Routledge
Taylor & Francis Group

Journal of Management Information Systems

ISSN: 0742-1222 (Print) 1557-928X (Online) Journal homepage: https://www.tandfonline.com/loi/mmis20

Smart Products for Sharing

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ABSTRACT: Sharing markets create mutual insurance for consumers who are unsure about their future needs for goods, thus rendering products more valuable both before and after the purchase. By embedding intelligence in their products, enabling them to sense, monitor, and authorize transfers between users, firms can economically participate in the collaborative consumption of their goods after they have been sold. Building on a dynamic model with overlapping generations of heterogeneous agents, we determine a firm's jointly optimal product price and sharing tariff. The active use of product intelligence as a gatekeeper for collaborative consumption can narrow the gap between the retail and the equilibrium price in the sharing market. Because of its tendency to decrease the demand for ownership, the use of smart products with a positive sharing tariff does not always maximize the firm's overall expected profits. A positive sharing tariff tends to be profitable with relatively high unit production cost and impatient consumers.

KEY WORDS AND PHRASES: aftermarket control, collaborative consumption, intelligent assets, Internet of things, product shareability, sharing economy, smart products.

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Journal of Management Information Systems / 2017, Vol. 34, No. 2, pp. 341–368.

ISSN 0742–1222 (print) / ISSN 1557–928X (online)
DOI: https://doi.org/10.1080/07421222.2017.1334466

The degree to which software, movie subscriptions, transportation tickets, and physical devices are shared among peers may be controlled by the producer or a seller of these items. For example, in order to authorize an owner of an annual rail pass to share it with another person, the ticket issuer can ask for a fee, referred to as a "sharing tariff." Similarly, for computer games delivered and accessed through online portals (e.g., Steam), ownership of a game could be shared, conditional on seller authorization. Even for physical devices, such as audio studio gear (e.g., by Antelope or United Audio), online registration may be required to gain access, which ultimately allows the producer to retain some control over peer-to-peer sharing and transfer of ownership. The limits of the control over the use of an item are due mainly to budget constraints or lack of imagination, rather than an intrinsic lack of feasibility.[1]

Examples and Motivation

Sharing allows agents to scale their use of the product up or down as their needs materialize (or not). This in turn enables consumers and society at large to make more efficient use of durable goods, especially when they are expensive to produce or costly to discard. The question addressed here is how the firms can possibly participate economically in the collaborative transactions that unfold as the users' heterogeneous needs realize over time. This may concern virtual goods, digital upgrades, and physical goods; see Figure 1 for an overview.

1. *Virtual Goods*. An annual rail pass remains unused while an agent is on a vacation. At that point economic value for the agent could be obtained by sharing this rail pass with someone else, especially if the asset is "intelligent."

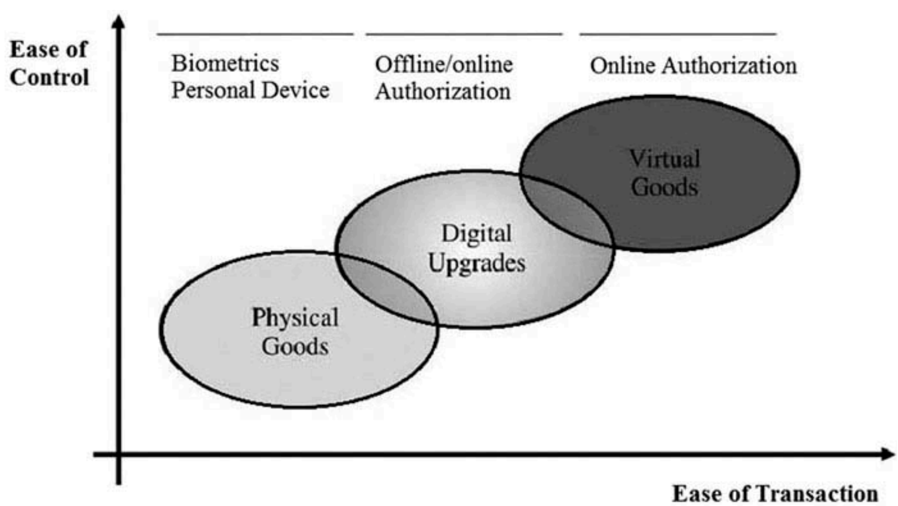

Figure 1. Sharing Control of Different Product Types

For this, instead of limiting the rail pass to the original owner's physical copy of the ticket, the rail operator or ticket issuer could transfer the ticket's validity for a defined length of time to a borrower who can use an authorized printout instead. In this way, the borrower and lender/owner would never need to meet for a physical exchange, thus reducing the frictional cost of sharing. In addition, the rail operator would be able to charge the owner of the ticket a sharing tariff before electronically transferring the rights of use. There are many other virtual goods that could be shared in this manner, such as software (via portals and license servers), memberships (e.g., in fitness or social clubs), subscriptions (e.g., to magazines or dance studios), and access permits (e.g., in the form of annual parking tickets or hunting licenses).

2. *Digital Upgrades*. As with virtual goods, users may share "digital enhancements" of physical or nonphysical goods in a quasi-frictionless way. For example, instead of producing and designing multiple hardware versions of a car engine, an automobile manufacturer may produce a single high-end version of the engine and then digitally select one of a number of different performance-graded versions. By paying for an upgrade, a user can increase the performance grade of his car. To decrease the commitment implied by the investment in an upgrade, the manufacturer may allow the user to share the upgrade with another car owner of the same brand, and allow for a temporary transfer of the engine-upgrade license. Similar mechanisms could be used for software upgrades, audio or photo equipment (e.g., in terms of ability to process higher-resolution files), power tools, and other goods for which the usage experience can be digitally controlled (including software, thus overlapping with virtual goods).
3. *Physical Goods*. The detection of different users for physical goods relies on product intelligence. For example, it may be possible to use biometric recognition to detect who is operating the device. Fingerprint readers and other biometric sensors have already been used to control access to computers, but they could also be used to control the operation of any physical device that contains key electronics components. Instead of direct user recognition, one can require that the product interact with a device that is difficult to transfer between agents. For instance, rather than a card to identify the user of a copy machine, one can require code entry and interaction with the user's personal phone, employing protocols that ensure authenticity.

The analysis in this study quantifies the economic benefits a firm can expect from being able to charge for the authorization of a peer-to-peer usage transfer in a collaborative economy.

Literature

The concept of an "Internet of things" as a communication platform for physical assets (such as household appliances) dates back more than a decade [14]. A connected “smart world” with sensing and control features embedded in products is likely to lead to a significant qualitative change of lifestyle of the humans “in the loop” [28]. On the one hand, connected products allow for item-level tracking across organizational boundaries [15], thus enabling visibility, for example, in supply chains [22]. The idea of embedding intelligence in products dates back at least to Ives and Vitale [17] who early on recognized the combination of maintenance and information technology as a “strategic opportunity,” providing the prospect of self-servicing intelligent assets up to the complexity of, say, an airplane [7]. In the context of sharing, the “intelligence” needed for a product to be smart enough to allow for effective aftermarket control of collaborative consumption is sensing of the user as well as the ability to allow or disallow access to the product’s functionality. The decision making and authorization may in many cases be provided by the network after confirmation and payment processing by a facility associated with the product-originating firm.

The control of the shareability with the help of smart products in a secondary market is reminiscent of market and aftermarket control through compatibility and technological complements. Manufacturers can try to limit the interoperability of devices in an attempt to limit access to a network and achieve customer lock-in to proprietary systems [1, 12]. Other options include tying the sale of one product to the purchase of another product [10], or controlling technological complements, for example, by requiring the use of proprietary cartridges with printers [27]. When controlling the shareability of an item, a producer effectively asks for the purchase of a “license to share” (i.e., a proprietary product) whenever the owner would like to “augment” the use of his item by lending it out to others. In our context, the term “lending” refers to the temporary transfer of usage rights for a given asset, which usually entails a monetary compensation of the owner.

Benkler defines shareable goods as (1) technically lumpy, that is, they provide “functionality in discrete packages rather than in a smooth flow,” and (2) systematically exhibiting “slack capacity relative to the demand of their owners” [5, p. 277], that is, there exists a natural excess capacity of such goods. With the emergence of peer-to-peer trading platforms, such as AirBnB or Eloue, many consumers’ preferences have been transformed to now favor access over ownership [3, 4, 23, 32]. Sharing intermediaries have been able to solve problems of asymmetric information [31], fueling the growth of peer-to-peer exchanges [11]. Sharing markets allow for mutual insurance of consumers’ ex post utilities derived from a combination of access to goods and the realization of ex ante uncertain needs, in the spirit of Arrow [2]. In this way they can correct, at least partially, for over- or undercommitment of an individual’s resources when owning a product or not. The economic rationality of collaborative consumption [6, 13], when paired with sharing markets, has given rise to a sharing economy that is set to disrupt traditional modes of consumption and production.

