically different properties seems important in the early design stage (de Bono [7]). The design process can be defined as a non-linear iteration (Grobman *et al.* [10]) between **divergent** and **convergent** processes on the one hand, and analytical and synthetic on the other hand (Brown [5], p.58). In this context, particularly important is the use of visual models and the designer's interaction with them during the design process (Kotnik and D'Acunto [13]).

Various numerical form-finding methods (such as the force density method and dynamic relaxation) have been initially formulated in the 1970s in order to find equilibrium forms of networks with purely axial tensile or compressive stresses. During the last years, various research projects have been presented to extend the force density method to mixed compression and tension structures (Kemmler [12], Lachauer [14], Miki [17] and Tachi [23]). Given a structural topology and specific boundary constraints, these methods are particularly effective in finding one exact equilibrium solution. However, most of them are not suited to quickly generate and explore alternative solutions across different design spaces. Conversely, the extended CEM framework here presented, supports both **lateral (divergent)** and **vertical (convergent)** design exploration. One of the hypothesis of the present research is that some metric boundary constraints such as the exact support locations can be seen as a secondary aspect in the early creative stage of an equilibrium-focused design process. This hypothesis, which enables lateral design explorations, can be supported by a quote of Jörg Schlaich [22], p.111: *“The architect will not get the best results by demanding a structure from an engineer under already fixed and constraining boundary conditions. The architect must be open to a contribution from the engineer, and the engineer must be willing to contribute by proposing alternative structural solutions”* Other than the multiplicity of solutions, the way the designer interacts with structural models is crucial. Most of the form-finding processes consists of alternating phases of modelling and computation, in which the role of the designer is generally limited to the construction of the initial model and the definition and modification of metric boundary constraints, in a rather closed set-up (Lachauer [14], p.27). Moreover, most of the available methods are heavily dependent on the mathematical expertise of the designers (Kemmler [12], p.124). In most of the form-finding techniques, the objective is to find a particular equilibrium state by minimizing the residual force at each node of the network for a given set of constraints. The present research is based on a different approach. Within the CEM-framework, the equilibrium condition is inherent to the model. As a result, contrary to other methods, any generated solution is valid in terms of static equilibrium. In other words, the equilibrium of the network is not the aim of an optimization process, but rather the starting point. Optimization processes can be here used to address specific design constraints.

## 6. Conclusion

This paper has presented an extension of the CEM, an equilibrium-based design framework that allows the exploration of divergent yet unknown equilibrium forms. Based on the latest developments, gradient-based optimization techniques can be used to adapt an equilibrium state towards desired constraints automatically. As shown by the case studies, the presented approach has the potential to stimulate designers' creativeness on their way to find novel structural forms, while allowing for the control of both structural behaviour as well as specific boundary constraints.

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