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To my parents, Nasrin & Parviz
To my husband, Manu
To my sons, Nathan & Liam
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Abstract

This thesis examines the optimal mode of financing for banks and financial institutions.

The first chapter, which is a joint work with Prof. Jean-Charles Rochet, investigates how Systemically Important Financial Institutions (SIFIs) should be financed. The main specificity of such firms is that their failure imposes negative externalities on the financial system and, more broadly, on the whole economy. Since their shareholders do not internalize these externalities, they tend to provide less capital than what would be socially optimal, generating too many failures. In jurisdictions where they receive tax exemptions, CoCo bonds are a simple means of increasing the resilience of SIFIs while providing large tax benefits to banks’ shareholders. However, we show that when tax revenues are properly accounted for, these CoCo bonds are detrimental to social welfare.

The second chapter provides a formal model of a bail-in plan, a pre-defined contract that results in self-recapitalization of a financial institution when it is in distress and has no access to equity financing. Bail-ins have the potential to eliminate inefficient bank liquidations and ease financing constraints in bad times. Using a theoretical model of a bail-in contract where banks face time varying financing frictions, taxation and liquidation costs, I show that banks’ capital structure decisions and optimal financing and pay-out policies are largely affected by the design of the contingent capital (specifically the conversion ratio). However independent of its design, for an optimal level of debt, bail-in contracts always decrease shareholders incentives to build up liquid buffers and to recapitalize in good times but also eliminate any risk-taking incentives.

The third chapter presents a model of bank optimal maturity structure when banks face systemic risk through correlated investments. Banks can privately affect the probability of success of their projects. Risky short-term debt can act as a disciplinary device if it is not rolled over when an adverse interim-date signal on the quality of banks’ assets is received. The optimal maturity structure is the result of the trade-off between the disciplinary benefits of short-term debt and the costs of inefficient early liquidations. I show that bank asset commonality affects this trade-off since it reduces the costs of exerting effort through positive information synergies and increases the inefficiency of early liquidations through negative fire-sale externalities. In particular, there is a more important role for the disciplinary effects of short-term debt in high correlation, high fire-sale externalities and low information synergies environments.
Abstract

Key words: Bail-in, CoCo bonds, SIFI, contingent capital, cash reserves, conversion trigger, conversion ratio, short-term debt, asset correlation, roll over risk
Résumé

Cette thèse examine le mode de financement optimal des banques et institutions financières.

Le premier chapitre qui a été écrit en collaboration avec Prof. Jean-Charles Rochet étudie la manière dont les institutions financières d’importance systémique (SIFIs) devraient être financées. Ce qui caractérise principalement ces sociétés est le fait que leur faillite engendre des externalités négatives sur le système financier et, de manière générale, l’économie. Etant donné que les actionnaires n’internalisent pas ces externalités, ils fournissent moins de capital que ce qui serait socialement optimal. Ceci a pour effet d’engendrer trop de faillites. Dans les juridictions qui permettent une exonération fiscale des obligations CoCo, celles-ci sont un moyen simple pour augmenter la résistance des SIFIs tout en fournissant des réductions fiscales importantes aux actionnaires des banques. Nous montrons toutefois que lorsque les recettes fiscales sont correctement prises en considération, les obligations CoCo portent préjudice au bien-être social.

Le deuxième chapitre présente un modèle formel d’un plan de bail-in. Il s’agit d’un contrat prédéfini qui a pour effet d’auto recapitaliser une banque en difficulté financière et qui n’a pas d’accès au financement par actions. Les bail-ins ont le potentiel d’éliminer des liquidations inefficientes des institutions financières et assouplir les contraintes financières durant les périodes difficiles. En utilisant un modèle théorique d’un contrat de bail-in dans lequel les banques font face à des frictions de financement, de taxation et de coûts de liquidation variables dans le temps, je montre que la décision de la structure de capital, du financement optimal et de la politique de redistribution des banques est principalement affectée par les termes du capital conditionnel (plus spécifiquement le ratio de conversion). Cependant et peu importe les termes, les contrats bail-in diminuent toujours les incitations des actionnaires de constituer des réserves de liquidités et de recapitaliser durant les bonnes périodes mais éliminent aussi toutes incitations à prendre du risque.

Le troisième chapitre présente un modèle de structure d’échéance de la dette pour les banques lorsque celles-ci font face à des risques systémiques par le biais d’investissements corrélés. Les banques peuvent de manière privée influencer la probabilité de succès de leurs projets. Lorsqu’un signal négatif sur la qualité des actifs de la banque est perçu à la date intermédiaire, des dettes à court terme risquées qui ne sont pas renouvelées peut agir comme un dispositif disciplinaire. La structure d’échéance optimale est le résultat d’un compromis entre les bénéfices disciplinaires de la dette à court terme et les coûts d’inefficience liés à des liquidations.
Abstract

 anticipées. La corrélation entre les actifs a des effets sur ce compromis car elle réduit les coûts des efforts des banquiers par le biais de synergies d’information et augmente le niveau d’inefficacité des liquidations anticipées à cause des externalités négatives des ventes en urgence. En particulier, les effets disciplinaires des dettes à court terme sont plus importants dans les environnements de corrélation élevée, d’externalités de ventes en urgence élevées et de synergies d’information basses.

Mots clefs : Bail-in, obligations CoCo, SIFI, capital conditionnel, réserve de liquidités, ratio de conversion, dette à court terme, corrélation d’actifs, risque de refinancement
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Introduction

The main focus of this thesis is the privately and socially optimal capital structure and policy choices of banks in a systemic and interconnected financial system. The financial crisis of 2007-2009 demonstrated that the distress of a bank or financial institution can threaten the overall stability of the financial system. Failure of a Systemically Important Financial Institution ("SIFI") has significant disruptive effects on other financial firms and on the financial system as a whole. SIFI's shareholders do not internalize the social costs associated with these negative externalities. Because of this, the private costs of bank's failure are significantly lower than its social costs and thus the privately optimal bank leverage can be higher than what would have been socially optimal. This generates too many bank failures. Consequently, the composition of bank capital structure and possible solutions to deal with the too-big-to-fail problem have been the subject of many discussions by policy makers and academics. These discussions have motivated the proposals for contingent capital "CoCo" financing and "bail-in" requirements for SIFIs.

CoCo bonds, which are the focus of Chapter 1, are defined as bonds that convert into equity or sustain automatic write-downs when some pre-defined threshold is reached. Such hybrid securities have gained increasing attention in the aftermath of the recent financial crisis since they correspond to a debt contract in good times whilst their conversion to equity provides an automatic recapitalization when banks face financial distress. In Chapter 1, I show that with tax deductions on debt, CoCo bonds are good both for shareholders and regulators because they reduce taxes while increasing the resilience of SIFIs due to their superior loss absorption capacity. However, social welfare which includes tax revenues is always lower with any form of market debt and equity financing is the socially optimal mode of financing for SIFIs. This main result is not affected by different design features of CoCo contract including conversion ratio (which determines burden sharing between shareholders and debt holders in the event of conversion), and conversion threshold (which determines the probability of conversion or conversion risk).

In Chapter 1, I discuss how SIFIs should be subject to capital requirements because their failure leads to significant social costs that are not internalized by their shareholders. The objective of bank's regulation is thus to decrease the probability of bank's failure or equivalently government bail-outs. In Chapter 2, I study a "bail-in" plan as an alternative solution to deal with the social costs of SIFI's failure. A bail-in plan is defined as a pre-determined contract that
results in an automatic recapitalization of the bank when it is in distress and has no access to equity financing. My objective in Chapter 2 is twofold. First by using the debt to equity conversion in the context of a restructuring contract that is set ex-ante, I entirely eliminate the risk of bank’s failure and its associated social costs without any equity support from the government. Second, I study bank’s optimal capital structure decisions and optimal pay-out and financing policies in the presence of such a plan. I show that although bank’s policy choices and capital structure decisions are largely affected by the design of the bail-in contract, for an optimal level of debt, bail-in contracts always decrease shareholders incentives to build up liquid buffers and to recapitalize in good times. So bail-in plans can potentially lead to a less capitalized, more levered banking system; however, they can create value from both private and social points of view by eliminating both the costs of liquidation and the costs of negative externalities associated with SIFIs’ failures.

Having discussed the choice of bank leverage in Chapters 1, and 2, Chapter 3 focuses on the choice of bank maturity structure. In current financial markets banks and other financial institutions extensively use short-term debt to fund their long-term assets. The maturity mismatch between bank’s assets and liabilities leads to roll over risk. This roll over risk can cause banks’ failures and lead to financial crises. The collapse of the asset-backed commercial paper market and the role it played in the current financial crisis, have highlighted the need to better understand the optimal maturity structure of banks. To this end, a substantial body of literature investigates the optimality of short-term debt as a disciplinary device that can help align the incentives of shareholders and debt holders. When investors are not willing to roll over their debt if they receive an interim adverse news on the quality of bank’s assets, short-term debt financing bears roll over risk. In a model where bankers can privately affect the probability of success of their projects by exerting costly effort, risky short-term debt acts as a disciplinary device. Short-term financing can become the optimal mode of financing if the disciplinary benefits of short-term debt overcome the costs associated with its roll over risk and inefficient early liquidations.

As the experience of the recent financial crisis shows, banks hold correlated assets. This asset commonality can become a source of systemic risk when one bank’s failure affect the other banks in the financial system. A representative bank framework that studies the optimal bank maturity structure assumes the disciplining effect of short-term debt at an individual level and excludes the effects of asset commonality and the systemic risks it exposes banks to. Recognizing this short-coming, Chapter 3 develops a set-up with multiple banks that are subject to negative and positive externalities because they invest in correlated assets. Bank asset commonality reduces the costs of exerting effort through positive information synergies and increases the inefficiency of early liquidations through negative fire-sale externalities. In Chapter 3, I seek to understand how the trade-off between the costs and benefits of short-term financing is affected by bank asset commonality. To do so, I study the optimal level of effort exerted by each banker and the social welfare as functions of the correlation between banks’ investments in both long-term debt and short-term debt cases. I show that short-term debt can discipline bankers to exert more effort, however whether or not this higher effort leads to
a higher social surplus depends on the level of the correlation between banks’ assets and the level of externalities this correlation leads to. In particular, there is a more important role for the disciplinary effects of short-term debt in high correlation, high fire-sale externalities and low information synergies environments.
1 How to Finance SIFIs?

*with Jean-Charles Rochet*

1.1 Introduction

The debate on how Systemically Important Financial Institutions (SIFIs) should finance themselves has become very polarized. On the one hand Admati and Hellwig (2014) claim that, contrarily to what bankers assert (IIF 2011), equity is not expensive and that regulatory capital requirements should be raised way above the levels contemplated by the Basel Committee.\(^1\) On the other hand, other academics (see for example DeAngelo and Stulz (2013)) argue that "high leverage is optimal for banks" and that raising capital requirements could have a large social cost because it would prevent banks from performing adequate liquidity provision to the financial system.

The objective of this paper is not to come up with quantitative recommendations on what should be the optimal level of capital for banks. Instead, it examines the qualitative question of what type of securities should be eligible as bank "capital". As such it belongs to the abundant recent literature that has tried to show why and how hybrid securities (contingent capital, CoCo bonds) can be useful.\(^2\) We use a stylized model of a SIFI, defined as a financial institution whose failure entails important costs for society, and abstract from adverse selection or risk shifting problems due to deposit insurance systems. The presence of this failure externality is the justification for regulating the financing mode of SIFIs. We study the question of the optimal way to finance SIFIs, both from private and public perspectives.

Our main results are as follows: first, in the absence of exogenous benefits of debt (such as a tax advantage, or a way to reduce adverse selection problems), any form of long-term debt

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\(^2\)This literature is discussed later on. Note, however, that it is dismissed by Admati and Hellwig (2012) as second order considerations compared with the need for a massive increase in banks’ equity.
(straight, CoCo, ...) is sub-optimal because it leads to too many defaults. Second, with tax deductions on debt, CoCo bonds are good both for shareholders and for regulators because they allow reducing taxes while decreasing the bank's probability of default, due to the superior loss absorption capacity of CoCo bonds. However social welfare (which includes tax revenues) is always lower with any form of long-term debt: equity financing of SIFIs is socially optimal. Therefore the main message of the paper is that CoCo bonds should be viewed as a bad compromise struck between regulators and bankers, at the expense of the taxpayers. Bankers only accepted to increase their loss absorbing capacity in exchange for a tax relief, i.e. another form of a Too Big to Fail subsidy. All the papers on CoCo bonds that use standard corporate finance models (which do not incorporate adverse selection or asset substitution), are potentially misleading because in such models any kind of debt financing is socially suboptimal.

Related literature. Our paper relates to the extensive research on contingent convertible bonds, their design features and their impacts on bank capital structure. First introduced by Mark Flannery in (2005) "CoCos" are defined as bonds that convert into common equity or sustain automatic write-downs once some threshold is reached. Such securities have gained increasing attention since they correspond to a debt contract in good states of the world whilst their conversion to equity provides an automatic recapitalization when banks face financial distress and their market access to equity capital is limited. Since their introduction, all versions of the CoCo proposals studied by academics and regulators have the common goal of establishing a contractual structure that increases bank capital in bad states of the economy (the bail-in feature); there are, however, numerous differences in the designs of such proposals. Specific proposals usually vary with regard to the conversion trigger (which determines the probability of conversion or conversion risk) and the conversion ratio (which determines burden sharing between shareholders and debtholders in the event of conversion). Contingent Capital valuation formulas along with its key design issues have been obtained by Pennacchi et al. (2010), Glasserman and Nouri (2010), Albul et al. (2013), Sundaresan and Wang (2015), McDonald (2010), Barucci ad Del Viva (2012, 2013) and Chen et al. (2013) among others.

McDonald (2010) prices CoCo with a dual trigger using both the bank’s stock price and a market index. In a framework based on the traditional capital structure model of Leland (1994), Albul et al. (2013) obtain closed form pricing expressions and show that if not required, shareholders of SIFIs whose straight debt is guaranteed by government will not willingly include any CoCo bonds in their capital structure since by doing so they will lose a fraction of the government subsidy. Glasserman and Nouri (2010) analyse the case of contingent capital with an accounting based capital ratio trigger and partial conversion. Using option pricing methods, Hilscher and Raviv (2014) evaluate how the introduction of CoCos into a bank’s capital structure reduces its probability of default. Pennacchi (2010) develops a structural model for contingent bank capital when bank’s assets follow a jump diffusion process with

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3The role of debt as a way to reduce adverse selection problems has been put forward in the early contributions of Myers and Majluf (1984) and Gorton and Pennacchi (1990).
1.1. Introduction

stochastic interest rate, and shows that the possibility of sudden declines in a bank's asset value has a qualitatively distinct impact on valuing contingent capitals.\(^4\) Barrucci and Del Viva (2013) show that an automatic conversion imposed by the regulatory authorities induces banks to issue less CoCo but more debt than in the case of a conversion barrier chosen by shareholders.\(^5\) So the intervention by the authorities is not necessarily positive if the goal is to minimize expected bankruptcy costs. Finally Chen et al. (2013) study the design features of CoCo's in a capital structure model with bank specific and market wide tail risk.

Another strand of literature analyses the incentives created by CoCo bonds. Chen et al. (2013) investigate how CoCos affect debt overhang, asset substitution, the firm's ability to absorb losses and the sensitivity of equity holders to various types of risk. They find that CoCos generally have positive incentive effects when the conversion trigger is not set too low. Hilscher and Raviv (2014) argue that incentives to shift risk can be eliminated when the profits and costs of conversion offset each other. Thus one can find a level of conversion at which shareholders are indifferent to the amount of risk and have no incentive to take excessive risk. Pennachi (2010) suggests that designing CoCo's as close to default free as possible instills incentives in shareholders similar to those which would occur under unlimited liability; thereby both risk-shifting incentives and debt overhang problem can be reduced when a bank issues appropriately designed CoCos. Berg and kaserer (2015) assume that asset value is only known to regulators who enforce conversion or default, and show that if a wealth transfer from CoCo bond holders to equity holders takes place at conversion, the CoCo bonds can magnify both the asset substitution as well as the debt overhang problem. Barucci and Del Viva (2012) study the effects of a counter-cyclical contingent capital on shareholders' risk taking incentives and bankruptcy costs.\(^6\) They conclude that the added counter-cyclical feature mitigates the asset substitution incentives, whilst at the same time does not help reducing the bankruptcy costs.\(^7\) Sundaresan and Wang (2015) suggest that the conversion ratio that achieves a unique equilibrium for market prices must produce no value transfer between equity holders and CoCo bond holders, and thus might not be able to generate the desired incentives for bank managers. Koziol and Lawrenz (2012) show that when contracts are incomplete in the sense that owners enjoy discretion over the risk of their investments, CoCo bonds always distort risk taking incentives, and thus can increase the bank's probability of default as well as its expected distress costs.

In an empirical study, Avdjiev et al. (2015) provide an overview of the CoCo bonds market, the issues and the participants. The paper shows that the volume of CoCo issues has been increasing since 2012 as regulatory pressure on banks to boost their Tier 1 capital has increased. The

\(^4\)Jump-diffusion processes allow for the possibility of sudden large declines in the bank asset value which can characterize a financial crisis.

\(^5\)Barrucci and Del Viva (2013) assume such conversion trigger to be an exogenously given rule that applies similarly to all banks.

\(^6\)Counter-cyclical contingent capital is defined as notes that are converted into equity in the bad state of the economy upon the decision of a regulatory authority.

\(^7\)Thus Barucci and Del Viva (2012) argue that depending on the priority of objectives (reducing bankruptcy costs or mitigating risk-shifting incentives) the counter-cyclical feature should be removed or added.
geographical distribution of CoCo issuance mainly demonstrates the way Basel III regulations are applied by national financial authorities. Whilst about 80% of the CoCo bonds have been issued by European banks, CoCo issuance is very small in the US, where CoCo bonds do not qualify for AT1 or AT2 capital.\textsuperscript{8} Moreover, by analysing the pricing of banks’ securities after CoCo issuance, Avdjiev et al. (2015) show that CoCo investors view CoCo bonds as risky investments that bear a significant possibility of conversion.

The literature so far studies the optimality of CoCo financing for shareholders or regulatory authorities. Our paper differentiates from the existing literature by including taxpayers and studying the optimality of CoCo bonds from a social point of view. We find that no matter how CoCo bonds are designed and whether or not they are optimal to include in a bank’s capital structure from a private or regulatory point of view, they are never socially optimal when their only benefits are tax subsidies. The impact of taxes on bank capital structure has been studied by Schepens (2015) among others.\textsuperscript{9} Schepens uses a natural experiment in the form of a fiscal change in Belgium in 2006 and suggests that decreasing the relative tax advantage of debt financing could be used to incentivize banks to build up their capital buffers. We build on this study by analysing the optimality of different kinds of debt when there is no tax advantage to debt like instruments. Our results show that in the absence of exogenous benefits of debt, no kind of debt is socially optimal. So the social optimal financing mode is 100% equity. If interest payments are tax deductible, CoCo bonds can be optimal for equity holders and regulators but not for social welfare.\textsuperscript{10}

The paper is organized as follows. Section 1.2 presents the model. Section 1.3 derives closed form solutions for the value of the bank from private and public points of view when there is no tax advantage to debt instruments and studies the optimal financing for SIFIs in such a framework. Section 1.4 studies the same question when there are tax subsidies to debt instruments. We compare straight debt and CoCo bonds. Section 1.5 is devoted to a numerical illustration. Section 1.6 concludes.

1.2 Model

Most of the papers on CoCos use models in the spirit of Merton (1974)/Leland (1992) where firms keep no cash reserves because they can continuously issue equity at no cost. In these

\textsuperscript{8}UK and Swiss institutions are the main issuers of CoCo bonds in Europe. Swiss banks are required to hold a minimum 9% of their risk-weighted assets in loss absorbing instruments. Similar loss-absorbing capital requirements have been adopted by the UK regulators since 2012.

\textsuperscript{9}Other papers relating taxes and bank capital structure are Admati et al. (2014) and Poole (2009).

\textsuperscript{10}In the US, in general, tax deductibility of the distributions on any financial instrument depends on whether or not the instrument is characterized as debt or equity. Under current US tax-law for an instrument to be treated as debt, it must include an unconditional promise to pay a certain sum either on demand or at a fixed maturity date. Whether or not this condition is fulfilled for any particular CoCo bond depends on the likelihood of the conversion and the sum accrued to CoCo holders upon conversion. Thus there is no reasonable certainty that the interest payments on CoCo bonds are tax-exempt in the US (see Hammer and Chen (2012)). As Avdjiev et al. (2015) shows CoCo issuance has been very small in the US. This supports our theory that in the absence of tax subsidies, CoCo bonds (or any debt-like instrument) are suboptimal.
1.2. Model

Models default only occurs for solvency reasons. However in practice, due to fixed transaction costs, banks only issue equity infrequently. Also, they typically have problems to access equity markets in distress situations. Moreover, banks default essentially for liquidity, not solvency, reasons. Given this, we believe that a Merton/Leland type model is not the appropriate set-up to represent a financial institution. We use instead the same set-up as Radner and Shepp (1996) and consider a profitable bank or financial firm that may be forced to close down because of liquidity problems.\textsuperscript{11} Fixed issuance costs prevent continuous injection of capital. In such a context, liquidity management becomes crucial.

We model a bank (or more generally a SIFI) as a firm that transforms a fixed volume $D$ of risk-free deposits into a fixed volume $A$ of risky assets.\textsuperscript{12} So the bank’s balance sheet is as follows

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Risky Assets $A$ & Deposits $D$
\hline
Cash Reserves $m$ & Long-term Finance
\hline
\end{tabular}
\end{table}

Note that in our model we isolate long-term financing from deposit financing. Unlike firms in other industries, a financial firm does not necessarily need external financing for its day-to-day activities. However the intermediation activity is risky, and the bank needs to manage its precautionary cash reserves to cover operating losses, otherwise it can face the risk of an immediate closure.

We model random cash flows generated over time by the SIFI’s operations (net earnings) as an arithmetic Brownian Motion with positive drift $\mu$ and volatility $\sigma$ defined over a complete probability space $(\Omega, \mathcal{F}, P)$. Specifically cumulated cash flows $R_t$ evolve according to

$$dR_t = \mu dt + \sigma dZ_t, \quad R_0 = 0.$$ \hfill (1.1)

The process $Z \equiv \{Z_t, t \geq 0\}$ is a standard Brownian motion with respect to the filtration $\mathcal{F}; t \geq 0$ that models the flow of information. Bank’s assets can involve operating losses $dR_t < 0$ as well as operating profits $dR_t > 0$.

All investors are risk-neutral and discount the future at rate $r$. When the bank is started, owners issue securities to finance cash reserves. We assume that there are no corporate governance problems such as moral hazard and asset substitution. However, two frictions are present in our set-up: first, issuance of securities is costly. Second, there is an opportunity cost of keeping cash in the bank; we assume internally held cash does not earn any interest.\textsuperscript{13}

\textsuperscript{11}Ideally, one would envisage a more general model incorporating both types of default and essentially nesting Merton/Leland and Radner/Shepp models.

\textsuperscript{12}The assumption that $A$ and $D$ are fixed is not crucial. It allows us to focus on the loss absorbing capacity of market finance (equity and debt-like instruments). It also allows us to focus on liquidity issues and on the loss absorbing capacity of liabilities.

\textsuperscript{13}The idea is, as in Décamps et al. (2011), that the managers of the bank can engage in wasteful activities when the bank holds cash or other liquid assets. The resulting agency costs effectively reduce the rate of return on
Chapter 1. How to Finance SIFIs?

In our model the SIFI defaults when it fails to cover its operating losses by drawing cash from its reserves or by issuing new securities. The SIFI character in our model is captured by the simple feature that its failure entails a social cost $\gamma$, that is not internalized by shareholders. The question we examine is the SIFI’s optimal security design problem: what is the best way to finance such an institution? We try to answer this question both from private and public perspectives. For the ease of exposition, we model this financing decision by a simple debt-equity choice.\footnote{Using a similar model as Radner and Shepp (1996), Jeanblanc and Shiryaev (1995) study more general forms of financing.} Debt pays a constant coupon $c$ per unit of time. The dividend policy is chosen so as to maximize the wealth of shareholders. Dividends are characterized by a non decreasing (cumulated) cash flow process $L \equiv \{L_t; t \geq 0\}$. We do not make any restrictions on $L$ apart from assuming that it is $\{\mathcal{F}_t; t \geq 0\}$-adapted and right continuous, and that it is non-decreasing, reflecting limited liability (non-negative payments to shareholders). For the sake of simplicity, the liquidation value of the SIFI is assumed to be zero.

1.3 No tax-advantage of debt

In this section we assume there is no tax-advantage to debt instruments and study the optimal financing methods for a SIFI. There is a fixed cost to issue new securities, thus new issuances are going to be lumpy and infrequent. In this section we consider the scenario where security re-issuance is so costly that it actually never happens. Under this assumption we characterize the optimal security design for both bank’s owners and regulators. The case of re-issuance is studied in the Appendix.

1.3.1 Privately optimal security design

The shareholders of the bank decide when to distribute dividends. The dividend policy is characterized by a non-decreasing (cumulated) dividend process $L_t$. At date zero, the bank also issues debt which pays a continuous coupon of $c$. We assume, in this section, that these coupon payments are not tax deductible. Initial owners maximize their wealth by choosing $c$, $L_t$, and $m_0$ (initial cash reserves) that maximize

$$E[\int_0^\tau e^{-rt} dL_t] - m_0,$$

where $\tau = \inf\{t \mid s.t. \ m_t < 0\}$ is the random default time and cash reserves $m_t$ evolve as

$$dm_t = \{[(1-\theta)\mu - c]dt + (1-\theta)\sigma dZ_t \} - dL_t.$$  \hfill (1.3)

$\theta$ is the tax rate. Since earnings are i.i.d the solution to (1.2) is Markovian with respect to the cash reserves process and can be obtained by recursive techniques. In particular, the
1.3. No tax-advantage of debt

decision to distribute dividends at date \( t \) only depends on \( m_t \). More precisely, no dividends are distributed (\( d_L = 0 \)) when \( V'(m_t) > 1 \) (the marginal value of the cash inside the firm is higher than one). The optimal policy is to distribute dividends when a target cash level \( m^* \) is reached, with \( V'(m^*) = 1 \). The total value of the bank \( V(m) \), and the target cash level \( m^* \) are the unique solution of the following ODE (ordinary differential equation) with boundary conditions

\[
\begin{align*}
  r V(m) & = c + [(1-\theta)\mu - c] V'(m) + \frac{1}{2} \sigma^2 (1-\theta)^2 V''(m) \\
  V(0) & = 0 \\
  V'(m^*) & = 1 \\
  V''(m^*) & = 0.
\end{align*}
\] (1.4)

The left hand side of (1.4) represents the return required by investors for investing \( V(m) \) (debt plus equity) in the bank. The right hand side consists of coupon payments, and the effects of cash savings and the volatility of the bank’s cash flows. Zero liquidation value and no re-issuance when cash reserves hit zero result in the first boundary condition. The second boundary condition has already been explained.

Before solving for the explicit value of the bank, we observe that the optimal coupon payment \( c \) must be zero. Indeed, since there is no tax advantage to interest payments, the only effect of such payments is to reduce the drift of the cash reserves process. This means that interest payments will increase the probability that the bank’s cash reserves hit zero and thus increase the probability of default. Since in this model we assume no other benefit to debt, the optimal amount of coupon payments to be paid out of the bank’s earnings should be zero. We prove in Appendix A.1.1 that this is indeed the case and that the optimal coupon payment \( c^* \) is zero.

Given \( c^* = 0 \), we can re-write (1.4)

\[
\begin{align*}
  r V(m) & = (1-\theta)\mu V'(m) + \frac{1}{2} \sigma^2 (1-\theta)^2 V''(m) \\
  V(0) & = 0 \\
  V'(m^*) & = 1 \\
  V''(m^*) & = 0.
\end{align*}
\] (1.5)

The additional boundary condition in (1.5) is the super contact condition that characterizes the target cash threshold that maximizes the value of equity which is here equal to the total value of the bank (see Dumas (1991)).

The closed form solution to the above ODE is

\[ V(m) = \frac{e^{z_1 m} - e^{z_2 m}}{z_1 e^{z_1 m^*} - z_2 e^{z_2 m^*}} \]

where \( z_1 > 0 > z_2 \) are the roots of the characteristic equation

\[ r = (1-\theta)\mu z + \frac{1}{2} \sigma^2 (1-\theta)^2 z^2, \] (1.6)
and the dividend pay-out threshold is given by

\[ m^* = \frac{\ln(\frac{z_2}{z_1})^2}{z_1 - z_2}. \]

Dividends are only paid when \( m > m^* \), that is when past performance has been good enough to accumulate cash reserves above \( m^* \). Since \( V'(m^*) = 1 \), \( m^* \) is also the optimal amount of cash to be invested when the bank is started. After this initial cash injection, the bank restrains from distributing any dividend until the level of its cash reserves hits again \( m^* \), and then any cash in excess of \( m^* \) is paid out. If the bank’s performance deteriorates such that its cash reserves hit zero, the bank defaults because we have assumed for the moment that re-issuance is too costly.

1.3.2 Optimal security design for regulators

In this section we study the optimal financing mode of a SIFI when regulators are in charge of making such a decision. As mentioned before, the main characteristic of the SIFI in our model is that its failure generates a social cost \( \gamma \), which is not internalized by the bank’s shareholders. The regulators’ objective function equals the private value \( V(m) \) of the bank minus the expected present value of default costs. Similar to the last case, it is not optimal to pay a coupon out of the bank’s earnings when these interest payments are not tax deductible.\(^{15}\) So we set \( c = 0 \). Regulators’ objective is to choose dividend distribution (cumulated cash flow process \( L_t \)) and initial cash level \( m_{R,0} \), that maximize the regulatory value of the SIFI

\[ R = \mathbb{E}\left[ \int_0^\tau e^{-rt} dL_t - \gamma e^{-rt} \right] - m_{R,0}, \]

where as before \( \tau = \inf\{t \mid s.t. \ m_t < 0\} \) is the failure time and cash reserves evolve according to (1.3). The solution to the regulators’ problem is obtained by the same recursive techniques. The regulatory value of the SIFI solves the same ODE as before but with different boundary conditions

\[
\begin{cases}
    rR(m) &= (1 - \theta)\mu R'(m) + \frac{1}{2}\sigma^2(1 - \theta)^2 R''(m) \\
    R(0) &= -\gamma \\
    R'(m^*_R) &= 1 \\
    R''(m^*_R) &= 0.
\end{cases}
\]

Similar to the case of shareholders, payments to shareholders only occur when the level of the bank’s cash reserves is higher than \( m^*_R \). So these payments are in the form of dividends that are only paid out when bank’s assets have been performing well. However, due to the social costs incurred in the event of bankruptcy, regulators impose a higher dividend threshold and only allow the distribution of dividends to bank’s shareholders when \( m > m^*_R > m^* \).

\(^{15}\)This is proven in Appendix A.1.1.
1.3. No tax-advantage of debt

In our simple framework where risky assets $A$ and retail deposits $D$ are fixed, the book value of equity $e = m + A - D$ varies one to one with cash reserves. Thus the higher dividend payout threshold translates into a higher capital buffer that regulators require the SIFI to keep within the bank. Given that the bank has to wait more and build a bigger cash buffer before distributing any dividend, the higher dividend threshold decreases the probability of default. In Appendix A.1.1 we provide a formal proof for $m^*_R > m^*$.

The closed form expression for the regulatory value of the SIFI is given by

$$R(m) = \frac{(1 + \gamma z_2 e^{z_2 m^*_R}) e^{z_2 m}}{z_1 e^{z_1 m^*_R} - z_2 e^{z_2 m^*_R}},$$

(1.7)

where $z_1$ and $z_2$ are given in (1.6), and $m^*_R$ is the dividend threshold chosen by regulators which is determined by the super contact condition.

$$R''(m^*_R) = 0 \implies z_1^2 e^{-z_1 m^*_R} - z_2^2 e^{-z_2 m^*_R} - \gamma z_2 z_1 (z_2 - z_1) = 0.$$

The following proposition summarizes our results so far.

**Proposition 1.1.** When issuance costs are high and there is no tax-advantage of debt the following holds

**a)**
- Privately optimal long-term financing mode is 100% equity (i.e. $c^* = 0$). Initial owners of the SIFI issue the number of stocks needed to finance productive assets and a target cash reserve of $m_0 = m^*$.
- Earnings are retained whenever cash reserves are below the target $m^*$. Excess cash is distributed as dividends. Bank is closed whenever cash reserves fall below zero.

**b)**
- Socially optimal long-term financing mode is also 100% equity.
- Regulators require more capital than what the owner would issue. The target cash reserve of regulators is $m^*_R > m^*$.
- This can be implemented by prohibiting dividend distribution if capital is below some minimum value.

Thus in the absence of tax advantage of debt, all market financing for SIFIs should be in the form of equity, both from shareholders’ and regulators’ points of view; however, there is a need for regulation because shareholders do not internalize the costs of failures. Regulation takes the form of a restriction on dividends: dividend distribution is forbidden if SIFI’s capital ratio is too low.

In this section we assumed that security issuance is so costly that no re-issuance happens in the future. In the Appendix we relax this assumption.
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1.3.3 CDS prices

A simple way to analyse the impact of regulation on the probability of default of the bank is to compute the following function

\[ p(m) = E[e^{-rT} \mid m_0 = m], \]

where cash reserves evolve according to (1.3). This "discounted probability of default" can be interpreted as the price of an infinite duration CDS that pays 1 dollar when the bank defaults. This price satisfies the following ODE

\[
\begin{aligned}
  rp(m) &= (1 - \theta) \mu p'(m) + \frac{1}{2} \sigma^2 (1 - \theta)^2 p''(m) \\
  p(0) &= 1 \\
  p'(m^*) &= 0.
\end{aligned}
\]

When the cash level is zero, probability of default is one (and the CDS pays one dollar). At the dividend threshold, since any additional cash is paid out of the bank, the marginal value of the CDS is zero. This equation has a closed form solution. The CDS price is given by

\[
p(m) = \frac{z_1 e^{z_1(m-m^*)} - z_2 e^{z_2(m-m^*)}}{z_1 e^{-z_1 m^*} - z_2 e^{-z_2 m^*}}, \tag{1.8}
\]

When regulators impose a higher dividend threshold \( m^*_R > m^* \), the CDS price is reduced for all values of \( m \). This is a consequence of the following result.

**Lemma 1.** The discounted probability of default \( p(m) \) is decreasing in \( m^* \).

The proof is provided in the Appendix. The following proposition is a direct consequence of Lemma 1.

**Proposition 1.2.** The bank’s discounted probability of default is reduced by regulation.

Figure 1.1 shows how the discounted probability of default decreases when the level of cash reserves increases, and is evaluated for two different values of dividend threshold \( m^* \) and \( m^*_R \). At any given level of cash reserves, the expected probability of default is higher when the dividend threshold is lower.

Given \( p(m) \), we can rewrite the expression for the regulatory value of the bank (1.7) as

\[
R(m) = \frac{e^{z_1 m} - e^{z_2 m}}{z_1 e^{z_1 m^*} - z_2 e^{z_2 m^*}} \gamma p_R(m),
\]

where \( p_R(m) \) is the CDS price given in (1.8) evaluated at regulatory dividend threshold \( m^*_R \). Thus the regulatory value of the SIFI is equal to its equity value when dividend threshold is set by regulators minus its expected external cost of failure.
1.4 Tax-advantage of debt

The property that 100% equity is the best long-term financing mode, is very general. It can be extended to other cash-flow processes, new security issuance, interest paid on cash, etc. The intuition is simple: because of financial frictions, cash is (strictly) more valuable inside the bank than outside, until the target cash reserve is attained. Any security that draws cash before the target is attained is suboptimal. Of course this reasoning only applies to long-term financing: collecting deposits is one of the core activities of banks. Our paper entirely focuses on the choice between equity and (various forms of) debt instruments for long-term financing. The next section studies the financing and pay-out behaviour of the SIFI in the presence of tax subsidies to debt instruments.

1.4 Tax-advantage of debt

In this section we assume that interest payments on any debt instruments are exempt from taxes. Assuming this tax advantage, we study the privately and publicly optimal financing contracts using straight debt and CoCo bonds.

1.4.1 Straight debt

Suppose the SIFI can issue some straight debt at time zero, paying a constant coupon of \( c \) as long as the SIFI is not in default. The SIFI is exempt from paying taxes on these interest payments. Default occurs when the SIFI’s level of cash reserves hits zero. When the SIFI defaults, both straight debt holders and equity holders receive nothing, since we assume a liquidation value of zero. The bank is systemically important, meaning that there are some negative externalities generated by its default, the costs of which are not internalized by bank’s shareholders. These costs are taken into account by regulatory authorities. Social welfare includes these negative externality costs, but also the value of taxes collected from the bank. In the following sections we study the optimal capital structure of the SIFI that can issue straight debt from the points of view of the SIFI’s equity holders, regulators and social welfare.