The present study addresses the last point by analyzing a producer’s (or retailer’s) options to use smart products to ration shareability in order to capture secondary market rents in a peer-to-peer sharing economy. Interestingly, our results show that exerting such control may not be optimal because the loss in sales (primary revenue stream) when introducing a sharing tariff may outweigh the additional income (secondary revenue stream). This study is an extended version of Weber [34].

Model

Each consumer in the dynamic discrete-time sharing economy lives for two periods in an overlapping generation, as in Weber [33]. At any time $t \in \{0, 1, \ldots\}$, a new generation $\mathcal{G}_t$ of consumers (or "agents") is born. Without loss of generality (except for the underlying assumption of stationarity), the number of consumers in each generation $\mathcal{G}_t$ is normalized to 1, so that at any time t the total number of agents in the economy is 2; see Figure 2 for a timeline. The first period (t) of an agent's life in generation $\mathcal{G}_t$ is called his "early consumption phase" ($\mathcal{C}_0$), while the last period ($t + 1$) is referred to as his "late consumption phase" ($\mathcal{C}_1$). We assume that at time $t = 0$, a consumer generation in its late consumption phase already exists, so that the economy always operates in steady state.

The good is available for purchase from a monopolist firm at the retail price r or else it can be obtained for a one-period loan from peers at the sharing price p, provided the sharing market is active. Using suitable product intelligence for recognizing the transfer of usership, sharing transactions are subject to a sharing transfer τ paid from the owner to the firm; see Figure 2 for an overview of the various financial transactions, including retail, sharing-control (with payment of the sharing tariff), and peer-to-peer. This implies the effective transaction price:

$$\hat{p} \equiv p - \tau,$$

for lenders in the sharing market; $\hat{p}$ represents the net rent (or absolute markup) of a supplier in the sharing market, while borrowers in the peer-to-peer market pay the nominal sharing price p.

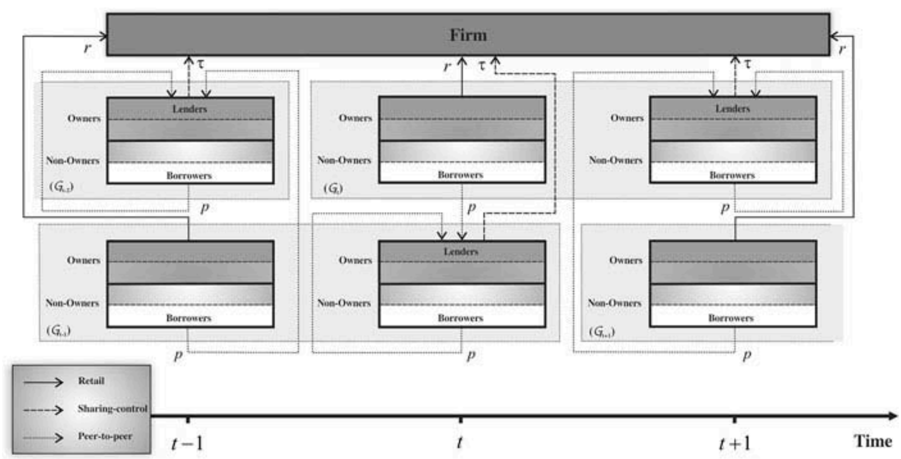

Figure 2. Timeline: Retail, Sharing-Control, and Peer-to-Peer Transfers

Remark 1. Without loss of generality, we assume that the borrower pays the sharing price p to the lender, and the lender pays the sharing tariff τ to the firm. A payoff-equivalent setting is that the borrower pays the effective transaction price $\hat{p}$ to the lender and the sharing tariff τ to the firm. In certain situations this may correspond to a more suitable flow of transactions, without changing any of the parties' surplus. The equilibrium price in the sharing market would then decrease by τ. We also note that this is similar to the fact that a sharing intermediary's two-sided commission structure tends to affect only the lenders [31]; this finding echoes an earlier neutrality result by Caillaud and Julien [9] for intermediated markets.

Agents have heterogeneous preferences for consuming a durable good. Each agent is characterized by his subjective likelihood of need $\theta \in [0, 1]$ and his expected use value $v \in [0, 1]$ conditional on a realized need. That is, agents are heterogeneous in two dimensions, and a consumer's "type" is a point (θ, v) in the type space $\mathcal{Q} = [0, 1] \times [0, 1]$. For simplicity, it is assumed that the type distribution for any generation is uniform on $\mathcal{Q}$ and that each consumer's type is persistent over his lifetime. Furthermore, the need realizations for the product are independently distributed across time and agents, so nothing can be learned from other consumers or his own current need about his future demand for the durable good.[2]

Dynamic Choice

Consumers observe their respective needs at the beginning of each period and then, contingent on this observation, they make decisions about transactions with the retailer or the sharing market. More specifically, at the beginning of consumption phase $\mathcal{C}_i$, any given agent of type $(\theta, v) \in \mathcal{Q}$ observes the realization s_i of his random need state $\tilde{s}_i \in \{0, 1\}$, which is distributed according to $\text{Prob}(\tilde{s}_i = 1) = \theta$, for all $i \in \{0, 1\}$.

Remark 2. In general, a consumer's need state may not be independent across the two consumption phases. For example, once a power drill has been used to do its job everywhere around the house, for many consumers it is unlikely that they will need a power drill again in the foreseeable future. On the other hand, for numerous agents a current high need for a car tends to imply a high car need in the near future. Here we assume serial independence to simplify the analysis and to model the fact that consumption phases during an agent's lifetime are spaced sufficiently far apart for random needs to become independent of each other. It also allows us to compare our results directly to those by Weber [33], which were obtained for uncorrelated need states in the absence of product intelligence and thus without the manufacturer's ability to extract rent from collaborative consumption on an aftermarket.

Contingent on a realized high need state ($s_i = 1$), to get access to the product the agent can *either* purchase it from a retailer at the price r>0 *or* borrow it on a peer-to-peer market at the sharing price p. Regarding the latter, the right for a one-time use of the product can be traded (i.e., acquired or relinquished) on a sharing market where owners are asked to pay a transfer $\tau \geq 0$ to the firm for the authorization to rent the good to a peer at the (nonnegative) price $p<r$.[3] At this point of the analysis, the sharing price p is exogenous because all market participants are price takers. This price becomes endogenous via clearing of the sharing market, examined later.

Thus, ex ante before making an ownership decision, a consumer considers the product both from the perspective of his benefits, as a function of his type (in terms of likelihood of need and contingent consumption value), his costs, and his opportunities ex post (in terms of p, r, τ). The retail price r and the sharing tariff τ are advertised with the firm's product offering, and p is (via rational expectations) the correctly anticipated price in the sharing market.

Because the price p for access in a sharing market cannot exceed the purchase price r, ownership decisions are made only in the early consumption phase $\mathcal{C}_0$ as long as the sharing market is active.[4] In the event of a sharing shutdown (discussed in detail below), high-value consumers who experience a need solely in their late consumption phase might still become owners. By contrast, with an enabled peer-to-peer economy consumers in their late consumption phase restrict attention to gaining and providing access to the good by taking the requisite borrowing and lending decisions on the sharing market. Given p, r, τ such that $0 \leq \tau \leq p \leq r$, we now analyze the agents' choice behavior in their two consumption phases using backward induction for a given generation of consumers; see Figure 3.

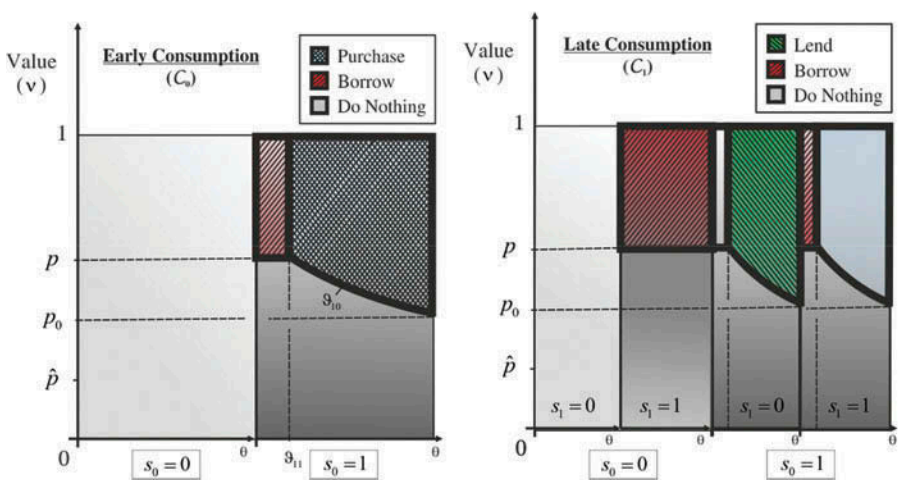

Figure 3. Dynamic Choice Behavior with Sharing

Late Consumption Phase

At the beginning of $\mathcal{C}_1$, a consumer of type (θ, v) observes the realization $s_1 \in \{0, 1\}$ of his need state $\tilde{s}_1$. If he is a nonowner, then he can either not consume the product at all or rent it on the sharing market at the price p. Given that the future is unimportant in his late consumption phase, renting dominates buying the product at the higher retail price $r>p$. For $p = r$, we assume that a consumer would prefer the residual benefits of ownership, thus leading to a shutdown of the sharing market, discussed below. The following result summarizes the state-contingent payoffs in $\mathcal{C}_1$.

