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Real-Time Control of an Ensemble of Heterogeneous Resources*

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***Abstract*— This paper focuses on the problem of controlling an ensemble of heterogeneous resources connected to an electrical grid at the same point of common coupling (PCC). The controller receives an aggregate power setpoint for the ensemble in real time and tracks this setpoint by issuing individual optimal setpoints to the resources. The resources can have continuous or discrete nature (e.g., heating systems consisting of a finite number of heaters that each can be either switched on or off) and/or can be highly uncertain (e.g., photovoltaic (PV) systems or residential loads). A naïve approach would lead to a stochastic mixed-integer optimization problem to be solved at the controller at each time step, which might be infeasible in real time. Instead, we allow the controller to solve a continuous convex optimization problem and compensate for the errors at the resource level by using a variant of the well-known error diffusion algorithm. We give conditions guaranteeing that our algorithm tracks the power setpoint at the PCC on average while issuing optimal setpoints to individual resources. We illustrate the approach numerically by controlling a collection of batteries, PV systems, and discrete loads.**

I. Introduction

The problem of aggregating and disaggregating heterogeneous resources connected to a portion of an electrical grid has recently become a major research interest because of high penetration of dispersed energy resources (DERs) at the distribution level, and it is closely related to the concept of a virtual power plant. Aggregation entails representing the collection of resources as a single resource to the upper-level grid; disaggregation amounts to dispatching individual power setpoints to the different resources such that the power flow at the point of common coupling (PCC) equals the aggregate power setpoint dispatched from the higher-level grid.

In the literature, one encounters two basic approaches to solve this problem. The first is a model-based approach, where typically a limited number of resource models are defined (each modeling a family of homogeneous resources), and the parameters of those models are estimated off-line. Examples of this line of research include [1]–[8]. The other approach is non-parametric; the behavior of the devices is learned online, from the interaction between the devices and the aggregator, e.g., [9], [10].

Most works focus on aggregation processes for electricity balancing markets in the context of demand-side management and demand response, i.e., aggregation time scales varying from several hours to a few minutes (e.g., [2], [4], [11]–[13]). The aggregation and disaggregation of heterogeneous resources on the second or subsecond timescale was recently proposed in [14]–[16]. Note, however, that the latter papers do not give theoretical guarantees on the tracking properties of the proposed controller. Finally, a disaggregation method in the context of real-time control using an online and decentralized optimization algorithm was proposed in [17]; however the methodology can be applied only to deterministic resources with feasible sets that are convex.

In this paper, we consider the problem of controlling an ensemble of heterogeneous resources connected to an electrical grid at the same PCC, having continuous or discrete nature (e.g., heating systems consisting of a finite number of heaters that each can be either switched on or off) and/or are highly uncertain (e.g., photovoltaic (PV) systems or residential loads). We focus on the problem of *real-time disaggregation*, and we propose a controller design that allows tracking the aggregate power setpoint for the ensemble in real time while issuing individual optimal setpoints to the resources. The design is based on formulating a convex deterministic optimization problem for computing the power setpoints while compensating for the errors at the resource level by using a variant of the well-known error-diffusion algorithm [18]–[20]. We provide conditions guaranteeing that our algorithm tracks the aggregate power setpoint at the PCC on average. Finally, we complement the theoretical analysis with an illustrative simulation.

II. Problem Formulation

Consider a generic distribution network and a given bus of that network. The bus can represent, e.g., the primary side of a distribution transformer. Assume that there are N resources connected at this bus, and suppose that we can aggregate the power injections of these resources as a sum of the individual power injections. For brevity, in this paper, we focus only on controlling the *active* power injections, and we ignore the network structure behind the considered bus; however, our approach can be easily extended to joint control of the active and reactive power (see Remark 3 below) and to explicit consideration of the network topology connecting the N resources as in [14], [15], [17].

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Let $k = 1, 2, \cdots$ denote the discrete time-step index and $i \in \{1, \cdots, N\} := \mathcal{I}$ denote the resource index. At each time step k, resource i is characterized by its performance cost function, $C_i^{(k)} : \mathbb{R} \to \mathbb{R}$, and the set of feasible power setpoints of that resource, $\mathcal{Y}_i^{(k)} \subseteq \mathbb{R}$. We find this representation useful as it conveniently decomposes the *shape* of the function (given by $C_i^{(k)}$) from its *domain of definition* (given by $\mathcal{Y}_i^{(k)}$). This decomposition stems from the following practical considerations:

- The shape of the function typically represents soft and slowly time-varying constraints of the resource, such as the state-of-charge of the battery or the temperature of a room.
- The domain of definition typically represents hard and fast time-varying constraints, such as the maximum power we can get from a PV or the switching frequency constraint of an ON-OFF device.