Shareholders

Given the assumptions previously discussed, bank’s cash reserves evolve according to

\[
dm_t = (1 - \theta)(\mu - c) dt + \sigma dZ_t - dL_t. \tag{1.9}
\]

Every time the level of bank’s cash reserves hits the target cash level, which for simplicity we still denote by \( m^* \), shareholders collect dividends. Default occurs when cash reserves hit zero. Debt holders collect coupon \( c \) for as long as the bank is not in default. For the moment, we take the target cash level \( m^* \) as given. Below we explain how it is chosen. The total value of
Chapter 1. How to Finance SIFIs?

the bank, \( V(m) \), solves the following ODE subject to the boundary conditions

\[
\begin{align*}
   r V(m) &= c + (1 - \theta)(\mu - c)V'(m) + \frac{1}{2}\sigma^2(1 - \theta)^2 V''(m) \\
   V(0) &= 0 \\
   V'(m^*) &= 1.
\end{align*}
\]

The first boundary condition reflects the zero liquidation value in case of default. The second boundary condition comes from the fact that at the optimal dividend threshold, the marginal value of cash in the bank is equal to one. This means that an additional dollar inside and outside the bank has the same value. Thus this is the target value at which the bank starts paying out dividends. At all levels of cash reserves below \( m^* \), the marginal value of cash is higher inside the bank (bigger than 1 which is the marginal value of cash outside the bank), thus all the earnings are retained within the bank.

The solution is in closed form. The total value of the bank is

\[
V(m) = \frac{c}{r} [1 - P(m, c)] + \frac{e^{y_1 m} - e^{y_2 m}}{y_1 e^{y_2 m^*} - y_2 e^{y_1 m^*}}, \tag{1.10}
\]

where \( y_1 > 0 > y_2 \) are the roots of the characteristics equation

\[
r = (1 - \theta)(\mu - c)y + \frac{1}{2}\sigma^2(1 - \theta)^2 y^2, \tag{1.11}
\]

and \( P(m, c) \), which is the discounted expected probability of default defined in Section 1.3.3, is given by

\[
P(m, c) = \frac{y_1 e^{y_2 (m - m^*)} - y_2 e^{y_1 (m - m^*)}}{y_1 e^{-y_2 m^*} - y_2 e^{-y_1 m^*}}. \tag{1.12}
\]

The first term on the right hand side of (1.10) is the value of debt \( D(m) \), which is the value of risk-free debt \( \frac{c}{r} \) times the discounted probability of the SIFI’s survival. The second term is the value of equity \( E(m) \). Closed form solutions for \( D(m) \) and \( E(m) \) are included in Appendix A.1.3.

When shareholders can not commit on their future dividend policy, the dividend pay-out threshold is determined by the super contact condition for the value of equity \( E(m) \), which is \( E''(m^*) = 0 \). So

\[
m^* = \frac{\ln(\frac{E}{H})^2}{y_1 - y_2}. \tag{1.13}
\]

This is because the maximization problem of bank owners is solved sequentially. Equity holders choose the dividend threshold that maximizes their wealth, after debt has been issued. So the dividend threshold is determined by (1.13). Given this optimal pay-out threshold, equity holders decide how much debt they want to include in their capital structure (determining coupon payments) to maximize the total value of the bank (their value as equity holders
plus the proceeds from debt issuance) at time zero minus the initial cash injection. Since $V'(m^*) = 1$, optimal initial cash injection is $m^*$. So $m_0 = m^*$, and the maximization problem equity holders face at time zero is given by

$$\max_c V(m^*) - m^*.$$ 

There is a trade-off between the costs and benefits of debt; whilst SIFI can save taxes by paying interests to debt holders, the interest payments draw cash from bank's cash reserves before the pay-out threshold is attained and when marginal value of cash is bigger inside the bank than outside. To find an optimal amount of debt, shareholders need to find the value of $c$ at which the costs and benefits of debt are equal. We show in the Appendix that when the tax rate $\theta$ is large enough, the optimal $c$ chosen by shareholders is positive.

**Proposition 1.3.** For $\theta$ large enough, it is optimal for shareholders to choose some debt financing (i.e. $c^* > 0$).

Figure 1.2 shows the graph of CDS prices $P(m, c)$ for different values of $c$. As could be expected, $P(m, c)$ increases in $c$ for all values of $m$.

As is typical in the corporate finance literature, we assume that shareholders decide on the level of dividend threshold after debt is issued; however it is interesting to look at the case where shareholders choose the dividend threshold ex-ante and commit to paying dividends at this threshold. In this case, shareholders choose optimal coupon payments and optimal dividend threshold simultaneously. This means that to find the optimal dividend threshold $m^*_\text{Com.}$, shareholders maximize the whole value of the bank at time zero. So the super contact condition that determines the optimal pay-out threshold is

$$V''(m^*_\text{Com.}) = 0.$$ 

To choose the optimal coupon payment, bank owners solve the following optimization problem.

$$\max_c V(m^*_\text{Com.}) - m^*_\text{Com.}.$$ 

In Appendix A.1.1, we show that this commitment increases the threshold above which dividends are distributed. When shareholders commit to receiving dividends later, they are able to issue more debt. Note that there is an interactive relationship between the dividend threshold and coupon payments. Dividends and coupons are both cash payments that are distributed outside the bank; however coupons are tax-deductible. If equity holders commit to distributing less dividends, they can issue more debt to benefit further from the tax subsidies. If they do not commit on their future dividend policy, they distribute more dividends and thus they can issue less debt.
Chapter 1. How to Finance SIFIs?

Regulators

As in Section 1.3, the bank in our model is systemically important, so we assume a social cost of failure equal to $\gamma$ that is not internalized by SIFI’s shareholders in the event of default. When regulators choose the dividend pay-out threshold and the amount of debt to include in SIFI’s capital structure, they take into account the social costs of failure; thus $R(m)$, the value function for regulators, is given by

$$R(m) = V(m) - \gamma E[e^{-rt} | m_0 = m],$$

where $V(m)$ is the total value of the bank. The ODE for $R(m)$ is given by

$$\begin{cases}
    r R(m) &= c + (1 - \theta)(\mu - c) R'(m) + \frac{1}{2} \sigma^2 (1 - \theta)^2 R''(m) \\
    R(0) &= -\gamma \\
    R'(m^*_R) &= 1 \\
    R''(m^*_R) &= 0.
\end{cases}$$

The first boundary condition comes from the external costs of SIFI’s failure. The second boundary condition is given by the fact that at optimal dividend threshold set by regulators, the marginal value of cash is equal to one. The third boundary condition is the super contact condition that determines the optimal dividend threshold for regulators.

Solving the above ODE gives the following closed form solution for the regulatory value of SIFI

$$R(m) = \frac{c}{r} [1 - P_R(m, c)] + \frac{e^{y_1 m} - e^{y_2 m}}{y_1 e^{y_1 m^*_R} - y_2 e^{y_2 m^*_R} - \gamma P_R(m, c)},$$

where $y_1$ and $y_2$ are given in (1.11), and $P_R(m, c)$ is the discounted probability of default given in (1.12) when the dividend threshold is $m^*_R$. The first and second terms represent debt value and equity value respectively and the last term is the present value of the social costs incurred at the time of default (costs times the discounted expected probability of default).

The dividend threshold set by regulators $m^*_R$ solves the super contact condition.

$$R''(m^*_R) = 0 \implies \left( \frac{c}{r} + \gamma \right)(y_1 - y_2) + \frac{y_1}{y_2} e^{-y_1 m^*_R} - \frac{y_2}{y_1} e^{-y_2 m^*_R} = 0.$$

To find the optimal amount of straight debt, regulators need to maximize $R(m)$ at time zero net of initial cash injection $m_0 = m^*_R$.

$$\max_c R(m^*_R) - m^*_R.$$

As was the case for shareholders, regulators trade off the benefits of interest tax savings with the costs of drawing cash from SIFI’s cash reserves before dividend threshold is attained. However in the case of regulators, default is more costly. Similar to Proposition 1.3, regulators choose $c^* > 0$ when the tax rate is high enough.
1.4. Tax-advantage of debt

We show in Appendix A.1.1 that for any given level of $c$, the optimal dividend threshold is higher for regulators than for bank owners. If shareholders commit on the dividend policy ex-ante, the threshold they choose is closer to the regulatory dividend threshold than when they choose it ex-post. However, this committed dividend threshold is still lower than the regulatory threshold. This is because shareholders do not internalize the costs of SIFI’s failure. So even if they could commit on future dividend threshold, their objective function would still be different from the objective function of regulators, implying a need for regulation.

To compare the level of debt that regulators find optimal with the level chosen by SIFI’s shareholders, the relationship between dividend threshold and coupon payment must be taken into account. Regulators decide on optimal dividend threshold and optimal coupon at time zero when debt is being issued. This is comparable to the case when shareholders can commit on future dividend threshold at debt issuance. We know that when shareholders commit on future dividend threshold, the target threshold they set is closer to the regulatory threshold. The difference between these two thresholds depends on the magnitude of the social cost of failure $\gamma$. The lower the social cost, the closer $m^*_{\text{Com.}}$ is to $m^*_{\text{R}}$. Since default is more costly to regulators, when dividend thresholds are set close enough, the regulatory optimal amount of debt is lower than the amount of debt shareholders would have chosen. Moreover higher social costs leads to lower levels of debt that is optimal from a regulatory point of view. On the other hand, when shareholders do not commit on the future dividend threshold, the dividend threshold they choose is much lower; for this low level of dividend threshold they can not issue as much debt as what they would have if they could commit. So it is possible for the regulatory amount of optimal debt to be higher than the amount which would have been issued by shareholders when they do not commit on future dividends. The next proposition summarizes our findings

**Proposition 1.4.** When there are tax subsidies to debt and the tax rate is high enough

- regulators also choose a positive coupon for a large enough tax rate.
- regulators choose a higher dividend threshold than shareholders.
- there is a need for regulation. Regulators need to restrict dividend distributions to shareholders and the amount of debt issued.

**Social welfare**

From a social point of view, in addition to the social costs of SIFI’s failure, the taxes collected from SIFI should be taken into account. So $W(m)$, the social value of the SIFI, can be written as

$$W(m) = V(m) + T(m) - \gamma E[e^{-rt} | m_0 = m],$$

The idea that regulation can serve as a commitment device for shareholders is discussed in Admati et al. (2014).
where $T(m)$ is the present value of all taxes collected until the time of default.

$$T(m) = E[\int_0^T \theta(\mu - c)e^{-rt} \, dt \mid m_0 = m].$$

We can calculate the closed form solution for the social value of the SIFI by solving the following ODE with boundary conditions

$$\begin{align*}
   rW(m) &= c(1 - \theta) + \theta \mu + (1 - \theta)(\mu - c)W'(m) + \frac{1}{2} \sigma^2 (1 - \theta)^2 W''(m) \\
   W(0) &= -\gamma \\
   W'(m^*_W) &= 1 \\
   W''(m^*_W) &= 0.
\end{align*}$$ (1.14)

The right hand side of this ODE includes the coupon payments to debt holders, net taxes paid by SIFI, and the dividend payments to equity holders plus any capital gain on the value. Boundary conditions are identical to the case of regulators with the only difference that the dividend threshold is now the level of cash reserves at which it is socially optimal to pay-out dividends to SIFI’s shareholders. This pay-out threshold is set to maximize the social value of the bank. Solving the above ODE, $W(m)$ is given by

$$W(m) = \frac{c}{r} (1 - P_W(m, c)) + \frac{e^{y_1 m} - e^{y_2 m}}{y_1 e^{y_1 m^*_W} - y_2 e^{y_2 m^*_W}} - (\gamma + \frac{\theta(\mu - c)}{r}) P_W(m, c),$$ (1.15)

where $P_W(m, c)$ is the discounted probability of default given in (1.12) when the dividend threshold is the socially optimal level $m^*_W$. The first and second terms of (1.15) are the value of debt and equity and the last term is the value loss in case of SIFI’s default which is the social costs incurred and the taxes collected from the bank.

The last boundary condition determines the socially optimal dividend threshold $m^*_W$.

$$W''(m^*_W) = 0 \implies \left( \frac{c}{r} + \frac{\theta(\mu - c)}{r} \right) (y_1 - y_2) + \frac{y_1}{y_2} e^{-y_2 m^*_W} - \frac{y_2}{y_1} e^{-y_1 m^*_W} = 0$$

To find the socially optimal amount of straight debt, $W(m)$ needs to be maximized at time zero when the initial cash injection into the bank is $m_0 = m^*_W$.

$$\max_c W(m^*_W) - m^*_W.$$ 

In Appendix A.1.1 we prove that the socially optimal $c$ is always zero. Indeed, in our model the only benefit to debt is the tax deductibility of interest payments. Since taxes are included in the social value of the SIFI, there is no social benefit to debt. No social benefit and the costs of drawing cash from SIFI’s reserves when the marginal value of cash is bigger than one make debt socially suboptimal. The following proposition formalizes our findings

**Proposition 1.5.** When the only benefits to debt instruments are tax subsidies, debt is socially
suboptimal and the socially optimal long-term financing mode for a SIFI is 100% equity.

1.4.2 CoCo bonds

The second case we study is the case in which SIFIs can issue CoCo bonds at time zero. A CoCo bond pays a constant coupon of $c$ until a pre-defined threshold of cash reserves $\bar{m}$ at which it converts to equity. This means that CoCo bond holders stop receiving any coupon payments but instead they get a fraction $\alpha$ of the bank’s equity. Thus $\bar{m}$, the conversion threshold, and $\alpha$, the conversion ratio are the pre-defined characteristics of the CoCo bond issued at time zero. They can be decided by shareholders or regulators. The bank is exempt from paying taxes on the interest paid on CoCo bonds. Similar to the last case, the cost of SIFI’s failure is not internalized by bank’s shareholders. This cost along with the tax deductibility of CoCo’s coupon payments lead to a different optimization problem from each different point of view: private, regulatory and public. This is what we study in the following sections.

Shareholders

Before conversion of CoCo bond, the level of cash reserves of the bank evolves according to

$$dm_t = (1-\theta)(\mu - c)dt + \sigma dZ_t - dL_t.$$

However as soon as CoCo bond converts to equity, there will be no more interest payments and the dynamics of the cash reserves go back to

$$dm_t = (1-\theta)\mu dt + \sigma dZ_t - dL_t.$$

This means that we have different value functions (for both equity and the total value of the bank): one after conversion (when the bank is 100% equity financed) and one before conversion when it has CoCo bonds in its capital structure. To solve for the total value of the bank at time zero we start from the total value of the bank after conversion and work backwards.

Since there is no debt after conversion, the total value $V_a(m)$ of the bank after conversion is equal to the value of equity $V_a(m) = E_a(m)$, and satisfies the following ODE with its boundary conditions

$$\begin{align*}
    r V_a(m) &= (1-\theta)\mu V_a'(m) + \frac{1}{2} \sigma^2 (1-\theta)^2 V_a''(m) \\
    V_a(0) &= 0 \\
    V_a'(m^*_a) &= 1 \\
    V_a''(m^*_a) &= 0. 
\end{align*}$$

The first boundary condition comes from the fact that after conversion of CoCo bonds there is no more cushion against losses and as soon as the level of cash reserves hit zero, the bank
defaults. With a liquidation value equal to zero, the total value of the bank will be zero at default. \( m^*_a \) is the dividend pay-out threshold after conversion. Shareholders decide on the dividend threshold after CoCo bonds are already in place, which gives the second and third boundary conditions. Solving (1.16) gives us the total value of the bank after conversion, which is also the value of equity after conversion.

\[
V_a(m) = e^{z_1 m} - e^{z_2 m},
\]

where \( z_1 \) and \( z_2 \) are given in (1.6), and the dividend pay-out threshold is determined by

\[
m^*_a = \frac{\ln(z_2/z_1)}{z_1 - z_2}.
\]

Before conversion the total value \( V_b(m) \) of the bank solves the following ODE with boundary conditions

\[
\begin{cases}
  r V_b(m) = c + (1 - \theta)(\mu - c) V'_b(m) + \frac{1}{2} \sigma^2 (1 - \theta)^2 V''_b(m) & \text{on } [\bar{m}, m^*_b), \\
  V_b(\bar{m}) = V_a(\bar{m}) \\
  V'_b(m^*_b) = 1.
\end{cases}
\]

(1.17)

The first boundary condition is the no arbitrage condition at conversion which comes from the fact that

\[
\begin{cases}
  E_b(\bar{m}) = (1 - \alpha) V_a(\bar{m}) \\
  CC(\bar{m}) = \alpha V_a(\bar{m}),
\end{cases}
\]

(1.18)

where \( E_b(m) \) and \( CC(m) \) represent the value of equity before conversion and the value of CoCo bonds respectively. In Appendix A.1.3, we calculate the closed form expressions of these two values. The second boundary condition states that before conversion, dividends are distributed at a target cash threshold of \( m^*_b \) where the marginal value of cash is equal to one.

Solving (1.17) gives the following closed form solution for the total value of the bank before conversion

\[
V_b(m) = \frac{c}{r} [1 - A(m)] + [V_a(\bar{m}) + \frac{e^{y_2 m + y_1 m} - e^{y_1 \bar{m} + y_2 m}}{y_1 e^{y_1 m^*_b + y_2 m} - y_2 e^{y_2 m^*_b + y_1 m}}] A(m),
\]

(1.19)

where \( y_1 \) and \( y_2 \) are given in (1.11), and \( A(m) \) is the discounted expected probability of conversion given by

\[
A(m) = \frac{y_1 e^{y_1 m^*_b + y_2 \bar{m}} - y_2 e^{y_2 m^*_b + y_1 \bar{m}}}{y_1 e^{y_1 m^*_b + y_2 \bar{m}} - y_2 e^{y_2 m^*_b + y_1 \bar{m}}},
\]

(1.20)

To maximize their claims shareholders need to decide on several parameters: the amount of CoCo bonds they issue, the conversion threshold \( \bar{m} \), and the dividend thresholds before and
1.4. Tax-advantage of debt

After conversion. As previously discussed, we assume that equity holders cannot commit on future dividend policy. The optimal dividend threshold before conversion is determined by the super contact condition on the equity value before conversion.

\[ E_p'(m^*_p) = 0, \]

and thus solves the following equation

\[ y_1^2 e^{y_1(m - m^*_p)} - y_2^2 e^{y_1(m - m^*_p)} + (1 - \alpha) E_d(m) y_1 y_2 (y_2 - y_1) = 0. \]

Conversion threshold and coupon payments are chosen by maximizing the total value of the bank at time zero minus the initial cash injection of \( m_0 = m_{b,Com} \).

\[
\max_{c,m} V_b(m^*_b) - m^*_b.
\]

The conversion ratio can then be obtained from the no arbitrage conditions at conversion in (1.18).

There is a trade-off between the costs and the benefits of CoCo bonds. Since interest payments on CoCo bonds are assumed to be tax deductible, issuing more CoCo bonds can save SIFI’s owners in taxes. On the other hand interest payments draw cash out of cash reserves when its marginal value inside the bank is higher than outside of it. Although this trade-off looks similar to the case of straight debt, with CoCo bonds SIFI’s owners have the chance of abandoning coupon payments when SIFI is in distress. This means that when SIFI’s cash reserves are below a pre-specified level and the marginal value of cash is very high inside the bank, coupon payments stop. This delays SIFI’s default. So CoCo bonds are less costly than straight debt because of cushioning the bank against default at the time of distress. This protection makes CoCo bonds more appealing to shareholders than straight debt. So shareholders choose a higher level of CoCo compared to straight debt. The dividend threshold after conversion is lower than the one before \( m^*_a > m^*_b \). This is because when there is no more debt, shareholders can get their dividends at a faster rate.

**Regulators**

Like before, the difference between the objective functions of shareholders and regulators is the cost of default of the SIFI, which is not internalized by shareholders but is borne by society.

Similar to the last subsection, we perform our analysis backwards, starting by the value to
regulators after conversion, which we denote $R_a(m)$

$$
\begin{align*}
\begin{cases}
  rR_a(m) &= (1-\theta)\mu R_a(m) + \frac{1}{2} \sigma^2 (1-\theta)^2 R'_a(m) \\
  R_a(0) &= -\gamma \\
  R'_a(m^*_R,a) &= 1 \\
  R''_a(m^*_R,a) &= 0.
\end{cases}
\end{align*}
$$

Comparing (1.21) to (1.16), we observe that the difference is in the first boundary condition: regulators incur the social cost of SIFI’s default $\gamma$ when SIFI’s cash reserves fall to zero. Solving (1.21), the closed form expression for $R_a(m)$ is

$$
R_a(m) = \frac{e^{z_1 m - z_2 m} - e^{z_2 m}}{z_1 e^{z_1 m^*_a} - z_2 e^{z_2 m^*_a} - \gamma P_{R,a}(m)},
$$

where $z_1$ and $z_2$ are given in (1.6), and $m^*_a$ is the dividend threshold chosen by regulators after conversion, which is determined by the super contact condition.

$$
R'_a(m^*_R,a) = 0 \implies z_2^2 e^{-z_2 m^*_a} - z_1^2 e^{-z_1 m^*_a} - \gamma z_2 z_1 (z_2 - z_1) = 0.
$$

$P_{R,a}(m)$ is the discounted probability of default in (1.8) evaluated at the regulatory dividend threshold of $m^*_R,a$. After conversion the total value of the SIFI to regulators solves the following ODE with boundary conditions

$$
\begin{align*}
\begin{cases}
  rR_b(m) &= c + (1-\theta)(\mu - c) R'_b(m) + \frac{1}{2} \sigma^2 (1-\theta)^2 R''_b(m) \\
  R_b(\bar{m}_R) &= R_a(\bar{m}_R) \\
  R'_b(m^*_R,b) &= 1 \\
  R''_b(m^*_R,b) &= 0.
\end{cases}
\end{align*}
$$

Similar to the case of equity holders the first boundary condition is the no arbitrage condition at conversion. Since regulators choose the (before conversion) dividend threshold that maximizes the regulatory value of the SIFI, the super contact condition in (1.22) applies. Solving (1.22) gives the following closed form solution for the total value of the bank to regulators before conversion

$$
R_b(m) = \frac{c}{r} [1 - A_R(m)] + [R_a(\bar{m}_R) + \frac{e^{y_1 \bar{m}_R + y_1 m} - e^{y_1 m^*_a + y_2 m}}{y_1 e^{y_1 m^*_b + y_1 \bar{m}_R} - y_2 e^{y_1 m^*_b + y_1 \bar{m}_R}}] A_R(m),
$$

where $y_1$ and $y_2$ are given in (1.11), and $A_R(m)$ which is the discounted expected probability of conversion in this case is given by

$$
A_R(m) = \frac{y_1 e^{y_1 m^*_b + y_1 m} - y_2 e^{y_1 m^*_b + y_1 \bar{m}_R}}{y_1 e^{y_1 m^*_b + y_2 \bar{m}_R} - y_2 e^{y_2 m^*_b + y_1 \bar{m}_R}}.
$$
1.4. Tax-advantage of debt

We can re-write (1.23) as follows

\[ R_b(m) = \frac{c}{r} [1 - A_R(m)] + [V_a(\bar{m}_R) - \gamma P_{R,b}(m, c) + \frac{e^{y_2 m_{R,b} + y_1 m} - e^{y_1 m_{R,b} + y_2 m}}{y_1 e^{y_1 m_{R,b} + y_2 m} - y_2 e^{y_2 m_{R,b} + y_1 m}}] A_R(m), \tag{1.24} \]

where \( P_{R,b}(m, c) \) is the expected probability of default given by (1.12) and evaluated at the regulatory dividend threshold of \( m_{R,b}^* \). Comparing the expression in (1.24) for the regulatory value of the bank before conversion with its private value before conversion (1.19), the difference between these two values comes from the external cost that is incurred by regulators in case of bank’s failure. Since the failure of the bank can only happen after conversion, the expected cost of default depends not only on the expected probability of default but also on the expected probability of conversion (\( P_{R,b}(m, c), A_R(m) \)).

Regulators maximize their objective at time zero by choosing the initial cash injection \( m_0 = m_{R,b}^* \), the conversion threshold \( \bar{m}_R \), and the coupon \( c \) of CoCo bonds.

\[ \max_{c, \bar{m}_R} R_b(m_{R,b}^*) - m_{R,b}^* \]

The conversion ratio \( \alpha \) can be obtained from one of the no arbitrage conditions at conversion (1.18).

Given the optimal dividend threshold, we have two parameters left with respect to which the value of SIFI is to be maximized: \( c \), and \( \bar{m} \). There is a relationship between the coupon payment \( c \), and the conversion threshold \( \bar{m} \). To be able to optimally issue a CoCo bond with a higher coupon payment (which allows for bigger tax savings), a higher conversion threshold is required. Since a bigger coupon payment draws out cash faster from the bank’s cash reserves, it increases the probability of default. To compensate for this, the CoCo bond should be converted into equity at higher levels of cash reserves. This means that a bank with higher \( c \) is to be in distress earlier. The bigger the coupon payments, the faster they should stop being paid. So shareholders can decide to pay out a big coupon payment and save more on taxes as long as the bank is doing good enough, but when the bank is not doing so well these coupon payments stop and shareholders face dilution as a result of the conversion of the CoCo bond. Alternatively shareholders can decide to pay a lower coupon payment on the CoCo bond but enjoy the benefits of interest tax savings longer since a lower coupon payment allows for a lower conversion threshold. The same argument holds for regulators.

When shareholders commit on the future dividend threshold by maximizing total firm value instead of equity value, dividends are paid-out at higher levels of cash reserve. In Appendix A.1.1, we show that for a given coupon and a given CoCo contract, shareholders choose a lower dividend threshold than regulators. Committed shareholders’ dividend threshold is somewhere in between since by committing on the future dividend threshold shareholders maximize the total value of the bank but they still do not internalize the SIFI’s cost of failure. The more costly SIFI’s failure, the higher the difference between the regulatory and the privately optimal dividend threshold.
Dividends are distributed out of the bank’s cash reserves, so their deferral increases the debt capacity of the bank. Regulators delay dividends distribution and thus can benefit from higher tax subsidies by issuing a CoCo bond with bigger coupon payments. This is why for any given CoCo contract, regulators choose a higher dividend threshold and a higher interest payment. This higher optimal level of debt creates value. The following properties emerge from our numerical simulations which we discuss in the next section.

- Issuing CoCo bonds rather than straight debt increases the debt capacity of the bank, which in turn increases the value of the bank to shareholders and regulators.
- Increasing the conversion threshold allows for a higher optimal coupon payment.
- For any given CoCo contract, the commitment on the future dividend threshold allows regulators to choose a much higher dividend threshold than shareholders and thus leads to higher regulatory optimal coupon payments on the CoCo bond.

Social welfare

From a social point of view, the value of taxes collected from a SIFI have to be added to its regulatory value. In Appendix A.1.4 we obtain closed form solutions for the social values of the bank before and after conversion, and the maximization problem that solves for the optimal conversion threshold and the optimal amount of CoCo to be issued.

Proposition 1.6. When the only benefits of debt are tax subsidies, CoCo bonds are socially suboptimal and the optimal long-term financing mode for a SIFI remains 100% equity.

Appendix A.1.1 provides the proof for the above proposition. The intuition is that although CoCo bonds can be converted into equity in times of distress, their only benefit comes from the tax subsidies they provide. Taxes that are paid by the bank are included in its social value, so there is no social benefit to CoCo bonds. This makes CoCo bonds socially suboptimal, even though they can be converted into equity in the event of SIFI’s distress.

1.5 Numerical illustration

In this section, we illustrate our results with an example. Table 1.1 reports the base case parameters we assume for our model. Specifically, we set $\mu = 0.15$, $\sigma = 0.09$, $r = 0.03$, $\theta = 0.35$, and $\gamma = 2$.

1.5.1 No tax-advantage of debt

When there are no tax subsidies to debt instruments, the optimal financing is 100% equity. Then the only optimizing parameter is the dividend threshold. Table 1.2 summarizes our
results for the privately and socially optimal dividend thresholds when there is no security re-issuance.

When there is no re-issuance, equity holders choose to distribute dividends faster than regulators. Thus the dividend threshold set by equity holders is not socially optimal. With our base case parameters the regulatory optimal dividend threshold is about 4.4% higher than the privately optimal threshold. As Figure 1.3 shows, the higher the social costs of default are, the higher the difference between the privately and the socially optimal dividend thresholds will be.

### 1.5.2 Tax-advantage of debt

**Straight debt**

When interest payments on straight debt provide tax subsidies to the bank, the two optimizing parameters are the dividend threshold and the amount of coupon payment. In Table 1.3 we summarize our results for these two values when they are chosen by different parties: shareholders (that can be committed or uncommitted on dividend policy), regulators, and social planner.

Table 1.3 is an illustration of our results in Propositions 1.3- 1.5. When shareholders cannot commit on their dividend policy, they choose the dividend threshold that maximizes the value of equity rather than the whole value of the bank. Table 1.3 shows that shareholders choose a much lower dividend threshold than regulators. Since dividends are distributed out of the bank much faster, the probability that the level of cash reserves hit zero increases and thus the debt capacity of the bank decreases. That is why, as demonstrated in Table 1.3, the optimal level of straight debt for shareholders is lower than the regulatory optimal level. Moreover, Table 1.3 shows that if shareholders were to commit on the future dividend threshold, they could substantially increase the debt capacity of the bank by choosing to distribute dividends at a much higher level of cash reserves. However since they do not internalize the social cost of default, the amount of debt they would like to issue is higher than what is optimal for regulators. Commitment, combined with the external cost of default, leads to an even higher dividend threshold and a lower coupon payment for regulators. Assuming higher expected costs of default, along with commitment, regulators can further increase the initial social value of the bank; however the main benefit of regulation is to allow shareholders to commit. The commitment increases the initial social value of the bank from 4.658 to 4.944, an increase of 6.2%. Assuming the social cost of default, increases the social value of the SIFI by a further 1% from 4.944 to 4.996.

When taking into account the value of the taxes paid by the SIFI, the optimal amount of straight debt is zero. The initial social value of 5.00 when the SIFI is 100% equity financed is higher than the initial social value in the cases of uncommitted shareholders, committed shareholders, and regulators by about 7.3%, 1.1% and 0.08% respectively. This shows that even
though debt can be optimal for shareholders and even regulators, it decreases the value of the SIFI dramatically from a social point of view. Since taxes that are paid by SIFI are included in the social value of the bank, there is no advantage in issuing debt.

Figure 1.4 shows how dividend threshold changes with the coupon. The dashed line represents the case of a regulated SIFI. The solid line and the dot-dashed line represent the cases of uncommitted shareholders and committed shareholders respectively. Interest payments draw cash out of SIFI’s cash reserves when the marginal value of cash is higher than one. On the other hand, tax savings on coupon payments help cash build up faster inside the bank. Figure 1.4 shows that for our benchmark case, the latter is dominated both for committed shareholders and regulators. Thus with higher coupon payments, dividend payments are postponed. However when shareholders cannot commit on the future dividend threshold, increasing the coupon payment first increases and then decreases the dividend threshold. This means that increasing the debt coupon payment beyond some point, makes the increased tax savings the dominant factor in deciding the dividend threshold. For any given level of \( c \) the dividend threshold chosen by shareholders is lower than the regulatory optimal threshold. Uncommitted equity holders distribute dividends faster than what would have been optimal for regulators because they do not internalize the social costs of SIFI’s failure and because they can not commit on future dividend threshold.

Figure 1.5 shows how dividend threshold changes when changing the main parameters of the model: profitability of the bank, volatility of the cash flows and tax rate. For each parameter we study three different levels of coupon payment (\( c = 5\% \), \( c = 7\% \), \( c = 9\% \)). As panel A shows higher profitability allows for faster distribution of dividends both for regulators and committed shareholders. When the bank is more profitable, cash reserves build up faster in the bank and thus dividends can be paid out at lower thresholds. For shareholders, the dividend threshold first increases and then decreases with profitability and for any given level of coupon payment. Panel B shows that the riskier the bank, the higher the probability of cash reserves hitting zero. This leads to a higher required dividend threshold for a riskier bank by shareholders and regulators for any given level of coupon payment. Higher tax rates decrease the dividend threshold as shown in panel C. This is the case for both regulators and shareholders and for any given level of coupon payment. In all cases, the dividend threshold for committed shareholders is higher than the dividend threshold for shareholders but lower than the one for regulators.

Figure 1.6 shows how optimal amount of straight debt changes with profitability of assets in place, volatility of cash flows and SIFI’s tax rate for shareholders (solid line), committed shareholders (dot-dashed line), and regulators (dashed line). Higher profitability and lower volatility increase the debt capacity of the bank. Higher tax rates increase the benefit of debt and thus increase the amount of debt that trades off its costs and benefits. As discussed commitment on the future dividend threshold along with the limited liability on the social cost of SIFI’s failure, allows committed equity holders to choose a higher coupon payment than both shareholders and regulators no matter the parameter values.
CoCo bonds

When a SIFI can issue CoCo bonds, coupon payments, conversion threshold and dividend pay-out thresholds before and after conversion have to be chosen. Table 1.4 summarizes the financing and pay-out policy of a SIFI with base case parameters of Table 1.1 chosen by shareholders, regulators and social planner. The table shows how interest payments delay the distribution of dividends before conversion. After conversion the dividend threshold is always lower than before conversion. Similar to the last case, since shareholders cannot commit on the future dividend threshold, they choose a much lower pay-out threshold than regulators by maximizing the value of equity after the CoCo bond has been issued. This decreases the debt capacity of the bank. The amount of coupon payment chosen by shareholders is 11.7% compared to 14.6% for regulators. Shareholders choose to distribute dividends out of the bank when cash reserves are at a level of 0.375 whilst regulators do not allow for any dividend distribution before cash reserves hit a level of 0.688. This shows how commitment postpones dividends and allows to save more on taxes by increasing the debt capacity of the bank.

Comparing Table 1.2 and Table 1.3 shows how the conversion feature of the CoCo bond increases the debt capacity of the bank both for shareholders and regulators. For our benchmark case parameters shareholders pay 11.7% on the CoCo bond whilst they only pay 10% on the straight debt. Regulators choose to pay a continuous coupon of 14.6% on the CoCo bond compared to the 13% they pay on the straight debt. Figure 1.7 shows that this statement holds for different levels of profitability, volatility of bank’s cash flows, and tax rate.

To see whether or not this increased debt capacity is beneficiary, we calculate the total value to shareholders at time zero (the sum of the value of equity and debt minus the initial cash injection into the bank) for both straight and CoCo bonds cases. The 11.7% CoCo bond gives a total initial value to shareholders of 4.11. The initial total value to shareholders with a 10% straight debt is 3.92, which shows a decrease of almost 5% compared to the CoCo case. We can also calculate the total initial value to regulators (the sum of the regulatory value of the bank and the value of debt minus the initial cash injection into the bank) for both straight debt and CoCo cases. The initial value to regulators is 4.26 with the 14.56% CoCo bonds, whilst the 12.95% straight debt leads to an initial value of only 4.14, which shows a decrease of almost 3% compared to the case of CoCo bonds. Figure 1.8 shows how the value to regulators remains greater with CoCo bonds compared to straight debt, by changing the values of the main model parameters (profitability, volatility, tax rate and external cost of default).

Figure 1.9 shows how the optimal conversion threshold changes with the main parameters of the model. It decreases with profitability and tax rate, and increases with volatility. Conversion occurs at a higher level of cash reserves when the bank is less profitable, more risky or when the tax subsidies of debt are lower. Moreover, the conversion threshold increases when SIFI’s failure is socially more costly. This means that regulators consider the SIFI to be in

17 To save space these results have only been demonstrated for regulators. The results for shareholders are qualitatively similar.

17 To save space these results have only been demonstrated for regulators. The results for shareholders are qualitatively similar.
distress at higher levels of cash reserves when its failure is more costly to the society.

Figure 1.10 shows both the dividend threshold and the amount of coupon payment that are optimally chosen by shareholders (solid lines) and regulators (dashed lines) for a given CoCo contract. In addition to our benchmark case, in Figure A.1 in the Appendix, we do the comparison between shareholders and regulators choices of coupon payment and dividend threshold for different values of the model parameters (profitability, cash flow volatility, tax rate and external cost of failure). Regulators choose a much higher dividend threshold than shareholders. Since they distribute dividends at a lower speed, they can issue a CoCo bond with a bigger coupon payment. This is why we observe that the level of privately optimal coupon payment is smaller than the regulatory one. Dividends and coupons are both cash that are distributed outside of the the bank, the difference is that interest payments are tax deductible whilst dividends are not. Postponing dividend payments, allows for bigger coupon payments which in turn allow for more tax savings and thus create value. This means that if shareholders could commit to receive dividends later, they could have benefited more from the tax subsidies by issuing a higher coupon CoCo bond.