Lemma 1. A type-(θ, v) agent's $\mathcal{C}_1$-payoffs are $U_0 = 0$, $U_1 = \max\{0, v - p\}$ as nonowner, and $V_0 = \max\{0, \hat{p}\}$, $V_1 = \max\{v, \hat{p}\}$ as owner, respectively.

Buying the product may make sense for a consumer if the expected difference in payoffs warrants it, given his current need state. In the low-need state ($s_1 = 0$), the payoff difference between owner and nonowner is:

$$V_0 - U_0 = \max\{0, \hat{p}\},$$

while in the high-need state ($s_1 = 1$), it is:

$$V_1 - U_1 = \begin{cases} \hat{p}, & \text{if } v < \hat{p}, \\ v, & \text{if } v \in [\hat{p}, p], \\ p, & \text{if } v > p. \end{cases}$$

As long as the firm charges a positive sharing tariff, consumers with a high value of use may care distinctly more about ownership than those with a low value of use. As a function of the agents' consumption value, the ownership benefit in the high-need state increases from $\hat{p}$ to p, reflecting the agents' opportunity costs for not choosing their respective second-best mode of consumption (own use vs. participation in the sharing market). Lastly, we note that for *participants* in the sharing market, the payoff is *independent* of the need state: indeed, $U_0 = U_1$ for $v \in [p, 1]$ and $V_0 = V_1$ for $v \in [0, \hat{p})$; owners with $v \in (\hat{p}, 1]$ lend only in the low-need state.

Early Consumption Phase

In $\mathcal{C}_0$, an individual of type (θ, v), who is in need state s_0, has the option to purchase the product from a retailer at the price r to become an owner. The individual can use a purchased item immediately, in that same consumption phase. Alternatively, the agent can rent the item on the sharing market at the price p, also for immediate use. Note that at this early stage in his life, the agent forms an expectation about his future utility: $\bar{V}$ as owner or $\bar{U}$ as nonowner. All agents discount future payoffs at the common per-period discount factor:

$$\delta \in (0, 1].$$

Choosing the best of his three alternatives (do nothing /borrow on the sharing market /buy from the retailer) an agent's discounted state-dependent total payoff is:

$$\bar{T}_{s_0} = \max\{\delta\bar{U}, vs_0 - p + \delta\bar{U}, vs_0 - r + \delta\bar{V}\},$$

where $\bar{U} = (1-\theta)U_0 + \theta U_1$ and $\bar{V} = (1-\theta)V_0 + \theta V_1$, and where U_{s_1}, V_{s_1}, for $s_1 \in \{0, 1\}$, are given by Lemma 1. Combining the first two decision options yields the total expected payoff of nonownership,

$$\bar{T}_{s_0}^{non-owner} \equiv (s_0 + \delta\theta) \max\{0, v - p\},$$

which needs to be compared against the total expected payoff of ownership,

$$\bar{T}_{s_0}^{owner} \equiv vs_0 - r + \delta(\hat{p} + \theta \max\{0, v - \hat{p}\}),$$

separately for the agents' possible current need states $s_0 \in \{0, 1\}$. As a nonowner, the agent's total payoff is his current net use value $v - p$ when getting access to the product by borrowing it on a sharing market plus the expected discounted value of the same net use value taking account of the uncertain prospect that the item is needed again in the future. As an owner, on the other hand, the agent would get the net use value $v - r$ immediately (in case of current high need) and the expected discounted value of either renting it out (to obtain the proceeds $\hat{p}$) or the use value v, depending on his beliefs about the future need.

Lemma 2. In $\mathcal{C}_0$, a type-(θ, v) agent in the high-need state $s_0 = 1$ becomes an owner if and only if

$$\theta \geq \frac{\max\{0, r - \min\{v, p\} - \delta\hat{p}\}}{\delta(\tau + \min\{0, v - p\})} \equiv \theta_0(p, v);$$

otherwise, if $v \geq p$, he borrows the item on the sharing market. In any other event (including when $s_0 = 0$), the agent does nothing.

To find ownership attractive, high-value agents need to anticipate a sufficiently high sharing price, in comparison to the retail price. In addition, the effective transaction price $\hat{p}$ as a lender (in case of low need in the future) should be sufficiently high.

Demand for Ownership

Combining the purchase decisions for high-value and medium-value agents in their early consumption phase yields the overall demand for the monopolist's goods in the presence of an active peer-to-peer market. Based on Lemma 2 the demand for ownership is:

$$\Omega = \int_{p_0}^{1} \left(\int_{\theta_0(p,v)}^{1} \theta d\theta \right) dv,$$

where $v = p_0$ is the lowest valuation of an agent (with $\theta = 1$) purchasing the item:

$$p_0 \equiv \frac{r}{1+\delta};$$

this lower bound is such that its present value ($(1+\delta)p_0$), when paid in every period, exactly equals the retail price (r). On the other hand, there exists an upper price bound,

$$p_1 \equiv \frac{r+\delta\tau}{1+\delta},$$

which, when taken in perpetuity, has the same present value ($(1+\delta)p_1$) as the maximum revenue obtainable from the asset ($r+\delta\tau$), that is, the retail price plus the sharing tariff in the next period. For now we conjecture that the sharing price p will be "moderate" in equilibrium, so it lies in the interval $[p_0, p_1]$; the validity of this assertion is formally established in Lemma 3 shortly after the next result. The aggregate demand for ownership is based on the agents' choice behavior (see Figure 3). Whether certain agents borrow in their early consumption phase (i.e., whether $\theta_0(p,p)>0$) depends on the price in the sharing market. For $p_0<p<p_1$, agents with low likelihood types θ prefer gaining access to the product via sharing to an ownership commitment. In the absence of a sharing tariff, that is, when $\tau = 0,$ the last condition becomes vacuous, and there is no sharing activity at all in the early consumption phase.

Theorem 1. Given p, τ, r with $p \in [p_0, p_1]$ and $0 \le \tau \le r,$ the aggregate demand for ownership in an active sharing economy is $\Omega = \Omega_{10}^A + \Omega_{11}^A,$ where

$$\Omega_{10}^A = \frac{p-p_0}{2} - \frac{p_0-\hat{p}}{2\delta^2}\left(\frac{p-p_0}{p_0-\hat{p}} + (1+\delta)\left[2\ln\left(\frac{p_0-\hat{p}}{p-\hat{p}}\right) + (1+\delta)\frac{p-p_0}{p-\hat{p}}\right]\right),$$

$$\Omega_{11}^A = \frac{1-p}{2}\left(1 - \left(\frac{(1+\delta)(p_1-p)}{\delta\tau}\right)^2\right).$$

The terms Ω_{10}^A and Ω_{11}^A correspond to the aggregate demand by consumers with consumption values v in the interval $(\hat{p}, p)$ ("medium-value agents") and in the interval $[p, 1]$ ("high-value agents"), respectively. The remaining "low-value agents" never purchase the product. Note that this trichotomy of agents in terms of their consumption values is determined endogenously because the sharing price p (as well as the effective transaction price $\hat{p}$ for the lender) is implied by the market equilibrium. It is interesting to observe that ownership by medium-value agents is an artifact produced by the existence of a positive sharing tariff as well as agents that care about the future.[5]

Sharing Equilibrium

Let $r>0$ be a given retail price and $\tau \in [0, p]$ be an implementable sharing tariff. Assuming that the sharing market clears, the price p in the sharing market must be such that demand for the shared product equals the supply.[6] The potential suppliers in the sharing market include all agents in their late consumption phase $\mathcal{C}_1$ who opted for ownership in their early consumption phase $\mathcal{C}_0$. As implied by our discussion of the demand for ownership, the nature of the equilibrium depends on whether the sharing price is "high" (with $p>p_1$) or "moderate" (with $p_0 \le p \le p_1$).[7] The following result helps focus the analysis of the sharing equilibrium, effectively excluding any high-price scenario in equilibrium.

> *Lemma 3.* For any $(r, \tau) \ge 0$, *a sharing market is active (i.e., it has positive trading volume) if and only if* $r \in (0, 1+\delta)$ *and* $\tau \le p$. The clearing price p of an active sharing market is moderate, that is, it lies in the interval $[p_0, p_1]$.