We make the following assumption.

Assumption 1. For each i and k, the function $C_i^{(k)} : \mathbb{R} \to \mathbb{R}$ is convex, and the set $\mathcal{Y}_i^{(k)}$ is compact.

Note that we are not assuming that the feasible sets $\mathcal{Y}_i^{(k)}$ are convex. Moreover, in Section V, we will explicitly consider certain nonconvex sets, such as finite discrete sets.

We assume that the aggregator knows the collection[1] $\{C_i^{(k)}\}_{i=1}^N$ at each time step. The collection $\{\mathcal{Y}_i^{(k)}\}_{i=1}^N$ can vary significantly with time, and in particular those variations might be on a shorter timescale than the typical communication and computation delays. Hence, we assume that the aggregator has access only to a *delayed version* of the feasibility sets, namely to $\{\mathcal{Y}_i^{(k-1)}\}_{i=1}^N$. Both collections can be communicated to the aggregator by the resources as in, e.g., [14]. Finally, the aggregator also receives the aggregate power setpoint $P_{\text{PCC}}^{(k)}$ from a higher-level optimization and control algorithm.

In this setting, the aggregator has two tasks to perform:

(i) **Aggregation:** Represent the collection of resources as a single resource to the higher-level control algorithm.

(ii) **Disaggregation:** Compute the vector of power setpoints $\mathbf{p}^{(k)} := (P_i^{(k)})_{i=1}^N$ based on the available information with the following twofold goal: (i) optimize the individual costs $C_i^{(k)}$; and (ii) keep the setpoint at the PCC as close as possible to $P_{\text{PCC}}^{(k)}$, namely minimize $\| \sum_{i=1}^N P_i^{(k)} - P_{\text{PCC}}^{(k)} \|$. The individual power setpoints $P_i^{(k)}$ are then communicated to the resources that implement feasible setpoints $\widehat{P}_i^{(k)} \in \mathcal{Y}_i^{(k)}$.

The aggregation task typically amounts to computing an approximate feasible set for the entire collection by approximating the Minkowski sum $\mathcal{Y}_{\text{PCC}}^{(k)} = \sum_{i=1}^N \mathcal{Y}_i^{(k)}$ and by computing an approximation to the aggregated cost function; see, e.g., [14], [16]. In this paper, we focus on the disaggregation task, and we consider controllers of the form

$$\mathbf{p}^{(k)} = f^{(k)} \left(\{C_i^{(k)}\}_{i=1}^N, \{\mathcal{Y}_i^{(k-1)}\}_{i=1}^N, P_{\text{PCC}}^{(k)} \right) \tag{1a}$$

$$\widehat{P}_i^{(k)} = g_i^{(k)} \left(P_i^{(k)}, \mathcal{Y}_i^{(k)} \right), \quad i = 1, \cdots, N, \tag{1b}$$

where (1a) represents the control function of the aggregator, whereas (1b) represents the implementation method employed at each resource. In the following two sections, we propose the design of the functions $f^{(k)}$ and $g_i^{(k)}$.

III. Aggregator Design

In this section, we propose an aggregator design that is based on formulating the corresponding optimization problem to compute the setpoints. We first consider the prototypical ("ideal") optimization problem:

$$\text{(P0)} \quad \min_{\mathbf{p}^{(k)}} \ \sum_{i=1}^N C_i^{(k)}(P_i^{(k)}) \tag{2a}$$

$$\text{subject to}$$

$$P_i^{(k)} \in \mathcal{Y}_i^{(k)}, \ i = 1, \cdots, N \tag{2b}$$

$$\sum_{i=1}^N P_i^{(k)} = P_{\text{PCC}}^{(k)}. \tag{2c}$$

However, (P0) is practically infeasible for our application because of the following reasons. First, note that the set $\mathcal{Y}_i^{(k)}$ in (2b) is not yet available at time step k. Second, the sets $\{\mathcal{Y}_i^{(k)}\}$ might be nonconvex or even discrete, thus leading to a nonconvex (or mixed-integer) optimization problem that is hard to solve in real time. Finally, the constraints (2b)-(2c) might not be feasible at every time step k.