Figure 1.10 also shows how the optimal coupon payment and dividend threshold change by changing the conversion threshold for both shareholders and regulators. A higher conversion threshold means that the CoCo bond is converted into equity at a faster rate (at higher levels of cash reserves). Since the cushion against the default is triggered faster, the bank can make bigger interest payments on the CoCo bond. This is the case for both shareholders and regulators. Coupon payments can be bigger if they stop faster. At the same time, the bigger coupon rates postpone dividend pay-outs, that is why both private and regulatory dividend thresholds increase by increasing the conversion threshold.

Comparing the total initial values with CoCo bonds and with straight debt, we observe that the conversion feature of CoCo bonds that is triggered when the bank is in distress and can delay SIFI’s failure, creates value to shareholders and regulators; however from a social point of view the optimal amount of CoCo bonds is zero.

1.6 Conclusion

In many jurisdictions, financial regulators have decided to allow large banks to count CoCo bonds as part of their regulatory capital. As a result, a vast academic literature has started to study the optimal design (and the pricing) of such hybrid securities. Our paper differs from this literature in two important ways: first, we consider liquidity problems as the major source of default for banks. Instead of using the Merton/Leland type of structural models of corporate finance, where these liquidity issues are neglected, we use a cash management model in the spirit of Radner and Shepp (1996). In that model, banks default not because they are insolvent, but because they cannot find enough liquidity in the market. A second difference with the

\footnote{Changing the external cost of failure only affects the regulators choice.}
previous literature is that we adopt a social welfare perspective, in which the tax advantage of
debt (a mere transfer between taxpayers and bank’s shareholders) does not create any value.

In this set up, we show that CoCo bonds, like any form of long-term debt, are a socially
inefficient way of financing SIFIs. For CoCo bonds to be socially useful, additional ingredients
have to be incorporated into the analysis, such as adverse selection (which may explain why
equity is more expensive than debt for banks), preference for liquidity (but it is not clear why
CoCo bonds should be more liquid than other securities), or an hypothetical disciplining
role of debt, whose empirical magnitude still needs to be established. The reason for which
CoCo bonds have been adopted in many jurisdictions may therefore be purely political. CoCo
bonds reduce both the probability of default (which is good for regulators) and the taxes paid
by banks (which is good for the financial industry). However this is done at the expense of
taxpayers, who do not seem to be aware of the situation.
Table 1.1: Base case parameter values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>The profitability of bank's operations</td>
<td>0.15</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>The volatility of bank's operations</td>
<td>0.09</td>
</tr>
<tr>
<td>$r$</td>
<td>The risk-free rate</td>
<td>0.03</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Corporate tax rate</td>
<td>0.35</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Social cost of SIFI's failure</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1.2: No tax advantage - optimal dividend threshold

<table>
<thead>
<tr>
<th>Privately optimal</th>
<th>Socially optimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>No re-issuance</td>
<td>0.182</td>
</tr>
</tbody>
</table>

Table 1.3: Optimal policy choices and capital structure with straight debt

<table>
<thead>
<tr>
<th>Shareholders</th>
<th>Regulators</th>
<th>Social welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Uncom.</td>
<td>Com.</td>
</tr>
<tr>
<td>Div. threshold</td>
<td>0.30</td>
<td>0.617</td>
</tr>
<tr>
<td>Coupon payment</td>
<td>10.0%</td>
<td>13.3%</td>
</tr>
<tr>
<td>Social value $^{19}$</td>
<td>4.658</td>
<td>4.944</td>
</tr>
</tbody>
</table>

Table 1.4: Optimal policy choices and capital structure with CoCo bonds

<table>
<thead>
<tr>
<th>Shareholders</th>
<th>Regulators</th>
<th>Social welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>After conv. div. thresh.</td>
<td>0.182</td>
<td>0.190</td>
</tr>
<tr>
<td>Coupon payment</td>
<td>11.7%</td>
<td>14.6%</td>
</tr>
<tr>
<td>Before conv. div. thresh.</td>
<td>0.375</td>
<td>0.688</td>
</tr>
<tr>
<td>Conv. thresh.</td>
<td>0.058</td>
<td>0.061</td>
</tr>
<tr>
<td>Social value</td>
<td>4.806</td>
<td>4.998</td>
</tr>
</tbody>
</table>

$^{19}$The initial social value is the social value of the SIFI at time zero given by (1.15) and evaluated at the dividend thresholds and coupon payments chosen by shareholders, committed shareholders, and regulators respectively. The level of cash reserves at time zero is equal to the relevant dividend threshold in each case.
Figure 1.1: CDS price - no tax-advantage of debt

The figure shows $p(m)$ (the CDS price in basis points) as a function of cash reserves $m$ for two different levels of dividend threshold: $m^*$ (solid line) and $m^*_R$ (dashed line). Parameter choice reported in Table 1.1.

Figure 1.2: CDS price - straight debt case

The figure shows $P(m, c)$ (CDS price in bps) as a function of cash reserves $m$ for different levels of straight debt coupon ($c = 0$, $c = 3\%$, $c = 7\%$, $c = 10\%$). Parameter choice reported in Table 1.1.
Chapter 1. How to Finance SIFIs?

Figure 1.3: Effect of social costs of SIFI’s failure

The figure shows the difference between privately and socially optimal dividend thresholds \((m^*_R - m^*)\) as a function of external cost of SIFI’s failure.

Figure 1.4: Dividend threshold - straight debt case

The figure shows the dividend thresholds as functions of coupon payment for: equity holders (solid line), committed equity holders (dot-dashed line), and regulators (dashed line).
The figures show the dividend threshold as functions of profitability (panel A), volatility (panel B), and tax rate (panel C) for different levels of coupon payment ($c = 5\%$, $c = 7\%$, $c = 9\%$). Dashed lines, solid lines, and dot-dashed lines represent regulated SIFI, committed shareholders and uncommitted shareholders respectively.
Chapter 1. How to Finance SIFIs?

The figure shows the optimal coupon payment on straight debt as functions of profitability, volatility, and tax rate for: equity holders (solid lines), committed equity holders (dot-dashed lines), and regulators (dashed lines).

The figure shows the optimal coupon payment on straight bond (solid lines) and on CoCo bond (dashed lines) as functions of profitability, volatility, and tax rate.
Figure 1.8: Optimal value to regulators

The figure shows the optimal regulatory value of the bank with straight debt (solid lines) and CoCo debt (dashed lines) as functions of profitability, volatility, tax rate and external cost of failure.
Figure 1.9: Optimal conversion threshold

The figure shows the optimal conversion threshold as functions of profitability, volatility, tax rate and external cost of failure.
Figure 1.10: Optimal capital structure and pay-out policy - CoCo bond

The figure shows how the optimal dividend threshold and the optimal coupon payment of the CoCo bond change with conversion threshold. Solid lines and dashed lines represent shareholders and regulators respectively.
2 Bail-in Plan and Macroeconomic Conditions

2.1 Introduction
Since the onset of the financial crisis of 2007-2009, the stability of the financial system has been the subject of many discussions by policy makers and academics. These discussions mostly focus on banks or financial institutions that are too big and too inter-connected to fail. As the experiences of the collapse of Bear Sterns and Lehman Brothers (among others) show, the failure of a Systemically Important Financial Institution (“SIFI”) has significant disruptive effects on other financial firms and on the financial system as a whole. The costs of these negative externalities are not borne by the SIFI’s shareholders. When banks do not or can not recapitalize, financial authorities have no option but to bail them out using public funds to assure the stability of the financial system.

The social costs and adverse incentive effects of bank’s bail-outs have motivated the proposals for a "bail-in" requirement as a possible solution to the too big to fail ("TBTF") problem. A bail-in plan is defined as a pre-determined contract that results in an automatic self-recapitalization of a financial institution when it faces financial difficulties. The objective of a bail-in plan is thus to eliminate the risk of banks’ failures without any equity support from the government. Under a bail-in plan, certain non-equity obligations of the bank transform into equity at the point of a pre-defined threshold so that the bank can absorb operating losses, recapitalize and continue its operations. This means that debt to equity conversion is an essential feature of any bail-in plan. Additionally for the bail-in plan to be effective, the bank should be able to repeat this procedure in any future cases of financial distress.

The bail-in mechanism, its possible designing options and its implications on banks’ policy choices have been the subject of many discussions including Coffee (2010), Anderson (2011), Huertas (2011), Ötker-Robe et al. (2011), De Grauwe (2013), and Zhou et al. (2012). However, to the best of my knowledge, no theoretical work has been done to analyse the quantitative aspects of a bail-in plan and its effects on banks’ optimal capital structure and policies. A large body of theoretical literature analyse the debt to equity conversion feature by studying different forms of contingent convertible ("CoCo") securities without addressing distress
Chapter 2. Bail-in Plan and Macroeconomic Conditions

situations that the firm can face post conversion. My objective in this paper is twofold. First by using the debt to equity conversion in the context of a restructuring contract that is set ex-ante and is triggered when banks are in distress and their access to outside financing is limited, I eliminate the risk of inefficient bank liquidations. Second, I study banks’ optimal capital structure decisions and pay-out and financing policies in the presence of such a plan. I also study the effects of the design of the bail-in contract on these decisions.

To do so, I formulate a dynamic structural model in which financial institutions face stochastic financing frictions, default costs and taxation. Banks hold a portfolio of risky assets and can build up risk-free liquid reserves. The failure of the bank in my model triggers significant disruptions in the financial markets and is thus to be prevented at all times. Committing to a bail-in plan, banks issue some contingent convertible debt that converts into equity only when they face financial distress and outside equity financing is not possible. Banks choose their capital structure and their pay-out, and refinancing policies to maximize shareholders value.

As customary in the contingent capital literature, I assume that the CoCo debt converts into a pre-determined fraction (which sets the conversion ratio) of the bank’s equity when a pre-determined threshold (which sets the conversion level) is triggered. There are, however, two main differences between my paper and this literature. First, unlike the prior contributions on CoCo debt, I assume that banks face fixed costs of outside financing, and thus continuous injection of capital inside the bank is not possible. In the presence of these financing frictions liquidity management becomes crucial: banks retain earnings to build up liquid buffers that they can use to absorb losses and to save on the costs associated with outside liquidity. Moreover, I assume financing frictions are stochastic. Banks adapt their policies to the fluctuations in the financing costs and thus their policy choices become time dependent. I assume that issuing CoCo debt in the bad state is costlier than issuing equity in the good state but is less costly than issuing equity in the bad state. Second, upon conversion which happens only when equity financing is not available, banks replace the converted debt with newly issued CoCo debt of the same terms. This issuance provides banks with new capital and a new cushion against future losses and eliminates the risk of inefficient failures.

The main results of my model are as follows. First, the optimal conversion threshold is zero. Since conversion is followed by costly reissuance of CoCo debt, it is optimal to postpone it as long as possible. Thus shareholders optimally wait until the bank runs out of cash to trigger conversion. If there is a change in equity financing conditions from bad to good before cash reserves are depleted, the bank can issue equity which is less costly. If not, the bank can convert and reissue CoCo debt at the last possible moment to save on the discounted expected financing costs and the carry cost of cash.

Second, bail-in plans affect the recapitalization behaviour of banks in good times. In my model, different states of the world are characterized by different investor demand for bank securities. Whilst in the good state of the world investors have an appetite for bank equity,
in the bad state CoCo investments provide an alternative for investors who shy away from equity markets. As Bolton et al. (2013) show when financing frictions are time varying, it may be optimal for shareholders to raise new equity funds before they run out of cash in the fear of the worsening of financing conditions. By providing a new source of outside liquidity when banks do not have access to equity financing, bail-in plans alleviate the severity of financing constraints in the bad state. This decreases shareholders incentives to recapitalize in the good state and drives down the threshold below which shareholders raise new funds. When choosing its recapitalization threshold, the bank balances the higher costs of liquidity in the bad state with the carry cost of cash and the refinancing costs in the good state. Therefore the recapitalization threshold increases with the difference between the costs associated with equity financing in the good state and CoCo debt financing in the bad state, and the probability of a jump to the bad state. Moreover the design of the CoCo contract has direct effects on the recapitalization policy in the good state. Notably for the same level of coupon payment, higher conversion ratios increase equity dilution at conversion and thus provide shareholders with more incentives to recapitalize in the good state.

A third result of the paper is to show that banks hold smaller liquid buffers when they commit to a bail-in plan. When outside financing is scarce, keeping cash inside the bank acts as a form of risk management. Shareholders choose the target level of cash in each state by balancing the costs of holding liquid buffer versus the refinancing and default costs. Bail-in plans eliminate the costs associated with inefficient liquidations, and thus lead to lower precautionary motives to build up cash buffers. Although the bank does not default when it runs out of cash in the bad state, shareholders still lose a fraction of their stake in the bank at conversion. When conversion is more dilutive, shareholders have incentives to build up more cash to avoid conversion. Thus for a given coupon payment, the target levels of cash increase with conversion ratio. Additionally the higher costs associated with CoCo debt financing compared to equity financing increase the value of the cash inside the bank in the bad state and thus increase the target level of cash in this state compared to the good state. Therefore banks hold countercyclical cash buffers.

Next, I show that depending on their design, bail-in plans can eliminate risk-taking incentives. When shareholders face financing frictions, the precautionary demand for cash leads to equity values which are increasing and concave in the level of cash reserves. This is the case in Décamps et al. (2011), Bolton et al. (2011, 2015), or Hugonnier and Morellec (2016) where shareholders do not have any risk-taking incentives. When financing conditions are time varying, such as in Bolton et al. (2013), for low levels of cash reserves in the good state of the world, the bank which is concerned about losing its access to outside equity may find it optimal to exercise its option to time equity markets. Since equity refinancing cost is fixed, equity issuance is lumpy. For a low enough level of cash the motive to time the equity market overcomes the precautionary need for cash and can lead to a local convexity in the value of equity in the good state. With a bail-in plan in place, shareholders can face conversion rather than liquidation if they do not recapitalize before the window of cheap equity financing is closed. Depending on the cost of CoCo issuance and the dilution they face upon conversion,
the precautionary motive can dominate and thus lead to a globally concave equity value in the good state. For a given coupon payment, the higher the conversion ratio the higher is the level of the cash reserves that separates the concave and convex regions of the equity value.

Lastly, I show that unless the conversion is significantly dilutive, shareholders adjust their optimal capital structure such that the optimal level of debt commands a zero recapitalization threshold in the good state. Moreover this optimal leverage leads to a globally concave equity value and thus entirely eliminates any risk-taking incentives. My results show that bail-in plans have the potential to decrease recapitalizations and cash buffers within the banking system. However since they eliminate the costs of liquidation and the private incentives for risk taking they can be socially optimal.

My paper relates to the literature that examines the design features of contingent capital bonds and their impacts on bank capital structure and policy choices. CoCo bonds have been first introduced by Mark Flannery (2005) as bonds that convert into common equity or are written-off when some pre-defined threshold is reached. CoCo bonds have gained growing attention during the financial crisis of 2007-2009 as a possible solution for the inadequacy of bank capital in bad times. Since their introductions, numerous studies have tried to formulate valuation models and address the key design issues of CoCo bonds. See Pennacchi et. al. (2010), Glasserman and Nouri (2010), Albul et al. (2013), Sundaresan and Wang (2015), McDonald (2010), Barucci and Del Viva (2012, 2013), Chen et al. (2013), Flannery (2009), and Calomiris and Herring (2013) among others. These proposals usually vary with regard to two main features of a CoCo bond: the conversion trigger (which determines the probability of conversion or conversion risk) and the conversion ratio (which determines burden sharing between shareholders and debt holders).

All of these models assume that firms do not keep cash reserves because they can continuously issue equity at no cost. In these models default occurs for solvency reasons. On the contrary in my model, I assume banks face time varying financing conditions and can have problems accessing equity markets. When outside financing is costly, banks have incentives to build up liquid buffers within the firm. In my model, if banks were to default it would have been for liquidity and not solvency reasons. Additionally prior studies do not allow for the reissuance of the CoCo bond and thus can not address the too big to fail problem after the existing CoCo debt has been fully converted. In my model, committing to a bail-in plan, the bank replaces its converted CoCo debt with new debt of the same terms to preserve its cushion against future losses. The bail-in plan in my model acts as a restructuring contract that is set ex-ante in order to eliminate the risk of inefficient liquidations and thus can address the TBTF problem.

The design of my CoCo contract is most closely related to the design of the countercyclical contingent capital introduced by Barucci and Del Viva (2012). They analyse the case of a contingent capital that is only converted to common shares in poor macroeconomic conditions. In their model, the operating profit of the bank is affected by macroeconomic conditions but its access to outside financing which is assumed to be costless is not. The recent financial
2.1. Introduction

crisis has shown that the appetite for equity investing dries up during poor macroeconomic conditions. In my model, unlike Barucci and Del Viva (2012) poor economic conditions are characterized by more severe financing frictions. Thus countercyclical CoCo bond is defined as a CoCo debt that converts into equity only when the distressed bank does not have access to outside equity financing.

Another strand of literature on CoCo debt analyses the incentives created by CoCo bonds. These include Chen et. al. (2013), Hilscher and Raviv (2014), Pennachi (2010), Berg and Kaserer (2015) Barucchi and Del Viva (2012), Sundaresan and Wang (2015), Koziol and Lawrenz (2012), and Martynova and Perotti (2015). Although different designs of these models give rise to different results, they all agree that the design of the contingent capital is detrimental to its effects on banks policies and risk-taking incentives. In my model, I show that when shareholders choose the optimal level of CoCo debt in their capital structure, for any conversion ratio the optimal capital structure significantly decreases or fully eliminates risk-taking incentives, but also decreases incentives to recapitalize and keep cash within the bank.

My model also relates to the papers that study firms’ capital structure and financial decisions when their access to outside liquidity is costly. Most of these papers including Décamps et al. (2011), Bolton et al. (2011), Hartman-Glaser and Milbradt (2014), Hugonnier et al. (2015), Décamps et al. (2016) and Hugonnier and Morellec (2016) assume financing frictions are constant. Equity value is globally concave in these models since inefficient liquidations give incentives to precautionary cash holdings. Moreover except for Hugonnier and Morellec (2016), firms in these models are all equity financed. Introducing time varying financing frictions à la Bolton et al. (2013) leads to local convexity and thus risk loving behaviour for shareholders. Della Seta et al. (2017) study another form of convexity that is observed in the value of equity close to financial distress due to the roll over losses induced by short-term debt financing. By introducing countercyclical CoCo bond into banks’ capital structure, I provide banks with another source of financing that can help ease their access to outside liquidity. So whilst in all other models with cash holdings firms face liquidation at some point, the bank in my model avoids liquidation by converting and reissuing the debt obligations when it faces financial distress and has no access to outside liquidity. The bank adapts its financial and risk management decisions depending on the design of the CoCo debt, i.e. the pay-off shareholders expect to receive at conversion.

The paper is organized as follows. Section 2.2 presents the model. Section 2.3 derives closed form solutions for the value of equity and debt and studies optimal bank policy choices and capital structure in a benchmark case where the bank has only access to a traditional straight debt and thus can default. Section 2.4 studies the design of a bail-in contract and characterizes the bank’s policies and capital structure decisions when it has committed to such a contract. Section 2.5 concludes.
Chapter 2. Bail-in Plan and Macroeconomic Conditions

2.2 Model

The subject of my study is a SIFI (I use the terms "SIFI", "bank", and "firm" interchangeably) that I model as a firm that transforms a fixed volume $D$ of risk-free deposits into a fixed volume $A$ of risky assets. Since the bank is systemically important its failure does not only entail losses to its own shareholders, but also triggers extreme negative externalities and major disruptions to the financial market as a whole. So the SIFI character in my model is very stylized; its closure entails a social cost that is not internalized by shareholders. This cost is so significant that it is optimal to avoid SIFI’s failure at all times.¹

I assume that the bank’s access to capital markets is not perfect, so a profitable bank holds cash in the fear of future liquidity problems. Specifically the bank needs to build a precautionary cash buffer to cover operating losses to avoid closure when outside financing is costly. No dividends are distributed outside the bank before the cumulative performance of the bank is sufficiently high and its total cash reserves add up to a target level. Thus the bank’s balance sheet is as follows:

<table>
<thead>
<tr>
<th>Risky assets $A$</th>
<th>Deposits $D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash reserves $w$</td>
<td>Market finance</td>
</tr>
</tbody>
</table>

Therefore the liability structure of the bank consists of market financing and deposits. Market financing can include equity and market debt. Bank’s deposits have a face value of $D$ and are insured against bank failure. I assume that the bank is required to make a continuous payment of $C_D$ to maintain its deposit accounts. $C_D$ includes the interest payment to depositors, the deposit insurance costs and the costs of servicing depositors.

I model random cash flows generated over time by the SIFI’s operations (net earnings) as an arithmetic Brownian Motion with positive drift $\mu$ and volatility $\sigma$ defined over a complete probability space $(\Omega, \mathcal{F}, P)$. Specifically the cumulative cash flow process $R$ evolves according to

$$dR_t = (1 - \theta)(\mu dt + \sigma dZ_t), \quad R_0 = 0.$$  \hfill (2.1)

The process $Z \equiv \{Z_t, t \geq 0\}$ is a standard Brownian motion with respect to the filtration $\{\mathcal{F}; t \geq 0\}$ that models the flow of information. $\theta$ is the rate at which the bank pays taxes on corporate income. Operations of the bank’s risky assets can involve losses $dR_t < 0$ as well as profits $dR_t > 0$.

All investors in my model are risk-neutral and the risk-free rate is $r$. I assume that management acts in the best interest of shareholders and that there are no corporate governance problems such as moral hazard and asset substitution. However two frictions are present in my set-up:

¹I do not explicitly model these external costs. However I implicitly assume that such costs are so high that it is socially optimal to avoid them.
first, there is an opportunity cost of keeping cash inside the bank; I assume internally held cash does not earn any interest.\footnote{The idea, as in Décamps et al. (2011), is that the managers of the bank can engage in wasteful activities when the bank holds cash or other liquid assets. The resulting agency costs effectively reduce the rate of return on internally held liquid assets from \( r \) to \( r - \rho \). For simplicity I assume \( r = \rho \) as in Hugonnier and Morellec (2016).} Second, the bank operates in an economy characterized by stochastic financing conditions. Specifically, I assume that there are two observable states of the world \( i = G, B \). Each state provides the bank with different financing opportunities. Similar to Bolton et al. (2013), and Della Seta et al. (2017), in the good state \( G \), shareholders have access to outside equity markets at a fixed cost \( \gamma_E \). In the bad state \( B \), the market for external equity financing shuts down or equivalently the cost of raising outside equity funds is too high. However unlike the previous models with stochastic financing frictions, in this state the bank in my model has access to a new source of outside liquidity in the form of countercyclical contingent capital debt ("CoCo debt"). Specifically when the bank has no access to equity markets, it can raise outside funds by issuing CoCo debt at a fixed cost \( \gamma_C > \gamma_E \). A CoCo debt is a hybrid security that automatically converts to equity when a predetermined threshold is triggered. In addition to this general characteristic, the CoCo debt in my model has a countercyclical feature: it only converts to equity when there is no access to outside equity markets, i.e. in the bad state \( B \).

I design a CoCo debt instrument as a subordinated debt with continuous coupon payments \( C_{CD} \geq 0 \) which is issued at par with a face value of \( CD \) and an infinite maturity. So CoCo debt holders are entitled to receive \( C_{CD} \) for as long as the CoCo is not converted. This means that if conversion was never triggered, the CoCo debt would act as a standard debt contract with infinite maturity. However, as soon as SIFI's cash reserves fall to/below a pre-determined level \( \bar{W} \), the coupon payments to current CoCo holders stop and they instead receive a fixed fraction \( \alpha \) of the bank's equity. I assume that the bank's capital structure consists of deposits, CoCo debt and equity. Upon conversion, depositors who are the most senior claim-holders do not bear any losses and continue their right to receive the constant stream of interest payments; a fraction \( \alpha \) of equity is allocated to CoCo debt holders, and equity holders are entitled to the remaining \( 1 - \alpha \) fraction of the bank's equity.

The state switches from \( i \) to \( j \) with probability of \( \pi_{ij} \) where \( i, j = G, B \) and \( i \neq j \). Except from the costs associated with security issuance, the characteristics of the bank remain the same in both states of the world. Thereby I can single out the effects of stochastic costs of outside financing on the bank's optimal policy choices. To choose the optimal level of liquid asset holdings the bank balances the lower returns on liquid reserves inside the bank with their liquidity benefits. The benefits of inside liquidity depends on the costs of accessing outside liquidity and thus is state dependant. Consequently the bank's optimal pay-out policy is also state dependant.

To deal with the high social costs associated with SIFI's failure, I model a default-free set-up. To achieve such a set-up in an economy with financing frictions, I define a pre-determined plan that keeps SIFI's cash buffer above zero at all times.\footnote{A typical approach for models studying the capital structure of a firm, that has been applied by Albul et al.}
issuance is not too high. Thus raising new equity funds at a cost can maintain the positivity of SIFI’s cash reserves and thus its survival. When banks recapitalize willingly, the conversion of debt into equity is not necessary. However, in the bad state of the world (e.g. during a financial crisis) equity investors with a positive demand for SIFI’s capital are scarce or alternatively the cost of equity issuance is very high. Under such circumstances, the SIFI might not be able to maintain its positive cash reserves if there is no alternative to equity financing. So the bank can be forced into liquidation following a series of negative shocks. To avoid this, I define a bail-in plan. Under this plan, in state B the conversion of the CoCo debt into equity is triggered as soon as the bank’s level of cash reserves hit some low threshold. Whenever the conversion is triggered the whole amount of debt is converted into equity. However, the SIFI reissues new CoCo debt with the same characteristics (coupon payment, conversion threshold and conversion ratio) to replace the converted debt, leading to a capital injection. Such a bail-in contract can alternatively be regarded as a form of insurance policy that pays off in the bad state of the world. In this case, the coupon payments are considered as continuous insurance premiums paid by the SIFI to the insurer. On the other hand when SIFI does not have access to equity markets, the insurer commits to providing liquidity for the bank in lieu of newly issued CoCo debt and a fraction of the equity of the bank.\footnote{This type of contingent capital contracts which is also called capital insurance is discussed in Kashyap et al. (2008) and Hanson et al. (2011).}

The conversion of the CoCo debt and the injection of new capital into the bank upon the reissuance of countercyclical CoCo ensure that the level of bank’s cash reserves stays positive even when it does not have access to outside equity markets. With this dynamic mechanism in place, the bank stays immune to any future events of failure even after the conversion of its initially issued CoCo debt.

Although the conversion of debt into equity is an essential feature of the bail-in plan, the converted debt has to be replaced by new debt in order to preserve the bank’s cushion against future losses. I consider that the bank commits to a stationary CoCo debt structure. Specifically, at conversion the bank replaces the converted CoCo debt with new CoCo debt of identical coupon, conversion ratio and conversion threshold. Since the cost of reissuing the CoCo debt is fixed, the bank finds it optimal to issue enough debt to restore its cash buffer to its target level after paying the reissuance costs.

The SIFI in my model can increase its internal liquidity either by retaining earnings or by raising new equity funds in the good state or new CoCo debt in the bad state. Given the model’s assumptions, the bank’s cash reserves $W \equiv \{w_t, t \geq 0\}$ evolve according to

$$dw_t = (1 - \theta)(\mu - C)d t + \sigma d Z_t) - d L_t + d H_t - d X_t.$$  \hspace{1cm} (2.2)

In this equation $C = C_{CD} + C_D$ is the combined interest payment that the bank has to pay on its...
2.3 Straight debt benchmark

Deposits and CoCo debt. $L_t$ is a non-decreasing process that represents cumulative dividend payments to equity holders. I do not make any restrictions on $L$ apart from assuming that it is $\{\mathcal{F}; t \geq 0\}$-adapted and right-continuous, and that it is non-decreasing, reflecting limited liability (non-negative payments to shareholders). $H_t$ and $X_t$ are also non-decreasing adapted processes that represent SIFI’s cumulative external financing and SIFI’s cumulative external issuance costs respectively. This equation shows that the bank’s liquid reserves increase with after-tax earnings and outside financing and decrease with coupon payments to debt holders, dividend payments to shareholders and the costs associated with external financing.

The bank may be subject to leverage requirements. I define the debt ratio of the bank as the bank’s book value of liabilities (including deposits and market debt) to its book value of assets (including the book value of its risky assets and its cash reserves)

$$\phi(w) = \frac{CD + D}{w + \frac{(1-\theta)\mu}{r}}. \quad (2.3)$$

For given face values of market debt and deposits, this debt ratio is decreasing in cash reserves, and the bank attains its maximum debt ratio at its minimum level of liquid buffer. The regulatory leverage of the bank (Tier 1 leverage ratio) is given by $1 - \phi(w)$ which represents the ratio of the bank’s tangible equity to its book value of assets. When the bank initially issues market debt it may be required by regulators to constrain its minimum Tier 1 leverage ratio to a fixed level $\Lambda$. This regulatory requirement is equivalent to

$$D + CD \leq (1 - \Lambda)(w_{min} + \frac{(1-\theta)\mu}{r}) \quad (2.4)$$

where $w_{min}$ is the minimum expected level of the bank’s cash reserves.

Management chooses the bank’s pay-out and refinancing policies after the debt has been issued to maximize the present value of the future dividends to shareholders net of the expected costs of security issuance and capital injections. In the following, to better understand the dynamic of the model, I first analyse the bank’s policy choices and its optimal capital structure when it has only access to straight debt and equity and thus can default in the bad state of the world. I will then build on the results from this benchmark case to analyse the bail-in plan that provides a default-free set-up for a financial institution. Moreover, I will study the pay-out and refinancing policies of a SIFI in the presence of such a plan.

2.3 Straight debt benchmark

In this section, I analyse a benchmark case in which the only source of market debt financing available to the bank is straight debt. In the good state the costs of equity issuance is low enough and the bank finds it optimal to recapitalize when inside liquidity drops to a certain
Chapter 2. Bail-in Plan and Macroeconomic Conditions

level. In the bad state, the bank has no access to outside equity markets. When the bank incurs losses, it uses its cash reserves to absorb these losses, thus the level of bank's cash reserves decreases and its debt ratio increases. Following a series of negative shocks, the bank’s cash reserves can drop down to zero. If the bank is constrained to meet leverage regulatory requirements, it can no longer borrow when it runs out of cash and thus is forced to liquidate. Under these assumptions, I first analyse the values of bank's equity and straight debt and then characterize the policy choices of the bank and its optimal capital structure.

2.3.1 Valuing corporate securities

In the presence of financing frictions it is optimal for SIFIs to retain their earnings to build-up inside liquidity. At the same time, keeping cash inside the bank is costly. I assume that the opportunity cost of holding liquidity is constant. When the cash reserves are sufficiently high, the marginal benefit of saving more cash is decreasing in the level of liquid reserves. This coupled with the constant marginal cost of holding cash leads to the existence of a threshold above which it is optimal to pay dividends, since the marginal benefits of cash inside and outside the bank become equal. Since the cost of financing is state dependant, the target pay-out threshold also depends on the state of the world. \( W^*_i \) denotes the dividend threshold in state \( i = G, B \). Because outside financing is costlier in the bad state, I expect \( W^*_B > W^*_G \). This means that the bank holds a bigger cash buffer in the bad state than in the good state, consistent with the evidence in Acharya et al. (2010) or Aspachs et al. (2005).

I assume that the straight debt pays a continuous coupon of \( C_{SD} \) and has a face value equal to \( SD \). So the bank’s total interest payment is \( C = C_D + C_{SD} \) which includes the interest payments to depositors as well as debt holders. I denote the values of bank's equity in good and bad states of the world as \( E_G(w) \) and \( E_B(w) \) respectively.\(^6\) Consider first state G in which recapitalization is possible. There is a positive probability \( \pi_{GB} \) that the state switches form \( G \) to \( B \) at any moment. Upon this switch, the bank loses its access to outside liquidity. Because of this, shareholders may have incentives to issue new equity before the cash buffer is depleted. Since I assume a fixed cost of financing, for any level of liquid reserves smaller than \( W \in [0, W^*_G] \), the bank issues new equity to restore its cash reserves to the target level \( W^*_G \). A positive \( W \) means it is optimal for the bank to issue new equity before it runs out of liquid reserves.\(^7\) On the other hand any cash in addition to \( W^*_G \) is paid out as a lump sum dividend to the bank’s shareholders, since there is no benefit in keeping it inside the bank. This means that in state \( G \), the cash reserves evolve in \( [W, W^*_G] \). In this region, it is optimal to retain earnings and thus

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5 In general, there exists a critical cost of issuance above which issuing securities is prohibitively costly and thus the bank prefers to default. If the issuance costs are lower than this critical cost, shareholders prefer to issue new shares and avoid bankruptcy. I assume here that the cost of issuing equity in the good (bad) state is lower (higher) than this critical cost. See Décamps et al. (2011) for more details.

6 For tractability I omit the dependence of equity value on \( (C, W^*_i, W) \).

7 If there was only one state of the world, the equity issuance would have been delayed as much as possible because of the costs associated with security issuance. So in a one state model with straight debt reissuance only happens when the bank runs out of cash.
equity value satisfies the following ODE:

\[ r E_G(w) = (1 - \theta)(\mu - C)E'_G(w) + \frac{1}{2}(1 - \theta)^2 \sigma^2 E''_G(w) + \pi_{GB}(E_B(w) - E_G(w)). \]

(2.5)

The left hand side of this equation represents the required rate of return for investing in the bank's capital. The right hand side represents the expected change in the value of equity. The first two terms represent the effects of cash savings and cash flow volatility. The last term captures the effects of time varying financing frictions. This term is the product of the probability of a state switch from good to bad and the change in the value of equity upon this switch from \( E_G(w) \) to \( E_B(w) \).

Consider now the bad state of the world. In this state, the bank retains earnings in \([0, W^*_B]\), where \( W^*_B > W^*_G \). Financing frictions are very severe in this state, and it is never optimal for shareholders to recapitalize. So the bank is forced into liquidation as soon as its cash reserves are depleted. I assume that liquidation is inefficient and that the liquidation value of the risky assets of the bank represent a fraction of their first best value. If \( \phi \in (0, 1] \) represents the costs associated with liquidation, the liquidation value is given by

\[ l = (1 - \phi) \frac{(1 - \theta)\mu}{r}. \]

Depositors are the most senior claim holders and in the event of liquidation get paid before subordinated market debt holders. If the liquidation value is enough to repay the face value of deposits, the residual is paid-out to debt holders. If not, the difference between the face value of deposits and the liquidation value is covered by deposit insurance. In this case, depositors are fully paid-out but debt holders receive nothing. I assume that the liquidation value of the assets is smaller than the face value of the total debt (deposits plus market debt). This means that the market debt is risky. When the bank is forced into liquidation, all proceeds accrue to depositors and debt holders, and shareholders receive nothing.

Due to the time varying financing conditions the boundaries of the cash region are state dependent. Whilst cash reserves evolve in \([0, W^*_B]\) in the bad state, they evolve only in \([W, W^*_G]\) in the good state. If the state switches from bad to good when the level of cash reserves is in the region of \([0, W]\), an immediate recapitalization restores the bank's cash buffer to \( W^*_G \). This is because in the good state it is optimal for the bank to issue equity when its cash buffer drops below \( W^*_B \). On the other hand if the state switches from bad to good when \( w \in [W^*_G, W^*_B] \), the bank distributes a lump sum of dividend equal to \( w - W^*_G \) since there is no benefit to keeping any cash in addition to \( W^*_G \) inside the bank. So the cash buffer goes down to \( W^*_G \). As a result, there are three different ODEs for the value of the equity in the bad state of the world depending on the level of cash reserves:
The equations in (2.6) are similar to (2.5) except for the third terms on the right hand side which capture the effects of a state switch from B to G. The difference comes from the fact that the variation in the value of equity upon this state switch depends on the level of cash at which this switch happens. Equity value changes from \( E_B(w) \) to \( E_G(W^*_G) + w - W^*_G - \gamma_E - E_B(w) \) when the switch occurs in the range of \([0, W]\) due to the immediate recapitalization. If the switch occurs when the cash reserves are in the range of \((W, W^*_G]\), the value of equity goes from \( E_B(w) \) to \( E_G(w) \). If the state switches when the cash buffer is in the range of \((W^*_G, W_B]\), the value of equity goes from \( E_B(w) \) to \( E_G(W^*_G) + (w - W^*_G - E_B(w)) \) due to the instantaneous dividend distribution in the good state.