The preceding result implies that in the special case without sharing tariff (for $\tau = 0$), the market price is equal to p_0, i.e., $p = r/(1+\delta)$ as in Weber [33]. It also justifies restricting attention to the interesting case of a moderate sharing price $p \in [p_0, p_1]$ in equilibrium.

Supply

The supply in the sharing market consists of owners who are happy to lend when they find themselves in a low-need state ($s_1 = 0$). In a high-need state ($s_1 = 1$), owners with values $v<\hat{p}$ (who are a priori willing to lend) do not exist, as owners' lowest value is $p_0 \ge \hat{p}$. Thus, the sharing supply becomes:

$$S = \int_{p_0}^{1} \left(\int_{\theta_0(p,v)}^{1} (1-\theta)\theta d\theta \right) dv.$$

> *Lemma 4.* Given p, τ, r with $p \in [p_0, p_1]$ and $0 \le \tau \le r$, the sharing supply in an active sharing economy is $S = S_{10}^A + S_{11}^A$, where

$$S_{10}^A = \frac{(1-\lambda)\tau}{6}\left(1 - \frac{3}{\delta^2} - \frac{2}{\delta^3}\right) - \frac{(1+\delta)^2\lambda\tau}{\delta^3}\left(\ln(\lambda) + (1-\lambda)\frac{5+2\delta-(1+\delta)\lambda}{6}\right),$$

$$S_{11}^A = (1-p)\left(\frac{1}{6} - \frac{1}{2}\left(\frac{(1+\delta)(p_1-p)}{\delta\tau}\right)^2 + \frac{1}{3}\left(\frac{(1+\delta)(p_1-p)}{\delta\tau}\right)^3\right),$$

> using the abbreviation $\lambda \equiv (p_0 - \hat{p})/\tau$.

The sharing supply collapses whenever the sharing tariff τ exceeds the equilibrium price p in the market because this would entail a negative absolute markup $\hat{p}$. Naturally, the equilibrium price also depends on τ, so that checking whether τ exceeds p in equilibrium involves solving the corresponding fixed-point problem, which is discussed below.

Demand

Nonowners like to access a product on the sharing market whenever they are in a high-need state (in either consumption phase) and their contingent use value exceeds the market price (so $v \geq p$). The corresponding sharing demand from the mature generation in $\mathcal{C}_1$ amounts to:

$$D_1 = \int_p^1 \left(\int_0^1 (1-\theta)\theta d\theta + \int_0^{\theta_0(p,v)} \theta^2 d\theta \right) dv,$$

while for the early generation in $\mathcal{C}_0$ it is:

$$D_0 = \int_p^1 \left(\int_0^{\theta_0(p,v)} \theta d\theta \right) dv.$$

Since the distributions of consumption values and likelihood types are by assumption uncorrelated, the effects of the two separate multiplicatively. Consider any time period $t \geq 1$. Consumers in the currently mature generation $\mathcal{G}_{t-1}$, who had a low need in their past consumption phase ($\mathcal{C}_0$) and a high need in the current consumption phase ($\mathcal{C}_1$), want to borrow in the sharing market, provided their consumption value exceeds the sharing price. Moreover, consumers in that generation who had a high need in their past consumption phase but a low likelihood type (with θ below $\theta_0(p, v)$) did not purchase the product and are therefore nonowners; given a current high need these agents also want to borrow, that is, they are also part of D_1. It is precisely this type of consumer in the currently early generation $\mathcal{G}_t$ that makes up the demand D_0. The sum of the demand from the two presently extant consumer generations accounts for the aggregate sharing demand D, which can be obtained in closed form.

Lemma 5. Given p, τ, r with $p \in [p_0, p_1]$ and $0 \leq \tau \leq r$, the sharing demand in an active sharing economy is:

$$D = D_0 + D_1 = (1-p)\left(\frac{1}{6} + \frac{1}{2}\left(\frac{(1+\delta)(p_1 - p)}{\delta\tau} \right)^2 + \frac{1}{3}\left(\frac{(1+\delta)(p_1 - p)}{\delta\tau} \right)^3 \right).$$

Based on the preceding result it is clear that, all else equal, the sharing demand is increasing in the retail price r and decreasing in both the sharing price p and the sharing tariff τ. Naturally, the sharing price depends in equilibrium on the firm's pricing scheme (r, τ).

Market Clearing

For a moderate sharing price $p \in [p_0, p_1]$, the sharing market clears when supply equals demand: $S = D$. Then the number of lenders equals the number of borrowers in Figure 3. As shown there, the lenders are agents from the mature generation in the late consumption phase, whereas the borrowers generally come from both generations (when the sharing tariff is positive).

Theorem 2. Given $r, \tau,$ with $0 \leq \tau \leq r,$ the equilibrium sharing price is $p^* = p_0 + (1 - \lambda)\tau,$ where $\lambda \in [1/(1+\delta), 1]$ solves the fixed-point problem

$$\lambda = 1 - \frac{1}{\tau}\left(1 - p_0 - \left(\frac{\delta}{(1+\delta)\lambda - 1}\right)^2 S_{10}^A\right),$$

where λ and S_{10}^A are as in Lemma 4.

A straightforward analysis reveals a monotone dependence of the equilibrium sharing price on the monopolist's choice variables. Provided there is an active sharing market, the equilibrium sharing price p^* is increasing in $(r, \tau) \geq 0$.

By increasing the sharing tariff *beyond* the equilibrium market price, the monopolist can effectively disable the sharing market (i.e., induce a "sharing shutdown") because in that case the absolute markup $\hat{p}$ becomes negative, thus negating any economic incentives peers may have had to make their goods available for collaborative consumption. The question of which precise bound this implies on the sharing tariff is answered by solving the fixed-point problem $\tau = p^*(r, \tau)$, discussed in the next section because it is central to the solution of the monopolist's profit-maximization problem.

Optimal Pricing

Active Sharing

With an active peer-to-peer market, the firm has two sources of revenue: the sales of the product and the rent from the sharing tariff on sharing transactions. Thus, its profits are of the form:

$$\Pi = (r - c)\Omega + \tau D,$$

subject to the equilibrium sharing price $p = p^*(r, \tau)$ in Theorem 2. As already pointed out, for the sharing market to stay liquid, the sharing tariff τ cannot exceed the equilibrium sharing price. This implies a discontinuity at the boundary $\tau = p^*(r, \tau)$ in the monopolist's profit function. We also note that for a zero sharing tariff, we can revert to simpler optimality conditions to obtain an optimal retail price.[8] The power of the sharing tariff is that it allows the monopolist to syphon off surplus from the sharing market at a point where established owners (in $\mathcal{C}_1$) have observed their need states. For any given sharing tariff τ, the firm's profit has an interior optimum $r^*(\tau) \in (c, 1+\delta)$ because a price at the boundary would lead to zero profits. Thus, $r^*(\tau)$ can be determined from a (modified) monopoly pricing rule (see, e.g., Tirole [29, p. 66]).

Theorem 3. In an active sharing market, for a given sharing tariff $\tau \in [0, p^*]$, the optimal retail price $r^*(\tau)$ satisfies the following inverse-elasticity rule for sharing:

$$\frac{r-c}{r} = \frac{1}{\varepsilon} + \frac{\tau D^*}{r\Omega^*}\left(\frac{\hat{\varepsilon}}{\varepsilon}\right),$$

where $\varepsilon \equiv -r\Omega_r^*/\Omega^*$ is the own-price elasticity of the equilibrium demand for ownership $(\Omega^*(r,\tau) \equiv \Omega(p^*(r,\tau), r, \tau))$ and $\hat{\varepsilon} \equiv rD_r^*/D^*$ is the cross-price elasticity of the equilibrium demand in the sharing market $(D^*(r,\tau) \equiv D(p^*(r,\tau), r, \tau))$ with respect to the retail price.[9]

Because an active peer-to-peer market requires the sharing price p to be below the retail price, the sharing tariff (which cannot exceed p) is also less than the retail price. In addition, the transaction volume in the sharing market is bounded from above by the number of goods owned in the economy (which is given by the equilibrium demand for ownership in our model). Hence, the factor $(\tau/r)(D^*/\Omega^*) < 1$, limiting the influence of the cross-price elasticity $(\hat{\varepsilon})$ of the sharing demand. The latter becomes irrelevant for $\tau = 0$ when the optimality condition specializes to the standard monopoly pricing rule for sharing markets (see note 8). Finally, we note that in any given period, the sales revenue must always exceed the firm's income from aftermarket sharing-control rents, since $r\Omega \geq \tau D$.

Substituting the retail price $r^*(\tau)$, the firm's equilibrium profit depends only on τ and needs to be maximized on the interval $[0, \bar{\tau}]$, where the upper bound for the sharing tariff is determined (implicitly) by the fixed-point problem:

$$\bar{\tau} = p^*(r^*(\bar{\tau}), \bar{\tau}).$$

Necessary and sufficient optimality conditions for globally maximizing any continuously differentiable function on an interval (here $[0, \bar{\tau}]$) have recently been developed by Weber [35].