To tackle these difficulties, we propose using a convex (and always feasible) version of (P0) in the aggregator and compensating for the introduced errors at the resource level by designing an appropriate function $g_i^{(k)}$ (see Section IV for details). In particular, we make the following approximations:

- We replace the feasible sets in (2b) by the *convex hull* of the *delayed versions thereof*, $\mathsf{ch}\mathcal{Y}_i^{(k-1)}$.
- We relax the constraint (2c) by using $P_{\text{PCC}}^{(k)} - \epsilon \leq \sum_{i=1}^N P_i^{(k)} \leq P_{\text{PCC}}^{(k)} + \epsilon$, and we penalize the value of ϵ in the objective function.

To summarize, the following convex optimization problem is solved at the aggregator at each time step k:

$$\text{(P1)} \quad \min_{\mathbf{p}^{(k)}, \epsilon} \ \sum_{i=1}^N C_i^{(k)}(P_i^{(k)}) + \mu\epsilon \tag{3a}$$

$$\text{subject to}$$

$$P_i^{(k)} \in \mathsf{ch}\mathcal{Y}_i^{(k-1)}, \ i = 1, \cdots, N \tag{3b}$$

$$P_{\text{PCC}}^{(k)} - \epsilon \leq \sum_{i=1}^N P_i^{(k)} \leq P_{\text{PCC}}^{(k)} + \epsilon, \tag{3c}$$

$$\epsilon \geq 0 \tag{3d}$$

where $\mu > 0$ is a weight parameter that influences the choice of the size of the tracking deviation ϵ. The larger μ is, the

[1] In this paper, we consider the centralized aggregation problem for brevity. Alternatively, decentralized schemes can be considered, as in [17], which do not require knowledge of $\{C_i^{(k)}\}_{i=1}^N$ and $\{\mathcal{Y}_i^{(k)}\}_{i=1}^N$ at the aggregator (see Remark 2 below).

more accurate the tracking will be, possibly on the expense of the optimality of the individual DERs.

Remark 1. Observe that (P1) is a time-varying optimization problem. Therefore, instead of solving (P1) exactly at each time step, one can use online optimization methods to track the solution of (P1) in real time, as in [14], [17], [21]. The development and analysis of such online methods in the context of aggregation is a subject of an ongoing work.

Remark 2. As mentioned, the centralized approach proposed here requires two-way communication between the resources and the aggregator. However, the convex optimization problem (P1) can be solved by using distributed optimization methods, such as the primal-dual decomposition method, as in [17], or the alternating direction method of multipliers (ADMM) as in, e.g., [22]. In such cases, only one-way communication is required, and the information that is passed to each resource is embedded in the Lagrange multiplier related to the coupling constraint (3c).

IV. Setpoint Implementation: The Error-Diffusion Algorithm

In this section, we propose a design of the control functions $g_i^{(k)}$ in (1) that compensates for the errors introduced by the aggregator (P1). As the main goal of the aggregator is to track the power setpoint at the PCC, we propose here a method that *provably* tracks a given setpoint *on average*. Tracking on average makes sense in our application because the average power setpoint at a given time interval represents the energy produced/consumed during this interval.

Our approach is based on an error-feedback technique that is known as *error diffusion* in image processing (and a specific variant is called Floyd–Steinberg dithering [19]). A similar technique is used in signal processing for sigma-delta modulation [18], [20] and some stability properties of this technique have been analyzed in that context [23], [24].

To start, we first introduce the concept of *total accumulated error*. Let $e_i^{(0)} := 0$, and, for $k = 1, 2, ...$, define

$$e_i^{(k)} := \sum_{\ell=1}^{k} (\widehat{P}_i^{(\ell)} - P_i^{(\ell)}) = e_i^{(k-1)} + \widehat{P}_i^{(k)} - P_i^{(k)}. \quad (4)$$

Recall that the power setpoint $P_i^{(k)}$ lies in $\mathsf{ch}\mathcal{Y}_i^{(k-1)}$; cf. (P1). We implement this setpoint using the following rule:

$$\widehat{P}_i^{(k)} = g_i^{(k)}(P_i^{(k)}, \mathcal{Y}_i^{(k)}) := \mathsf{Proj}_{\mathcal{Y}_i^{(k)}} \left(P_i^{(k)} - e_i^{(k-1)} \right), \quad (5)$$

where $\mathsf{Proj}_{\mathcal{Y}}(\cdot)$ is the *closest-point operator*, namely $\mathsf{Proj}_{\mathcal{Y}}(x) \in \arg\min_{y \in \mathcal{Y}} |x - y|$ and the ties can be broken arbitrarily.