Equations (2.5) and (2.6) are solved subject to the following boundary conditions. First, in both states of the world, at dividend threshold, the marginal value of the cash inside the bank is equal to the marginal value of the cash outside the bank, so there is no benefit in retaining more cash. Thus the bank distributes \( w - W^*_i \) as a lump sum of dividend, and for all \( w \geq W^*_i \) we have

\[
E_i'(w) = E_i(W^*_i) + w - W^*_i.
\]

After subtracting \( E_i(W^*_i) \) from both sides, dividing by \( w - W^*_i \), and taking the limit where \( w \) goes to \( W^*_i \), the following condition holds

\[
E_i''(W^*_i) = 1.
\]

The dividend threshold that maximizes the value of equity is determined by the smooth pasting condition (as in Dumas (1991)):

\[
E_i''(W^*_i) = 0.
\]

When the bank incurs losses, the level of cash reserves decreases and the marginal value of cash inside the bank increases. Below a threshold, it can be optimal for the bank to issue new equity. This only happens in the good state of the world since in the bad state the cost of equity
issuance is too high or there is no demand for bank’s equity. So for any \( w \leq W \) in the good state, the bank raises new funds. When the financing cost is fixed, it is optimal for the bank to raise enough to reset its cash buffer to \( W^*_G \). So

\[
E_G(W) = E_G(W^*_G) - (W^*_G - W) - \gamma_E.
\]

To recapitalize, shareholders wait until the marginal value of cash inside the bank is equal to the marginal cost of raising new funds which is one. Thus

\[
E'_G(W) = 1
\]

should hold when \( W > 0 \).

In state B, outside financing is not available to the bank. Thus when the cash buffer is depleted, the bank is forced into liquidation. Since I assume that the liquidation value of the bank is smaller than the face value of the bank’s total debt, shareholders receive nothing in liquidation and

\[
E_B(0) = 0
\]

should hold.

In addition to these boundary conditions, I need to impose the continuity and smoothness conditions at \( W \) and \( W^*_G \) to ensure that the different regions for the value of equity in the bad state are smoothly pasted:

\[
\lim_{w \downarrow W^*_G} E_B(w) = \lim_{w \uparrow W^*_G} E_B(w),
\]

\[
\lim_{w \downarrow W^*_G} E'_B(w) = \lim_{w \uparrow W^*_G} E'_B(w),
\]

and

\[
\lim_{w \downarrow W} E_B(w) = \lim_{w \uparrow W} E_B(w),
\]

\[
\lim_{w \downarrow W} E'_B(w) = \lim_{w \uparrow W} E'_B(w),
\]

Consider next the value of straight debt \( SD_i(w) \) (where I have dropped the dependence of debt value on other arguments \((C, W^*_i, W)\) for tractability). Debt holders receive a continuous coupon payment of \( C_{SD} \) in both states of the world for as long as the level of bank’s cash reserves is positive. In state G the bank retains earning for \( w \in [W, W^*_G] \) and optimally recapitalizes when its cash buffer hits \( W \). So in this state, the bank is never liquidated and the
value of the straight debt in the earnings retention region satisfies:

\[ rSD_G(w) = C_{SD} + (1 - \theta)(\mu - C)SD'_G(w) + \frac{1}{2}(1 - \theta)^2 \sigma^2 SD''_G(w) + \pi_{GB}(SD_B(w) - SD_G(w)). \]  

(2.7)

The left hand side of equation (2.7) is the rate of return required by straight debt holders. The right hand side of the equation captures the total change in the value of the straight debt in the good state. The first term is the coupon payment to debt holders. The second and the third terms represent the effects of a change in cash reserves and cash flow volatility. The last term captures the change in the value of straight debt due to the time varying financing conditions.

Similar to equity value, in the bad state of the world the value of straight debt solves three different equations depending on the level of the bank’s cash buffer:

\[
\begin{align*}
    rSD_B(w) &= \begin{cases} 
        C_{SD} + (1 - \theta)(\mu - C) & SD'_B(w) + \frac{(1 - \theta)^2 \sigma^2}{2} SD''_B(w) + \pi_{BG}(SD_G(w_B^*) - SD_B(w)) & w \in (0, W] \\
        C_{SD} + (1 - \theta)(\mu - C) & SD'_B(w) + \frac{(1 - \theta)^2 \sigma^2}{2} SD''_B(w) + \pi_{BG}(SD_G(w) - SD_B(w)) & w \in [W, W_G^*] \\
        C_{SD} + (1 - \theta)(\mu - C) & SD'_B(w) + \frac{(1 - \theta)^2 \sigma^2}{2} SD''_B(w) + \pi_{BG}(SD_G(w_G^*) - SD_B(w)) & w \in (W_G^*, W_B^*)
    \end{cases}
\end{align*}
\]

(2.8)

The last term on the right hand side of each of these three equations is the change in the debt value following a jump from the bad to the good state. If the state switches when cash reserves are in \([0, W]\), the bank immediately issues new shares and takes the level of cash buffer back to \(W_G^*\). So the value of the straight debt goes from \(SD_B(w)\) in the bad state to \(SD_G(w_G^*)\) in the good state. When cash reserves are in \([W, W_G^*]\), earnings are retained in both states, and if the state jumps from \(B\) to \(G\), the value of debt goes from \(SD_B(w)\) to \(SD_G(w)\). When the cash reserves are in \((W_G^*, W_B^*)\) the level of the bank’s cash buffer is higher than the pay-out threshold in the good state. Because of this, a switch from \(B\) to \(G\) results in an immediate lump-sum dividend distribution. Since all of this dividend accrues to shareholders the value of straight debt goes from \(SD_B(w)\) in the bad state to \(SD_G(w_G^*)\) in the good state.

Equations (2.7), and (2.8) are solved subject to the following boundary conditions. First, in the good state issuing new shares every time cash reserves hit \(W\) restores the bank’s cash buffer to \(W_G^*\). Thus the following holds:

\[ SD_G(W) = SD_G(W_G^*). \]

Second, the value of the straight debt does not change when dividends are distributed since
2.3. Straight debt benchmark

Dividends are only paid out to shareholders. So

$$SD_i'(W_i^*) = 0$$

holds. Lastly, the bank is liquidated in the bad state of the world when it runs out of cash. Thus at zero, straight debt holders receive the residual of the liquidation value of the bank’s risky assets after depositors are paid-out. If the liquidation value of the assets is smaller than the face value of deposits, debt holders receive nothing in liquidation. Thus the following holds

$$SD_B(0) = (1 - \phi)(1 - \theta)\frac{(1 - \theta)\mu}{r} - D)^+.$$ 

In addition to this, for the value of the debt to be continuous, conditions similar to the case of equity value should hold at $W$ and $W^*_G$:

$$\lim_{w \downarrow W_G} SD_B(w) = \lim_{w \uparrow W_G} SD_B(w)$$

$$\lim_{w \downarrow W_G} SD_B'(w) = \lim_{w \uparrow W_G} SD_B'(w),$$

and

$$\lim_{w \downarrow W} SD_B(w) = \lim_{w \uparrow W} SD_B(w)$$

$$\lim_{w \downarrow W} SD_B'(w) = \lim_{w \uparrow W} SD_B'(w),$$

Closed form expressions for the values of equity and straight debt in both states of the world are provided in the Appendix.

2.3.2 Optimal bank policies

Having determined the values of the bank’s equity and debt in both states of the world, I now turn to the analysis of the optimal bank policies. In the straight debt benchmark, there is no conversion of debt and management chooses the bank’s pay-out, default and financing policies that maximize the present value of the future dividends to shareholders after the debt has been issued. I first analyse the shareholders optimal policy choices assuming baseline values for the parameters of the model. I will then investigate how these policies are affected in different economic environments.

Table 2.1 reports the baseline values of the model parameters, as well as the endogenous values implied by the model. I set the risk free rate of return to $r = 0.035$, the tax rate to $\theta = 0.20$, the average cash flow rate to $\mu = 0.15$, and the cash flow volatility to $\sigma = 0.08$. I assume that the probability of a jump from the bad to the good state is $\pi_{BG} = 0.20$ and the probability of a jump from the good to the bad state is lower and equal to $\pi_{GB} = 0.18$. The issuance costs of
equity in the good state is set to $\gamma_E = 0.01$. I set the value of the liquidation costs to $\phi = 0.30$, which means the liquidation value of the bank's assets is equal to 2.4. I assume that the bank takes deposits (D) in an amount equal to 50% of the first best value of its risky assets. I set the costs of deposits to $C_D = 2\%$. Assuming the model parameters, the bank holds 1.71 in deposits. So the value to straight debt holders in liquidation is equal to 0.69. Shareholders choose the value maximizing bank policies after they have issued the debt. Since debt holders have rational expectations, the value of the debt reflects these policies. In this section, I set the coupon rate to $C_{SD} = 8.24\%$. In the next section I show that this is the optimal coupon level that shareholders choose to maximize the bank's value minus the cost of capital injection at debt issuance. I also study the case where the coupon payment is set to $C_{SD} = 5.79\%$. In the next section, I discuss that this is the coupon payment that shareholders choose when the bank is regulated and thus is subject to a Tier 1 leverage ratio requirement of $\Lambda$ which is set to 4\%.

Following the literature that studies firms' optimal dividend policies when outside financing is costly (including Décamps et al. (2011), Bolton et al. (2011, 2013), Hugonnier et al. (2015), and Hugonnier and Morellec (2016) among others), the optimal pay-out policy for shareholders in both states of the world is of barrier type: the bank distributes dividends to maintain its cash buffer at or below a constant threshold. Since financing frictions depend on the state of the world, it is natural to expect that the bank's policy choices are also state dependant. Shareholders have more incentives to build inside liquidity when the cost of accessing outside liquidity is higher, so dividend threshold is increasing in the issuance costs (see Décamps et al. (2011) for more details). Thus the higher issuance cost in the bad state compared to the good state, leads to a higher dividend threshold in this state.

In the bad state the bank is forced into costly liquidation following a series of negative operating shocks since it cannot raise new funds to cover operating losses when it runs out of internal funds. On the other hand, bank optimally raises new funds in the good state to avoid liquidation because the costs of refinancing is not too high. Although shareholders would like to delay this refinancing due to the costs associated with security issuance, they might find it optimal not to wait until the last moment (when the cash buffer is depleted) to raise new funds. This is due to the threat of liquidation that they face if the state switches from G to B when cash reserves are low. Thus unlike the one state model when the reissuance threshold is always set to zero, a positive refinancing threshold can be optimal.

Table 2.1 shows the optimal target levels of cash buffer in both states of the world. For the baseline parameters and $C_{SD} = 5.79\%$, shareholders distribute dividends to maintain liquid reserves at or below 0.210 (5.78\% of the total asset value) in the good state and 0.253 (6.94\% of the total asset value) in the bad state. Thus the bank's liquid holdings are countercyclical, in line with the evidence in Acharya et al. (2010) and Aspachs et al. (2005). In the good state shareholders find it optimal to tap equity markets when the level of the cash buffer drops to 0.060 which represents 1.64\% of the total asset value. Upon this issuance shareholders will refinance the bank to restore its cash holdings to the target level in the good state by injecting
2.3. Straight debt benchmark

an amount equal to 0.150 \((W^*_G - W)\). When the coupon payment is set to \(C_{SD} = 8.24\%\), the bank's target level of cash in the good and the bad states are equal to 0.238 and 0.285 which represent 6.50\% and 7.77\% of the total asset value of the bank respectively. For this level of coupon payment, in the good state shareholders raise new equity funds when the level of cash reserves drop to 0.069 (1.89\% of bank's total asset value).

Table 2.1 also shows the ranges for the debt ratio of the bank as defined in (2.3) but for the face value of straight debt \(SD\). Even though the value of deposits and the coupon payment on the debt are fixed, the bank's debt ratio fluctuates within a band since the cash buffer of the bank fluctuates in \([0.060, 0.210]\) in the good state and in \([0, 0.253]\) in the bad state for \(C_{SD} = 5.79\%\). As the table shows for this level of coupon payment, the debt ratio in the good state fluctuates between 92.03\% at the target level of cash and 96.00\% at the point where the bank raises new funds. In the bad state, the debt ratio fluctuates between 90.96\% at the target level of cash and 97.67\% at the point where the bank runs out of cash and defaults.

Figure 2.1 top panel shows the values of straight debt in both good and bad states as functions of the bank's cash reserves and for \(C_{SD} = 5.79\%\). The value of the debt in the good state is the highest at the boundaries of the region in which cash reserves evolve \([W, W^*_G]\). At \(W\) the value of debt is \(SD_G(W^*_G)\) since new equity is issued to restore the bank's cash reserves to the target level. After \(W\), increasing cash reserves has two opposing effects on the value of debt: first it decreases the probability of a reissuance. Second increasing the cash buffer decreases the probability of default which can occur if the state switches to the bad state and the bank runs out of cash. For lower levels of cash buffer the first effect dominates and the value of debt decreases the further cash buffer gets from the reissuance threshold. For higher levels of cash buffer the lower probability of default in the bad state dominates and the value of debt increases with cash buffer. When cash reserves reach the target level, any additional cash accrues to shareholders as dividend and the value of debt remains at its maximum value of \(SD_G(W^*_G)\). In the bad state the value of debt is increasing in cash reserves since the probability of an inefficient liquidation is lower for higher levels of cash. Note that compared to the value of debt in the bad state, the value of debt is less sensitive to the level of cash buffer in the good state because the risk of default in this state is zero.

The middle panel of Figure 2.1 shows the value of equity in states G and B. The bottom panel shows the marginal value of cash to equity holders in these two states. In the bad state since the bank has no access to outside financing, it has a precautionary motive to hoard cash. The value of equity is increasing and globally concave in cash reserves. On the other hand, the value of equity in the good state of the world is increasing in cash reserves but not globally concave. In models featuring time invariant financing constraints, limited access to outside liquidity gives rise to inefficient liquidations and provides a motive for precautionary cash savings. In these models, shareholders behave in a risk averse manner and the value of equity becomes a concave function of cash reserves. Introducing time dependant financing conditions provides incentives for bank's shareholders to time equity markets in the good state because if they do not do so, they may lose the opportunity of accessing cheap equity.
financing in the future. So shareholders may optimally issue new equity before the bank runs out of cash. Equity issuance is lumpy when it happens because the associated cost is fixed. Upon this issuance, shareholders incur the costs of security issuance and the carry costs of the additional cash holdings since the bank's cash reserves immediately increase. On the other hand by issuing equity faster they make sure to take advantage of cheap financing option before it vanishes. When choosing their financing policy, shareholders balance these costs and benefits. When the level of cash reserves is high, the option to issue equity is not so valuable since the bank is not in immediate need of external funds. For high enough levels of cash reserves the precautionary motive to keep cash leads to an increasing and concave equity value. On the other hand, when cash reserves are low, the option to tap equity markets before financing conditions worsen becomes very valuable. The time varying financing constraints along with the fixed costs associated with security issuance, can generate a local convexity in the value of equity for low levels of cash holdings in the state with more favourable financing conditions. This convexity gives rise to risk-taking incentives. In that, shareholders may find it optimal to increase asset risk when the bank is close to financial distress. The bottom panel of Figure 2.1 shows that in-line with this discussion, the marginal value of cash to equity holders is positive but not monotonic in the good state.\(^8\)

To study the effects of the main model parameters on bank's policy choices, Figure 2.2 plots the reissuance threshold, and the dividend thresholds in the bad and good states of the world as functions of the coupon payment \(C_{SD}\), the financing cost of equity \(\gamma_E\), and the switching intensities \((\pi_{BG}, \pi_{GB})\).

Coupon payments are cash outflows which drive down the bank's cash reserves. So for a given profitability \(\mu\), a higher coupon payment results in a lower drift. For the lower levels of coupon payment, the continuation value of the bank to equity holders is high enough to induce the shareholders to keep the bank alive by waiting longer before distributing any dividends. This is why increasing coupon payments increases the dividend thresholds in both states of the world for lower levels of coupon payment. When coupon payments are very high, continuation value to shareholders become very small and they lose their incentives to keep the bank alive. Because of this, for very high levels of coupon payment shareholders start distributing dividends faster, when interest payments increase. There is a similar pattern for reissuance threshold. When increasing the interest payments on debt, shareholders first find it optimal to tap equity markets faster in anticipation of worsening financing conditions. At the point where interest payments become so big that shareholders lose their interest in the bank survival, they delay any equity issuance further to avoid the costs associated with such an issuance.

To optimally choose their dividend policy shareholders balance the carry cost of cash with the external cost of financing. As a result, when the cost of equity issuance increases shareholders wait longer before they recapitalize. Additionally it becomes optimal to delay costly recapitalizations by retaining more cash inside the bank. This is why both dividend thresholds increase

\(^8\)For a more detailed discussion of the convexity of the value of equity in the good state, see Bolton et al. (2013).
whilst reissuance threshold decreases when the cost of equity issuance in the good state rises. When in the good state, an increase in the risk of switching to the bad state (a higher $\pi_{GB}$) where outside financing is too costly increases the precautionary demand for cash. This gives incentives to shareholders to build up more cash inside the bank. They do so by both delaying dividend payments and accelerating equity reissuance. Thus both reissuance and dividend threshold in the good state increase with $\pi_{GB}$. On the other hand when in the bad state, an increased probability of recovery from this state (a higher $\pi_{BG}$) leads to a higher likelihood of future access to outside financing and thus decreases the value of the cash inside the bank. As a result, the bank’s cash inventory drops through a combination of increased dividend pay-outs and delayed equity financing. Thus both reissuance threshold and dividend threshold in the bad state decrease with $\pi_{BG}$.

Figure 2.1 also shows that changing different parameters of the model does not change the fact that the target level of cash in the bad state is higher than the one in the good state because financing constraints are more severe in this state.

I have so far characterized the bank policy choices for a given coupon payment. In the next section, I study the optimal capital structure that bank shareholders choose to maximize bank’s value at the time of issuance.

### 2.3.3 Optimal capital structure

Having discussed bank’s optimal policies, I now investigate the privately optimal financing structure of the bank. I consider that the bank which is initially in the good state of the world issues some perpetual debt with coupon $CSD$ and infinite maturity to take advantage of the tax exemption of the interest payments. Paying coupon payments to debt holders depletes the bank’s cash reserves. This increases the default risk in the bad state. Shareholders receive zero upon bank’s liquidation. At the same time in the good state of the world, the level of coupon payments affects the reissuance and thus expected refinancing costs. The value maximizing level of debt is determined by balancing its tax benefits with its default and refinancing costs.

Shareholders choose the optimal level of the bank’s debt by maximizing the value of equity after debt issuance plus the proceeds from this issuance net of the costs of providing the required capital. Assuming that the bank has no initial liquid reserves before debt issuance ($w_0^- = 0$), the following static maximization problem decides the optimal capital structure of the bank

$$\max_{CSD \in \mathbb{R}^+} E_G(W^*_G(C_{SD})) + SD_G(W^*_G(C_{SD})) - W_G^*(C_{SD}).$$

I assume that the failure of the bank imposes negative externalities on the financial system and thus leads to significant social costs that are not internalized by bank’s shareholders. Higher levels of debt increase the probability of bank’s default and thus the expected social costs.
associated with its failure. This means that from a regulatory point of view the cost of debt does not only include its liquidation and refinancing costs, but also the social costs associated with the disruptive effects of the bank’s failure on the whole economy. So debt is more costly for regulators compared to shareholders. Shareholders who do not include the social costs of debt in their optimization problem tend to take on more debt than what would be socially optimal.\footnote{In this context see also Sigrist and Rochet (2017) who study the optimal mode of financing for SIFIs from both private and public points of view and argue that the presence of SIFIs’ failure externalities is the justification for regulating their financing mode.}

For this reason, I assume that the bank is constrained by regulatory leverage requirements as described in Section 2.2. In the good state, when the bank initially chooses its optimal capital structure, the bank’s maximum debt ratio (which is attained at equity reissuance threshold) should not exceed the regulatory required level of $1 - \Lambda$. So I append the constraint defined in (2.4) when $w_{min} = W$ to shareholders’ optimization problem in (2.9).

With the baseline parameters reported in Table 2.1, shareholders choose a coupon payment equal to 8.24% if the bank is not regulated and 5.79% when the bank is required to maintain a Tier 1 leverage ratio of at least 4%. Although I assume that the level of bank’s deposits and the coupon payment on straight debt remain constant, the dynamic of the bank’s cash reserves results in a dynamic capital structure. When the bank is regulated, for the baseline parameters, the leverage constraint is binding and the bank is constrained to choose a smaller level of coupon payment. For this level of $C_{SD}$, the bank’s debt ratio fluctuates in the range of $[92.03, 96.00]$ in the good state and $[90.96, 97.67]$ in the bad state. When the bank runs out of cash in the bad state of the world, its debt ratio increases to 97.67 which is more than the maximum regulatory level of 96%. The bank does not have access to outside equity in this state and thus can not maintain its minimum regulatory capital to asset ratio. As a result, the bank can not issue additional debt to inject new capital inside the firm and thus defaults.

I investigate the effects of varying the main parameters of the model on the optimal capital structure and policies of the regulated bank by varying these parameters around their base case values and thus considering different cases reported in Table 2.2.

I study the effects of asset risk by considering a low volatility environment where $\sigma$ is set to 0.06 and a high volatility environment with $\sigma$ equal to 0.10. When the bank’s leverage constraint is binding, shareholders can choose a higher level of coupon payment when the bank is riskier. This is because the face value of debt is decreasing in the volatility of the bank’s risky assets. Panels B and C of Table 2.2 show that shareholders can issue debt with a coupon payment equal to 5.95% when the volatility of the bank’s cash flows is 0.10, but can only choose a coupon payment equal to 5.66% when the volatility is set to 0.06. If the leverage constraint was not binding or the bank was unregulated, shareholders would choose more conservative debt levels when assets are more risky since higher volatility leads to an increase in default probability and thus increases the cost of market debt. In Table A.1 in the Appendix, I report the optimal capital structure and financing policies of an unregulated bank when varying the main parameters of the model. As the table shows shareholders of an unregulated bank
choose a lower level of debt \( (C_{SD} = 7.65\%) \) when bank's assets are riskier \( (\sigma = 0.10) \). Table 2.2 also shows that an increase in asset risk leads to an increase in the frequency of refinancing and a decrease in pay-outs to shareholders in both states of the world. Changing \( \sigma \) from 0.10 to 0.06, decreases the reissuance threshold from 0.090 to 0.032, the target level in state G from 0.283 to 0.138 and the target level in state B from 0.342 to 0.167.

I also study the bank's behaviour when the probability of getting shut out of financial markets is higher \( (\pi_{GB} = 0.20) \) or lower \( (\pi_{GB} = 0.16) \). When \( \pi_{GB} \) is higher, the expected duration of the good state is shorter. In response, the bank increases its liquid reserves by both raising new funds sooner and paying dividends later. Panels F and G of Table 2.2 show that by increasing the risk of deteriorating financing conditions from 0.16 to 0.20, \( W \) and \( W^*_G \) increase to 0.063 and 0.212 from 0.056 and 0.207 respectively. Moreover for higher levels of \( \pi_{GB} \), probability of default that happens in the bad state of the world increases and thus the value of straight debt is decreasing in \( \pi_{GB} \), this allows shareholders to issue more debt when \( \pi_{GB} \) is higher and when the leverage constraint is binding. Panels F and G of Table A.1 in the Appendix show that when the bank is not regulated, shareholders choose less debt for higher levels of \( \pi_{GB} \) because of the increased probability of default.

Finally, I investigate the effects of the severity of financing frictions by changing the reissuance costs of equity in the good state \( \gamma_E \) around its base case value and set \( \gamma_E \) to 0.005 and 0.02.\(^{10}\) The lower the costs associated with equity financing, the earlier the bank taps equity markets to raise new funds. Panels D and E of Table 2.2 show that when outside financing cost decreases from 0.02 to 0.005, the reissuance threshold rises from 0.042 to 0.075. When outside financing becomes less costly in the good state, the precautionary incentives of hoarding cash inside the bank are weaker and thus dividends are paid-out faster. Panels D and E demonstrate that the magnitude of the effects of the refinancing costs is smaller on pay-out boundaries than reissuance threshold. Coupon payments are cash outflows that drive down the level of the bank's liquidity buffer and thus raise its need to external liquidity. When external liquidity is more costly, it is optimal to issue less debt. Table 2.2 shows that the optimal level of coupon decreases from 5.85 to 5.74 when the cost of issuing equity increases from 0.005 to 0.02.

### 2.4 Bail-in plan: the case of countercyclical CoCo

Having studied the effects of time varying financing conditions on bank's policies with straight debt, I now turn to a set-up where the bank commits to a pre-determined bail-in plan. When the bank is systemically important to the financial system, and recapitalization is too costly to be privately optimal, financial authorities are forced to bail-out the bank using public funds to avoid the disruptive effects of the SIFIs collapse on the whole financial system. A bail-in plan is an alternative solution to deal with this too big to fail problem. In my simplified bail-in framework, in the bad state of the world the bank has access to another source of

\(^{10}\) For both these cases, the costs associated with security issuance in the bad state are assumed to be so high that security issuance in this state remains suboptimal.
outside liquidity in the form a countercyclical CoCo debt. The main feature of this security is its automatic conversion to equity when the bank faces financial distress and when equity financing is not possible (optimal) for bank's shareholders. Moreover since the bank's debt obligation converts into equity when it faces financial difficulties, the post-conversion debt ratio of the bank drops down. This in turn allows the bank to be able to borrow again and inject new capital inside the bank. This is in contrast with the case of straight debt in which higher than regulatory acceptable debt ratios constrain the bank from issuing new debt when it runs out of cash. Having access to this mode of financing in state B, shareholders update their optimal choices of pay-out and financing policies and their optimal capital structure.

The objective of a bail-in plan is to eliminate the possibility of bank's failure and its associated social costs. If shareholders could commit to a bail-in plan, the costs of debt would be fully internalized and the level of debt that maximizes the value to shareholders would also be regulatory optimal. Thus if the execution of the bail-in plan is efficient, one can argue that it can replace (at least partially) banking regulation. In this section, I first study the case where the bank does not face any regulatory leverage requirements because it has committed to a bail-in plan. I will then study the case of a bank with a bail-in plan that faces regulatory leverage requirements and compare the two cases.

2.4.1 Valuing corporate securities

When in state B, losses due to negative operating shocks are absorbed by the bank's internal cash buffer until the bank's cash inventory drops to a given threshold $\bar{W}$. At this point, the bank's outstanding debt is converted into equity. Upon this conversion current equity holders give up a fraction $\alpha$ of the bank's equity to CoCo debt holders.\footnote{As previously noted, bail-in plan acts as a pre-determined restructuring plan. When conversion is triggered, depositors who are the most senior claim holders continue to receive the same fixed interest payments as before and thus do not bear any losses. The interest payments to current CoCo debt holders stop and they receive a fraction $\alpha$ of the bank's equity. Equity holders receive the residual fraction $1 - \alpha$ of the bank.} I keep the assumption that the equity financing cost in state B is too high for any equity issuance to be optimal. However, unlike the benchmark case of straight debt where no financing is possible (or optimal) in state B, the bank can reissue CoCo debt by paying a fixed cost of $\gamma_C$. This cost is higher than the costs associated with equity issuance in the good state. Nonetheless, it is still lower than the costs of issuing equity in the bad state when investors are reluctant to invest in bank's equity. Reissuance of CoCo debt upon conversion in the bad state along with equity refinancing in the good state provides a default free set-up.

Although with a bail-in plan the bank never faces liquidation, it still faces higher costs of refinancing with CoCo debt in state B. This means that state B is still characterized by more severe financing constraints. In state G, the bank has a finite window of opportunity to issue equity which is the cheaper source of financing. For low levels of cash reserves, if shareholders do not exercise this option, they can face higher refinancing costs if the state switches to B and conversion is triggered. Depending on the relative refinancing costs (equity in state G
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versus CoCo debt in state B), and the probability of a jump from G to B, shareholders may find it optimal to tap equity markets to raise new equity funds in the good state before the bank runs out of cash. So the optimal reissuance threshold in state G can be strictly positive. Similar to the case of straight debt, more severe financing frictions in the bad state lead to a larger level of target cash buffer in this state and thus $W_B^*$ is expected to be bigger than $W_G^*$.\(^\text{12}\) So bank's cash holdings remain countercyclical.

Consider now the value of the bank's equity. I keep the same notation $E_i(w)$ for the equity value in state $i = G, B$ (where I omit the dependence on $(C, a)$). In the good state the cash reserves evolve in the region of $[W, W_G^*]$. In this region the bank does not deliver any cash flows to shareholders. Above $W_G^*$, any additional cash is paid out as dividends and below $W$, the bank recapitalizes to restore the cash buffer to its target level. So the value of equity in the good state satisfies the same ODE as the value of equity with straight debt which is given in (2.5).

In the bad state of the world, the bank reissues CoCo debt upon the conversion of the existing debt at a given threshold $\bar{W}$. To do so, shareholders incur a fixed cost $\gamma_C$. Since this cost is higher than the equity issuance costs in the good state, it is optimal for shareholders to delay the conversion and the reissuance of the CoCo debt for as much as possible. This means that shareholders wait until internal cash reserves are depleted before they convert the existing debt. To better understand this, consider a series of negative operating shocks. The bank draws down on its cash reserves to cover operating losses. When the level of cash reserves are low, at any point in time shareholders can either convert the existing CoCo debt and replace it with new debt that restores the cash buffer to its target level or wait for a jump to the good state of the world. If the state switches from B to G before CoCo debt is converted, shareholders have the option to raise new funds at a lower cost in the good state. If the state does not switch before the bank runs out of cash, shareholders convert and reissue the CoCo debt at the last moment, i.e. when the level of cash buffer hits zero. Given that the bank has access to a cheaper source of outside liquidity in the good state and that carrying cash inside the bank is costly, shareholders optimally postpone the conversion of CoCo debt to when all internal funds are exhausted. This leads to the following proposition:

**Proposition 2.1.** The optimal conversion threshold is zero.

A direct implication of Proposition (2.1) is that, the optimal bail-in plan is a contract that acts as a resolution mechanism. Such a plan is considered to be an automatic restructuring contract that is set in place ex-ante in order to prevent the costly restructuring process of a SIFI in the event of financial distress.\(^\text{13}\) Given the results in Proposition (2.1), I set $\bar{W} = 0$ in

\(^{12}\)For the sake of simplicity, I have kept the same notations for reissuance and dividend thresholds. Their values are, however, different in straight debt and CoCo debt cases.

\(^{13}\)There are costs associated with both chapter 11 and private negotiation debt restructuring. These costs include direct (out of pocket transaction costs such as charges for legal and investment banking services) or indirect costs (time and effort negotiating with different parties involved such as banks, creditors, and authorities). See Gilson et al. (1990), Betker (1997), and Hotchkiss et al. (2008) for more on distressed debt restructuring. See James (1991)
my analysis. So cash reserves evolve in \([0, W_B^*]\) in the bad state of the world and the value of equity in this state \(E_B(w)\) follows the same system of equations as in (2.6).

The equations for the values of equity in both states of the world (which are included in Appendix A.2.2) are solved subject to the following boundary conditions. The first boundary condition

\[ E_i'(W_i^*) = 1 \]

represents the fact that at target level of cash, the marginal benefit of cash inside the bank is equal to its marginal cost which is one since the costs associate with security issuance are assumed to be fixed. The second boundary condition

\[ E_i''(W_i^*) = 0 \]

gives the value maximizing target level of cash (see Dumas (1991)).

In the good state of the world, if the bank's cash buffer decreases to a sufficiently low level, the bank raises new equity by paying the issuance cost of \(\gamma_E\). The threshold below which shareholders find it optimal to raise new funds \(W\) is the level of cash reserves at which the marginal value of cash to shareholders is equal to the marginal cost of raising new equity which is one due to the fixed reissuance costs. So when \(W > 0\), the following should hold

\[ E_G'(W) = 1. \]

When there is no \(W\) that satisfies this condition, it is not optimal for the bank to raise new equity funds before it runs out of cash. In this case, \(W = 0\) and the bank only taps equity markets when its cash reserves are depleted. The threat of the tightening of financing conditions when cash buffer is close to zero, can lead to non-zero reissuance thresholds in the good state. A positive \(W\) means that the bank raises new funds in the equity market before it runs out of cash.

Up until now, the analysis of the value of equity closely follows the benchmark case of straight debt. The main difference in the solutions to the values of equity in the straight debt and CoCo debt cases comes from the boundary conditions on the value of equity in the bad state of the world. In the case of straight debt, in the bad state of the world the bank is forced into liquidation as soon as its cash buffer is exhausted. Shareholders receive nothing in the event of liquidation. By contrast, in the case of CoCo debt as soon as the level of cash reserves hit the conversion threshold, the bank converts its existing CoCo debt and issues new debt of identical terms \((C_{CD}, \alpha)\) to restore its cash buffer back to \(W_B^*\). The issuance of new CoCo debt injects new capital in to the bank which avoids bankruptcy and helps maintain the bank's

and Flannery (2011) for bank-specific cost of failures. A bail-in mechanism provides a pre-determined structuring plan that eliminates the need of private or court-organized negotiations and thus avoids the costs associated with these negotiations.
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cushion against future losses. So the value to shareholders when the bank runs out of cash in
the bad state is not zero any more.

To fully characterize the value to equity holders when cash buffers hit zero in state B, note that
in the bad state of the world, CoCo debt is the only source of outside financing available to the
bank. At the same time it is optimal for the bank to inject cash inside the firm to take its cash
reserves back to the target level because refinancing costs are fixed. So the proceeds from the
CoCo reissuance should be high enough to cover the cash injection into the bank and the cost
of reissuance. I assume that the bank commits to a stationary debt structure and keeps the
same level of coupon payment and the conversion ratio at each conversion. So the following
budget constraint should hold

\[ CD_B(W_B^*) \geq W_B^* + \gamma_C; \]  

(2.10)

where \( CD_B(W_B^*) \) is the value of CoCo debt at the target level of cash in the bad state of the
world which I will solve for later on.

Even though no equity is issued in the bad state, when conversion of the CoCo debt is triggered,
current shareholders of the bank face dilution since they give up a fraction \( \alpha \) of the new bank
to the current CoCo debt holders. The value to shareholders at conversion has two components:
first, current shareholders get a fraction \( (1 - \alpha) \) of the equity of the newly capitalized bank
\( (E_B(W_B^*)) \). Second, shareholders also get a fraction \( (1 - \alpha) \) of the proceeds from debt issuance
net of cash injection and reissuance costs. If the proceeds from the new debt issuance is
higher than what the bank requires to restore its cash buffer to the target level and cover the
reissuance costs, the budget constraint in (2.10) is not binding. The bank places no premium
on internal funds in addition to the target level, thus distributes the additional cash to the
bank's new and old equity holders. The old shareholders now own a fraction \( (1 - \alpha) \) of the
bank, so they are entitled to a fraction \( (1 - \alpha) \) of the surplus from the debt issuance. As a result,
the value of equity in the bad state of the world should satisfy

\[
E_B(0) = (1 - \alpha) \begin{cases} 
\text{value of equity ex-dividend} & : E_B(W_B^*) \\
\text{value of dividend} & : CD_B(W_B^*) - W_B^* - \gamma_C \\
\end{cases} 
= (1 - \alpha)(E_B(W_B^*) + CD_B(W_B^*) - W_B^* - \gamma_C)
\]

In addition to the boundary conditions discussed above, continuity and smoothness con-
ditions at \( W \) and \( W_B^* \) for the value of equity in the bad state should hold. The boundary
conditions along with the ODEs for the values of equity in both states of the world are summa-
rized in Appendix A.2.2.

Consider next the value of the CoCo debt \( CD_i(w) \), \( i = G, B \) (I again omit the arguments
\( (C, \alpha) \)). In the good state, CoCo debt acts as an ordinary straight debt with continuous coupon
payments and infinite maturity since the conversion into equity only happens in the bad
state of the world. Thus \( CD_G(w) \) satisfies the same ODE as in (2.7) where \( CSD, SD_B(w), \) and
SDG(w) are replaced with CC_D, CD_B(w), and CD_G(w) respectively.

On the other hand, in the bad state of the world CoCo debt holders receive a continuous coupon payment of CC_D up to the conversion threshold at which point, their debt gets converted into equity and they own a fraction α of the newly capitalized bank. The value of CoCo debt in this state CD_B(w) satisfies a similar system of ODEs as in (2.8) where CS_D, SD_B(w), and SD_G(w) are replaced with CC_D, CD_B(w), and CD_G(w) respectively.

The equations for CD_G(w) and CD_B(w) (which are included in the Appendix) are solved subject to the following boundary conditions. First, the value of the CoCo debt does not change when the level of cash reserves hits the target threshold. This is because above the dividend threshold any additional dollar of earnings is distributed to the bank’s shareholders. So the following should hold

\[ CD_G'(W^*_G) = 0. \]

Second, in the good state the bank recapitalizes every time its cash reserves hit W to bring back the cash buffer to W^*_G. This means the following holds

\[ CD_G(W) = CD_G(W^*_G). \]

Finally, in the bad state the CoCo bond is converted whenever cash reserves are depleted. Current CoCo debt holders receive a fraction α of the bank’s equity after new CoCo debt is issued. Additionally they get a fraction α of the proceeds from the new debt issuance net of the costs of capital injection and security reissuance. So the following holds

\[ CD_B(0) = \alpha(E_B(W^*_G) + CD_B(W^*_G) - W^*_B - \gamma) \]

Given the budget constraint (2.10), the market value of the new CoCo debt to be issued should be at least enough to take the cash reserves back to the dividend pay-out threshold after paying for the reissuance cost γ_C. So the term \( CD_B(W^*_G) - W^*_B - \gamma_C \) is either positive or zero. This boundary condition characterizes the main difference between the value of the straight debt and CoCo debt. Whilst straight debt holders receive the liquidation value of the bank when the bank runs out of cash in state B, CoCo debt holders receive a fraction of the bank value upon conversion which includes a fraction of the equity value and net proceeds of new debt issuance.