Theorem 4. The monopolist's optimal product offering (r^{**}, τ^{**}) is such that $r^{**} = r^*(\tau^*)$, and τ^{**} solves:

$$\Pi^{**} \equiv \max_{\tau \in [0,\bar{\tau}]} \{(r^*(\tau) - c)\Omega^*(r^*(\tau), \tau) + \tau D^*(r^*(\tau), \tau)\}.$$

The preceding result implies the equilibrium price on the (by construction active) sharing market:

$$p^{**} = p^*(r^{**}, \tau^{**}) \in [p_0, p_1].$$

For sufficiently small production cost it has recently been shown that without sharing tariff, that is, for $\tau = 0$ and $c \geq 0$ small, the firm prefers no sharing to sharing [33], even when it has control over a product's durability [25]. Accordingly, we examine next the possibility of deliberate sabotage of the sharing market by the monopolist's leveraging its control of shareability.

Sharing Shutdown

In the event the sharing tariff is too high (so it would exceed the equilibrium sharing price), the sharing market becomes illiquid and breaks down. The peer-to-peer market is therefore not accessible, and consumers need to make "isolated" consumption decisions. This situation reverts to the case without sharing examined in [33]. One can backward-induct ownership decisions over the agents' life cycles and derive the ownership demand $\hat{\Omega}_i$ in $\mathcal{C}_i$ for $i \in \{0, 1\}$:

$$\hat{\Omega}_0 = \frac{(r-\delta)^2 - 1}{2\delta^2} - \left(\frac{\ln(r) - \ln(1+\delta)}{\delta}\right)\frac{r}{\delta},$$

and

$$\hat{\Omega}_1 = \max\left\{0, \frac{1-r}{6}\right\}.$$

The total demand for ownership in the absence of a sharing market is $\hat{\Omega} = \hat{\Omega}_0 + \hat{\Omega}_1$, for any given retail price $r \in (0, 1+\delta)$. Correspondingly, the firm's no-sharing profit is:

$$\hat{\Pi} = (r - c)\hat{\Omega}.$$

For sufficiently small cost, $c \in [0, 1/2]$, the optimal retail price without sharing can be obtained in closed form,

$$\hat{r}^* = \frac{1}{3\rho} + \frac{c}{2},$$

resulting in the optimal no-sharing profit:

$$\hat{\Pi}^* = \frac{(2 - 3\rho c)^2}{36\rho},$$

where $\rho \equiv \frac{1}{6} + \frac{1}{\delta}\left(1 - \frac{\ln(1+\delta)}{\delta}\right) \in [(7/6) - \ln(2), 2/3]$ is a decreasing function of $\delta \in (0, 1]$. This implies that the firm's no-sharing profits increase in the consumers' level of patience δ.

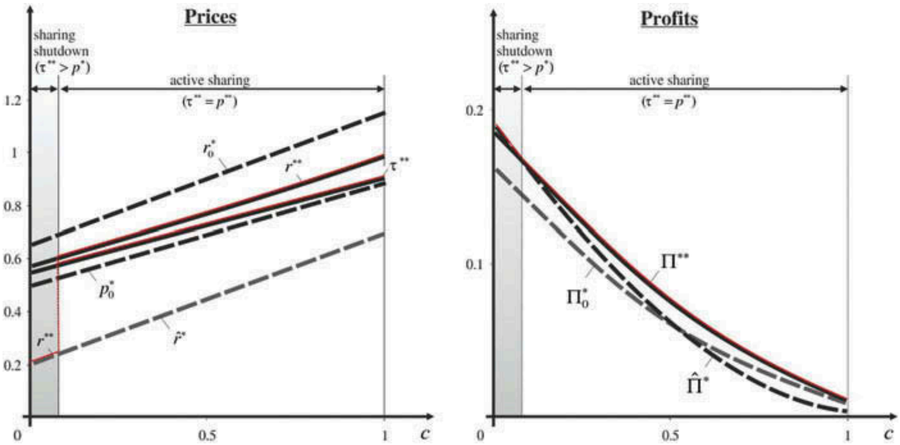

Figure 4. Equilibrium Prices and Profits ($\delta = 0.3$)

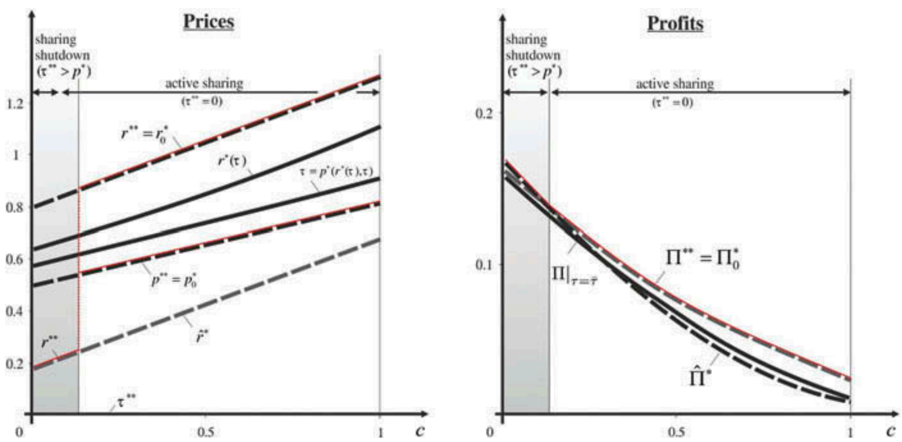

Figure 5. Equilibrium Prices and Profits ($\delta = 0.6$)

Implementation

With an active peer-to-peer market, the firm has two sources of revenue: the sales of the product and the income from authorizing sharing transactions. Figures 4 and 5 show the optimal profits and prices for $\delta = 0.3$ and $\delta = 0.6$, where $\Pi_0^* \equiv \Pi(r_0^*, 0)$ denotes the firm's optimal profits conditional on a *zero* sharing tariff ($\tau = 0$), that is, without any sharing-control technology. In that case, the optimal retail price is $r_0^* \equiv r^*(0)$ and the equilibrium sharing price is $p_0^* \equiv p^*(r_0^*, 0)$. On the other hand, a *positive* sharing tariff ($\tau \in (0, \bar{\tau}]$) does improve the monopolist's profit for impatient (low-δ) customers over the technology-free profit Π_0^* and the no-sharing profit $\hat{\Pi}$. In this situation, the extra revenue from the secondary market provides a positive incentive for the firm to support collaborative consumption of its products. By contrast, for impatient (high-δ) customers and high-cost products the firm is best off with sharing but without a sharing tariff. For low-cost products, $\tau \in (0, \bar{\tau}]$ is still not optimal, but the firm can use an excessive sharing tariff to trigger a sharing shutdown, using its control over the shareability of its products to disable the peer-to-peer secondary market.

Investing in Product Intelligence for Sharing

Given the potential for extracting rents at the point of collaborative consumption, the question arises how much a company should be willing to invest to make its products "sharing-smart." Based on our model we can provide an upper bound for an investment, but this view is narrow because it neglects the manifold of other possibilities to interact with intelligent assets [18], once the connectivity and other intelligence-enabling factors have been added to a product [21]. The more modes of beneficial interaction with a smart product, for example, related to its maintenance and the various contexts of its use, the more revenue streams it offers to the originating company, provided the firm can retain sufficient control.

Limiting ourselves to the revenue stream from a smart product's control over its shareability, the monopolist has the option (using the sharing tariff τ) to extract contingent rents from low-need owners or else to disable the sharing market altogether. As shown in Figure 4, the benefits from extracting "shareability rents" can be significant, possibly justifying an investment in proprietary technology that enables aftermarket control.[10] The firm would want to invest in technologies that ensure its control over the owner's sharing activities. Thus, to seek the extra rent a monopolist would want to make a one-time investment I of (at most) the net present value of the perpetuity of the excess profit. On the other hand, any per-period cost γ (e.g., monitoring, accounting, and administration) related to gaining and maintaining shareability control would need to be compared to the per-period benefit, so that in total:

$$I + \frac{\gamma}{1-\delta} \leq \frac{\Pi^{**} - \Pi_0^*}{1-\delta},$$

for the applicable discount factor $\delta \in (0, 1)$.[11] In order to fully appropriate the shareability rent the firm may need to run its own secondary sharing market. Yet, because of the critical mass already attained by existing sharing platforms, it may need to limit its take so as to accommodate the commissions charged by third-party intermediaries and still leave sufficient incentives for owners on the table to share their products.

Conclusion

Controlling the shareability of sufficiently smart products allows a company to align its pricing instruments more closely with consumer needs, as they arise. Depending on its production cost and the nature of consumers, it can be optimal to omit a sharing tariff ($\tau = 0$), or else to use $\tau > 0$ in order to *either* extract a shareability rent from peer-to-peer transactions ($\tau = p$) *or* to shut down the sharing market altogether ($\tau > p$), removing owners' economic incentive to offer their products to peers. A sharing shutdown transposes the manufacturer to a situation without sharing that tends to maximize profits for low-cost products and sufficiently patient (nonmyopic) customers. Indeed, without sharing, the product price is lower and demand for ownership is relatively high.