We next analyze the tracking properties of (5). To that end, we introduce the following definitions. First, let

$$\mathcal{D}_i := \mathsf{ch} \left(\bigcup_{k=0}^{\infty} \mathsf{ch}\mathcal{Y}_i^{(k)} \right) \quad (6)$$

denote the minimal convex set (interval) that contains the sequence $\{\mathcal{Y}_i^{(k)}\}_{k=1}^{\infty}$. Also, let

$$\Delta_i^{(k)} = \max_{x \in \mathsf{ch}\mathcal{Y}_i^{(k)}} \left| \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(x) - x \right| \quad (7)$$

denote the maximum approximation error introduced by using the convex hull of the feasible set, and define

$$\Delta_i^{\max} := \sup_{k \geq 1} \Delta_i^{(k)}. \quad (8)$$

Finally, let

$$\mathcal{S}_i := \mathcal{D}_i + [-\Delta_i^{\max}, \Delta_i^{\max}] \quad (9)$$

denote the inflation of $\mathcal{D}_i$ by a ball with radius $\Delta_i^{\max}$ centered at the origin; $+$ denotes the Minkowski sum.

Theorem 1. *Suppose that the set $\mathcal{S}_i$ is compact. Then, under algorithm* (5)*, the total accumulated error is bounded for all k. Specifically,*

$$\left| e_i^{(k)} \right| \leq \mathsf{diam}\mathcal{S}_i, \; k = 1, 2, \cdots \quad (10)$$

where $\mathsf{diam}\mathcal{S}$ is the diameter of a set $\mathcal{S}$. Consequently, the average tracking error converges to zero:

$$\lim_{k \to \infty} \left| \frac{1}{k} \sum_{\ell=1}^{k} \widehat{P}_i^{(\ell)} - \frac{1}{k} \sum_{\ell=1}^{k} P_i^{(\ell)} \right| = 0 \quad (11)$$

Corollary 1. *Suppose that for every resource $i = 1, \cdots, N$, the sets $\mathcal{S}_i$ are compact. Let*

$$\widehat{P}_{PCC}^{(k)} = \sum_{i=1}^{N} \widehat{P}_i^{(k)}$$

denote the implemented setpoint at the PCC at time step k. Then, the proposed disaggregation method defined by (P1) and (5) *guarantees*

$$\limsup_{k \to \infty} \left| \frac{1}{k} \sum_{\ell=1}^{k} \widehat{P}_{PCC}^{(\ell)} - \frac{1}{k} \sum_{\ell=1}^{k} P_{PCC}^{(\ell)} \right| \leq \limsup_{k \to \infty} \frac{1}{k} \sum_{\ell=1}^{k} \epsilon^{(\ell)}, \quad (12)$$

where $\epsilon^{(\ell)}$ is the optimal value of the tracking deviation ϵ in (P1) at time step ℓ.

To prove the above results, we need the following auxiliary results. We assume that $\mathcal{D}_i$ and $\mathcal{S}_i$ are compact throughout.

Lemma 1. *For any $x \in \mathsf{ch}\mathcal{Y}_i^{(k)}$ and $y \in \mathcal{S}_i$, we have that*

$$x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) \in \mathcal{S}_i.$$

The proof of Lemma 1 follows by straightforward verification and appears in the appendix.

Lemma 2. *For all i and k, we have that $P_i^{(k)} - e_i^{(k-1)} \in \mathcal{S}_i$.*

Proof. Let $i \in \{1, \cdots, N\}$. Define $x^{(k)} := P_i^{(k)} - e_i^{(k-1)}$. We prove by induction on k that $x^{(k)} \in \mathcal{S}_i$ for all k. First, trivially, $x^{(1)} = P_i^{(1)} \in \mathsf{ch}\mathcal{Y}_i^{(0)} \subseteq \mathcal{D}_i \subseteq \mathcal{S}_i$. Assume that $x^{(k)} \in \mathcal{S}_i$. We have that

$$\begin{aligned} x^{(k+1)} &:= P_i^{(k+1)} - e_i^{(k)} \\ &= P_i^{(k+1)} - e_i^{(k-1)} - \widehat{P}_i^{(k)} + P_i^{(k)} \\ &= P_i^{(k+1)} + x^{(k)} - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}} \left(x^{(k)} \right), \end{aligned} \quad (13)$$

where the second equality follows by the error recursion (4), and the last equality follows by (5). Note that in (13),