Lastly, for the value of the CoCo debt to be continuous and smoothly pasted over different regions of the cash buffer, continuity and smoothness conditions similar to the case of equity value should hold at W and W^*_G. A summary of the ODEs for CD_G(w) and CD_B(w) along with the relevant boundary conditions is provided in Appendix A.2.2.

I can solve for the values of equity and CoCo debt in states B and G similar to the values of equity and straight debt in the benchmark case assuming the same ODEs (thus the same analytical solutions as in (A.10), (A.14),(A.15), and (A.16) in Appendix A.2.1) but with different
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boundary conditions (so with different coefficients).

2.4.2 Optimal bank policies

With a bail-in plan in place, the bank replaces inefficient liquidations with conversion and costly reissuance of CoCo debt. The conversion ratio, the proceeds from the debt reissuance, and the costs of CoCo reissuance at conversion threshold determine the value to current equity holders upon conversion. As a result, the design of the CoCo contract, including the coupon payment, conversion ratio and conversion threshold, affects both the values of equity and CoCo debt. Moreover the refinancing policy of the bank in the good state is directly affected by the characteristics of the conversion in the bad state. For any given CoCo contract, management chooses the bank’s pay-out and financing policies in both states of the world to maximize the present value of future dividends to shareholders.

Proposition (2.1) characterizes the optimal financing policy of the bank in the bad state of the world. To save on the discounted expected refinancing costs and the carry cost of cash, shareholders optimally choose a zero conversion and refinancing threshold. So the bank waits until cash reserves are depleted to convert and reissue CoCo debt. On the other hand, in the good state of the world shareholders can optimally choose to issue equity when the bank’s cash buffer is low but not zero because they are concerned that they could possibly face higher financing costs in the future. Equity issuance in the good state of the world is still a cheaper source of outside liquidity. As long as there is a positive probability of a jump to the bad state of the world, it can be optimal to raise equity at $W > 0$. Whether the optimal equity issuance threshold is bigger than zero or not depends on the costs associated with security issuance in state B compared to those costs in state G. When this difference is larger, the option to raise equity in the good state becomes more valuable, so it is more probable for the bank to find it optimal to raise equity funds before it runs out of cash. Additionally higher probability of a state switch from the good to the bad state of the world makes it more likely for shareholders to time the equity market in the good state. As previously discussed the optimal equity reissuance threshold solves $E_i'(W) = 1$. When there is no $W$ that satisfies this condition, a corner solution $W = 0$ characterizes the optimal equity reissuance threshold. When this is the case, the option to recapitalize before cash reserves are depleted has no value to shareholders and they only recapitalize when it is absolutely necessary to do so. This can happen when the expected increase in the costs of financing due to a jump to the bad state is not too high or the probability of such a jump is small.

Table 2.1 summarizes the model implied financing and pay-out policies of the bank in the CoCo debt case for a given conversion ratio of $\alpha = 0.50$ and a coupon payment of $C_{CD} = 5.60\%$ and with the base case parameters.\(^\dagger\) Given this CoCo contract, in state G shareholders rely on inside equity for as long as the bank’s cash reserves are not depleted, and only raise new funds

\(^\dagger\) I assume in this section that the coupon payment $C_{CD} = 5.60\%$ is given. In the next section, I show that this is indeed the value maximizing coupon payment that shareholders choose given $\alpha = 0.5$.  

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when the bank runs out of cash.

Since CoCo debt financing alleviates financing frictions in the bad state of the world, it is expected that for a given level of $C_{CD}$, the reissuance threshold in the good state is higher with straight debt financing compared to the CoCo debt financing. With straight debt, the lack of access to any outside liquidity, makes the option to time equity markets in the bad state very valuable. CoCo financing, although more expensive than equity in the good state, provides a new source the bank can turn to in the bad state and thus decreases shareholders’ incentives to tap equity markets in the good state. For the sake of comparison, Table 2.1 also includes shareholders policies when the level of coupon payment is set to $C_{CD} = 5.79\%$, the optimal coupon in the case of straight debt. For this level of coupon payment, shareholders find it optimal to tap equity markets well before the bank runs out of cash ($W = 0.060$) in the straight debt case. For the same coupon payment, shareholders in the bail-in case choose to abstain from equity financing until absolutely necessary, i.e. $W = 0$. For this level of coupon payment, the reissuance threshold stays zero for as long as the cost of CoCo issuance is smaller than 0.55 which represents about 35\% of the face value of CoCo debt with $C_{CD} = 5.79\%$. For $\gamma_C \geq 0.55$, it becomes optimal for the bank to raise equity funds in the good state before its cash buffer is depleted. Therefore for shareholders to optimally tap equity markets before the bank runs out of cash, the costs of CoCo reissuance in the bad state should be fairly high.

In line with models with costly external financing, the optimal payout policy for shareholders is to distribute dividends to maintain bank’s cash buffer at or below a target level. More severe financing frictions in the bad state increase the value of the cash inside the bank and lead to a higher target level of cash in this state compared to the good state. So similar to the case of straight debt, bank’s cash holdings are countercyclical. As Table 2.1 shows the target level in the bad state is 0.158 (4.48\% of the total asset value) which is higher than the target level in the good state 0.116 (3.28\% of the total asset value). By committing to a pre-defined bail-in plan shareholders eliminate the risk of inefficient liquidations. Because of this the optimal target levels of cash in both states are lower with CoCo debt compared to the straight debt. Table 2.1 shows that for $C_{CD} = 5.79\%$, both pay-out thresholds are significantly lower than pay-out thresholds with straight debt financing.

The pay-off to shareholders when the bank runs out of cash in the good state depends directly on the level of conversion ratio. For a given level of $C_{CD}$ a higher conversion ratio means lower expected pay-off to shareholders if the state switches to B and conversion is triggered. This provides stronger incentives for shareholders to raise equity funds in the good state when cash reserves are low and thus increases the threshold at which shareholders tap equity markets. At the same time, for a higher conversion ratio, shareholders wait more before distributing dividends outside the bank to postpone conversion since the value they receive at conversion is lower. Figure 2.3 shows how reissuance threshold and target levels of cash in both good and bad states increase when the conversion ratio increases. For $C_{CD} = 5.60\%$, reissuance threshold becomes positive for conversion ratios bigger than 0.55, and increases to 0.059 when conversion ratio increases to 1. Figure 2.3 also shows that the dividend threshold in the bad
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(good) state increases from 0.062 (0.020) to 0.250 (0.208) when conversion ratio increases from 0 to 1.

Figure 2.4 plots the reissuance threshold and the target levels of cash in states B and G as functions of coupon payment for straight debt and for two different levels of conversion ratio \( \alpha = 0.5 \) and \( \alpha = 0.80 \). For higher conversion ratios, the expected pay-off to shareholders at conversion is closer to their expected pay-off in liquidation with straight debt (which is zero). Because of this, shareholders choose financing and dividend thresholds closer to those thresholds with straight debt. For the fully dilutive case of \( \alpha = 1 \) the optimal reissuance and pay-out thresholds coincide with the case of straight debt since shareholders receive nothing in conversion. Moreover, Figure 2.4 demonstrates that the patterns identified in the case of straight debt remain and that both target levels of cash and reissuance threshold first increase and then decrease with \( C_{CD} \). The following statement summarizes my results:

Result 2.1. For a given coupon payment shareholders choose lower reissuance and pay-out thresholds with CoCo debt financing compared to straight debt financing.

A direct implication of Result (2.1) is that, CoCo debt financing can lead to lower frequencies of recapitalization in the good state and lower cash holdings in both states compared to straight debt financing. As long as conversion ratio is less than one, shareholders get more in conversion than in bankruptcy, and thus are less willing to recapitalize the bank in the good state of the world. Moreover keeping cash inside the bank has a precautionary motive and thus is a form of risk management when outside financing is scarce. When committed to a bail-in plan, shareholders have weaker precautionary incentives to hoard cash inside the bank since the access to CoCo debt financing reduces the severity of the financing frictions in the bad state.

Figure 2.5 plots the values of equity \( E_i(w) \), marginal value of cash to shareholders \( E_i'(W) \) and the value of debt \( CD_i(w) \) for \( i = G, B \) as functions of the bank’s cash reserves and for the base case parameters of Table 2.1 and \( C_{CD} = 5.60\% \). The values of CoCo debt in states G and B display patterns analogous to the case of straight debt. Value of CoCo debt in the good state is highest at reissuance threshold (here zero) and at the target level of cash. Since no default or conversion happens in this state, the value of CoCo debt is quite insensitive to the level of cash reserves. Value of the CoCo debt in the bad state of the world is increasing with the level of cash buffer since higher cash holdings bear lower risks of conversion.

Figure 2.5 also shows that the value of equity is increasing in cash reserves in line with other dynamic models with financing frictions. Additionally, the marginal values of cash to equity holders show that equity values in both states of the world are now concave. This means that for this given bail-in contract, the option to time the cheaper equity market before the bank runs out of cash has no value to shareholders.

To study the effects of the terms of the CoCo contract on the convexity of equity value in the good state of the world, Figure 2.6 plots the marginal value of cash to shareholders for
different cases. To see how the levels of interest payment and conversion ratio affect this convexity, I fix the coupon payment (at \( C_{CD/SD} = 4.00\% \), \( C_{CD/SD} = 5.60\% \), \( C_{CD/SD} = 6.00\% \), and \( C_{CD/SD} = 7.00\% \)) and plot the marginal value of cash for straight debt and CoCo debt with \( \alpha = 0.5 \) and \( \alpha = 0.8 \). The marginal value of cash to shareholders starts decreasing when the option to time equity markets loses its value to shareholders. When the bank has no access to external liquidity with straight debt, the level of cash reserves that separates the convex and concave regions of equity value is quite high. With CoCo debt, bank’s shareholders have access to an alternative source of financing in the bad state but they lose a fraction of the bank upon conversion. If the conversion is not too dilutive, CoCo financing becomes a very attractive alternative source of liquidity. The precautionary motive dominates the motive to time the market and thus the value of equity in the good state becomes globally concave. On the other hand when conversion ratio is high, CoCo refinancing becomes less appealing to shareholders. Thus for low levels of cash the option to time the cheaper equity markets in the good state of the world dominates the precautionary motive. This leads to a local convexity in the value of equity.

Figure 2.5 shows that for a given coupon payment the higher the conversion ratio, the larger the inflection point that separates the convex and concave regions. The maximum happens for \( \alpha = 1 \) which coincides with the case of straight debt. Additionally for a given conversion threshold, higher coupon payments decrease the continuation value of the bank to shareholders and thus their incentives to tap equity markets in the good state before the bank runs out of cash. This decreases the inflection point at which equity value becomes concave. For example, when \( C_{CD/SD} = 4.00\% \) equity value in the good state becomes concave for cash reserves as low as 0.04 when conversion ratio is 0.5 but stays convex up to levels as large as 0.07 for a conversion ratio equal to 0.8. When \( C_{CD/SD} = 5.60\% \), the value of equity is globally concave when \( \alpha = 0.5 \), but is convex up to a level of 0.065 when \( \alpha = 0.8 \). For both cases the cash level that separates the convex and concave regions is the highest for straight debt financing. The following statement summarizes the results on the convexity of equity value in the good state:

**Result 2.2.** With CoCo debt financing, the level of cash reserves that separates the convex and concave regions of the equity value in the good state is lower than with straight debt financing and is increasing in conversion ratio and decreasing in the level of coupon payment.

Result (2.2) has direct implications on the risk-taking incentives of shareholders when they commit to a bail-in plan. Appropriate choice of the CoCo bail-in contract (the level of interest payments and the conversion ratio) can eliminate the local convexity in the value of the equity in the good state and thus discourage shareholders from risk taking.

Results (2.1) and (2.2) imply that the level of dilution shareholders face upon conversion and the level of interest they pay to CoCo debt holders are key determinants of the effects of CoCo financing on bank’s payout and refinancing policies. Shareholders keep less cash inside the bank and recapitalize less frequently when they are committed to a bail-in contract. To what extent they do so depends largely on the conversion ratio. Moreover appropriate choice of
2.4. Bail-in plan: the case of countercyclical CoCo

the terms of the bail-in contract can decrease or fully eliminate shareholders’ risk-taking incentives.

2.4.3 Optimal CoCo design and bank capital structure

Having analysed the optimal bank policy choices and the effects of the design of CoCo contract on these policies, I now study the optimal capital structure of the bank. When the conversion threshold is optimally set to zero, the level of coupon payments and the conversion ratio determine the value of the CoCo debt. For any given level of conversion ratio, the optimal coupon is such that the sum of the equity value and the proceeds from debt issuance net of the costs of capital injection is maximized when CoCo debt is issued for the first time. Bank’s shareholders choose the level of interest payments that maximizes

\[
\max_{C_{CD}(\alpha) \in \mathbb{R}^+} E_G(W^*_G(C_{CD}(\alpha))) + C_{CD}(W^*_G(C_{CD}(\alpha))) - W^*_G(C_{CD}(\alpha)).
\]

(2.11)

Since the bail-in plan is designed to eliminate the possibility of the bank’s failure, I assume that the bank who commits to such a plan does not face regulatory leverage requirements. I will discuss the case where the bank with a bail-in plan is also regulated in the next section.

Under the baseline parametrization of Table 2.1, the optimal coupon payment for the bank’s shareholders is \(C^*_C = 5.60\%\). This is the level of coupon payment reported in Table 2.1 and used for the analysis in Section 2.4.2. For this level of coupon, shareholders set the optimal target level of cash to 0.116 (3.28\% of the total asset value) in the good state and 0.158 (4.48\% of the total asset value) in the bad state. Equity reissuance threshold is zero which means that the option to time equity markets in the good state has no value to shareholders.

Table 2.3 reports the value maximizing capital structure and the implied bank policies as functions of conversion ratio. The optimal reissuance threshold is zero for all conversion ratios. The table shows that higher conversion ratios increase the debt capacity of the bank. When the conversion ratio is high, the price of the CoCo debt is high and so are the proceeds from CoCo issuance. The two features of the bail-in plan that eliminate the risk of inefficient liquidations include conversion of the debt into equity and reissuance of new CoCo debt to replace the converted one. I assume that conversion is efficient, in that there is no cost in converting debt into equity shares of the bank. However CoCo refinancing is costly. Higher interest payments decrease the bank’s cash reserves faster and thus increase the probability of hitting the conversion and reissuance threshold. So even though a higher coupon payment cannot lead to higher costs of default (since these costs do not exist in this set-up), they can still increase the CoCo refinancing costs. The increased frequency of CoCo refinancing leads to higher expected costs. So to find the value maximizing coupon, the tax benefits of the debt are balanced against its refinancing costs. Table 2.3 shows that for high levels of conversion

\[\text{Depositors are senior claim holders that do not bear any losses in the event of conversion and continue their right to receive a fixed stream of interest. Since deposits and issuance costs are fixed, I omit them from the maximization problem.}\]
ratios, this trade-off happens at higher level of coupon payments, and shareholders choose higher levels of debt that lead to big tax savings. At the same time, coupon payments are cash outflows that decrease the bank’s cash reserves and increase the probability of conversion. To compensate for the higher cash outflows, shareholders optimally wait longer before distributing any dividends outside the bank.

If shareholders set the conversion ratio to one, they commit to a restructuring plan that transfers the full ownership of the bank from shareholders to debt holders in the event of bankruptcy. This means that by defining how to deal with the bank’s failure ex-ante, shareholders avoid the costs of liquidation and create value.\(^\text{16}\) The following statement formalizes these results:

**Result 2.3.** *Higher conversion ratios lead to higher optimal leverage ratios and create more value for the shareholders of the bank.*

To study the effects of asset risk and the severity of financing conditions in the bad state, Table (2.5) reports the optimal capital structure, the value of the bank net of cash reserves (as calculated in (2.11) at the optimal level of \(C^*_C\)), the reissuance threshold, the target levels of cash in both states of the world, and the debt ratio ranges in both states for different model parameters. The table reports these values for two different conversion ratios: \(\alpha = 0.5\), and \(\alpha = 1\) and for the case of regulated straight debt. Panel (A) reports the values obtained in the base case environment. Whether or not CoCo financing for a bank that does not face regulatory leverage requirements leads to higher debt ratios depends on the level of dilution shareholders face upon conversion. However, as the panel shows no matter how dilutive the conversion is, since bail-in plan eliminates the costs of liquidation and decreases the severity of financing constraints in the bad state, it increases the bank value. Moreover shareholders are less willing to recapitalize before they have to in the good state of the world since they have access to an alternative source of outside liquidity in the bad state. Lastly, shareholders have less incentive to build up liquidity buffers since they do not face inefficient liquidations when the bank runs out of cash. Shareholders face dilution and incur the cost of refinancing when the bank depletes its cash reserves, however for as long as these costs are lower than the liquidation costs, the optimal target levels of cash in both states are smaller with CoCo debt compared to straight debt.

Panels (B) and (C) report the bank’s optimal choices for high \((\sigma = 0.10)\) and low \((\sigma = 0.06)\) levels of asset risk. The results demonstrate that the patterns identified with straight debt financing remain and here again riskier assets decrease the optimal leverage ratios and increase the pay-out thresholds for an unregulated bank. An additional result is that no matter how risky

\(^{16}\) There is an additional benefit to more dilutive CoCo bonds. As Calomiris and Herring (2013) discuss, higher conversion ratios make CoCo debt financing more interesting to investors and thus increase the demand from investors. In my model, CoCo refinancing cost is independent of the design of the CoCo. However if there is a higher demand for more dilutive (thus more debt-holder friendly) CoCo debts from investors, one can assume the refinancing cost of such debt obligation to be lower. This will only re-enforce the fact that higher conversion ratios create more value.
2.4. Bail-in plan: the case of countercyclical CoCo

Bank’s assets are, shareholders always choose a level of debt that eliminates any incentives to time the equity markets in the good state, and thus only recapitalize at the last moment. The optimal level of debt results in a globally concave value of equity in the good state and eliminates any risk-taking incentives. These results hold for both levels of conversion ratio when the bank is not regulated.

Panels (D) and (E) show the effects of CoCo reissuance costs (which is an indication of the severity of financing constraints in the bad state). Higher refinancing costs lead to lower optimal leverage ratios, and higher target levels of cash. Since refinancing is more costly, cash becomes more valuable inside the bank and dividends are paid out later. It is also optimal to issue CoCo debt with lower coupon payments since coupon payments are cash outflows that can increase the probability of conversion and thus expected refinancing costs. Shareholders respond to the more severe financing constraints in the bad state through a combination of lower leverage and higher cash buffers inside the bank. For the case of \( \alpha = 0.5 \), shareholders choose to recapitalize only when the bank runs out of cash in the good state for both levels of reissuance costs. When conversion is fully dilutive, the higher CoCo issuance cost of \( \gamma_C = 0.20 \) in the bad state leads to a positive reissuance threshold in the good state. The combination of high dilution and high reissuance cost makes conversion very costly for shareholders and thus provides them with higher incentives to raise new equity funds before the bank runs out of cash in the good state.

Finally, panels (F) and (G) demonstrate the effects of the higher probability of a jump from the good to the bad state. Results are similar to the ones obtained in panels (D) and (E). Since higher \( \pi_{GB} \) increases the probability of facing more severe financing constraints, shareholders decrease the coupon payments and increase the target levels of cash to retain larger cash buffers inside the bank.

Overall, the comparison of the results obtained in different panels for both CoCo cases and for straight debt case, shows that as long as liquidation is more inefficient than conversion and refinancing, CoCo debt financing creates value for shareholders. The following statement summarizes my results in this section:

**Result 2.4. A bank who commits to a bail-in plan**

- can choose lower or higher leverage ratios depending on how dilutive CoCo debt is.
- chooses to recapitalize less in the good state and keep lower cash buffers in both states of the world, independent of the conversion ratio.
- for any given conversion ratio, chooses a debt level that leads to globally concave equity value and eliminates risk-taking incentives when it is not regulated.

The above discussion implies that bail-in plans have the potential to increase leverage ratios, and decrease recapitalizations and cash buffers within the banking system. However since
they eliminate the costs of negative externalities associated with SIFIs' failures they can be socially optimal, for as long as their execution is efficient.

Regulated bank

In this section, I study the effects of banking regulation on the bank's optimal capital structure and policy choices in the presence of a bail-in plan. When the bank is regulated, I append the leverage constraint in (2.4) evaluated at \( W \) to shareholders optimization problem in (2.11). When shareholders initially raise some debt in the good state of the world, they are constrained to choose a coupon payment for which their maximum debt ratio (which is attained at equity reissuance threshold in the good state) does not exceed the regulatory level of \( 1 - \Lambda \).

Under the baseline parametrization of Table 2.1, the leverage constraint is not binding and the optimal coupon payment \( C_{CD} = 5.60\% \) satisfies the regulatory leverage requirements since the maximum debt ratio for this level of coupon payment is 95.79 which is less than 96\%. Table 2.4 reports the optimal capital structure and the implied bank policy choices as functions of conversion ratio for a regulated bank. The table shows that the results in the case of regulated bank generally follow the same pattern as the results for the case of unregulated bank. Specifically, higher conversion ratios allow shareholders to choose higher coupon payments. Additionally both dividend thresholds and the reissuance threshold increase when conversion ratio increases. The variation in the optimal level of debt is, however, much smaller than the case of unregulated bank. Increasing conversion ratio from 0.40 to 1 increases the unregulated optimal coupon from 4.49\% to 11.61\%, whilst it only increases the regulatory optimal coupon from 4.49\% to 5.62\%. Apart from \( \alpha = 0.4 \) and \( \alpha = 0.5 \) cases, the leverage constraint is binding and the debt ratio fluctuates over a fairly similar range. The small variation in the optimal coupon payment when the leverage ratio is binding is due to the fact that the face value of the CoCo debt that is issued at par when the bank is in the good state and its capital buffer is at its maximum (target level) is quite insensitive to the level of conversion ratio. When the leverage constraint is binding the value of the bank net of capital injections decreases with the conversion ratio. This is because the increase in capital injection (which is equal to the target level of cash in the good state of the world) is more significant than the increase in the coupon payment when conversion ratio increases.

Comparing the results for regulated and unregulated bank shows that for any level of conversion ratio for which the regulatory constraint is binding, regulation decreases the debt capacity of the bank and its debt ratio. It also increases the dividend thresholds in both states of the world and the reissuance threshold. Banks that are subject to Tier 1 leverage requirements, issue less debt, keep bigger cash buffers and recapitalize more often in the good state. Notably, reissuance threshold that is zero for all conversion ratios for an unregulated bank increases to 0.059 for a regulated bank with a fully dilutive CoCo debt. However, regulation decreases the value of the bank net of capital injection when leverage constraint is binding.

SIFIs are subject to capital requirements since their failure can lead to significant social costs
2.5 Conclusion

I develop a formal model of a bail-in plan to eliminate the inefficient liquidations of SIFIs. To do so, I introduce a countercyclical CoCo debt as a hybrid security that converts into equity when the bank is in distress and recapitalization or reissuance of additional debt is not possible. The bank in my model faces stochastic financing frictions and it can raise equity at a cost only in the good state of the world. In the bad state when the conversion is triggered, the bank issues new CoCo debt to replace the converted debt. Issuing CoCo debt in the bad state is more costly than issuing equity in the good state. I study the optimal financing and pay-out
policies of the bank and its value-maximizing capital structure when it has committed to a bail-in plan. I also examine how the design of the bail-in plan (specifically the conversion ratio) affects the bank’s policy choices and optimal leverage ratios.

With this model, I first show that higher conversion ratios increase the debt capacity of the bank and lead to higher potential leverage ratios. At the same time by providing a new source of outside liquidity and eliminating liquidations costs, bail-in plans decrease shareholders’ incentives to build up cash buffers within the bank and to recapitalize when they can. Independent of the conversion ratio, shareholders always choose a level of coupon for which it is optimal to abstain from recapitalization until absolutely necessary in the good state. This level of debt leads to globally concave value of equity and thus eliminates any risk-taking incentives.

My results shows that although bail-in plans potentially lead to a less capitalized, more levered banking system, they can create value from both private and social points of view since they eliminate both the costs of liquidation and the costs of negative externalities associated with SIFIs’ failures.
Table 2.1: Base case parameters and implied variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Parameter values</td>
<td></td>
</tr>
<tr>
<td>Profitability of bank's operations</td>
<td>$\mu$</td>
</tr>
<tr>
<td>Volatility of bank's operations</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Risk-free rate</td>
<td>$r$</td>
</tr>
<tr>
<td>Cost of deposits</td>
<td>$C_D$</td>
</tr>
<tr>
<td>Corporate tax rate</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Cost of equity issuance in G</td>
<td>$\gamma_E$</td>
</tr>
<tr>
<td>Cost of CoCo issuance in B</td>
<td>$\gamma_C$</td>
</tr>
<tr>
<td>Liquidation cost</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Probability of moving from G to B</td>
<td>$\pi_{GB}$</td>
</tr>
<tr>
<td>Probability of moving from B to G</td>
<td>$\pi_{BG}$</td>
</tr>
<tr>
<td>Regulatory Tier 1 leverage ratio</td>
<td>$\Lambda$</td>
</tr>
<tr>
<td>Conversion ratio</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regulated</th>
<th>Unregulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{SD} = 5.79%$</td>
<td>$C_{SD} = 8.24%$</td>
</tr>
</tbody>
</table>

| B. Implied variables for straight debt |
| Dividend threshold in G | $W^*_G$ | 0.210 | 0.238 |
| Dividend threshold in B | $W^*_B$ | 0.253 | 0.285 |
| Equity reissuance threshold | $\bar{W}$ | 0.060 | 0.069 |
| Debt ratio range in G(%) | $[\varphi(W^*_G), \varphi(W)]$ | [92.03,96.00] | [108.78,114.03] |
| Debt ratio range in B (%) | $[\varphi(W^*_B), \varphi(\bar{W})]$ | [90.96,97.67] | [107.41,116.33] |

| C. Implied variables for CoCo debt |
| Dividend threshold in G | $W^*_G$ | 0.116 | 0.104 |
| Dividend threshold in B | $W^*_B$ | 0.158 | 0.147 |
| Equity reissuance threshold | $\bar{W}$ | 0 | 0 |
| Conversion threshold | $\bar{\bar{W}}$ | 0 | 0 |
| Debt ratio range in G(%) | $[\varphi(W^*_G), \varphi(W)]$ | [92.65,95.79] | [93.74,96.58] |
| Debt ratio range in B (%) | $[\varphi(W^*_B), \varphi(\bar{W})]$ | [91.55,95.79] | [92.62,96.58] |
### Table 2.2: Optimal bank capital structure and policy choices - regulated straight debt

<table>
<thead>
<tr>
<th></th>
<th>Optimal debt $C_{SD}^*(%)$</th>
<th>Bank value</th>
<th>Reissuance threshold (W)</th>
<th>Target level in G ($W_{G}^*$)</th>
<th>Target level in B ($W_{B}^*$)</th>
<th>Debt ratio band in G (%)</th>
<th>Debt ratio band in B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Base case environment</strong></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>5.79</td>
<td>3.052</td>
<td>0.060</td>
<td>0.210</td>
<td>0.253</td>
<td>[92.03,96.00]</td>
<td>[90.96,97.67]</td>
</tr>
<tr>
<td><strong>B. Higher cash flow volatility ($\sigma = 0.10$)</strong></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td></td>
<td>5.95</td>
<td>2.966</td>
<td>0.090</td>
<td>0.283</td>
<td>0.342</td>
<td>[91.01,96.00]</td>
<td>[89.56,98.52]</td>
</tr>
<tr>
<td><strong>C. Lower cash flow volatility ($\sigma = 0.06$)</strong></td>
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<tr>
<td></td>
<td>5.66</td>
<td>3.133</td>
<td>0.032</td>
<td>0.138</td>
<td>0.167</td>
<td>[93.14,96.00]</td>
<td>[92.40,96.88]</td>
</tr>
<tr>
<td><strong>D. Higher reissuance costs ($\gamma_E = 0.02$)</strong></td>
<td></td>
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<tr>
<td></td>
<td>5.74</td>
<td>3.048</td>
<td>0.042</td>
<td>0.214</td>
<td>0.252</td>
<td>[91.49,96.00]</td>
<td>[90.54,97.21]</td>
</tr>
<tr>
<td><strong>E. Lower reissuance costs ($\gamma_E = 0.005$)</strong></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.85</td>
<td>3.056</td>
<td>0.075</td>
<td>0.206</td>
<td>0.252</td>
<td>[92.45,96.00]</td>
<td>[91.34,98.09]</td>
</tr>
<tr>
<td><strong>F. More frequent jumps to state B ($\pi_{GB} = 0.20$)</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>5.80</td>
<td>3.050</td>
<td>0.063</td>
<td>0.212</td>
<td>0.253</td>
<td>[92.05,96.00]</td>
<td>[91.04,97.75]</td>
</tr>
<tr>
<td><strong>G. Less frequent jumps to state B ($\pi_{GB} = 0.16$)</strong></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.78</td>
<td>3.055</td>
<td>0.056</td>
<td>0.207</td>
<td>0.253</td>
<td>[92.02,96.00]</td>
<td>[90.88,97.58]</td>
</tr>
</tbody>
</table>

This table reports the optimal coupon payment, the optimal bank value net of capital injection, the equity reissuance threshold, the target levels of cash in the good and bad states, and the optimal debt ratio bands in the good and bad states under different parametric assumptions for the case of regulated straight debt.
Tables and Figures of Chapter 2

Table 2.3: Effect of conversion ratio - unregulated bank

<table>
<thead>
<tr>
<th>Conversion ratio $\alpha$</th>
<th>Optimal debt $C_{CD}^*$ (%)</th>
<th>Bank value</th>
<th>Reissuance threshold $W$</th>
<th>Target level in G $W_G^*$</th>
<th>Target level in B $W_B^*$</th>
<th>Debt ratio band in G (%)</th>
<th>Debt ratio band in B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>4.49</td>
<td>3.074</td>
<td>0</td>
<td>0.108</td>
<td>0.149</td>
<td>[83.96,86.59]</td>
<td>[82.99,86.59]</td>
</tr>
<tr>
<td>0.50</td>
<td>5.60</td>
<td>3.127</td>
<td>0</td>
<td>0.116</td>
<td>0.159</td>
<td>[92.65,95.79]</td>
<td>[91.55,95.79]</td>
</tr>
<tr>
<td>0.60</td>
<td>6.74</td>
<td>3.180</td>
<td>0</td>
<td>0.126</td>
<td>0.170</td>
<td>[101.54,105.28]</td>
<td>[100.29,105.28]</td>
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<tr>
<td>0.70</td>
<td>7.92</td>
<td>3.232</td>
<td>0</td>
<td>0.139</td>
<td>0.185</td>
<td>[110.58,115.05]</td>
<td>[109.16,115.05]</td>
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<tr>
<td>0.80</td>
<td>9.13</td>
<td>3.284</td>
<td>0.014</td>
<td>0.154</td>
<td>0.202</td>
<td>[119.70,125.07]</td>
<td>[118.10,125.07]</td>
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<td>0.90</td>
<td>10.36</td>
<td>3.332</td>
<td>0</td>
<td>0.172</td>
<td>0.224</td>
<td>[128.82,135.30]</td>
<td>[127.01,135.30]</td>
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<tr>
<td>1</td>
<td>11.61</td>
<td>3.378</td>
<td>0</td>
<td>0.196</td>
<td>0.251</td>
<td>[137.81,145.68]</td>
<td>[135.76,145.68]</td>
</tr>
</tbody>
</table>

Table 2.4: Effect of conversion ratio - regulated bank

<table>
<thead>
<tr>
<th>Conversion ratio $\alpha$</th>
<th>Optimal debt $C_{CD}^*$ (%)</th>
<th>Bank value</th>
<th>Reissuance threshold $W$</th>
<th>Target level in G $W_G^*$</th>
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<tr>
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<td>3.127</td>
<td>0</td>
<td>0.116</td>
<td>0.159</td>
<td>[92.65,95.79]</td>
<td>[91.55,95.79]</td>
</tr>
<tr>
<td>0.60</td>
<td>5.51</td>
<td>3.121</td>
<td>0.014</td>
<td>0.162</td>
<td>0.205</td>
<td>[92.03,96.00]</td>
<td>[90.96,96.39]</td>
</tr>
<tr>
<td>0.70</td>
<td>5.55</td>
<td>3.112</td>
<td>0.032</td>
<td>0.180</td>
<td>0.223</td>
<td>[92.05,96.00]</td>
<td>[90.98,96.89]</td>
</tr>
<tr>
<td>0.80</td>
<td>5.58</td>
<td>3.104</td>
<td>0.043</td>
<td>0.192</td>
<td>0.235</td>
<td>[95.00,96.00]</td>
<td>[90.99,97.21]</td>
</tr>
<tr>
<td>0.90</td>
<td>5.60</td>
<td>3.098</td>
<td>0.052</td>
<td>0.201</td>
<td>0.243</td>
<td>[92.06,96.00]</td>
<td>[90.99,97.45]</td>
</tr>
<tr>
<td>1</td>
<td>5.62</td>
<td>3.093</td>
<td>0.059</td>
<td>0.208</td>
<td>0.251</td>
<td>[92.06,96.00]</td>
<td>[91.00,97.64]</td>
</tr>
</tbody>
</table>

These tables report the optimal coupon payment, the optimal bank value net of capital injection, the target levels of cash in the good and bad states, and the optimal debt ratio bands in the good and bad states for different conversion ratios, and for both regulated and unregulated banks.
### Table 2.5: Optimal bank capital structure and policy choices

<table>
<thead>
<tr>
<th></th>
<th>Optimal debt $C_{CD/SD}^*$ (%)</th>
<th>Bank value</th>
<th>Reissuance threshold ($W^*$)</th>
<th>Target level in G ($W^*_G$)</th>
<th>Target level in B ($W^*_B$)</th>
<th>Debt ratio band in G (%)</th>
<th>Debt ratio band in B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Base case environment</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight debt (regulated)</td>
<td>5.79</td>
<td>3.052</td>
<td>0.060</td>
<td>0.210</td>
<td>0.253</td>
<td>[92.03,96.00]</td>
<td>[90.96,97.67]</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
<td>5.62</td>
<td>3.093</td>
<td>0.059</td>
<td>0.208</td>
<td>0.251</td>
<td>[92.06,96.00]</td>
<td>[91.00,97.64]</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
<td>11.61</td>
<td>3.378</td>
<td>0</td>
<td>0.196</td>
<td>0.251</td>
<td>[137.81,145.68]</td>
<td>[135.76,145.68]</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>5.60</td>
<td>3.127</td>
<td>0</td>
<td>0.116</td>
<td>0.159</td>
<td>[92.65,95.79]</td>
<td>[91.55,95.79]</td>
</tr>
<tr>
<td><strong>B. Higher cash flow volatility ($\sigma = 0.10$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight debt (regulated)</td>
<td>5.95</td>
<td>2.966</td>
<td>0.090</td>
<td>0.283</td>
<td>0.342</td>
<td>[91.01,96.00]</td>
<td>[89.56,98.52]</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
<td>5.68</td>
<td>3.030</td>
<td>0.089</td>
<td>0.280</td>
<td>0.338</td>
<td>[91.06,96.00]</td>
<td>[89.65,98.49]</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
<td>11.42</td>
<td>3.315</td>
<td>0</td>
<td>0.227</td>
<td>0.300</td>
<td>[135.03,143.96]</td>
<td>[132.38,143.96]</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>5.48</td>
<td>3.067</td>
<td>0</td>
<td>0.153</td>
<td>0.211</td>
<td>[90.65,94.69]</td>
<td>[89.21,94.69]</td>
</tr>
<tr>
<td><strong>C. Lower cash flow volatility ($\sigma = 0.06$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight debt (regulated)</td>
<td>5.66</td>
<td>3.133</td>
<td>0.032</td>
<td>0.138</td>
<td>0.167</td>
<td>[93.14,96.00]</td>
<td>[92.40,96.88]</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
<td>5.57</td>
<td>3.156</td>
<td>0.031</td>
<td>0.137</td>
<td>0.165</td>
<td>[93.15,96.00]</td>
<td>[92.41,96.87]</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
<td>11.74</td>
<td>3.437</td>
<td>0</td>
<td>0.164</td>
<td>0.202</td>
<td>[140.50,147.24]</td>
<td>[139.05,147.24]</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>5.69</td>
<td>3.179</td>
<td>0</td>
<td>0.080</td>
<td>0.109</td>
<td>[94.60,96.82]</td>
<td>[93.82,96.82]</td>
</tr>
<tr>
<td><strong>D. Higher CoCo reissuance costs ($\gamma_C = 0.20$)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Straight debt (regulated)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
<td>5.62</td>
<td>3.092</td>
<td>0.059</td>
<td>0.208</td>
<td>0.250</td>
<td>[92.06,96.00]</td>
<td>[91.00,97.65]</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
<td>11.33</td>
<td>3.348</td>
<td>0.014</td>
<td>0.214</td>
<td>0.268</td>
<td>[135.19,143.05]</td>
<td>[133.21,143.64]</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>5.59</td>
<td>3.113</td>
<td>0</td>
<td>0.124</td>
<td>0.167</td>
<td>[92.33,95.68]</td>
<td>[91.24,95.68]</td>
</tr>
</tbody>
</table>

This table reports the optimal coupon payment, the optimal bank value net of capital injection, the equity reissuance threshold, the target levels of cash in the good and bad states, and the optimal debt ratio bands in the good and bad states under different parametric assumptions for regulated straight debt, regulated $\alpha = 1$, unregulated $\alpha = 1$, and $\alpha = 0.5$. 
<table>
<thead>
<tr>
<th>A. Base case environment</th>
<th>Optimal debt $C^*_{CD/SD}$(%)</th>
<th>Max. value</th>
<th>Reissuance threshold ($W$)</th>
<th>Target level in G ($W^*_G$)</th>
<th>Target level in B ($W^*_B$)</th>
<th>Debt ratio band in G (%)</th>
<th>Debt ratio band in B (%)</th>
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<td>Straight debt (regulated)</td>
<td>5.79</td>
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<td>0.060</td>
<td>0.210</td>
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<td>3.378</td>
<td>0</td>
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<td>[91.55,95.79]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>E. Lower CoCo reissuance costs ($\gamma_C = 0.10$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight debt (regulated)</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>F. More frequent jumps to state B ($\pi_{GB} = 0.20$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight debt (regulated)</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>G. Less frequent jumps to state B ($\pi_{GB} = 0.16$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight debt (regulated)</td>
</tr>
<tr>
<td>$\alpha = 1$ (regulated)</td>
</tr>
<tr>
<td>$\alpha = 1$ (unregulated)</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
</tr>
</tbody>
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This table reports the optimal coupon payment, the optimal bank value net of capital injection, the equity reissuance threshold, the target levels of cash in the good and bad states, and the optimal debt ratio bands in the good and bad states under different parametric assumptions for regulated straight debt, regulated $\alpha = 1$, unregulated $\alpha = 1$, and $\alpha = 0.5$. 