Using smart products and charging a positive sharing tariff is optimal for relatively impatient customers. In that case, the manufacturer should ideally charge a sharing tariff $\tau = p$ that extracts all rent from the aftermarket without shutting it down altogether. Because this option relies, in our theoretical model, on the willingness of owners to share in a low-need state even when obtaining a zero absolute markup, the recommendation is not exactly robust with respect to even small misperceptions or perturbations of the model. The conclusion in this regime is more of an indication that a positive sharing tariff—while maintaining a liquid sharing market—can be profitable for the firm (see Figure 4), despite the fact that the primary effect of the positive sharing tariff is to reduce demand for ownership and decrease the retail price.[12] Indeed, its secondary effect is to increase the sharing price narrowing the gap between retail price r (i.e., the price on the primary market) and the sharing price p (i.e., the price on the secondary market). Finally, for high-cost products and sufficiently patient customers it is interesting to note that the firm's optimal sharing tariff τ is zero (see Figure 5): even though it has full control over the shareability of its products it is in its own best interest to not exercise it at all.

Naturally, a question arises as to whether smart products are likely to find sufficient acceptance by consumers, given the well-known unresolved concerns and issues in the context of the Internet of things [16, 19, 28]. Ultimately, this may be an empirical issue [20], but from a purely economic viewpoint, a company needs to offer enough tangible benefits to offset agents' concerns that may arise as a consequence for the company's retention of control and the information it obtains from their products' networked sensing abilities. This amounts to a higher-resolution version of the ownership concept, which instead of all usage rights, only allocates some of them to a buyer, which is then exactly what is paid for. The multioptionality when designing the product together with its intelligence (not necessarily limited to the context of sharing) will then allow for a finer adjustment of the level of benefits and the rents different consumers pay for different versions of smart products, opening up an area for future research.

We note that a sharing tariff may not be the only method to align consumption flows with payment flows. Another option the firm may have at its disposal is to lease the product to the consumer instead of selling it outright.[13] Because the firm's rental option tends to conflict with the sharing market, the coexistence of a leasing option and a selling option with sharing markets may critically depend on consumer preferences as shown in a model without consumption-value heterogeneity by Razeghian and Weber [26]. An analysis of selling versus leasing versus smart products in a general setting promises to be an interesting topic for further investigation.

Acknowledgments: The author would like to thank Eric Clemons, Rajiv Dewan, Robert Kauffman, Maryam Razeghian, and participants of the 2017 Hawaii International Conference on System Sciences (HICSS) for helpful comments and suggestions.

NOTES

1. For instance, to restrict the use of a power drill to the original buyer, a manufacturer (or seller) could—if money is of no consideration—require biometric recognition (e.g., via fingerprint or voice) to operate the machine.
2. The difference $\varepsilon \equiv P(\tilde{s}_1 = 1|s_0 = 1) - P(\tilde{s}_1 = 1|s_0 = 0)$ increases from –1 to 1 when the serial correlation of an agent's need realizations across the consumption phases $\mathcal{C}_0$ and $\mathcal{C}_1$ varies from from –1 to 1. Allowing for heterogeneity across agents with respect to ε would be an interesting extension of the model. Here we assume independence, so $P(\tilde{s}_1 = 1|s_0) = P(\tilde{s}_1 = 1) = \theta$ for $s_0 \in \{0, 1\}$ and $\varepsilon = \theta - \theta = 0$ for all $\theta \in [0, 1]$.
3. A situation where $p \geq r$ implies that the sharing market is inactive.
4. There is no constraint preventing agents from purchasing in any consumption phase, but it would be irrational for consumers to acquire ownership in their late consumption phase when the sharing market is active because then access to a good can be obtained at a lesser price by renting it from others.
5. For $\delta \to 0^+$ or $\tau \to 0^+$, one obtains that $\Omega_{10}^A \to 0^+$.
6. In practice, there may be supply–demand imbalances [24, 31].
7. For "low" sharing prices (with $p<p_0$), the demand for ownership vanishes, thus also disabling any peer-to-peer aftermarket.
8. For $\tau = 0$, the optimal retail price is $r_0^* = (1 + \delta + c)/2$, resulting in the firm's optimal profit without sharing tariff, $\Pi_0^* = (1 + \delta - c)^2/(8(1 + \delta))$; see [33].
9. The subscripts denote partial derivatives.
10. As usual with "rent seeking," the expected benefit from sharing control justifies spending some (or even all) of it in order to attain it [30], resulting possibly in the dissipation of the entire shareability rent.
11. For $\delta = 1$, that is, when there is no discounting, the firm would want to spend whatever it takes ($I \leq \infty$) to gain control of the shareability of its products, as long as it yields a positive per-period benefit (net of the per-period cost γ). To keep notation simple, we assume here that the firm uses the same discount factor as the agents, which does not have to be the case; it is possible to simply substitute the firm's discount factor for this portion of the analysis.
12. In practice, a sharing tariff strictly below the sharing price (leaving a "robustness margin") may help avoid an inadvertent sharing shutdown. As suggested in the preceding section, the firm may want to operate its own secondary sharing market to avoid sharing shutdown.
13. In the literature, a discussion of selling vs. leasing is usually connected to product durability as a choice variable [8, 25, 29].
14. Alternatively, for a type-(θ, v) agent to purchase, $r \leq \delta[\hat{p} + \theta \max\{0, \tau + \min\{0, v - p\}\}]$ needs to hold, contradicting $r>p$.
15. Since $p = r - \delta\hat{p}$ does imply that $p - p_0 = \delta(p_0 - \hat{p})$ and $p - \hat{p} = (1 + \delta)(p_0 - \hat{p})$, one obtains the demand $\Omega_{10}^B = \frac{p - p_0}{2} - \frac{p_0 - \hat{p}}{\delta}\left(1 + \frac{\delta}{2} - \frac{1+\delta}{\delta}\ln(1 + \delta)\right)$.
16. Further analytical details (e.g., on obtaining S_{10}^B), are provided in the proof of Lemma 3.

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Appendix A: Proofs

Proof of Lemma 1. Consider the agents' decisions in their late consumption phase ($\mathcal{C}_1$), for nonowners and owners, respectively.

Nonowners. As pointed out in the main text, with an *active* sharing market, the retail price must *strictly* exceed the sharing price, so $r>p$. As a consequence, borrowing dominates buying in the late consumption phase. Specifically, a nonowner borrows if and only if the net payoff $vs_1 - p$ exceeds the (zero) payoff from doing nothing, resulting in the optimal state-dependent payoff:

$$U_{s_1} = \max\{0, vs_1 - p\}, \;\; s_1 \in \{0, 1\}.$$

That is, all types $(\theta, v) \in \mathcal{Q}$ with $v \in [p, 1]$ borrow in state $s_1 = 1$; otherwise, in state $s_1 = 0$, nonowners do nothing.

Owners. An owner has the option to keep the item for his own use or else pay the sharing tariff τ to the firm to lend it out at the price $p \in [\tau, r)$ for a net revenue of $\hat{p} = p - \tau \in [0, r - \tau)$, resulting in the optimal state-dependent payoff:

$$V_{s_1} = \max\{vs_1, \hat{p}\}, \;\; s_1 \in \{0, 1\}.$$

All types $(\theta, v) \in \mathcal{Q}$ with $v \in [0, \hat{p})$ therefore lend in state $s_1 = 1$, and all types with $v \in (0, 1]$ lend in state $s_1 = 0$; otherwise, owners take no action.□

Proof of Lemma 2. Consider the agents' consumption decisions in the early consumption phase ($\mathcal{C}_0$). By comparing the expected total payoffs $\bar{T}_{s_0}^{non-owner}$ and $\bar{T}_{s_0}^{owner}$ we first show that in the low-need state ($s_0 = 0$) no agent would ever purchase the product, and then we examine agents' purchasing behavior in the high-need state ($s_0 = 1$).

Low-need state ($s_0 = 0$). Consider individuals with contingent consumption values $v \geq p$ (and therefore also $v \geq \hat{p}$). Of those, agents with likelihood type

$$\theta \geq \frac{r - \delta\hat{p}}{\delta\tau} \equiv \vartheta_{01}$$

would purchase the product. For contingent consumption values $v \in (\hat{p}, p)$*, agents* with likelihood type

$$\theta \geq \frac{r - \delta\hat{p}}{\delta(v - \hat{p})} \equiv \vartheta_{00}$$

would like to buy the item. Lastly, no agent type with $v \leq \hat{p}$ would ever buy the item because they could not benefit from ownership more than by accessing the product as a borrower through the sharing market. For either threshold, ϑ_{00} or ϑ_{01}, to not exceed 1, that is, for *some* types to be willing to buy the product, its retail price r would have to be less than $\delta \min\{p, v\}$. But the latter is not possible, since $r>p$ is required for a functioning sharing market.[14] This implies that no agent in the low-need state would ever purchase the product.