$P_i^{(k+1)} \in \mathsf{ch}\mathcal{Y}_i^{(k)}$ (see (3b)) and $x^{(k)} \in \mathcal{S}_i$ by the induction assumption. Therefore, by Lemma 1, $x^{(k+1)} \in \mathcal{S}_i$ as well, which completes the proof. □

We are now ready to prove Theorem 1 and Corollary 1.

Proof of Theorem 1. Note that $e_i^{(k)} = \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}\left(x^{(k)}\right) - x^{(k)}$, where $x^{(k)} := P_i^{(k)} - e_i^{(k-1)} \in \mathcal{S}_i$ for all k by Lemma 2. Now, as $\mathsf{Proj}_{\mathcal{Y}_i^{(k)}}\left(x^{(k)}\right) \in \mathcal{Y}_i^{(k)} \subseteq \mathcal{S}_i$, we have that $|e_i^{(k)}| \leq \mathsf{diam}\mathcal{S}_i$ for all k as required. Finally, (11) follows by the definition of $e_i^{(k)}$ in (4). In particular,

$$\left|\frac{1}{k}\sum_{\ell=1}^{k}\widehat{P}_i^{(\ell)} - \frac{1}{k}\sum_{\ell=1}^{k}P_i^{(\ell)}\right| = \frac{|e_i^{(k)}|}{k} \leq \frac{\mathsf{diam}\mathcal{S}_i}{k}.$$

This completes the proof of the theorem. □

Proof of Corollary 1. The proof follows by using Theorem 1 for individual resources and applying triangle inequality. □

Remark 3. Compared to the existing literature on error diffusion, we analyzed the general nonconvex and delayed case, hence our results are new in this context. Moreover, our focus on the one-dimensional error-diffusion algorithm is for brevity only. Indeed, the approach can be extended to a multidimensional case using methods similar to [25], [26] (see more details in our extended paper on generalized error diffusion for control applications [27]).

V. Application Example

In this section, we illustrate numerically the proposed aggregation method. To this end, consider a collection of three prototypical resources[2]: (i) a PV system; (ii) a heating, ventilating, and air-conditioning (HVAC) system; and (iii) a battery. These three types of devices cover most modern DERs. Indeed, the PV system represents a volatile renewable power generator, the HVAC system represents a nonconvex (discrete) controllable load, and the battery represents a bidirectional energy-storage resource. This collection will allow us to illustrate how different exogenous variables (such as solar irradiance variation and the switching frequency constraint of the HVAC system) influence the decision making of the aggregator and its ability to track the aggregate power setpoint. Note that we consider here a small collection of heterogeneous resources as it is the most interesting case for real-time aggregation. Indeed, increasing the number of resources of the same type makes the aggregation smoother and hence tracking easier.

For a PV system with an available active power $P_{i,\mathrm{av}}^{(k)}$ at time step k, the feasible set is given by the interval $\mathcal{Y}_i^{(k)} = [0, P_{i,\mathrm{av}}^{(k)}]$. We assume that the PV system always wants to maximize its power production, hence the cost function is given by $C_i^{(k)}(P) = -c_{\mathrm{PV}}P$ for some $c_{\mathrm{PV}} > 0$, and it is independent of k.

For an HVAC system, we assume that it can be in M discrete states, depending on the settings of the system (motor speed, heating setting, etc). When at state $m \in \{1, \cdots, M\}$, it consumes $P_{i,m}$ active power, with $0 := P_{i,1} < P_{i,2} < \cdots < P_{i,M} := P_{i,\max}$. Moreover, the system can be "locked" in one of the states due to, e.g, switching frequency constraint or because the user manually locked the system. Let $\ell_i^{(k)} \in \{0,1\}$ denote a binary state variable that equals 1 if the system is locked at time step k. In the simulation experiments below, we lock the system after each change of the setpoint for a predefined number of time steps, $T_{i,\mathrm{lock}}$. The feasible set is then given by

$$\mathcal{Y}_i^{(k)} = \begin{cases}\{-P_{i,M}, -P_{i,M-1}, \cdots, -P_{i,1}\}, & \text{if } \ell_i^{(k)} = 0,\\ \{-\widehat{P}_i^{(k-1)}\}, & \text{if } \ell_i^{(k)} = 1.\end{cases}$$