The figure plots the values of equity and straight debt and the marginal values of cash to shareholders as functions of the bank’s cash reserves in the good and bad states of the world.
Panels A and B plot the target level of cash and the reissuance threshold in state G as functions of C, $\gamma_E$, and $\pi_{BG}$. Panel C plots the target level of cash in state B as functions of C, $\gamma_E$, and $\pi_{BG}$.
Figure 2.3: Effect of conversion ratio

The figure plots the reissuance threshold, and the target levels of cash in states B and G as functions of conversion ratio.
Figure 2.4: Straight debt versus CoCo debt

The figure plots the reissuance threshold, and the target levels of cash in states B and G as functions of coupon payment for straight debt, $\alpha = 0.5$, and $\alpha = 0.8$. 
The figure plots the values of equity and CoCo debt and the marginal values of cash to shareholders as functions of the bank’s cash reserves in the good and bad states of the world.
Figure 2.6: Marginal value of cash

The figure plots the marginal values of cash to shareholders in the good state for straight debt, $\alpha = 0.5$, and $\alpha = 0.8$ as functions of the bank’s cash reserves and for different levels of coupon payment ($C_{\text{CD}/\text{SD}} = 4.00\%$) top left, ($C_{\text{CD}/\text{SD}} = 5.60\%$) top right, ($C_{\text{CD}/\text{SD}} = 6.00\%$) bottom left, and ($C_{\text{CD}/\text{SD}} = 7.00\%$) bottom right.
3 Debt Maturity and Systemic Risk

3.1 Introduction

One of the most prominent features of current financial markets is the extensive use of short-term debt. Banks and other financial institutions use short-term debt as a main source to fund their long-term assets. When funding long-term assets with short-term debt, banks face roll over risk: investors might not be willing to roll over their debt if they receive an interim adverse news on the quality of banks’ assets, leading to inefficient early liquidation. This makes short-term debt an attractive disciplinary device that can help align the incentives of shareholders and debt holders. So although short-term debt can increase roll over and bankruptcy risks, banks may still find it optimal if the disciplinary benefits of short-term financing overcome the costs associated with roll over risk and early liquidations. Consistent with this view, several studies analyse the optimal maturity structure of a bank (see e.g. Huberman and Repullo (2013), and Repullo et al. (2013)).

The standard theoretical approach to study the optimal maturity structure of banks considers a "representative" bank framework. In this framework bankers face a trade-off between the costs of early liquidation of short-term debt and its disciplinary benefits. As the experience of the financial crisis of 2007-2009 shows, banks hold correlated assets, and this asset commonality can become a source of systemic risk since one bank’s failure can affect the other banks in the financial system. Thus a representative bank framework that studies the optimal maturity structure of banks, has a major short-coming since it only assumes the discipling effect of short-term debt at an individual level and excludes the effects of asset commonality and the systemic risks it exposes banks to. Recognizing this short-coming, this paper develops a set-up with multiple banks that subject themselves to negative and positive externalities by investing in correlated assets. The trade-off between the costs and benefits of short-term financing takes into account these externalities. The purpose of this paper is therefore to understand how the optimal maturity structure changes when banks invest in correlated assets and thus are subject to systemic risk as well as their own individual risk. To this end, I build a model in which two banks suffer fire-sale externalities but benefit from information synergies because they invest...
in correlated assets. I then examine the optimal bank liability structure and compare it to the representative bank framework.

Banks in my model invest in correlated risky assets. The expected pay-off of each asset depends on the effort exerted by each banker. The correlation between these two investments is exogenously given and is known to all agents. The effort exerted by bankers is costly and its costs are increasing in its level. However, the costs of exerting effort are lower when assets are more correlated because the correlation between assets generates information synergies that bankers can benefit from. To focus on the choice of maturity structure, I abstract from the choice of leverage and I assume that banks finance their risky investments entirely with debt. Bankers act in the best interest of shareholders and choose between long-term and short-term debt financing to maximize shareholders' value. In addition to the optimal maturity, bankers need to decide how much effort to exert. Since bankers decide on the level of effort after the required funds have been raised and they are the residual claimants behind investors, there is a misalignment of incentives between bankers and investors who can not observe how diligently bankers work.

My model has three dates: $t = 0$ is the initial date when bankers raise funds to invest in risky projects and decide on the level of effort they exert. $t = 1$ is the interim date at which a public signal on the quality of banks' projects is received. When banks are financed with short-term debt they have to roll over their debt at $t = 1$; if short-term debt is not rolled over at this interim date, banks are subject to early liquidation. $t = 2$ is the final date when each bank's investment's return is realized if the bank has not been liquidated before. The return of each investment directly depends on the level of effort each banker decides to exert.

To study the disciplinary effects of short-term debt, my model abstracts from liquidity risk and focuses on roll over decisions of investors based on the signal they receive on the quality of banks' projects. This unique binary signal received at the interim date, indicates whether the investments are likely to fail or succeed. Short-term investors can decide not to roll over their debt if the signal is sufficiently bad. In this case the bank has to liquidate its assets to pay-off short-term debt holders. Any banker who faces early liquidation loses a private benefit of control. Early liquidation is inefficient. However, unlike representative bank models in which the cost of early liquidation is fixed, in my model the level of inefficiency of early liquidation depends on how much assets are liquidated in total, i.e. whether one or both banks are subject to liquidation, and how correlated banks' assets are. This is because if the investments of the two banks are highly correlated, fire-sale of the assets of one bank in the market decreases the selling price of the assets of the other bank. So the failure of one bank affects the other bank since it increases its cost of liquidation. In my model, similar to the representative bank models, the threat of early liquidation along with the loss of the private benefit of control can discipline bankers when they choose their levels of effort. However, the fact that the roll over

1 A liquidity shock to investors is another potential reason for which a bank might fail to roll over its short-term debt. This means that investors might not be willing to roll over their debt because they face a more urgent or an alternative need for their money. The early liquidations caused by such liquidity shocks have no disciplinary effects on bankers and are thus excluded from my analysis.
cost of short-term debt for each bank depends on the performance of the other bank and the correlation of its assets with the other bank, is a key feature that distinguishes my model from representative bank frameworks. In my model asset commonality leads to negative fire-sale externalities and positive information synergies and through these two channels it affects the costs of exerting effort and the level of inefficiency of early liquidations. So asset commonality plays an important role in the trade-off between the disciplinary benefits and the early liquidation costs of short-term debt.

Using this systemic model of banks, I first solve for the optimal effort level each banker chooses to exert in both long-term and short-term debt cases. I show that when private benefit of control is large enough or early liquidation value is small enough, short-term debt can induce each banker to choose a higher level of effort. Given the optimal effort level, I study banks’ optimal maturity structure by comparing the social surplus given the optimal effort level chosen by each banker in both long-term and short-term debt cases. I show that even though a higher effort level can be achieved with the use of short-term financing, the inefficiency of early liquidations, costs of exerting effort and the loss of the private benefit of control do not guarantee the optimality of short-term financing.

Next, I examine the role of banks’ assets correlation on the optimal effort level and maturity structure. Since banks can invest in correlated assets, the liquidation of one bank’s assets can have significant consequences on the other one. When more than one bank is forced to liquidate correlated assets, liquidation values depress even further. This means that the failure of one bank to roll over its debt has negative externalities on the other one. On the other hand, correlation can be beneficial since it leads to information synergies which bank managers can benefit from to decrease their costs of effort. For example bankers can share information when they lend to the same sector or to the same geographical area. I show that in the presence of these positive and negative externalities, the optimal liability structure of each bank is significantly dependant on the correlation of banks’ assets.

When the level of asset correlation is fixed, the costs of exerting effort depends on the level of information synergies. Additionally the level of inefficiency of early liquidations depends on how severe fire-sale externalities are. For a given level of correlation, high information synergies decrease the cost of exerting effort and provide incentives for bankers to exert more effort even with long-term debt. In this case, the level of effort with long-term debt becomes closer to the level of effort with short-term debt and the disciplinary benefits of short-term debt decreases. Fire-sale externalities can affect the maturity structure in two different ways. First, for a given level of correlation, when fire-sale externalities are high early liquidations become more costly. This increases the costs of short-term debt. Second, with high fire-sale externalities the threat of early liquidation becomes stronger and thus the disciplinary benefits of short-term debt increase. So when fire-sale externalities are high, early liquidations associated with short-term debt become more costly but less probable. Short-term debt becomes the optimal mode of financing when the second effect dominates the first one.
How changing asset commonality affects the optimality of short-term financing depends on the level of negative and positive externalities that investment correlation leads to. My analysis show that when information synergies are low enough or fire-sale externalities are high enough, increasing correlation can lead to higher disciplinary effects of short-term debt that induce higher efforts exerted by bankers. In this case the disciplinary benefits of short-term debt can eventually overcome its costs and short-term debt can become the optimal mode of financing. Thus there is a more important role for short-term debt in high correlation, high fire-sale externalities and low information synergies environments.

The main contribution of this paper is to show that each bank's optimal mode of financing depends not only on its individual characteristics, but also on the positive and negative externalities that exist in the banking systems where banks invest in correlated assets. Thus studying the optimal maturity structure in a representative bank framework can lead to misleading results since it ignores the role that the correlation between banks' assets play on the trade-off between the disciplinary effects of short-term debt and its roll over risk.

**Related Literature** The literature on the maturity structure of bank's liabilities and the role of short-term debt is divided into two main strands. One branch of this literature studies the liquidity services banks provide through short-term financing. This literature assumes that investors can be hit by liquidity shocks and thus decide not to rollover their debt; see for example Diamond and Dybvig (1983) and Jacklin and Bhattacharya (1988). The second branch focuses on the disciplinary effects of short-term debt in the presence of incentive problems that arise when the interests of bankers and creditors are not aligned. Some examples include Calomiris and Kahn (1991), Diamond (1991), Rajan (1992), Diamond and Rajan (2001), Diamond and He (2013), Cheng and Milbradt (2012), Huberman and Repullo (2013), and Repullo et al. (2013). My paper belongs to the latter since investors in my model have no demand for liquidity.

Among the literature that considers the role of short-term debt in the presence of some sort of frictions, Calomiris and Kahn (1991) is one of the first ones. They consider the role of short-term debt in a model where the banker can abscond with funds ex-post. Absconding is socially inefficient and is more tempting to the banker when the realized returns are lower. So it can be optimal to use short-term debt since investors are able to force liquidation to prevent absconding. The focus of this paper is thus on the ex-post incentive effect of the maturity structure and not ex-ante risk taking. Diamond (1991) has an adverse selection set-up where at an interim date investors can upgrade or downgrade their initial imperfect rating of the bank according to some new information received. It is more likely for the good type banks to be upgraded. There is a preference for short-term financing when borrower expects their rating to improve. This comes at the cost of liquidation risk of short-term debt. The choice of maturity structure has to trade-off the two. Rajan (1992) focuses on a borrower's choice between short-term or long-term bank debt and long-term arm's-length lender. The optimal bank debt maturity depends on the bargaining power of the bank after the true state of the project is revealed. This paper focuses mostly on the effect of lender's type on ex-ante level of effort.
chosen by the borrower. Diamond and Rajan (2001) model short-term demandable debt as an effective tool that can incentivize the banker not to attempt to extort rents. Such short-term debt can help overcome the commitment problem between the banker and depositors.

More recently, Diamond and He (2013) study how the debt maturity affects debt overhang problem i.e. the reduced incentives for equity holders to undertake investments, because some value accrue to more senior claimants. The paper studies under what conditions the debt overhang problem is more severe in case of short-term versus long-term debt. Cheng and Milbradt (2012) study the optimal maturity decision in a continuous time setting where the banker can choose the riskiness of the assets. Also in another recent approach, Della Seta et al. (2017) formulate a dynamic model with financing frictions, and show that short-term debt can increase shareholders’ incentives for risk-taking.

The papers closest to mine in this literature are Huberman and Repullo (2013) and Repullo et al. (2013). Huberman and Repullo (2013) use an ex-ante moral hazard framework to compare the benefits and costs of short-term and long-term financings. They show that under some conditions risky short-term debt can be an effective incentive device. Using a similar framework Repullo et al. (2013) characterize the optimal mix of short-term and long-term debt maturities as a function of the project’s profitability. They show that using short-term debt is optimal only when the investment’s profitability is low and that the gains from short-term financing are generally small. Both Huberman and Repullo (2013) and Repullo et al. (2013) model a moral hazard problem within a representative bank framework. My paper is built on the models used in these two papers; however an additional bank with correlated assets adds systemic externalities that banks are subject to which affect the costs of exerting effort and the level of inefficiency of early liquidations and consequently banks’ optimal maturity decisions.

The paper is organized as follows. Next section includes the model and the cases of long-term and short-term debt. Section 3.3 is devoted to model implications. Section 3.4 concludes.

### 3.2 Model

I consider an economy with three dates \((t = 0, 1, 2)\), two banks, \(A\) and \(B\), managed by two bankers and a large number of investors. All agents are risk-neutral and the risk free rate is normalized to zero. There are also two risky assets that each for an investment of one unit of funds at time zero yields stochastic cash flow \(\tilde{R}\) at date \(t = 2\). \(\tilde{R}\) can take two values

\[
\tilde{R} = \begin{cases} 
R_H, & \text{with probability } p_i \\
R_L, & \text{with probability } 1 - p_i,
\end{cases}
\]

where \(i = A, B\). Each banker chooses the parameter \(p_i \in [0, 1]\), i.e. the probability of success, by deciding how much effort to exert. So the distributions of the cash flows of the two risky
investments depends on the effort of each banker. Each banker incurs a cost when exerting effort. This cost depends not only on the level of effort but also on the correlation of the banks’ investments. If banks are invested in correlated projects they can benefit from sharing information about the projects, i.e. information synergies. So for the same amount of effort they incur lower costs. The cost function is thus assumed to be given by \( g(\rho)C(p) \) where \( g(\rho) \) is a decreasing function of the correlation of the two risky assets with \( g(0) = 1 \) and \( C(p) \) is a twice differentiable, increasing and convex function of \( p \). \( C(p) \) and \( g(\rho) \) are identical for both banks. I define the cost function as a logarithmic function

\[
C(p) = \gamma \ln \frac{1}{1-p},
\]

which is increasing and convex and satisfies

\[
C'(1) = \infty.
\]

I assume the simplest form for \( g(\rho) \) as follows

\[
g(\rho) = 1 - bp,
\]

where \( 0 \leq b \leq 1 \) shows the level of information synergies bankers can benefit from. For a higher \( b \) the information synergies are greater and the cost of exerting effort is lower for a given correlation.

The correlation of banks’ investments is public information and is available to both bankers and investors. Since \( g(\rho) \) is a decreasing function of the correlation, more correlated investments can be appealing to bankers since they can benefit from information synergies that decrease their costs of efforts. The bankers decide simultaneously how much effort to exert after they have raised the necessary funds and observed the correlation between the risky assets.

To focus on the choice of maturity structure, I abstract from the choice of leverage. I assume that banks have no capital and can only fund their investments by borrowing from the outside lenders. I also assume that \( R_L < 1 \leq D_i < R_H \) where \( D_i \) is the face value of the debt that bank \( i \) takes. This means that each bank can only repay its debt when the higher cash flow is realized and defaults on its debt obligation when the lower outcome is realized. \( p \) is not observable by banks’ investors. However these outside investors observe a signal \( \tilde{s} \) on the pay-off of the risky assets at \( t = 1 \). Depending on the signal observed at interim date and the maturity structure of the bank’s debt, the bank can either get liquidated or continue to \( t = 2 \), when the final pay-offs

---

2 Although I do not assume any explicit correlation in the distribution of the two risky investments, they are indeed implicitly correlated. This is because the outcome of each project depends on the level of the effort exerted by each banker which in turn depends on the correlation between the risky projects. As I will discuss in more details later-on, the level of the effort each bankers optimally choose to exert depends on banks’ investment correlation through both the costs of exerting effort and the liquidation value of the risky project.

3 Synergies from sharing information has also been the subject of arguments used to motivate loan syndication.
are realized. If the bank finances itself with long-term debt, it continues its operations until the cash flows are realized at \( t = 2 \). If the bank finances itself with short-term debt, depending on the signal received at interim date \( t = 1 \), debt holders can decide to roll over their debt or not. If they roll over the debt, cash flows are realized at date \( t = 2 \). Otherwise the bank gets liquidated at \( t = 1 \). Early liquidation is inefficient. If any of the banks continues its operations until the final date \( t = 2 \), its banker earns a private benefit of control \( B \). Bankers lose this private benefit of control in the event of an early liquidation at the interim date.

The effect of one bank’s failure on the other bank is shown in the form of a drop in the liquidation value of the risky assets if both banks default or get early liquidated simultaneously. This depressed liquidation value, through which the failure of one bank affects the other bank, is the source of systemic risk in my model. At the interim date \( t = 1 \), each individual bank faces inefficient liquidation if it fails to roll over its debt. Additionally if early liquidation is triggered, its level of inefficiency depends on the ability of the other bank to roll over its debt. Thus in my model the cost of liquidating banks’ assets depends on how much assets are being liquidated in total and how correlated these assets are. When only one bank liquidates its assets it can get a higher price for its liquidated assets while when two banks liquidate their assets at the same time, their liquidation values drop due to the fire-sale externalities of the simultaneous asset sales; to what level this happens depends on how correlated these assets are.

To model the fire-sale externalities, I define \( f(\rho) \) as the coefficient of fire-sale externalities. \( f(\rho) \) is a decreasing function of the correlation of the two investments with \( f(0) = 1 \). I assume \( f(\rho) \) is given by

\[
f(\rho) = 1 - a\rho, \]

where \( 0 \leq a \leq 1 \) shows the severity of fire-sale externalities. When more than one bank liquidate their assets, the liquidation value is a fraction \( f(\rho) \) of the liquidation value in the case where only one bank faces liquidation. This means that in the case of joint bank failures, the liquidation value of assets depends both on the level of the correlation between banks’ assets and the magnitude of fire-sale externalities (\( a \)). The bigger \( a \) is, the less the liquidation value when both banks default at the same time will be. This also holds for \( \rho \). In the following sections I study the systemic effects of asset commonality in more details in both long-term and short-term debt cases.

### 3.2.1 Long-term debt

Suppose that each bank funds one unit of risky investment by issuing long-term debt that matures at date \( t = 2 \). Each banker then decides on the level of effort \( p_i \) she wants to exert given that she has to incur a cost \( g(\rho)C(p_i) \) and that the probability of the good outcome \( R_H \) depends on her effort. So the expected pay-off of each project depends on the level of the effort that the banker decides to exert which in turn depends on the correlation between banks’ investments.
Since the debt is long-term, there is no roll over decision at $t = 1$ and the debt matures when the final payments of the projects are realized. This means that each banker gets $B$ regardless of her project’s outcome since she does not lose control of the bank till final date. If investment is successful and $R_H$ is realized, debt holders get the face value $D_i$ of their claims and each banker gets the remaining cash flows of $R_H - D_i$. If investment fails, it only yields a lower cash flow of $R_L$ which is not enough to repay debt and thus the bank defaults. Bankruptcy which is costly on its own, becomes even more costly when two banks default at the same time. This is because if the investments of the two banks are highly correlated, the fire-sale of the assets of one bank in the market decreases the selling price of the assets of the other bank.

To model this, I assume the liquidation value at $t = 2$ is a fraction $\lambda \leq 1$ of the cash flows of the investment if only one bank defaults, but is a fraction $f(\rho)\lambda$ of the cash flows if two banks default at the same time. $f(\rho)$ which is defined as $f(\rho) = 1 - ap$ is a decreasing function of the correlation of the two investments with $f(0) = 1$.

The pay-offs to the different claimants of bank $i$ are summarized in Table 3.1. Banker $i$’s pay-off is computed as

$$\Pi_i = p_i(R_H - D_i) - g(\rho)C(p_i) + B.$$

The optimal contract for bank $i$, $(p_i, D_i)$, is the solution to the following problem

$$\max_{(p_i, D_i)} \Pi_i, \quad (3.1)$$

subject to the incentive compatibility constraint (ICC)

$$p_i = \arg\max_{p_i'} p_i'(R_H - D_i) - g(\rho)C(p_i') + B, \quad (3.2)$$

and the participation constraint

$$p_iD_i + (1 - p_i)\lambda R_L[p_j + f(\rho)(1 - p_j)] \geq 1. \quad (3.3)$$

The ICC determines the bankers choice of effort $p$ given the promised debt repayment of $D$, and the participation constraint ensures that investors get at least their required rate of return on what they invest. The ICC is equivalent to

$$g(\rho)C'(p_i) = (R_H - D_i). \quad (3.4)$$

Solving for $D_i$ from (3.4) and plugging it into the participation constraint (3.3), I get

$$p_i[R_H - \lambda R_L[p_j(1 - f(\rho)) + f(\rho)] - g(\rho)C'(p_i)] \geq 1 - \lambda R_L[p_j(1 - f(\rho)) + f(\rho)].$$

The participation constraint is binding at the optimum since investors just break even. So each banker’s pay-off is equal to the total surplus (the expected pay-off of the debt holders
plus the expected pay-off of the banker minus the cost of effort and the cost of investment).  
\[ SW_{LT} = B + p_i R_H + (1 - p_i) \lambda R_L [p_j (1 - f(\rho)) + f(\rho)] - g(\rho) C(p_i) - 1 \]
\[ = B + p_i [R_H - \lambda R_L (p_j (1 - f(\rho)) + f(\rho))] + \lambda R_L [p_j (1 - f(\rho)) + f(\rho)] \]
\[ - g(\rho) C(p_i) - 1. \]

If I define \( h(p) \) as  
\[ h(p) = R_H - \lambda R_L (p(1 - f(\rho)) + f(\rho)), \]
I can re-write \( SW_{LT} \) as  
\[ SW_{LT} = R_H - (1 - p_i) h(p_j) + B - g(\rho) C(p_i) - 1. \] (3.5)

\( h(p) \) represents the difference between the total cash flows available in the good state (no default case) and the bad state (default case). In the good state the investment yields \( R_H \). In the bad state the investment yields \( R_L \) but since the bank defaults and default is costly, the total amount of cash flows available is only \( \lambda R_L [p(1 - f(\rho)) + f(\rho)] \) which depends on whether or not the other bank defaults as well. Since there is no early liquidation, \( B \) is available in both states so it does not appear in the expression for \( h(p) \). The first two terms on the RHS of (3.5) are the cash flows available from the project, the third term \( B \) is the private benefit of control to the banker, and the last two terms represent the cost of investment and the cost of exerting effort.

Given \( h(p) \), the optimization problem of banker \( i \) is equivalent to  
\[ \max_{(p_i)} p_i h(p_j) - g(\rho) C(p_i), \] (3.6)
subject to  
\[ p_i [h(p_j) - g(\rho) C(p_i)] \geq 1 - \lambda R_L [p_j (1 - f(\rho)) + f(\rho)]. \] (3.7)

### 3.2.2 Short-term debt

With short-term debt, investments have the same pay-off structure of the previous case, but the debt matures at the interim date \( t = 1 \). Whether short-term debt is rolled over to the final period or not depends on the information about the return of the banks’ investments which is revealed at the date of the roll over decision \( t = 1 \). In this case, I assume there is a unique signal \( s \) observed by investors for both banks. Depending on the signal investors receive, they decide whether or not to roll over their debt for another period. If investors of bank \( i \in \{A, B\} \) decide not to re-finance the bank, the bank will be liquidated at \( t=1 \) and investors will get the liquidation value of the banks’ assets. This early liquidation is inefficient in that investors can only receive a fraction of the expected pay-offs. Additionally, liquidation value depends on
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whether or not the other bank gets liquidated at \( t = 1 \). So the liquidation value of bank \( i \)'s assets, \( L_{1,i} \), at the interim date \( t = 1 \) satisfies

\[
L_{1,i} = \begin{cases} 
\psi E[\tilde{R}|\tilde{s} = s], & \text{if bank } j \text{ is not liquidated at } t = 1 \\
f(\rho)\psi E[\tilde{R}|\tilde{s} = s], & \text{if bank } j \text{ is liquidated at } t = 1,
\end{cases}
\]

where \( \psi \) is the fraction of the assets that can be recovered in the event of early liquidation. I assume

\[\psi < \lambda < 1,\]

meaning that early liquidation is more inefficient than bankruptcy at the final pay-off date. This is because there are deadweight costs associated with closing down the project early. If the assets were to be sold to an outside buyer (second-best owner) at \( t = 1 \), the valuation that this buyer assigns to these assets are lower at \( t = 1 \) compared to \( t = 2 \). This assumption means that if there were no disciplinary benefits to short-term debt, it would have never been optimal for banks to finance their risky assets with short-term debt. Assuming that early liquidation is always inefficient allows me to focus on the disciplinary benefits of short-term debt.

If investors of bank \( i \) decide to refinance the bank, the bank continues its operations until the final date when pay-offs are realized. In this case, the banker gets the private benefit of control \( B \). If investment yields the high cash flow, investors get the face value of their debt. If the low cash flow is realized, the bank defaults. The liquidation value at time \( t = 2 \) depends on whether or not the other bank has already been liquidated at \( t = 1 \) or is defaulting at \( t = 2 \). Thus the liquidation value of bank \( i \)'s assets at \( t = 2 \), \( L_{2,i} \) satisfies

\[
L_{2,i} = \begin{cases} 
\lambda R_L, & \text{if bank } j \text{ is not liquidated at } t = 1 \text{ neither at } t = 2 \\
f(\rho)\lambda R_L, & \text{if bank } j \text{ is liquidated at } t = 1 \text{ or } t = 2.
\end{cases}
\]

At interim date \( t = 1 \) lenders observe a public signal \( \tilde{s} \) on the banks’ risky assets which can take two values \( \tilde{s} \in \{s_G, s_B\} \) such that

\[
Pr(s_G|\tilde{R} = R_H) = Pr(s_B|\tilde{R} = R_L) = q,
\]

where parameter \( q \in \left[ \frac{1}{2}, 1 \right] \) describes the quality of the signal. By Bayes’ law for bank \( i \)

\[
Pr(\tilde{R} = R_H | s_G) = \frac{Pr(\tilde{R} = R_H)Pr(s_G|\tilde{R} = R_H)}{Pr(s_G)} = \frac{p_i q}{p_i q + (1 - p_i)(1 - q)},
\]

and

\[
Pr(\tilde{R} = R_H | s_B) = \frac{Pr(\tilde{R} = R_H)Pr(s_B|\tilde{R} = R_H)}{Pr(s_B)} = \frac{p_i (1 - q)}{p_i (1 - q) + q(1 - p_i)}.
\]
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Similarly for low realization

\[ Pr(\tilde{R} = R_L | s_G) = \frac{Pr(\tilde{R} = R_L)Pr(s_G | \tilde{R} = R_L)}{Pr(s_G)} = \frac{(1 - p_i)(1 - q)}{p_i q + (1 - p_i)(1 - q)}, \]

and

\[ Pr(\tilde{R} = R_L | s_B) = \frac{Pr(\tilde{R} = R_L)Pr(s_B | \tilde{R} = R_L)}{Pr(s_B)} = \frac{(1 - p_i)q}{p_i (1 - q) + q (1 - p_i)}. \]

When \( q = \frac{1}{2} \), the signal is uninformative since \( Pr(\tilde{R} = R_H | s_G) = Pr(\tilde{R} = R_H | s_B) = p_i \) and \( Pr(\tilde{R} = R_L | s_G) = Pr(\tilde{R} = R_L | s_B) = 1 - p_i \). The closer to 1 \( q \) is, the more informative the signal will be. For short-term debt to induce the banker to choose a higher level of effort, it needs to be risky. This means that for a binary signal, short-term debt holders should necessarily take two different actions: liquidating when the bad signal is observed and rolling over when the good signal is observed. If this is not the case and investors roll over their debt even when the bad signal is observed, short-term debt is safe and thus does not induce any disciplinary effects on bankers. Since my model focuses on the disciplinary effects of short-term debt, I assume the case where each bank is liquidated at \( t = 1 \) if and only if the bad signal \( s_B \) is observed. In this case, the pay-offs to different claimants of bank \( i \) are summarized in Table 3.2.

Each Banker’s pay-off is computed as

\[ \Pi_i = qp_i (R_H - D_i) - g(\rho)C(p_i) + [qp_i + (1 - q)(1 - p_i)] B. \]

The optimal contract for bank \( i, (p_i, D_i) \), is the solution to the following problem

\[
\max_{(p_i, D_i)} \Pi_i, \quad (3.8)
\]

subject to the incentive compatibility constraint (ICC)

\[
p_i = \arg \max_{p_i} qp_j (R_H - D_i) - g(\rho)C(p_j) + [qp_j + (1 - q)(1 - p_j)] B, \quad (3.9)
\]

and the participation constraint

\[
q p_i D_i + (1 - q)(1 - p_i) \lambda R_L [qp_j (1 - f(\rho)) + f(\rho)] + \psi [q (1 - p_i) R_L + (1 - q) p_i R_H] u(p_j) \geq 1, \quad (3.10)
\]

where \( u(p) = [qp + (1 - q)(1 - p)](1 - f(\rho)) + f(\rho) \) captures the effects of correlation on the early liquidation value. In the representative bank framework where \( f(\rho) = 1, u(p) \) is equal to 1 and the early liquidation value of one bank is independent of the other bank’s roll over
outcome. The ICC is equivalent to
\[ g(\rho)C'(p_i) = q(R_H - D_i) + (2q - 1)B, \] (3.11)
and shows that for any given level of \( D \) if the signal is informative enough (\( q \) close to 1) and the private benefit of control is big enough, the threat of early liquidation can induce each banker to exert a higher level of effort. Comparing the ICC in both cases of long-term and short-term debt (3.4) and (3.11), for a given level of \( D, B \), and \( q \), the higher the correlation, the bigger the difference between the effort levels in the short-term and the long-term financing will be. This is because \( g(\rho) \) is a decreasing function of \( \rho \).

I can do the same comparison for the participation constraints in the long-term and short-term debt cases ((3.3) and (3.10)), when \( q \) is close to 1, i.e. the signal is informative enough. Since early liquidation is less efficient than the liquidation at the final date (\( \psi < \lambda \)), for any given level of \( D \) and the effort level of the other bank \( p_j \), the participation constraint in the case of short-term financing is stricter than the one in the case of long-term financing. Moreover since for any given level of \( p_j \) and \( q, u(p_j) \) is increasing in \( f(\rho) \) and thus decreasing in \( \rho \), higher investment correlations make it even more difficult to satisfy the participation constraint for short-term debt (to see this better consider (3.3) and (3.10) for the completely informative signal \( q = 1 \)).

Solving for \( D_i \) from (3.11) and plugging it into the participation constraint (3.10), I get
\[
\begin{align*}
p_i[qR_H + (2q - 1)B - g(\rho)C'(p_i)] &+ (1 - q)(1 - p_i)\lambda R_L[qp_j(1 - f(\rho)) + f(\rho)] + \\
\psi[(1 - q)p_iR_H + (1 - q)p_i p_j + (1 - q)p_i R_H | u(p_j) | u(p_j)] &\succeq 1,
\end{align*}
\]
grouping terms in \( p_i \) gives
\[
\begin{align*}
p_i[qR_H + (2q - 1)B - (1 - q)\lambda R_L[qp_j(1 - f(\rho)) + f(\rho)] &+ \psi[(1 - q)R_H - qR_L] u(p_j) ] \\
-g(\rho)C'(p_i)] &+ (1 - q)\lambda R_L[qp_j(1 - f(\rho)) + f(\rho)] + \psi q R_L u(p_j) \succeq 1.
\end{align*}
\]
Since the participation constraint is binding at the optimum, each banker’s pay-off is equal to the social surplus.
\[
SW_{ST} = qp_i(R_H + B) + (1 - q)(1 - p_i)B + (1 - q)(1 - p_i)\lambda R_L[qp_j(1 - f(\rho)) + f(\rho)] \\
+ \psi[(1 - q)p_i R_H + q(1 - p_i)R_L] u(p_j) - g(\rho)C(p_i) - 1. \tag{3.12}
\]
The first three terms on the RHS of (3.12) represent the total pay-offs when the debt is rolled over (when the good signal is observed). The fourth term is the total pay-off when the debt is not rolled over and thus the project is liquidated at \( t = 1 \). The last two terms capture the cost of exerting effort and the cost of investment.
Regrouping terms in $p_i$ and simplifying, I get
\[
SW_{ST} = p_i [q R_H + (2q - 1)B - (1 - q)\lambda R_L [qp_j (1 - f(\rho)) + f(\rho)] \\
+ \psi[(1 - q)R_H - q R_L]u(p_j) + (1 - q)\lambda R_L [qp_j (1 - f(\rho)) + f(\rho)] \\
+ (1 - q)B + \psi q R_L u(p_j) - g(\rho)C(p_i) - 1.
\]

Let $w(p)$ denote
\[
w(p) = q R_H + (2q - 1)B - (1 - q)\lambda R_L [qp_j (1 - f(\rho)) + f(\rho)] \\
+ \psi[(1 - q)R_H - q R_L]u(p),
\]
then I can re-write the social surplus as
\[
SW_{ST} = p_i w(p_j) + \psi q R_L u(p_j) + (1 - q)\lambda R_L [qp_j (1 - f(\rho)) + f(\rho)] \\
+ (1 - q)B - g(\rho)C(p_i) - 1.
\]

So banker $i$’s optimization problem is equivalent to
\[
\max_{(p_i)} p_i w(p_j) - g(\rho)C(p_i)
\]
subject to
\[
p_i [w(p_j) - g(\rho)C'(p_i)] \geq 1 - (1 - q)\lambda R_L [qp_j (1 - f(\rho)) + f(\rho)] - \psi q R_L u(p_j).
\]

Similar to $h(p)$ in long-term debt case, $w(p)$ represents the difference between the total cash flows available in the good and bad states, when the signal is informative enough. When $q$ is very close to 1, investors decide not to roll over their debt when they receive a bad signal which is when investment is going to fail with a high probability. When the good signal is received, the project is going to succeed and investors roll over their debt. In the event of early liquidation total cash flows available is $\psi R_L [p(1 - f(\rho)) + f(\rho)]$ which depends on whether or not the other bank’s assets are liquidated at $t = 1$. In the good state, total cash flows of the project $R_H$ is available and since there is no early liquidation the banker can collect the private benefit of control $B$.\textsuperscript{4}

Since $\psi < \lambda$ and $B$ is positive, $w(p) > h(p)$ for any given level of $p$. This means that the difference between the cash flows available in the good and bad states is more significant with short-term financing compared to long-term financing.