High-need state ($s_0 = 1$). Consider first "high-value" agents with values $v \geq p$. Acquisition of the product is then interesting for the likelihood types

$$\theta \geq \max\left\{0, \frac{r - p - \delta\hat{p}}{\delta\tau}\right\} \equiv \vartheta_{11}.$$

Hence, there are *some* buyers (i.e., $\vartheta_{11} \leq 1$) if and only if the sharing price is bounded from below:

$$p \geq \frac{r}{1 + \delta} \equiv p_0.$$

Conversely, *all* high-value agents buy (i.e., $\vartheta_{11} = 0$) if and only if:

$$p \geq \frac{r + \delta\tau}{1 + \delta} \equiv p_1.$$

Consider now “medium-value” agents with values between the lower price bound p_0 and the clearing price p on the sharing market, so $v \in (\hat{p}, p)$. To them purchasing the item is attractive if and only if:

$$\theta \geq \max\left\{0, \frac{r - v - \delta\hat{p}}{\delta(v - \hat{p})}\right\} \equiv \vartheta_{10}.$$

Some of the medium-value agents purchase (i.e., $\vartheta_{10} \leq 1$), as long as $p > p_0$, while *all* likelihood types are interested in ownership (i.e., $\vartheta_{10} = 0$) if and only if $v \geq r - \delta\hat{p}$, which also requires that $p \geq p_1$. We note that “low-value” agents with values $v < \hat{p}$ would never want to purchase the item because it would imply that $p > p_1 + \tau/(1 + \delta)$. The fact that the sharing price is “moderate,” that is, $p \leq p_1$, is established in Lemma 3. Finally, combining the preceding analyses for high-value and medium-value consumers it is:

$$\theta_0(p, v) \equiv \frac{\max\{0, r - \min\{v, p\} - \delta\hat{p}\}}{\delta(\tau + \min\{0, v - p\})} = \begin{cases} \vartheta_{11}, & \text{if} v \geq p, \\ \vartheta_{10}, & \text{if} v \in (\hat{p}, p), \end{cases}$$

which completes the proof.□

Proof of Theorem 1. Using the definitions of ϑ_{10} and ϑ_{11} in the proof of Lemma 2, the high-value agents’ (with $v \geq p$) aggregate demand for ownership is:

$$\Omega_{11}^{A} = \begin{cases} 0, & \text{if} p < p_0, \\ (1 - p)\left(1 - \vartheta_{11}^{2}\right)/2, & \text{otherwise.} \end{cases}$$

The expression for Ω_{11}^{A}, for $p \in [p_0, p_1]$, follows from the fact that then:

$$\vartheta_{11} = \frac{(1 + \delta)(p_1 - p)}{\delta\tau}.$$

On the other hand, the medium-value agents’ (with $v \in (\hat{p}, p)$) *aggregate* demand for ownership is:

$$\Omega_{10} = \begin{cases} 0, & \text{if} p \leq p_0, \\ \Omega_{10}^{A}, & \text{if} p_0 < p < p_1, \\ \Omega_{10}^{B}, & \text{if} p \geq p_1, \end{cases}$$

where we set:

$$\Omega_{10}^{A} \equiv \int_{p_0}^{p} \left(\int_{\vartheta_{10}}^{1} \theta d\theta \right) dv = \int_{p_0}^{p} \frac{1 - \vartheta_{10}^{2}}{2} dv$$

and

$$\Omega_{10}^{B} \equiv \frac{p - (r - \delta\hat{p})}{2} + \int_{p_0}^{r - \delta\hat{p}} \frac{1 - \vartheta_{10}^{2}}{2} dv = \frac{p - p_0}{2} - \frac{1}{2}\int_{p_0}^{r - \delta\hat{p}} \vartheta_{10}^{2} dv.$$

Taking into account that

$$\int_{p_0}^{p} \vartheta_{10}^2 dv = \frac{p_0 - \hat{p}}{\delta^2}\left(\frac{p - p_0}{p_0 - \hat{p}} + (1+\delta)\left[2\ln\left(\frac{p_0 - \hat{p}}{p - \hat{p}}\right) + (1+\delta)\frac{p - p_0}{p - \hat{p}}\right]\right),$$

the expressions for Ω_{10}^A and Ω_{10}^B can be computed explicitly.[15]□

Proof of Lemma 3. To establish that the sharing price p needs to lie in the interval $[p_0, p_1]$, assume that $p > p_1$, which implies, by virtue of $\Omega_{10} = \Omega_{10}^B$, the demand for ownership:

$$\Omega = \frac{1}{2}\left(1 - (1-p)\vartheta_{11}^2 - p_0 - \beta(p_0 - \hat{p})\right),$$

with the constant coefficient

$$\beta \equiv \frac{2}{\delta}\left(1 + \frac{\delta}{2} - \frac{1+\delta}{\delta}\ln(1+\delta)\right) \in (0, 3 - 4\ln(2)).$$

The sharing supply by medium-value owners is:

$$S_{10}^B = \frac{p - p_0}{6} - \int_{p_0}^{r-\delta\hat{p}} \left(\frac{\vartheta_{10}^2}{2} - \frac{\vartheta_{10}^3}{3}\right) dv.$$

To compute the relevant integrals, it is convenient to set:

$$\Lambda \equiv \frac{r - (1+\delta)\hat{p}}{v - \hat{p}} = (1+\delta)\left(\frac{p_0 - \hat{p}}{v - \hat{p}}\right),$$

whence (omitting the constants in indefinite integrals):

$$\int \frac{\vartheta_{10}^2}{2} dv = \frac{v}{2\delta^2} - \frac{v - \hat{p}}{\delta^2}\left(\frac{\Lambda^2}{2} + \Lambda \ln(v - \hat{p})\right),$$

and

$$\int \frac{\vartheta_{10}^3}{3} dv = -\frac{v}{3\delta^3} + \frac{v - \hat{p}}{\delta^3}\left(\Lambda^2 - \frac{\Lambda^3}{6} + \Lambda \ln(v - \hat{p})\right).$$

Note that $\Lambda|_{v=r-\delta\hat{p}} = 1$ and $\Lambda|_{v=p_0} = 1 + \delta$. The two preceding integrals, taken between the bounds of p_0 and $r - \delta\hat{p}$, evaluate to:

$$\int_{p_0}^{r-\delta\hat{p}} \frac{\vartheta_{10}^2}{2} dv = \frac{p_0 - \hat{p}}{\delta}\left[1 + \frac{\delta}{2} - (1+\delta)\frac{\ln(1+\delta)}{\delta}\right]$$

and

$$\int_{p_0}^{r-\delta\hat{p}} \frac{\vartheta_{10}^3}{3} dv = \frac{p_0 - \hat{p}}{\delta^2}\left[-1 - \frac{\delta}{2} + \frac{\delta^2}{6} + (1+\delta)\frac{\ln(1+\delta)}{\delta}\right],$$

respectively. We therefore obtain:

$$S_{10}^B = \alpha(p - p_0) - \left(\alpha - \frac{1}{6}\right)\tau = \frac{\tau}{6} - \alpha(p_0 - \hat{p}),$$

for $p \geq p_1$, where

$$\alpha \equiv \frac{1+\delta}{\delta^2}\left[1 + \frac{\delta}{2} - (1+\delta)\frac{\ln(1+\delta)}{\delta}\right].$$

It is $1/6 < \alpha \leq 3 - 4\ln(2) \approx 0.2274$ for $\delta \in (0,1]$. Combining this with the earlier findings yields the sharing supply:

$$S = \frac{1-p_0}{6} - \hat{\alpha}(p_0 - \hat{p}) = \frac{1}{6} - \frac{\alpha r}{1+\delta} + \hat{\alpha}(p - \tau),$$

where $\hat{\alpha} \equiv \alpha - (1/6) > 0$ and $p \geq p_1$. The supply in the sharing market is decreasing in the retail price and the sharing tariff, and it is increasing in the sharing price. On the other side of the market, the demand in a high-price regime stems exclusively from agents with need-state realizations $s_0 = 0$ and $s_1 = 1$, so:

$$D = \int_p^1 \left(\int_0^1 (1-\theta)\theta d\theta\right) dv = \frac{1-p}{6}.$$

Equating supply and demand ($S = D$) implies a linear dependence of the market price on retail price and sharing tariff:

$$p^*(r,\tau) = \frac{r}{1+\delta} + \left(\frac{\hat{\alpha}}{\alpha}\right)\tau = p_0 + \left(\frac{\hat{\alpha}}{\alpha}\right)\tau,$$

where $\hat{\alpha} \equiv \alpha - (1/6) > 0$. However, the assumption $p^*(r,\tau) > p_1$ implies that $(\hat{\alpha}/\alpha) > \delta/(1+\delta)$, which does not hold for any $\delta \in (0,1]$. Thus, by contradiction of the counterfactual we obtain that the clearing price of an active sharing market must be "moderate" in the sense that it lies in the interval $[p_0, p_1]$.□