Now, the cost of being in one of the M states is system-dependent and reflects, for example, the current temperature and its distance from the desired setting. In the simulation experiments, we fixed the cost as a quadratic function that is minimized at a given optimal operating point $P_{i,\mathrm{opt}}^{(k)}$, that is $C_i^{(k)}(P) = c_{\mathrm{HVAC}}(P - P_{i,\mathrm{opt}}^{(k)})^2$ for some $c_{\mathrm{HVAC}} > 0$.

Finally, consider a battery with state-of-charge at time step k given by $\mathrm{SoC}_i^{(k)} \in [0,1]$. Let $P_{i,\min}^{(k)}$ and $P_{i,\max}^{(k)}$ denote, respectively, the lower and upper bounds on the active power production. These are time-varying quantities that depend on operating conditions of the battery, such as $\mathrm{SoC}_i^{(k)}$ and the DC voltage; see, e.g., [15]. Then, the feasible set is given by $\mathcal{Y}_i^{(k)} = [P_{i,\min}^{(k)}, P_{i,\max}^{(k)}]$. The associated cost function can be designed based on the current state-of-charge and the desired value for the state-of-charge, SoC_i^* (e.g., $\mathrm{SoC}_i^* = 0.5$). In our experiments, we use

$$C_i^{(k)}(P) = \begin{cases} c_{\mathrm{B}}(P - P_{i,\min}^{(k)})^2, & \text{if } \mathrm{SoC}_i^{(k)} \geq \mathrm{SoC}_i^*,\\ c_{\mathrm{B}}(P - P_{i,\max}^{(k)})^2, & \text{otherwise}\end{cases}$$

for some $c_{\mathrm{B}} > 0$.

For the purpose of simulation, we consider a time step in the *subsecond* order. As shown in [14], [17], the available power from a PV system, $P_{i,\mathrm{av}}^{(k)}$, might vary significantly within one second, hence we consider here an extreme scenario where $P_{i,\mathrm{av}}^{(k)}$ is uniformly distributed between 0 and 30 kW, which is the rated power of the PV system. As for the HVAC system, we assume a rated power of 70 kW, discretization step of 10 kW, and locking time $T_{i,\mathrm{lock}} = 5$ time steps. Finally, the battery's rated power is 50 kW, and $P_{i,\max}^{(k)} = -P_{i,\min}^{(k)} = 50$ kW during the simulation run. These parameters are on the order of magnitude of realistic systems, such as those used in, e.g., [14], [17].

We consider a scenario wherein the requested power at the PCC, $P_{\mathrm{PCC}}^{(k)}$, has typical step and ramp changes. This is shown in Fig. 1 (a), labeled as "PCC req". Fig. 1 (c) shows the maximum power available from the PV system at every time step (labeled "PV max"). The battery starts with the state-of-charge below the target value, hence it is willing to consume power; at time step $k = 150$, the state-of-charge goes above the target value, and therefore the battery is willing to produce power from that time step on (labeled "Battery opt" in Fig. 1 (b)). Finally, the optimal operating

[2]For simulation experiments on a realistic microgrid benchmark, see our extended paper [27].