As mentioned earlier, in order for short-term debt to have a disciplinary effect on the banker, it needs to be risky, i.e. it should not be rolled over when the bad signal is received. For this to be the case and the threat of early liquidation to be credible, the following condition should

\textsuperscript{4}To see this better, look at the case of the completely informative signal, i.e. $q=1$. In this case $w(p)$ simplifies to $w(p) = R_H + B - \psi R_L [p(1 - f(\rho)) + f(\rho)]$. [101]
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hold:
\[ E[\tilde{R}|\tilde{s} = s_B] \leq D \]  
(3.17)

When the bad signal is observed, debt holders do not roll over their debt if their expectation of the value of the bank at \( t = 2 \) is lower than their face value. Solving for \( D_i \) from (3.10), and given that
\[ E[\tilde{R}|\tilde{s} = s_B] = \frac{p_i(1-q)R_H + (1-p_i)qR_L}{p_i(1-q) + q(1-p_i)}, \]
(3.17) simplifies to
\[ [p_i(1-q)R_H + (1-p_i)qR_L][q^i + \psi(p_i(1-q) + q(1-p_i))u(p_j)] \]
+ \[ [p_i(1-q) + q(1-p_j)][(1-q)(1-p_i)\lambda R_L r(p_j) - 1] \leq 0, \]
where \( r(p) = q^p(1 - f(\rho)) + f(\rho) \).

3.3 Model implications

In this section, I examine the implications of the model for the level of the effort optimally exerted by each banker and the resulting total surplus in both short-term and long-term debt cases. I will then study the trade-off between the disciplinary effects of short-term debt and its roll over risk and the effects of investment correlation on this trade-off.

3.3.1 Optimal effort level

According to the optimization problems given by (3.6) and (3.7) for long-term debt and (3.15) and (3.16) for short-term debt, each banker chooses an optimal level of effort to exert. The participation constraints for both short-term and long-term financings (3.7), and (3.16) are binding at optimal levels of effort \( p_{i,LT}^* \) and \( p_{i,ST}^* \). So for any given level of bank \( j \)'s effort we have
\[ p_{i,LT}^* h(p_j) - p_{i,ST}^* w(p_j) + g(p)[p_{i,ST}^* C'(p_{i,ST}) - p_{i,LT}^* C'(p_{i,LT})] = \lambda R_L [(q(1-q) - 1)p_j(1 - f(\rho)) + f(\rho)(2 - q)] + \psi q R_L u(p_j). \]  
(3.18)

If the signal is informative enough, i.e. \( q \to 1 \), both \( u(p_j) \) and \( [(q(1-q) - 1)p_j(1 - f(\rho)) + f(\rho)(2 - q)] \) go to \( p_j(1 - f(\rho)) + f(\rho) \). This means that I can re-write (3.18) as:
\[ [p_{i,LT}^* h(p_j) - p_{i,ST}^* w(p_j)] + [g(p)[p_{i,ST}^* C'(p_{i,ST}) - p_{i,LT}^* C'(p_{i,LT})]] = (\psi - \lambda) R_L (p_j(1 - f(\rho)) + f(\rho)). \]
\( \psi < \lambda, \) and \( f(\rho) \leq 1, \) so \( (\psi - \lambda) R_L [p_j(1 - f(\rho)) + f(\rho)] \) is negative. When short-term debt is
used as a disciplinary device, the effort level it induces has to be higher than the effort level with long-term financing \( p_{1,ST}^* > p_{1,LT}^* \). When this is the case, and since the cost function \( C(p) \) is convex, \( g(\rho)[p_{1,ST}^* C'(p_{1,ST}^*) - p_{1,LT}^* C'(p_{1,LT}^*)] \) is positive. If (3.18) were to hold, the sum of the two brackets has to be negative. This means that the first bracket has to be negative enough to compensate for the positivity of the second bracket. For this to happen \( w(p_j) \) should be bigger enough than \( h(p_j) \). In this case, when the banker finances the assets with short-term debt, she experiences a more significant loss in the bad state relative to the good state compared to when she finances the assets by long-term debt. Thus, she has more incentives to exert a higher level of effort to compensate for this greater loss. This happens either when the private benefit of control \( B \) is big or when the early liquidation value \( \psi \) compared to the late liquidation value \( \lambda \) is small. A bigger loss of value with short-term debt either in the form of private benefit of control or inefficient early liquidation provides incentives for bankers to exert more effort.

Proposition 3.1. If private benefit of control is large enough and/or early liquidation value is small enough, short-term debt can induce each banker to choose a higher level of effort.

A higher level of effort induced by short-term debt does not necessarily lead to the optimality of short-term debt. This is due to the inefficiency of early liquidations. To study the optimality of short-term debt, one needs to compare the social surpluses resulting from the effort levels induced by short-term and long-term debt financing. This is what I study in detail in the next section.

3.3.2 Optimal maturity contract

To study banks’ optimal financing contracts, I focus on symmetric equilibrium where both bankers choose the same level of effort, i.e. \( p_i = p_j \). In this case I can re-write the optimization problem in the case of long-term debt (3.6) and (3.7) as follows

\[
\max_{(p)} p h(p) - g(p) C(p),
\]

subject to

\[
p[h(p) - g(p) C'(p)] \geq 1 - \lambda R_L[p(1 - f(\rho)) + f(\rho)].
\]

Since the constraint is binding at the optimal effort level \( p_{LT}^* \), we have

\[
p_{LT}^*[R_H - \lambda R_L[p_{LT}^*(1 - f(\rho)) + f(\rho)] - g(p) C'(p_{LT}^*)] - 1 + \lambda R_L[p_{LT}^*(1 - f(\rho)) + f(\rho)] = 0. \tag{3.19}
\]

\(^5\)When \( q \to 1 \), \( w(p_j) - h(p_j) = B + (\lambda - \psi) R_L[p_j(1 - f(\rho)) + f(\rho)].\)
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The total surplus at this optimal effort level is given by

\[
SW_{LT}^* = B + p_{LT}^* [R_H - \lambda R_L (1 - f(\rho)) + f(\rho)] \\
+ \lambda R_L [p_{LT}^*(1 - f(\rho)) + f(\rho)] - g(\rho)C(p_{LT}^*) - 1.
\]

From (3.19) and (3.20) we have

\[
SW_{LT}^*(\rho) = B + g(\rho)[C'(p_{LT}^*(\rho))p_{LT}^*(\rho) - C(p_{LT}^*(\rho))].
\]  

Equation (3.21) shows that in the case of long-term financing, the social surplus at any given level of correlation and private benefit of control depends on the optimal level of effort, and on the cost and the marginal cost of exerting effort. Both cost and marginal cost of exerting effort depend on the level of effort and also the correlation between banks investments as it is obvious from the presence of the term \( g(\rho) \). Since in the case of long-term financing bankers stay in control of the bank till the final date, total social surplus also includes the private benefit of control.

Using the same arguments, we can re-write the equation for the optimal effort in the short-term debt case \( p_{ST}^* \) as follows

\[
p_{ST}^* \left[ w(p_{ST}^*) - g(\rho)C'(p_{ST}^*) \right] - 1 + (1 - q) \lambda R_L [qp_{ST}^*(1 - f(\rho)) + f(\rho)] \\
+ \psi q R_L [qp_{ST}^* + (1 - q)(1 - p_{ST}^*)](1 - f(\rho)) + f(\rho)] = 0.
\]

The total surplus at this optimal effort level is given by

\[
SW_{ST}^* = p_{ST}^* w(p_{ST}^*) + (1 - q)B + (1 - q) \lambda R_L [qp_{ST}^*(1 - f(\rho)) + f(\rho)] \\
+ \psi q R_L [qp_{ST}^* + (1 - q)(1 - p_{ST}^*)](1 - f(\rho)) + f(\rho)] - g(\rho)C(p_{ST}^*) - 1.
\]

From (3.22) and (3.23) we have

\[
SW_{ST}^*(\rho) = (1 - q)B + g(\rho)[C'(p_{ST}^*(\rho))p_{ST}^*(\rho) - C(p_{ST}^*(\rho))].
\]

The expression for the social surplus for short-term debt (3.24) is very close to the one for long-term debt (3.21). It is evaluated at the optimal effort level in case of short-term debt and since the banker loses control in case of early liquidation, private benefit of control is only added with a probability of \( 1 - q \). The optimal level of effort with short-term debt depends on the private benefit of control since \( p_{ST}^* \) depends on \( w(p_{ST}^*) \) which in turn depends on \( B \) as shown in (3.13). Social welfare includes the private benefit of control in case of bank’s survival which happens with a probability of \( p_{ST}^*(2q - 1) + (1 - q) \). Although the term \( p_{ST}^*(2q - 1) \) is accounted for in the second term on the RHS of (3.24), \( (1 - q)B \) which is independent of \( p_{ST}^* \) is added to \( SW_{ST}^*(\rho) \) separately. By contrast, in the case of long-term financing the optimal level of effort \( p_{LT}^* \) is independent of the private benefit of control and thus \( B \) is added as a separate term to \( SW_{LT}^*(\rho) \) in (3.21).
3.3. Model implications

Given (3.21) and (3.24), for short-term debt to dominate long-term debt we should have

\[
\Delta SW(\rho) = SW^*_{ST}(\rho) - SW^*_{LT}(\rho)
\]

\[
= g(\rho) \left[ C'(p^*_ST(\rho))p^*_ST(\rho) - C'(p^*_LT(\rho))p^*_LT(\rho) + C(p^*_ST(\rho)) - C(p^*_LT(\rho)) \right] - qB \geq 0,
\]

which means that

\[
[C'(p^*_ST(\rho))p^*_ST(\rho) - C'(p^*_LT(\rho))p^*_LT(\rho)] \geq \frac{qB}{g(\rho)} + [C(p^*_ST(\rho)) - C(p^*_LT(\rho))].
\]

Equation (3.26) shows that whether or not short-term debt dominates long-term debt depends not only on the level of effort induced by each type of maturity, but also on the cost and the marginal cost of effort, private benefit of control, and the information synergies function (the benefit of having correlated investments since \( \frac{1}{g(\rho)} \) is increasing in correlation).

In the previous section, I analysed the conditions under which the optimal effort level is higher with short-term financing. If short-term debt is to be the optimal financing instrument for banks, the higher optimal effort it induces, should generate a higher total social surplus. To see whether or not this is the case, I study \( \Delta SW(\rho) \) from (3.25). When \( p^*_ST(\rho) > p^*_LT(\rho) \), the first difference inside the bracket on the RHS of (3.25) is positive whilst the second one is negative. This is because the cost function is increasing and convex. Moreover since \( B \) is positive, the sign of \( \Delta SW(\rho) \) is not necessarily positive. This means that inducing a higher optimal level of effort with short-term debt does not necessarily lead to a higher social surplus. Thus even though short-term debt can potentially increase the optimal level of effort, whether or not it is the optimal financing option depends on the level of private benefit of control, the cost function and the correlation between banks’ investments. The following proposition summarizes the results of this section.

**Proposition 3.2.** Under the conditions specified in Proposition 3.1, short-term debt financing can induce higher efforts. However the level of inefficiency of early liquidation, costs of exerting effort and the loss of the private benefit of control can lead to a lower total social surplus achieved with short-term debt. When this is the case, long-term debt becomes the optimal mode of financing despite the lower level of effort it leads to.

### 3.3.3 Role of correlation

Having studied the conditions under which short-term debt induces bankers to exert more effort and whether or not this higher effort leads to a higher social welfare, I now study the role of bank’s asset correlation on the trade-off between the disciplinary effects of short-term debt and its roll over losses. Specifically, I study how the optimality of short-term debt can be affected in the presence of positive and negative externalities induced by correlated investments. To do so, I assume a benchmark case where banks are indifferent between
short-term and long-term financing. Assume there exists a level of correlation $\rho_0$ such that at $\rho = \rho_0$ total surplus in the case of long-term debt is equal to the total surplus in the case of short-term debt. Thus for $\rho = \rho_0$, bankers are indifferent between issuing long-term and short-term debts and the following holds

$$\Delta SW(\rho_0) = \frac{g(\rho_0)}{g(\rho_0)}|C'(p^*_ST(\rho_0))p^*_ST(\rho_0) - C'(p^*_LT(\rho_0))p^*_LT(\rho_0) + C(p^*_LT(\rho_0)) - C(p^*_ST(\rho_0))| - qB = 0.$$  \hspace{1cm} (3.27)

Given that $\Delta SW(\rho_0) = 0$, I can study how the difference between total surpluses $\Delta SW(\rho)$ changes by changing the correlation from the original level of $\rho_0$. To do this, I calculate the derivative of $\Delta SW$ with respect to correlation

$$\Delta SW'(\rho) = g'(\rho)|C'(p^*_ST(\rho))p^*_ST(\rho) - C'(p^*_LT(\rho))p^*_LT(\rho) + C(p^*_LT(\rho)) - C(p^*_ST(\rho))|$$

$$+ g(\rho)|C''(p^*_ST(\rho))p^*_ST(\rho)p^*_ST(\rho) - C''(p^*_LT(\rho))p^*_LT(\rho)p^*_LT(\rho)|.$$  \hspace{1cm} (3.28)

Since at $\rho = \rho_0$, (3.27) holds we can evaluate the derivative of $\Delta SW(\rho)$ at $\rho_0$

$$\Delta SW'(\rho_0) = \frac{g'(\rho_0)}{g(\rho_0)}|C''(p^*_ST(\rho_0))p^*_ST(\rho_0)p^*_ST(\rho_0) - C''(p^*_LT(\rho_0))p^*_LT(\rho_0)p^*_LT(\rho_0)|$$

$$+ \frac{g'(\rho_0)qB}{g(\rho_0)}.$$  

For this derivative to be positive we need to have

$$C''(p^*_ST(\rho_0))p^*_ST(\rho_0)p^*_ST(\rho_0) > C''(p^*_LT(\rho_0))p^*_LT(\rho_0)p^*_LT(\rho_0) - \frac{g'(\rho_0)qB}{g(\rho_0)}.$$  \hspace{1cm} (3.29)

If (3.29) holds, $\Delta SW'(\rho_0)$ is positive and $\exists \rho > \rho_0$ for which $\Delta SW'(\rho)$ will be positive. Therefore, short-term debt becomes the optimal mode of financing for at least some level of correlation bigger than the benchmark level of $\rho_0$. When $\Delta SW(\rho)$ is a monotonic function of correlation (which is the case in the numerical example that I study in the next section), $\Delta SW(\rho)$ is positive for all $\rho > \rho_0$ and short-term debt is the optimal mode of financing for all levels of correlation bigger than the benchmark level of $\rho_0$.

So whether or not short-term debt is the optimal mode of financing depends not only on each bank’s individual characteristics, but also on how systemic banks’ investments are. Increasing or decreasing the correlation from a benchmark case where bankers are indifferent between short-term and long-term financing, can make short-term debt the dominant or dominated mode of financing depending on the level of existing positive and negative externalities. A representative bank framework does not include these negative and positive externalities and thus can lead to misleading results when banks invest in correlated assets.

As a special case, assume that $\rho_0 = 0$. This means that bankers are indifferent between short-term and long-term debt exactly when their investments are not correlated, i.e. the representative bank framework studied in Repullo et al. (2013). When (3.29) holds at $\rho_0 = 0$, $\Delta SW'(0)$ is positive and bankers would no longer be indifferent between short-term and
3.3. Model implications

long-term financing if they were to make correlated investments. In fact in this particular case, moving away from a representative bank framework to a case where banks’ assets are correlated, can make short-term debt the optimal mode of financing. In other words as soon as banks start investing in correlated assets, short-term debt becomes more appealing. By contrast, if \( \Delta SW'(0) \) is negative long-term financing becomes the optimal mode of financing as soon as we move away from the representative bank framework to a set-up where banks’ investments are correlated.

If \( \rho_0 > 0 \) and (3.29) holds, decreasing correlation from \( \rho_0 \) can make long-term debt the optimal mode of financing whilst increasing it beyond \( \rho_0 \) can make short-term financing optimal. This means that for at least some given investment correlation of \( \rho > \rho_0 > 0 \), short-term debt is the optimal mode of financing. However when banks’ maturity structure is studied in a representative bank framework which is equivalent to the case where \( \rho = 0 \), the optimal choice of maturity will appear to be long-term debt. When \( \Delta SW'(\rho) \) given by (3.28) is zero at \( \rho = 0 \), a representative bank framework gives an accurate result of the optimality of short-term versus long-term debt. In this case the difference between social surpluses is indifferent to the correlation between banks’ assets and thus the optimal choice of debt maturity is independent of the level of banks’ asset commonalities. I formalize my results from this section in the following proposition.

**Proposition 3.3.** When banks make correlated investments,

- their optimal mode of financing depends not only on their individual characteristics but also on the positive and negative externalities induced by the correlation between assets.
- studying the optimality of short-term debt in a representative bank framework, ignores these externalities and can lead to misleading results.

How correlation affects the optimality of short-term debt depends on the level of negative (fire-sale) and positive (information synergies) externalities. To see this better, I study a numerical example in the next section where I can show how the optimal mode of bank financing can change from short-term debt to long-term debt and vice versa by changing the correlation of banks’ investments.

3.3.4 A numerical example

To study a numerical case, I use the model parameters reported in Table 3.3, and I calculate the optimal effort levels in both long-term and short-term debt cases and the resulting total social surpluses. Calculating \( \Delta p \) (the difference between the optimal effort levels with short-term financing versus long-term financing), I study whether or not short-term debt induces bankers to exert more effort. When the threat of early liquidation with short-term debt provides more incentives for bankers to exert effort, \( \Delta p \) becomes positive. Next, I compare the total surpluses at the optimal effort level for both maturities by calculating \( \Delta SW \) to decide which maturity is
optimal for this set of parameters. A positive $\Delta SW$ means the total social surplus with short-term debt is higher than with long-term debt and thus it is optimal for bankers to finance their risky projects with short-term debt. For the set of parameters in Table 3.3, condition (3.17) is satisfied. So the short-term debt used to finance each bank's project is indeed risky and is only rolled over to the final date if the good signal is received at $t = 1$. Thus short-term debt financing has the potential to act as a disciplinary device.

Since investment correlation affects the optimal level of effort and the optimal maturity structure through negative and positive externalities, in addition to the benchmark case I study the effort level and the total social surplus in different environments with low, medium and high levels of fire-sale externalities and information synergies. In each of these cases, I study the effects of changing investment correlation, projects' outcome, and private benefit of control on the optimal level of effort and the optimal debt maturity. Additionally, I analyse the effects of changing externality parameters $(a, b)$ on the level of effort and the total social surplus in three cases with high, medium and low investment correlations.

Given the base case parameters value in Table 3.3, the correlation at which $\Delta SW$ is zero and thus bankers are indifferent between short-term and long-term debt is around 0.65. Moreover, at a zero investment correlation level (in a representative bank framework) $\Delta SW$ is negative and thus long-term debt is the optimal mode of financing. This optimal maturity structure changes to short-term debt when the level of correlation increases to more than 0.65. Below I summarize how the optimal maturity structure is affected by the main parameters of the model, specially the investment correlation, fire-sale externalities and information synergies.

**Investment correlation ($\rho$)**

The benchmark case I study has average levels of fire-sale externalities and information synergies. The base case values for fire-sale externalities parameter $(a)$ and information synergies parameter $(b)$ are set to 0.5 and 0.05 respectively. Figure 3.1 shows that in this benchmark case both $\Delta p$ and $\Delta SW$ increase by increasing investment correlation. This means that when bankers invest in more correlated assets, the difference between effort levels induced by short-term and long-term financings increases. Moreover, with the baseline parameters of fire-sale externalities and information synergies, when increasing investment correlation, the increase in $\Delta p$ leads to an increase in $\Delta SW$ until it eventually becomes positive, i.e. short-term debt eventually becomes the optimal mode of financing at $\rho = 0.65$. At correlation levels above 0.65 short-term debt is the optimal maturity structure whilst at lower levels of correlation, long-term debt is the optimal mode of financing.

As discussed in the last section, the effects of increasing correlation between banks' assets on $\Delta p$ and $\Delta SW$ depend on how severe fire-sale externalities and how beneficial information synergies are. To see this, I study different environments with different levels of fire-sale externalities and information synergies. For each case, I keep one of the two parameters at the medium level, whilst I change the other parameter to a "high" or "low" level. Thus I have 4 additional cases: high and low fire-sale externalities cases and high and low information synergies cases.
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As the top panel of Figure 3.1 shows, increasing correlation induces a much higher effort level with short-term debt compared to long-term debt when fire-sale externalities are big or information synergies are small. On the contrary, for low fire-sale externalities or high information synergies, short-term and long-term financing effort levels become closer when the correlation between banks’ investments is increased.

Note that in all cases \( \Delta p \) is positive for any given level of investment correlation, i.e. short-term debt always induces bankers to exert more effort regardless of how systemic banks are. This is true for different levels of information synergies and fire-sale externalities. However when fire-sale externalities are very high or information synergies are very low, a more significant difference in effort level for any given level of correlation is observed. In other words, the optimal effort level with short-term debt financing is closer to the optimal effort level with long-term debt financing when information synergies are high or fire-sale externalities are low.

When fire-sale externalities are high, increasing correlation results in a significant drop in early liquidation value when both banks default at the same time. This lower liquidation value provides more incentives for bankers to exert more effort with short-term debt. Moreover, for a given level of correlation, higher fire-sale externalities result in lower expected early liquidation values and thus a higher effort level is to be exerted with short-term debt. The opposite holds for high information synergies. When information synergies are high, increasing correlation results in a significant drop in the costs of exerting effort. When exerting effort is less costly, bankers find it optimal to exert more effort even with long-term debt so \( \Delta p \) decreases. Additionally for a given level of correlation, when information synergies are lower the cost of exerting effort is higher and so bankers are more reluctant to exert effort. In this case, there is a stronger disciplinary role for short-term debt financing.

Although \( \Delta p \) remains positive when changing investment correlation in different environments, the social surplus in case of short-term financing is not necessarily higher than the social surplus with long-term financing. This is demonstrated in the bottom panel of Figure 3.1 which shows that the level of \( \Delta SW \) can change from negative to positive depending on the level of correlation, fire-sale externalities and information synergies. This is in line with both Proposition 3.2, and Proposition 3.3. It is only for a high enough level of difference in effort levels that higher effort in short-term debt can lead to a higher total social surplus. \( \Delta SW \) does not only depend on \( \Delta p \) but also on the level of private benefit of control, cost of exerting effort and the level of inefficiency of early liquidation. The cost of exerting effort and the early liquidation value both depend on the correlation between banks assets and the level of inefficiency.

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6 For high and low fire-sale externalities cases I keep the information synergies at a medium level \( b = 0.05 \) and I study the cases where there are high fire-sale externalities \( a = 0.9 \) or low (no) fire-sale externalities \( a = 0 \). For high and low information synergies cases I keep the fire-sale externalities at a medium level \( a = 0.5 \) and I study the cases where the information synergies are high \( b = 0.1 \) or where there are low (no) information synergies \( b = 0 \).
fire-sale externalities and information synergies this correlation leads to. Thus correlation affects the optimal maturity both indirectly through its effects on $\Delta p$ and directly through its effects on the cost of effort and the early liquidation value. In the benchmark case with an average level of fire-sale externalities and information synergies, increasing correlation increases the disciplinary benefits of short-term debt relative to its early liquidation costs and at one point short-term debt becomes optimal. This shows that in this case, although when banks’ investments are uncorrelated, the benefits of short-term debt can not overcome its costs, increasing correlation beyond a point (here $\rho = 0.65$) changes this result and the disciplinary benefits of short-term debt eventually dominate its costs and make it the optimal choice of financing.

The patterns observed in the benchmark case remain for both high fire-sale externalities and low information synergies cases. However in both of these cases the increase in $\Delta SW$ is more significant compared to the benchmark case. When fire-sale externalities parameter is as high as 0.9, short-term debt becomes optimal for correlations as low as 0.3 and when there is no information synergies, short-term debt dominates long-term debt for levels of correlations as low as 0.4. On the other hand when there is no negative externalities and so the liquidation values do not depend on the correlation of banks’ investments, not only short-term debt never becomes optimal, its benefits decrease relative to its costs by increasing correlation. This is because the threat of early liquidation is not strong enough and thus only results in a small variation in the level of effort. As the top panel of Figure 3.1 shows in this case $\Delta p$ is very small and decreasing with correlation. The small increase in the level of effort with short-term debt is not enough to overcome the costs of early liquidations short-term financing leads to and thus short-term financing is never optimal. In the case where there is a high level of information synergies the benefits of short-term debt increase relative to its costs by increasing correlation. However similar to the previous case this increase is never enough for short-term debt to become optimal. As demonstrated in the top panel of Figure (3.1), here again $\Delta p$ is very small and decreasing with investment correlation. High information synergies decrease the costs of exerting effort and motivates bankers to exert more effort even with long-term financing. Increasing correlation decreases the costs of effort levels and thus $\Delta p$ even further. The increase in $\Delta p$ is not big enough to overcome the costs of short-term debt and thus long-term debt remains the optimal mode of financing for all levels of investment correlation.

My results so far show that the relative benefits of short-term versus long-term debt significantly depend on the level of correlation of banks’ investments and how this correlation affects their cost of exerting effort and their liquidation values. The following statement summarizes these results

**Result 3.1.** There is a more important role for short-term debt in high correlation, high negative fire-sale externalities and low information synergies environments.

The disciplinary effects of short-term debt can make bankers exert more effort and thus increase not only the probability of each individual bank’s success but also the probability
3.3. Model implications

of a joint success. However, only when the correlation is high and the negative fire-sale externalities are higher relative to the positive information synergies that these disciplinary benefits overcome the costs of early liquidation induced by short-term debt, and short-term financing becomes optimal.

**Negative and positive externalities** ($a$, $b$)

Figure 3.2 shows $\Delta p$ and $\Delta SW$ as functions of fire-sale externalities $a$ and information synergies $b$. Since investment correlation, and negative and positive externalities affect bankers’ effort levels and social welfare simultaneously, in addition to my benchmark case where correlation is set to an average level of ($\rho = 0.5$), I study the effects of $a$ and $b$ in a high correlation environment with $\rho = 0.9$ and a low correlation environment with $\rho = 0.2$.

The effects of fire-sale externalities and information synergies on $\Delta p$ is demonstrated in the top panel of Figure 3.2 in three different cases with low, medium and high investment correlations. $\Delta p$ is positive in all three cases and remains so when changing the level of positive and negative externalities. This means that short-term debt always induces a higher effort level. However $\Delta p$ decreases (the level of effort with short-term debt becomes closer to the level of effort with long-term debt) when information synergies are increased and when fire-sale externalities are decreased. This means that the disciplinary effects of short-term debt increase for higher levels of negative externalities and lower levels of positive externalities. This is because higher levels of fire-sale externalities lead to lower expected early liquidation values in the case of short-term debt financing and provides incentives for bankers to exert more effort. On the other hand, when information synergies are high bankers have more incentives to exert effort in both short-term and long-term debt cases because exerting effort is less costly. Since bankers are willing to exert more effort on their own, the disciplinary effects of short-term debt become less important and $\Delta p$ decreases.

High fire-sale externalities make early liquidations more costly and thus induce bankers to exert more effort with short-term debt, the higher effort increases the expected pay-off of the project and thus the social welfare. At the same time, fire-sale externalities increase the cost of early liquidations. So when fire-sale externalities increase, early liquidation becomes more costly but at the same time less probable. The bottom panel of Figure 3.2 shows that the first effect dominates and $\Delta SW$ increases when increasing $a$. Whether or not short-term debt becomes dominant for some higher levels of fire-sale externalities depends on the level of correlation of the banks’ investments. In high correlation cases the increase in $\Delta SW$ when increasing the fire-sale externalities is more significant and can eventually lead to the optimality of short-term debt. However, in low correlation cases $\Delta SW$ increases at a lower rate when increasing fire-sale externalities. When the correlation is low enough the increase in $\Delta SW$ is never big enough to make short-term debt optimal.

Increasing the level of information synergies decreases the benefits of short-term debt compared to its costs and makes it less and less attractive. High information synergies decrease the cost of exerting effort and thus $\Delta p$. When the level of effort induced by short-term debt is not
significantly higher than the effort level with long-term debt, the early liquidation costs with short-term debt become the more important factor and $\Delta SW$ decreases. For low correlation cases $\Delta SW$ starts at negative levels and decreases even further when increasing information synergies, meaning that short-term financing is never optimal. For higher correlation cases although short-term debt starts as the optimal mode of financing, by increasing information synergies it loses its relative advantage over long-term debt and it eventually becomes dominated by long-term debt. The following statement summarizes the key results of this discussion.

**Result 3.2.** With benchmark case parameters of Table 3.3

- increasing fire-sale externalities or decreasing information synergies, increases the relative benefits of short-term debt and leads to higher $\Delta SW$'s.
- whether or not this increase is enough to lead to the optimality of short-term debt depends on the level of investment correlation.

Although these results are obtained in my numerical example, it is clear that in general whether or not short-term debt is optimal depends on the level of investments’ correlation and the negative and positive externalities this correlation implies. Thus when banks invest in correlated assets, studying the optimality of short-term debt in a representative bank framework is not conclusive.

**Projects bad outcome ($R_L$)**

In addition to the benchmark case, I study the effects of changing projects’ bad outcome in high and low correlations, fire-sale externalities and information synergies environments. In each case I keep two of the parameters $\rho$, $a$, and $b$ at their medium levels and change the remaining parameter to a high or low value.\(^7\)

The top panel of Figure 3.3 shows that increasing the bad outcome of banks’ projects decreases $\Delta p$. This holds for all cases of low, medium and high investment correlation and positive and negative externalities. A higher bad outcome of the project increases the expected pay-off of the project. This higher expected pay-off leads to higher pay-offs in case of liquidation. Thus the banker has less incentives to exert more effort with short-term debt compared to long-term debt.

As the bottom panel of Figure 3.3 shows increasing $R_L$ decreases $\Delta SW$ rapidly from a positive level where short-term debt dominates long-term debt to a negative level where short-term debt is dominated by long-term debt. This means that by increasing $R_L$ the benefits of short-term debt decreases relative to its costs. This happens at a faster rate for cases in which there

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\(^7\)To be precise, in addition to the benchmark case ($\rho = 0.5, a = 0.5, b = 0.05$) I study the following cases: low correlation ($\rho = 0.2, a = 0.5, b = 0.05$), high correlation ($\rho = 0.9, a = 0.5, b = 0.05$), low (no) fire-sale externalities ($\rho = 0.5, a = 0, b = 0.05$), high fire-sale externalities ($\rho = 0.5, a = 0.9, b = 0.05$), low (no) information synergies ($\rho = 0.5, a = 0.5, b = 0$), and high information synergies ($\rho = 0.5, a = 0.5, b = 0.1$).
is low correlation between banks’ investments, low fire-sale externalities and high information synergies. So the level of $R_H$ above which short-term debt financing is not optimal depends on the level of the correlation and the magnitude of the positive and negative externalities this correlation leads to.

**Projects good outcome ($R_H$)**

The top panel of Figure 3.4 shows that increasing the good outcome of banks’ projects has a similar effect to increasing the bad outcome of the project and decreases $\Delta p$. This holds for all cases of low, medium and high investment correlation and positive and negative externalities. Similar to the last case, a higher $R_H$ increases the expected pay-offs in case of early liquidation. Thus the banker has less incentives to exert more effort with short-term debt compared to long-term debt.

As the bottom panel of Figure 3.4 shows, the effects of increasing the good outcome of banks’ investments on $\Delta SW$ are similar to the effects of increasing the bad outcome. By increasing $R_H$ benefits of short-term debt decreases rapidly relative to its costs and it eventually becomes dominated by long-term debt. This means that the disciplinary effect of short-term debt is only present for quite low levels of investment’s good outcome. This is in line with the results obtained in Repullo et al. (2013). However what is interesting is how the level of $R_H$ above which short-term debt is not optimal changes when the correlation between banks’ investments changes. For a high level of correlation the disciplinary effect of short-term debt remains the dominant factor for higher levels of $R_H$, whilst in the low correlation case short-term debt becomes dominated by long-term debt quite rapidly. Keeping correlation at a medium level, I can study how these results are affected by changing the level of negative and positive externalities. In the presence of very high negative fire-sale externalities the disciplinary benefits of short-term debt dominates its costs of early liquidation for a wider range of $R_H$, whilst in the case where there is no negative externalities short-term debt becomes quickly dominated by long-term debt at very low levels of $R_H$. Positive externalities have the opposite effects; lower information synergies leads to a bigger range of $R_H$ over which short-term debt is optimal whilst higher information synergies decrease the benefits of short-term debt faster and make it become the suboptimal mode of financing at lower levels of $R_H$.

**Private benefit of control ($B$)**

When bank projects are financed by short-term debt, bankers risk losing their private benefit of control if short-term debt leads to early liquidation. If the level of this private benefit of control is higher, bankers have more incentives to prevent early liquidations in the fear of losing this benefit of control. This is why $\Delta p$ increases by increasing $B$. The lower the private benefit of control, the closer the effort level with short-term debt will be to the effort level with long-term debt. The top panel of Figure 3.5 shows that decreasing the private benefit of control beyond a point, can actually induce a lower level of effort with short-term debt compared to long-term debt. This argument holds for all investment correlations and all levels of positive and negative externalities.
Increasing private benefit of control can affect $\Delta SW$ in two opposite directions. An increase in private benefit of control motivates bankers to exert higher levels of effort with short-term debt to avoid early liquidation and collect this higher benefit of control; this decreases the costs of short-term debt. On the other hand increasing private benefit of control makes early liquidation more costly in the case of short-term debt compared to long-term debt where the probability of early liquidation is zero; this implies a higher cost of short-term debt. The bottom panel of Figure 3.5 shows how $\Delta SW$ first increases and then decreases by increasing the private benefit of control. This means that increasing $B$ to some levels implies a net decrease in the cost of short-term debt (the first effect dominates) but a further increase beyond this level implies a net increase in the costs of short-term debt and reduces its relative advantage (the second factor dominates).

High and low correlation, fire-sale externalities and information synergies do not change the general pattern. However with average levels of positive and negative externalities, the increase in $\Delta SW$ is only big enough to result in the optimality of short-term debt when the level of correlation is high. For average correlations short-term debt becomes optimal when we increase private benefit of control only in high fire-sale externalities or low information synergies cases.

### 3.4 Conclusion

I study the optimal debt maturity in a framework where banks are subject to systemic risk through correlated investments. Fire-sale externalities and information synergies in the presence of correlated assets can affect the trade-off between the disciplinary effects of short-term debt and its roll over risk.

I show that whether or not short-term financing increases the total social surplus does not only depend on each bank's individual risk, but also on the systemic risk banks are exposed to. Specifically there is a more important role for the disciplinary effects of short-term debt in high correlation, high fire-sale externalities and low information synergy environments. Analysing the optimality of short-term debt versus long-term debt in a representative bank framework abstracts from the externalities correlated investments lead to and thus can be misleading.
The table shows the long-term debt pay-offs for bank $i$ where $i$ and $j \in \{A,B\}$ and $i \neq j$. The cost of effort for bank $i$ is $g(\rho)C(p_i)$.

### Table 3.1: Long-term debt pay-offs

<table>
<thead>
<tr>
<th>Investment at $t = 0$</th>
<th>Cash flow at $t = 2$</th>
<th>Pay-off at $t = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Investors Banker</td>
</tr>
<tr>
<td>$R_H$ $(\text{Pr. } p_i)$</td>
<td>$D_i$</td>
<td>$R_H - D_i + B$</td>
</tr>
<tr>
<td>$R_L$ $(\text{Pr. } 1 - p_i)$</td>
<td>$\lambda R_L$ $(\text{Pr. } p_j)$</td>
<td>$B$</td>
</tr>
<tr>
<td></td>
<td>$f(p)\lambda R_L$ $(\text{Pr. } 1 - p_j)$</td>
<td>$B$</td>
</tr>
</tbody>
</table>

$115$
The table shows the short-term debt pay-offs for bank $i$ where $i$ and $j \in \{A,B\}$ and $i \neq j$. The cost of effort for bank $i$ is $C(p_i)g(\rho)$. 