Proof of Lemma 4. Aggregating owners with $v \in [p, 1]$ yields the supply:

$$S_{11}^A = \int_p^1 \left(\int_{\vartheta_{11}}^1 (1-\theta)\theta d\theta\right) dv = (1-p)\left(\frac{1}{6} - \frac{\vartheta_{11}^2}{2} + \frac{\vartheta_{11}^3}{3}\right),$$

while owners with $v \in [p_0, p]$ constitute a supply of:

$$S_{10} = \int_{p_0}^p \left(\int_{\vartheta_{10}}^1 (1-\theta)\theta d\theta\right) dv = \begin{cases} 0, & \text{if} p \leq p_0, \\ S_{10}^A, & \text{if} p_0 < p < p_1, \\ S_{10}^B, & \text{if} p \geq p_1, \end{cases}$$

where, using the abbreviation $\lambda \equiv (p_0 - \hat{p})/\tau$, [16]

$$S_{10}^A = \tfrac{p-p_0}{6} - \int_{p_0}^{p} \left(\tfrac{\vartheta_{10}^2}{2} - \tfrac{\vartheta_{10}^3}{3}\right) dv$$
$$= \tfrac{(1-\lambda)\tau}{6}\left(1 - \tfrac{3}{\delta^2} - \tfrac{2}{\delta^3}\right) - \tfrac{(1+\delta)^2\lambda\tau}{\delta^3}\left(\ln(\lambda) + (1-\lambda)\tfrac{5+2\delta-(1+\delta)\lambda}{6}\right).$$

This yields the sharing supply, $S = S_{10} + S_{11}$, for any combination of retail price and sharing tariff that allow for an *active* (i.e., positive-trading-volume) peer-to-peer exchange in the secondary market. At the moderate sharing price $p \in [p_0, p_1]$, the supply from medium-value owners is:

$$S_{10}^A = \frac{p - p_0}{6} - \int_{p_0}^{p} \left(\frac{\vartheta_{10}^2}{2} - \frac{\vartheta_{10}^3}{3}\right) dv.$$

Using the expressions for the indefinite integrals in the proof of Lemma 3, between the bounds p_0 and p, one obtains:

$$\int_{p_0}^{p} \tfrac{\vartheta_{10}^2}{2} dv = \tfrac{p-p_0}{\delta^2}\left[\tfrac{1}{2} + \tfrac{(1+\delta)^2}{2}\left(\tfrac{p_0-\hat{p}}{\tau}\right) + (1+\delta)\left(\tfrac{p_0-\hat{p}}{p-p_0}\right)\ln\left(\tfrac{p_0-\hat{p}}{\tau}\right)\right]$$
$$= \tfrac{\tau}{\delta^2}\left[\tfrac{1-\lambda}{2} + (1+\delta)\lambda\ln(\lambda) + \tfrac{(1+\delta)^2}{2}\lambda(1-\lambda)\right],$$

and

$$\int_{p_0}^{p} \frac{\vartheta_{10}^3}{3} dv = \frac{\tau}{\delta^3}\left[-\frac{1-\lambda}{3} - (1+\delta)\lambda\ln(\lambda) - (1+\delta)^2\lambda(1-\lambda)\left(1 - \frac{1+\delta}{6}(1+\lambda)\right)\right],$$

where we recall that $\lambda = (p_0 - \hat{p})/\tau$. This implies the expression for the supply S_{10}^A.□

Proof of Lemma 5. Consider a "moderate" sharing price $p \in [p_0, p_1]$, so $\theta_0(p, v) = \vartheta_{11}$ as noted in the proof of Lemma 2. The sharing demand from consumers in their late consumption period amounts to:

$$D_1 = (1-p)\left(\frac{1}{2} - \frac{1}{3} + \frac{\vartheta_{11}^3}{3}\right),$$

while the sharing demand from agents in their early consumption phase is:

$$D_0 = (1-p)\frac{\vartheta_{11}^2}{2}.$$

Hence, the sharing demand at any given time $t \geq 0$ is:

$$D = D_0 + D_1 = (1-p)\left(\frac{1}{6} + \frac{\vartheta_{11}^2}{2} + \frac{\vartheta_{11}^3}{3}\right),$$

which concludes the proof, since $\vartheta_{11} = (1+\delta)(p_1 - p)/(\delta\tau)$.□

Proof of Theorem 2. The market clears if and only if the sharing supply S equals the sharing demand D. By Lemma 4 and Lemma 5 this is equivalent to:

$$S_{10}^{A} = (1-p)\left(\frac{(1+\delta)(p_1-p)}{\delta\tau}\right)^2 = (1-p)\vartheta_{11}^2.$$

Since, by its definition in the proof of Lemma 2, the type threshold ϑ_{11} is:

$$\vartheta_{11} = 1 - \frac{(1+\delta)(1-\lambda)}{\delta} = \frac{(1+\delta)\lambda - 1}{\delta},$$

the sharing price becomes a function of λ and τ:

$$p = 1 - \left(\frac{\delta}{(1+\delta)\lambda - 1}\right)^2 S_{10}^{A} \equiv h(\lambda, \tau).$$

From this we can determine $\lambda = \varphi(r, \tau)$ by solving a fixed-point problem (using the definition of λ):

$$\lambda = 1 - \frac{1}{\tau}(h(\lambda, \tau) - p_0) \equiv \varphi(r, \tau);$$

this finally yields the equilibrium sharing price p^* as a function of r and τ.□

Proof of Theorem 3. For any admissible sharing tariff τ (which does not exceed the equilibrium price in the sharing market), an optimal retail price $r^*(\tau)$ maximizes the firm's profit $\Pi = (r-c)\Omega + \tau D$. The corresponding first-order necessary optimality condition,

$$\Omega^* + (r-c)\Omega_r^* + \tau D_r^* = 0,$$

is equivalent to the (generalized) inverse-elasticity rule provided in Theorem 3.□

Proof of Theorem 4. The upper bound $\bar{\tau}$ for the sharing tariff is such that τ cannot exceed the sharing price in equilibrium. Thus, the monopolist's optimal sharing tariff is found by maximizing the profit $\Pi(r^*(\tau), \tau)$ with respect to $\tau \in [0, \bar{\tau}]$, where $r^*(\tau)$ can be found using the (generalized) inverse-elasticity rule in Theorem 3.□

Appendix B: Notation

Table B1 Summary of Notation

Symbol	*Description*	*Domain/Definition*
c	Marginal production cost	$\mathbb{R}_+$
$\mathcal{C}_0$ / $\mathcal{C}_1$	Early / late consumption phase	–
D	Sharing demand	$[0, 2]$
$\mathcal{G}_t$	Consumer generation born at time *t*	–
i	Index for early / late consumption phase	$\{0, 1\}$
I	Technology investment	$\mathbb{R}_+$
p	Sharing price	$\mathbb{R}_+$
$\hat{p}$	Effective transaction price in the sharing market	$\hat{p} = p - \tau$
p_0 / p_1	Lower / upper bound for ("moderate") sharing price	$p_0 = \frac{r}{1+\delta} = p_1 - \frac{\tau}{1+\delta}$
$\mathcal{Q}$	Type space: contains all consumer types (θ, v)	$[0, 1] \times [0, 1]$
r	Retail price	$\mathbb{R}_+$
S	Sharing supply	$[0, 2]$
s_0 / s_1	Need state in early / late consumption phase	$\{0, 1\}$
t	Time	$\{0, 1, \ldots\}$
$\bar{T}_{s_0}$	Total expected payoff conditional on s_0	$\bar{T}_{s_0} = \max\left\{\bar{T}_{s_0}^{non-owner}, \bar{T}_{s_0}^{owner}\right\}$
U_{s_1} / V_{s_1}	Payoff in $\mathcal{C}_1$ conditional on s_1 for non-owners / owners	$\mathbb{R}_+$
$\bar{U}$ / $\bar{V}$	Expected payoff in $\mathcal{C}_1$ for non-owners / owners	$\mathbb{R}_+$
γ	Per-period monitoring and implementation cost	$\mathbb{R}_+$
δ	Per-period discount factor (for consumers and firm)	$(0, 1]$
θ	Likelihood of need	$[0, 1]$
λ	Lower effective price-gap-sharing-tariff ratio (τ>0)	$\lambda = (p_0 - \hat{p})/\tau$
v	Consumption value (contingent on high need)	$[0, 1]$
Π	Firm's profit	$\Pi = (r - c)\Omega + \tau D$
τ	Sharing tariff	$[0, p]$
Ω / $\hat{\Omega}$	Demand for ownership with / without sharing market	$[0, 2]$