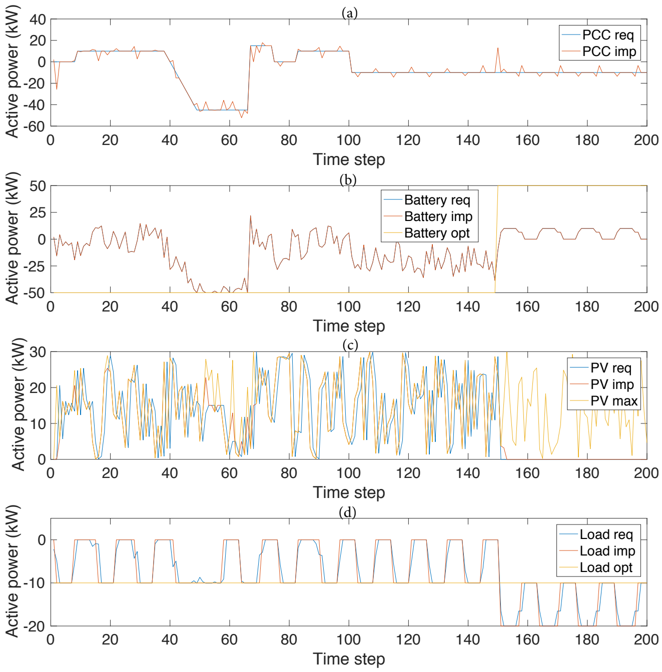

Fig. 1: Results with error-diffusion: instantaneous values. (a) Power setpoint at the PCC (“PCC req”) and the implemented power at the PCC (“PCC imp”). (b) Requested and implemented power of the battery (“Battery req”, “Battery imp”) and the optimal power based on state-of-charge (“Battery opt”). (c) Requested and implemented power of the PV (“PV req”, “PV imp”) and the maximum available power from the PV (“PV max”). (d) Requested and implemented power of the HVAC system (“Load req”, “Load imp”) and the optimal power of the HVAC (“Load opt”).

point of the HVAC system is at $P_{i,\text{opt}}^{(k)} = -10$ kW, as shown in Fig. 1 (d) (labeled “Load opt”).

Fig. 1 (a) shows that the proposed method is able to accurately track the requested power at the PCC even in the face of a very uncertain profile for the PV system and the discrete nature of the HVAC system. Moreover, the resources jointly contribute to the tracking task while operating in their preferred region most of the time. Particularly, the battery is mostly consuming power before $k = 150$, and is producing power after that; the PV system maximizes its power production before $k = 150$, whereas it is curtailed after that because of a higher cost incurred by the battery that needs to be discharged; finally, the HVAC system is close to its optimal value on average, but it also contributes to the requested changes in the PCC power, and it helps the battery to charge itself after $k = 150$ by increasing its power consumption. The corresponding average values are shown in Fig. 2 and corroborate numerically the results of Theorem 1 and Corollary 1.

For the purpose of comparison, we also ran the same scenario wherein instead of using the error-diffusion algorithm (5), a simple projection is used at the resource level to produce a feasible setpoint, that is: $\widehat{P}_i^{(k)} = \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}\left(P_i^{(k)}\right)$. The results are shown in Figs. 3 and 4. Observe that the tracking capability of the method is significantly worse, and the average requested and implemented setpoints do not converge to each other. Moreover, the PV system is not fully

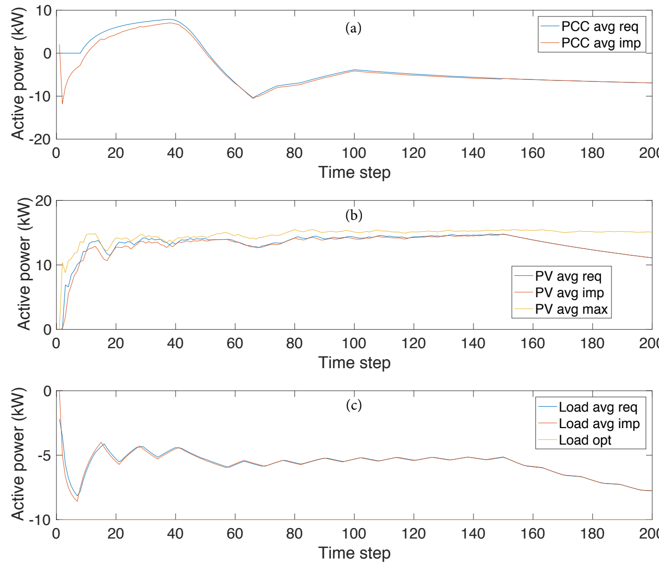

Fig. 2: Results with error-diffusion: average values. (a) Time-average of the power setpoint at the PCC and the implemented power at the PCC. (b) Time-average of the requested and implemented power of the PV and the maximum available power from the PV. (c) Time-average of the requested and implemented power of the HVAC system. The legend is similar to that of Fig. 1.

utilized, as shown in particular in Fig. 4 (b).

VI. Conclusion

This paper proposed a simple method to control a collection of heterogeneous resources in real time by dispatching approximate optimal setpoints to the resources that they track on average. The method makes it possible to accurately track the power setpoint at the PCC to the main grid. Several directions for future work include: (i) employing online and distributed optimization methods in the aggregator; (ii) extending the methods and results to the multidimensional setpoint control (active and reactive power, and multiphase buses); and (iii) extending the methodology to a network-cognizant case by explicitly considering the power-grid structure interconnecting the N resources.

Appendix

Proof of Lemma 1. Denote $\mathcal{D}_i := [a, b]$ and $\mathsf{ch}\mathcal{Y}_i^{(k)} := [c, d]$ with $a \leq c \leq d \leq b$. Then, $\mathcal{S}_i = [a - \Delta_i^{\max}, b + \Delta_i^{\max}]$; cf. (9). Assume first that $y \notin \mathsf{ch}\mathcal{Y}_i^{(k)}$. If $y > d$, we have that $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) \leq d + b + \Delta_i^{\max} - d = b + \Delta_i^{\max}$ and $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) = x + y - d \geq x \geq a - \Delta_i^{\max}$. Similarly, if $y < c$, we have $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) = x + y - c \leq x \leq b + \Delta_i^{\max}$ and $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) \geq c + a - \Delta_i^{\max} - c = a - \Delta_i^{\max}$.

Now, suppose that $y \in \mathsf{ch}\mathcal{Y}_i^{(k)}$. Clearly, $|y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y)| \leq \Delta_i^{\max}$; cf. (8). Hence $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) \leq x + \Delta_i^{\max} \leq b + \Delta_i^{\max}$ and $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) \geq x - \Delta_i^{\max} \geq a - \Delta_i^{\max}$. Therefore, $x + y - \mathsf{Proj}_{\mathcal{Y}_i^{(k)}}(y) \in \mathcal{S}_i$ as required. □

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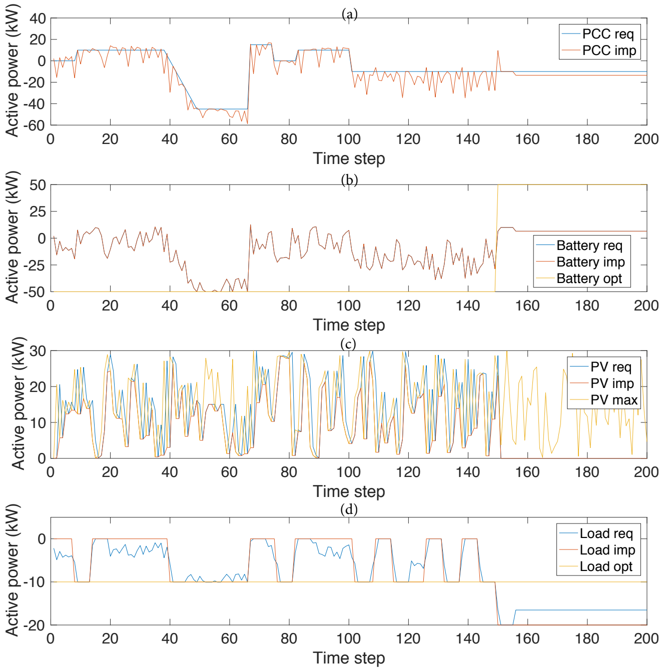

Fig. 3: Results without error-diffusion: instantaneous values. The legend interpretation is similar to that of Fig. 1.

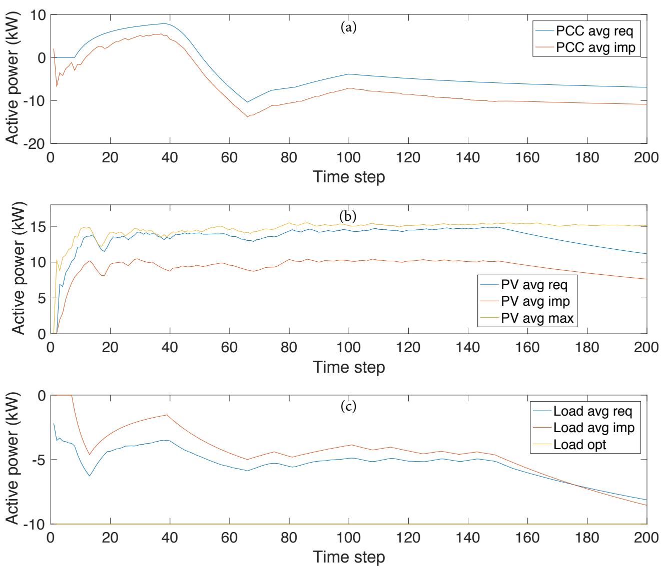

Fig. 4: Results without error-diffusion: average values. The legend interpretation is similar to that of Fig. 1.

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