<table>
<thead>
<tr>
<th>Investment at $t=0$</th>
<th>Cash flow at $t=2$</th>
<th>Signal observed at $t=1$</th>
<th>Pay-off at $t=2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_H$</td>
<td>$S_G$</td>
<td>$D_i$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($Pr. q p_i$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R_H$</td>
<td>$S_B$</td>
<td>($Pr. (1-q)p_i$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>($Pr. (1-q)p_j + q(1-p_j)$)</td>
</tr>
<tr>
<td></td>
<td>$R_L$</td>
<td>$S_B$</td>
<td>($Pr. (1-q)p_j$) $f(\rho)\psi R_H$ 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>($Pr. (1-q)p_j + q(1-p_j)$)</td>
</tr>
<tr>
<td></td>
<td>$R_L$</td>
<td>$S_G$</td>
<td>($Pr. (1-q)(1-p_i)$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($Pr. (1-q)(1-p_j)$)</td>
<td>$f(\rho)\lambda R_L$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>($Pr. (1-q)(1-p_j)$)</td>
<td>($Pr. 1-qp_j)$)</td>
</tr>
</tbody>
</table>
### Table 3.3: Base case parameter values

<table>
<thead>
<tr>
<th>Interpretation</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project’s good outcome</td>
<td>$R_H$</td>
<td>2.0</td>
</tr>
<tr>
<td>Project’s bad outcome</td>
<td>$R_L$</td>
<td>0.5</td>
</tr>
<tr>
<td>Liquidation parameter at $t = 2$</td>
<td>$\lambda$</td>
<td>1</td>
</tr>
<tr>
<td>Liquidation parameter at $t = 1$</td>
<td>$\psi$</td>
<td>0.8</td>
</tr>
<tr>
<td>Information synergy parameter</td>
<td>$b$</td>
<td>0.05</td>
</tr>
<tr>
<td>Fire-sale externalities parameter</td>
<td>$a$</td>
<td>0.5</td>
</tr>
<tr>
<td>Marginal cost parameter</td>
<td>$\gamma$</td>
<td>0.25</td>
</tr>
<tr>
<td>Private benefit of control</td>
<td>$B$</td>
<td>0.2</td>
</tr>
<tr>
<td>The quality of signal</td>
<td>$q$</td>
<td>0.95</td>
</tr>
<tr>
<td>Investment correlation</td>
<td>$\rho$</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Figure 3.1: $\Delta p$ and $\Delta SW$ versus investment correlation ($\rho$)

The figure shows how $\Delta p$ (top panel), and $\Delta SW$ (bottom panel) change with respect to the changes in banks' investments' correlation. The left hand panel shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) fire-sale externalities ($a = 0, 0.5, 0.9$) and the right hand panel shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) information synergies ($b = 0, 0.05, 0.1$).
Figure 3.2: $\Delta P$ and $\Delta SW$ versus positive and negative externalities ($a$ and $b$)

The figure shows how $\Delta p$ (top panel), and $\Delta SW$ (bottom panel) change with respect to both positive (the left panel) and negative externalities (the right panel). High ($\rho = 0.9$), medium ($\rho = 0.5$), and low ($\rho = 0.2$) cases of correlation are shown by dotted red, solid blue, and dashed black lines respectively.
Figure 3.3: ∆\(P\) and ∆\(SW\) versus bad project outcome (R\(L\))

The figure shows how \(\Delta P\) (top panel) and \(\Delta SW\) (bottom panel) change with respect to the changes in the bad outcome of banks’ projects. The panel on the left shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) correlation (\(\rho = 0.2, 0.5, 0.9\)). The middle panel shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) fire-sale externalities (\(a = 0, 0.5, 0.9\)). The panel on the right shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) information synergies (\(b = 0, 0.05, 0.1\)).
The figure shows how $\Delta p$ (top panel) and $\Delta SW$ (bottom panel) change with respect to the changes in the good outcome of banks’ projects. The panel on the left shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) correlation ($\rho = 0.2, 0.5, 0.9$). The middle panel shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) fire-sale externalities ($a = 0, 0.5, 0.9$). The panel on the right shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) information synergies ($b = 0, 0.05, 0.1$).
The figure shows how $\Delta P$ (top panel) and $\Delta SW$ (bottom panel) change with respect to the changes in the private benefit of control. The panel on the left shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) correlation ($\rho = 0.2, 0.5, 0.9$). The middle panel shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) fire-sale externalities ($\alpha = 0, 0.5, 0.9$). The panel on the right shows the cases of high (dotted red line), medium (solid blue line) and low (dashed black line) information synergies ($\beta = 0, 0.05, 0.1$).
Conclusion

This thesis makes three main contributions to the research on bank liability structure. First, in a cash management model in which liquidity problems are the major source of default for banks, the first chapter of this thesis studies SIFI’s optimal capital structure from a social point of view and when it has access to CoCo debt financing. The social welfare perspective adopted in this chapter has two features: first, SIFI’s failure entails significant social costs that are not borne by its shareholders. Second, the tax advantage of debt (a mere transfer between taxpayers and bank’s shareholders) does not create any value. In this set-up, we show that CoCo bonds like any other form of market debt are a socially inefficient way of financing SIFIs if their only advantage is to provide tax subsidies. CoCo bonds reduce both the probability of default (which is good for regulators) and the taxes paid by banks (which is good for financial industry). However this is done at the expense of tax payers for whom the optimal mode of financing for SIFIs is 100% equity.

Second, developing a theoretical model of a bail-in contract that eliminates the inefficient liquidations of SIFIs and their subsequent social costs, this thesis seeks to address the too-big-to-fail problem. To this end, the second chapter of this thesis introduces a countercyclical CoCo debt as a hybrid security that converts into equity in the context of a pre-defined restructuring plan that the bank commits to ex-ante. The conversion which is triggered when the bank is in distress and recapitalization is too costly and is followed by the reissuance of CoCo debt, helps ease bank’s financing constraints in bad times. With this model, I study the effects of the bail-in contract and its design features on bank’s optimal financing and pay-out policies and its optimal choice of capital structure. I show that higher conversion ratios increase the debt capacity of the bank and lead to higher potential leverage ratios. At the same time by providing a new source of outside liquidity and eliminating liquidation costs, bail-in plans decrease shareholders’ incentives to build up cash buffers within the bank and to recapitalize when they can. So bail-in plans can potentially lead to a less capitalized, more levered banking system, however, they can create value from both private and social points of view since they eliminate both the costs of liquidation and the costs of negative externalities associated with SIFIs’ failures.

Third, this thesis contributes to the literature studying the optimal debt maturity of banks. In particular the third chapter investigates the optimality of short-term debt financing in a framework where banks are subject to systemic risk through correlated investments. Asset
commonality leads to fire-sale externalities and information synergies which in turn can affect the trade-off between the disciplinary effects of short-term debt and its roll over risk. I show that, similar to the representative bank framework, short-term debt can induce bankers to exert more effort to achieve a higher investment return. Whether or not this higher level of effort increases the total social surplus depends on the costs of exerting effort and the costs of early liquidations. Higher investment correlations make exerting effort less costly but make early liquidations more costly. As a result the optimality of short-term debt is directly affected by the level of banks investment correlation and the magnitude of the positive and negative externalities this correlation leads to. Specifically there is a more important role for short-term debt financing in high correlation, high fire-sale externalities and low information synergies environments. My results in the third chapter conclude that analysing the optimality of short-term debt in a representative bank framework abstracts from the externalities correlated investments lead to and thus can provide misleading results.

Further research

In this thesis, I have sought to answer some of the concerns about the stability of the financial system. My focus is specifically on the liability structure of banks whose distress affects other banks and financial institutions. In the first two chapters I study the effects of CoCo debt financing and bail-in requirements on the optimal capital structure and optimal pay-out and financing policy choices of banks. To do so, I have assumed that banks hold a fixed level of assets. This means that I do not explore how CoCo debt financing or bail-in contract can affect bank's choice of risky assets, i.e. loan portfolios. Further research can be conducted to study how access to CoCo debt financing or commitment to a bail-in contract can also affect the level and composition of banks' portfolio of risky assets.

Additionally, in the first chapter I assume that banks default due to liquidity and not solvency problems. Specifically I consider a profitable financial firm that may be forced to close down because of liquidity problems. A more general model that incorporates both solvency related and liquidity related defaults can be the ideal framework for future research studying the optimal mode of financing for SIFIs.

Another possible area for further research is related to the cost structure of CoCo reissuance in the second chapter. In this chapter, I assume that banks incur a fixed cost when reissuing CoCo debt that replaces the converted debt. This cost is assumed to be independent of the terms of the bail-in contract. Equivalently, investors' appetite for CoCo bonds does not depend on how these bonds are designed. However, in practice, the design of the CoCo debt specially the conversion trigger can affect the investor demand for such securities and thus the costs associated with its issuance. Investors may be more willing to hold "investor friendly" CoCo bonds with conversion ratios closer to 1. This means that the design of the bail-in contract affects shareholders’ capital structure and policy choices both directly through the dilution upon conversion and indirectly through the costs of CoCo reissuance. Moreover, I assume conversion of CoCo debt into equity is efficient and costless. Further research can be
conducted to study bank’s optimal capital structure and policy choices when the cost of CoCo issuance depends on the design of the CoCo debt and/or when conversion is inefficient and costly.

In the third chapter of this thesis I focus on the debt maturity of banks when they invest in correlated assets. To do so, I abstract from the choice of leverage and assume banks have no capital. Moreover in my model I assume banks can either choose to finance their assets with short-term debt or long-term debt. Further research can deviate from these assumptions and allow for a mix of long-term and short-term debt and equity financing. I also assume the correlation between banks’ assets is exogenously given. A model in which banks can choose the level of correlation between their assets at the same time as their optimal maturity structure can be the subject of further studies.
Appendices

A.1 Appendix to Chapter 1

A.1.1 Proofs of Propositions 1.1 to 1.6

Proposition 1.1 We need to prove that the optimal coupon payment when interest payments are not tax deductible is zero, i.e. $c^* = 0$.

Proof. Shareholders maximization problem is equivalent to

$$\max_{m^*, c} V(m^*; c, m^*) - m^*.$$ 

Defining a new function $h(m; c, m^*) = V_2(m; c, m^*)$, and taking the derivative of (1.4) with respect to $c$, we have

$$\begin{cases} r h(m; c, m^*) = 1 - V_1(m; c, m^*) + [\mu(1-\theta) - c]h_1(m; c, m^*) + \frac{1}{2}(1-\theta)\sigma^2 h_{11}(m; c, m^*) \\ h(0; c, m^*) = 0 \\ h_1(m^*; c, m^*) = 0, \end{cases}$$

(A.1)

where $V_1(m; c, m^*)$ is the derivative of $V(m; c, m^*)$ with respect to $m$, and $h_1(m; c, m^*)$ and $h_{11}(m; c, m^*)$ are the first and second derivatives of $h(m; c, m^*)$ with respect to $m$.

Given that $h(m; c, m^*)$ is continuous, it has a maximum on the interval $(0, m^*)$. We prove below that this maximum is non-positive which will guarantee that $\forall m, c, m^* : h(m; c, m^*) \leq 0$.

- if the maximum is attained at $m = 0$, $\forall m, c, m^* : h(m; c, m^*) \leq 0$, because of the first boundary condition of (A.1).
- if the maximum is interior, $h_1(m; c, m^*) = 0$, and $h_{11}(m; c, m^*) < 0$ at the maximum. Given (A.1) and the fact that $V_1(m; c, m^*) > 1: \forall m, c, m^* : h(m; c, m^*) < 0$. 

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- if the maximum is attained at \( m^* \), since \( h_1(m^*;c,m^*) = 0 \) then \( h_{11}(m^*;c,m^*) \leq 0 \).\(^1\)

Given (A.1) and the fact that \( V_1(m^*;c,m^*) = 1: \forall m, c, m^* : h(m;c,m^*) \leq 0 \).

So \( h(m;c,m^*) = V_2(m;c,m^*) \) is always negative or zero; which means that \( V(m;c,m^*) \) is non-increasing in \( c \). Therefore the maximum value of the bank is attained when \( c \) is set to zero.

For the case of regulators the proof of \( c^* = 0 \) is identical. \( \square \)

We also need to prove that privately optimal capital ratio is lower than regulatory optimal capital ratio, i.e. \( m^* < m^*_R \).

**Proof.** We define \( \varphi(m) = z_1^2 e^{z_1 m} - z_2^2 e^{z_2 m} \). Dividend threshold is determined by the super contact condition in both equity holders and regulators cases.

\[
V''(m^*) = 0 \implies \varphi(m^*) = 0,
\]

and

\[
R''(m^*_R) = 0 \implies \varphi(m^*_R) = y z_2 z_1 e^{(z_1+z_2) m^*_R} (z_2 - z_1) > 0.
\]

Since \( z_1 > 0 > z_2 \), \( \varphi(m) \) is an increasing function of \( m \), and thus \( m^*_R \) has to be bigger than \( m^* \). \( \square \)

**Lemma 1** We need to prove that the discounted probability of default \( p(m) \) is decreasing in \( m^* \).

**Proof.** Taking the derivative of \( p(m) \) given by (1.8) gives

\[
\frac{\partial p(m)}{\partial m^*} = \frac{z_1 z_2 (z_2 - z_1) e^{-m^* (z_1+z_2)} (e^{z_2 m} - e^{z_1 m})}{(z_1 e^{-z_2 m^*} - z_2 e^{-z_1 m^*})^2}
\]

which is negative since \( z_2 < 0 < z_1 \). \( \square \)

**Proposition 1.3** We need to prove that for a large enough tax rate, it is optimal for shareholders to have some debt financing.

\(^1\)Indeed if \( h_{11}(m^*;c,m^*) \) was positive, the Taylor expansion around \( m^* \):

\[
h(m;c,m^*) \sim h(m^*;c,m^*) + \frac{h_{11}(m^*;c,m^*)}{2} (m - m^*)^2
\]

would contradict the fact that \( h(\cdot;c,m^*) \) is maximum at \( m^* \).
Proof. We first define

\[ TV(c) = V(m^*(c), c) - m^*(c) \]

as the function to be optimized by shareholders at time zero. We need to show that the max value of \( TV(c) \) is attained when \( c \) is positive, in other words \( TV'(0) > 0 \).

Since

\[ V(m^*(c), c) = \frac{c}{r} [1 - P(m^*, c)] + E(m^*(c), c), \]

and

\[ E(m^*(c), c) = \frac{(1 - \theta)(\mu - c)}{r}, \]

\[ TV'(0) = \frac{d}{dc} [V(m^*(c), c)]_{c=0} - \frac{dm^*}{dc}(0) \]

can be calculated as

\[ TV'(0) = \frac{1 - P(m^*(0), 0)}{r} - \frac{1 - \theta}{r} - \frac{dm^*}{dc}(0) \] \hspace{1cm} (A.2)

where \( P(m^*(0), 0) \) is given by

\[ P(m^*(0), 0) = \frac{z_1 e^{z_1(m - m^*)} - z_2 e^{z_2(m - m^*)}}{z_1 e^{-z_1 m^*} - z_2 e^{-z_2 m^*}}. \]

\( m^*(c) \) is given by

\[ m^*(c) = \frac{\ln y_2^2(c, \theta) - \ln y_1^2(c, \theta)}{y_1(c, \theta) - y_2(c, \theta)}, \]

or equivalently

\[ m^*(c, \theta) = \frac{\ln y_2^2(c, 0) - \ln y_1^2(c, 0)}{y_1(c, 0) - y_2(c, 0)} [1 - \theta], \]

where

\[ y_1(c, 0) = (1 - \theta) y_1(c, \theta) \]

Given (A.2), the condition for a positive coupon payment to be optimal becomes

\[ \theta > P(m^*(0), 0) + r \frac{dm^*}{dc}(0). \] \hspace{1cm} (A.3)

Taking the derivative of \( m^* \) with respect to \( c \) evaluated at \( c = 0 \), we get

\[
\frac{dm^*}{dc}(0) = \frac{\frac{2}{z_1} \frac{dy_1}{dc}(0) - \frac{2}{z_2} \frac{dy_2}{dc}(0)}{z_1 - z_2} - \frac{\frac{dy_1}{dc}(0) - \frac{dy_2}{dc}(0)}{(z_1 - z_2)^2} \ln \frac{z_2}{z_1} \]
where
\[
\frac{dy_i}{dc}(0) = \frac{z_i}{\mu + \sigma^2(1-\theta)z_i}.
\]

Since the super contact condition holds at \(m^*(0)\), \(e^{-z_i m^*(0)} = \frac{z_i^2}{z_1^2} e^{-z_i m^*(0)}\), we can rewrite \(P(m^*(0), 0)\) as
\[
P(m^*(0), 0) = \left| \frac{z_2}{z_1} \right|^{\frac{z_1+z_2}{z_1-z_2}} (A.4)
\]

Using the expression for \(\frac{dy}{dc}(0)\), \(\frac{dm^*}{dc}(0)\) can be simplified as
\[
\frac{dm^*}{dc}(0) = \frac{2\sigma^2(1-\theta) + \mu m^*(0)}{\left| \mu + \sigma^2(1-\theta)z_2 \right| \left| \mu + \sigma^2(1-\theta)z_1 \right|} (A.5)
\]

Given (A.4) and (A.5), we can rewrite (A.3) as
\[
\theta > \left| \frac{z_2}{z_1} \right|^{\frac{z_1+z_2}{z_1-z_2}} + \frac{r(2\sigma^2 + \mu m^*(0))}{\left| \mu + \sigma^2(1-\theta)z_2 \right| \left| \mu + \sigma^2(1-\theta)z_1 \right|} (A.6)
\]

When the tax rate is high enough such that (A.6) is satisfied, it is optimal for shareholders to choose some debt financing (i.e. \(c^* > 0\)).

We also need to prove that commitment on future dividend threshold delays dividend distribution.

**Proof.** Define \(\varphi(m) = y_1 e^{y_1 m} - y_2 e^{y_2 m}\). Dividend threshold is determined by super contact condition on the value of equity and the total value of the bank in the cases of equity holders and committed equity holders respectively.

\[E''(m^*) = 0 \implies \varphi(m^*) = 0,\]

and

\[V''(m^*_{Com.}) = 0 \implies \varphi(m^*_{Com.}) = \frac{c}{r} (y_2 - y_1) y_1 y_2 e^{(y_1 + y_2)m^*_{Com.}} > 0.\]

Since \(\varphi(m)\) is an increasing function of \(m\), \(m^*_{Com.}\) has to be bigger than \(m^*\) for any given level of \(c\).

Similarly for regulatory dividend threshold we can define \(\phi(m) = y_1^2 e^{-y_1 m} - y_2^2 e^{-y_2 m}\). So

\[V''(m^*_{Com.}) = 0 \implies \phi(m^*_{Com.}) = \frac{c}{r} (y_2 - y_1) y_1 y_2.\]

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\[ R''(m^*_R) = 0 \implies \phi(m^*_R) = \left( \frac{c}{r} + \gamma \right) (y_2 - y_1) y_1 y_2. \]

For as long as \( \gamma > 0 \), \( m^*_R \) has to be bigger than \( m^*_{Com.} \). So \( m^*_R > m^*_{Com.} > m^* \) for any given level of \( c \). \( \square \)

**Proposition 1.5** We need to prove that the socially optimal financing mode for a SIFI is 100% equity when the only benefits to debt are tax subsidies.

**Proof.** Similar to Section 1.3.1 we can define a function \( h(m; c, m^*_W) = W_2(m; c, m^*_W) \). By taking the derivative of (1.14) with respect to \( c \), we have

\[
\begin{align*}
\{ & rh(m; c, m^*_W) = (1 - \theta)(1 - W_1(m; c, m^*_W)) + |\mu(1 - \theta) - c| h_1(m; c, m^*_W) \\
& \quad + \frac{1}{2}(1 - \theta) \sigma^2 h_{11}(m; c, m^*_W) \\
& h(0; c, m^*_W) = 0 \\
& h_1(m^*_W; c, m^*_W) = 0. \tag{A.7}
\end{align*}
\]

Given that \( h(m; c, m^*_W) \) is continuous, it has a maximum on the interval \((0, m^*_W)\):

- if the maximum is attained at \( m = 0 \), \( \forall m, c, m^*_W : h(m; c, m^*_W) \leq 0 \), because of the first boundary condition of (A.7).
- if the maximum is interior, \( h_1(m; c, m^*_W) = 0 \), and \( h_{11}(m; c, m^*_W) < 0 \) at the maximum. Given (A.7) and the fact that \( W_1(m; c, m^*) > 1 \) and \( \theta < 1 \), \( \forall m, c, m^*_W : h(m; c, m^*_W) < 0 \).
- if the maximum is attained at \( m^*_W \), since \( h_1(m^*_W; c, m^*_W) = 0 \), then \( h_{11}(m^*_W; c, m^*_W) < 0 \). Given (A.7) and the fact that \( W_1(m^*_W; c, m^*_W) = 1 \), \( \forall m, c, m^*_W : h(m; c, m^*_W) < 0 \).

So \( h(m; c, m^*_W) = W_2(m; c, m^*_W) \) is always negative or zero; which means that \( W(m; c, m^*_W) \) is non-increasing in \( c \). Therefore the maximum social value of the bank is attained when \( c \) is set to zero. \( \square \)

**Results of Section 1.4.2** We need to prove that commitment increases the dividend threshold and that the socially optimal dividend threshold when there are tax subsidies to CoCo bonds is higher than the privately optimal dividend threshold, i.e. \( m^*_{R,b} > m^*_{Com.,b} > m^*_b \).

**Proof.** Define \( \varphi(m) = y_2^2 e^{\lambda (\bar{m} - m)} - y_1^2 e^{\lambda (\bar{m} - m)} \). The equations to find the dividend threshold for different cases are given by:

**for equity holders**

\[
E''(m^*_b) = 0 \implies \varphi(m^*_b) = y_1 y_2 (y_2 - y_1)(1 - \alpha) E(a(\bar{m})),
\]

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for committed equity holders
\[ V_b''(m_{Com,b}) = 0 \implies \varphi(m_{Com,b}) = y_1 y_2 (V_a(\tilde{m}) - \frac{c}{r}), \]

and for regulators
\[ R_b''(m_{R,b}) = 0 \implies \varphi(m_{R,b}) = y_1 y_2 (V_a(\tilde{m}) - \frac{c}{r}). \]

Since \( y_2 < 0 < y_1 \), \( \varphi(m) \) is a decreasing function of \( m \). Given that regulators incur the cost of failure \( R_a(\tilde{m}) < V_a(\tilde{m}) \). Moreover \( V_a(\tilde{m}) - \frac{c}{r} = E_a(\tilde{m}) - \frac{c}{r} < E_a(\tilde{m}) - CC(\tilde{m}) = (1 - \alpha)E_a(\tilde{m}) \), since the value of CoCo bond is smaller than the risk-free debt with the same coupon payment \( ((CC(\tilde{m}) = \frac{c}{r} (1 - A(\tilde{m}) < \frac{c}{r}, as \ A(\tilde{m}) \geq 0). \) Therefore we have

\[ R_a(\tilde{m}) - \frac{c}{r} < V_a(\tilde{m}) - \frac{c}{r} < (1 - \alpha)E_a(\tilde{m}). \]

This means that \( m_{R,b}^* > m_{Com,b}^* > m_b^* \).

**Proposition 1.6** We need to prove that CoCo bonds are socially suboptimal when the only benefits of debt are tax subsidies.

**Proof.** Similar to Proposition 1.5 we can define a function \( h(m; c, m_W^*) = W_{b,2}(m; c, m_W^*) \). By taking the derivative of (A.9) in Section A.1.4 with respect to \( c \), we have

\[
\begin{align*}
\{ & r h(m; c, m_W^*) = (1 - \theta)(1 - W_{b,1}(m; c, m_W^*)) + [\mu(1 - \theta) - c] h_1(m; c, m_W^*) \\
& + \frac{1}{2} (1 - \theta) \sigma^2 h_{11}(m; c, m_W^*) \\
& h(0; c, m_W^*) = 0 \\
& h_1(m_W^*; c, m_W^*) = 0. \\
\end{align*}
\]

(A.8)

As before, we can prove that \( h(m; c, m_W^*) = W_{b,2}(m; c, m_W^*) \) is always negative or zero; which means that \( W_b(m; c, m_W^*) \) is non-increasing in \( c \). Therefore the maximum social value of the bank is attained when no CoCo bonds are issued. \( \square \)

A.1.2 Security re-issuance

In this section we relax the assumption that the cost of issuance is so high that the bank never finds it optimal to re-issue any security. As Décamps et al. (2011) discuss, when there is no uncertainty of funds availability in the secondary market and the cost of re-issuance is not too high, the firm will always find it optimal to re-issue new securities as soon as it runs out of cash. In this case, the firm never defaults. Since our aim is to study a SIFI whose default imposes high social costs on the whole society, a default-free set-up is not ideal. To make our model more representative of a SIFI, we introduce capital supply uncertainty into our
model. In the same esprit of Hugonnier et al. (2015) we assume that the SIFI needs to search
for investors in the capital markets who are willing to purchase its securities. If this is the case,
there might be instances where the bank would like to issue new securities but it has no access
to investors that are willing to purchase its securities. In such a set-up, the bank can run out of
cash and thus faces a positive probability of default. In the next section we study the optimal
capital structure of a SIFI when capital supply is uncertain.

Uncertain capital supply

To incorporate the capital supply uncertainty, we assume, as in Hugonnier et al. (2015), that
the bank meets new investors at the jump times of a Poisson process $N_t$ with intensity of $\lambda \geq 0$.
Under this additional assumption, the cash reserves of the bank evolve according to

$$dm_t = (1 - \theta)(\mu dt + \sigma dZ_t) + (f_t - i)dN_t - dL_t,$$

where $f_t$ is the funds raised through the equity issuance upon arrival of the new investors.\(^2\) $f_t$ is a non-negative predictable process. $i$ is the fixed cost of issuance. We assume that $i$
is low enough for a new security issuance to be sometimes optimal.\(^3\) Given that there is a
fixed cost of issuance even when $\lambda = \infty$ (which is when there is no capital supply uncertainty),
the capital markets are not frictionless and the bank has a need to keep precautionary cash.
If $\lambda = 0$, no matter how small $i$ is, the bank never meets new investors and thus can never
re-issue any securities and we are back to the case of no security reissuance.

Bank owners need to decide on the financing and liquidation policy of the SIFI. As before, there
exists a target level of cash reserves $m^{**}$ above which cash is distributed outside the bank. This
is where the marginal value of cash inside and outside the bank is equal. In addition, note that
when current shareholders decide to issue new equity, they will get $V(m^{**}) - (m^{**} - m) - i$.
Shareholders would only be willing to do so if their abandonment value $V(m)$, is below the
value they would get upon re-issuance. This is equivalent to

$$V(m) - m \leq V(m^{**}) - m^{**} - i.$$

Since the left hand side of the above equation is decreasing in $m$, there exists a level of cash
reserves $0 < m < m^{**}$, below which it is optimal to reissue whenever possible, i.e. whenever
one can find new investors. When the bank's cash reserves are below $m$, the bank issues new
equity as soon as it meets new investors which allows to bring its cash holdings back to the
target level $m^{**}$. Finally, the bank gets liquidated when its cash holdings reach zero.

\(^2\)We abstract from introducing a continuous coupon payment $c$ into our cash reserves process, since it can be
proven easily and similar to Section 1.3, that in the absence of tax subsidies, the optimal $c$ chosen by both bank
owners and regulators is zero.

\(^3\)See Décamps et al. (2011) for more details. If $i$ is small enough, we are in the case where secondary issuance
is optimal and the firm would always reissue capital as soon as it runs out of cash. However in our set-up since
capital supply is uncertain, the timing of the re-issuance changes. This is discussed later-on.
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We thus conjecture that we have three different regions for the value of the bank.

- First in the region \((0, m)\), the bank retains earnings and issues new equity as soon as a new investor is found. Thus in this region, the value of the bank follows the following ODE

\[
r V(m) = (1 - \theta)\mu V'(m) + \frac{\sigma^2}{2}(1 - \theta)^2 V''(m) + \lambda[V(m^{**}) - (m^{**} - m) - i - V(m)]
\]

with boundary conditions

\[
\begin{align*}
V(0) &= 0 \\
V(m^-) &= V(m^+) \\
V'(m^-) &= V'(m^+)
\end{align*}
\]

As before, the left hand side of this equation represents the required rate of return for investing in the bank (the risk-free rate). The right hand side consists of the effects of cash savings and the volatility of the bank’s cash flows (captured in the first and second terms). The additional last term on the right hand side represents the effects of capital supply uncertainty and fixed cost of issuance. As previously discussed, since the cost of equity issuance is fixed, the optimal amount to issue when new investors are found, is the exact amount that takes the cash reserves back to the target level \(m^{**}\). So whenever the bank meets a new investor, which happens with a probability of \(\lambda\), the bank issues new equity to take its cash reserves from \(m\) to \(m^{**}\) and thus the value of the bank from \(V(m)\) to \(V(m^{**})\). To do so, the bank incurs a fixed cost of \(i\).

The first boundary condition \(V(0) = 0\), shows that when the bank’s cash reserves hit zero, it defaults. This means that the bank has not been able to meet enough investors who are willing to inject new capital into the bank before it runs out of cash. The second and third boundary conditions are required for the value of the bank to be continuous.

There is an additional condition from which \(m\), the lower barrier level below which equity issuance is optimal can be solved for

\[
V(m) = V(m^{**}) - (m^{**} - m) - i.
\]

This condition holds because when the level of the bank’s cash reserves is exactly at \(m\), the bank is indifferent between issuing and not issuing new equity.

- The second region is \((m, m^{**})\), when cash reserves are between the lower equity issuance level \(m\) and the target cash level \(m^{**}\). In this region, the bank’s cash reserves are high enough that it is not optimal for the bank to issue new securities and pay the cost of re-issuance upon the arrival of new investors. Thus in this region the bank does not re-issue equity and the total value of the bank is given by the following ODE and its
boundary conditions
\[
\begin{align*}
  r V(m) &= (1-\theta)\mu V'(m) + \frac{\sigma^2}{2} (1-\theta)^2 V''(m) \\
  V'(m^{**}) &= 1 \\
  V''(m^{**}) &= 0.
\end{align*}
\]

Since there is no re-issuance of the equity in this region, the last term of the ODE for the first region is not included here. The first boundary condition states that the marginal value of cash at the target level of \(m^{**}\) is equal to one. The last boundary condition is the super contact condition which determines \(m^{**}\), and can also be represented as \(V(m^{**}) = \frac{(1-\theta)\mu}{r}\).

- The third region is \([m^{**}, \infty)\), where cash reserves are bigger than the target level of \(m^{**}\). In this region the bank pays-out any additional cash to keep its cash reserves at \(m^{**}\). There is obviously no equity re-issuance. Thus the value of the bank is given by
  \[V(m) = V(m^{**}) + m^{**} - m.\]

What is important in our discussion is not the explicit value of the bank, but the fact that there exists a distress region where it is optimal for the bank to issue equity and increase its cash reserves. However since investors are not always available, there is a positive probability that bank’s cash reserves hit zero and it defaults. There is no dividend distribution in this region. Second, there exists a distribution region where it is not optimal for the bank to re-issue equity even upon the arrival of investors, but it is optimal to distribute cash outside the bank when cash reserves are above a target level.

The possibility of equity re-issuance does not change the optimal capital structure of the bank of 100% equity. Indeed since the marginal value of the funds is decreasing, for as long as it is bigger than one, any kind of payment that draws funds from the bank’s cash reserves would be suboptimal.

From the point of view of regulators the value function is similar to the value function for equity holders with one major difference: the costs of SIFI’s failure that are not internalized by SIFI’s equity holders. This means that the three regions of the value function stay the same, but the thresholds below which equity re-issuance happens and the pay-out threshold are not the same as before. So we can characterize the social value of the SIFI as follows

- In the region \((0, m_g]\), SIFI retains earnings and issues new equity whenever new investors are found. When the bank meets no new investors before it runs out of cash, it can default. There is a social cost of SIFI’s default \(\gamma\), which is not internalized by its shareholders. Thus the regulatory value of the bank solves the following ODE with its
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boundary conditions

\[
\begin{align*}
   r R(m) &= (1 - \theta) \mu R'(m) + \frac{\sigma^2}{2} (1 - \theta)^2 R''(m) \\
   &\quad + \lambda [R(m^*_R) - (m^*_R - m) - i - R(m)] \\
   R(0) &= -\gamma \\
   R(m^*_R) &= R(m^*_R) \\
   R'(m^*_R) &= R'(m^*_R).
\end{align*}
\]

The following condition determines the re-issuance threshold

\[V(m_R) = V(m^*_R) - (m^*_R - m_R) - i.\]

- In the region \((m^*_R, m^*_R],\) the bank does not re-issue equity and its social value solves the following ODE which is identical to the case of equity holders, with a different pay-out threshold \(m^*_R\)

\[
\begin{align*}
   r R(m) &= (1 - \theta) \mu R'(m) + \frac{\sigma^2}{2} (1 - \theta)^2 R''(m) \\
   R'(m^*_R) &= 1 \\
   R''(m^*_R) &= 0.
\end{align*}
\]

- In the region \([m^*_R, \infty),\) the bank pays-out any cash in excess of \(m^*_R\) and its value to regulators is equal to

\[R(m) = R(m^*_R) + m^*_R - m.\]

The positive probability of failure and its social cost lead to different objective functions for equity holders and regulators. Equity holders, if left alone, would not choose a high enough pay-out threshold, and thus choose a too low book equity target: they would distribute dividends too often. Regulators would like SIFI's to build up more cash before they start distributing any cash outside the bank. Our results from this section are formalized in the following proposition.

**Proposition A.1.** In the presence of issuance costs and capital supply uncertainty

- privately optimal financing mode is still 100% equity.

- the only difference with no-re-issuance case and the always re-issuance case is that there is a positive probability of equity re-issuance and a positive probability of default.

- since default is possible, the objective function of the regulator is not the same as the objective function of equity holders and thus there is a need for capital regulation.
A.1. Appendix to Chapter 1

A.1.3 Closed form values of equity and debt

Straight debt case

In this appendix, we calculate the closed form solutions for the value of equity and debt where the debt to be included in the capital structure is in the form of a straight bond. We start from the case where shareholders are in charge of the optimization problem.

Every time the level of bank's cash reserves hit \( m^* \), shareholders collect dividends and they get zero when the cash reserves hit zero. Thus the value of the bank's equity solves the following ODE subject to its boundary conditions.

\[
\begin{align*}
\frac{d}{dm} E(m) &= (1-\theta)(\mu - c)E'(m) + \frac{1}{2}\sigma^2 (1-\theta)^2 E''(m) \\
E(0) &= 0 \\
E'(m^*) &= 1 \\
E''(m^*) &= 0.
\end{align*}
\]

The above ODE along with its boundary conditions results in

\[
E(m^*) = \frac{(1-\theta)(\mu - c)}{r}.
\]

It is then easy to solve for the value of equity in the closed form solution.

\[
E(m) = \frac{e^{y_1 m} - e^{y_2 m}}{y_1 e^{y_1 m^*} - y_2 e^{y_2 m^*}}
\]

where \( y_1 \) and \( y_2 \) are given in (1.11), and the dividend pay-out threshold is given explicitly by

\[
m^* = \frac{\ln(y_2)^2}{y_1 - y_2}.
\]

Debt holders receive the coupon payment of \( c \) continuously till the time of default when they also get zero since the liquidation value is set to zero. Thus the value of debt solves the following ODE.

\[
\begin{align*}
\frac{d}{dm} D(m) &= c + (1-\theta)(\mu - c)D'(m) + \frac{1}{2}\sigma^2 (1-\theta)^2 D''(m) \\
D(0) &= 0 \\
D'(m^*) &= 0.
\end{align*}
\]

Since every dollar in addition to \( m^* \) is distributed to shareholders, at dividend boundary threshold the marginal value of debt is zero. Solving the above ODE gives the closed form solution of the value of the straight debt.

\[
D(m) = \frac{c}{r} [1 - P(m, c)],
\]
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where \( P(m, c) \) is the expected probability of default and is given in (1.12). The expressions for the value of equity and debt stay the same in the case of committed equity holders and regulators except that dividend threshold \( m^* \) is to be replaced by the appropriate dividend thresholds in each case (\( m_{Com}^* \) for committed equity holders and \( m_R^* \) for regulators.)

In the social welfare case since no debt is issued, the value of equity is equal to the total value of the bank.

**CoCo bonds case**

In this appendix, we calculate the closed form solutions for the value of equity and debt where the debt to be included in the capital structure is in the form of a CoCo bond. We start from the case where shareholders are in charge of the optimization problem. Similar to calculating the value of the bank, we start from the period during which the conversion has already been triggered and we work backwards to date zero.

After the conversion of the CoCo bond and since there is no more debt included in the capital structure the value of equity is equal to the whole value of the bank.

\[
E_a(m) = z_1 e^{z_1 m} - z_2 e^{z_2 m},
\]

where \( z_1 \) and \( z_2 \) are given in (1.6), and the dividend threshold \( m^*_a \) is given explicitly by

\[
m_a^* = \frac{\ln(z_2/z_1)^2}{z_1 - z_2}.
\]

Before conversion the value of equity solves the following ODE with its boundary conditions.

\[
\begin{align*}
r E_b(m) &= (1 - \theta)(\mu - c)E'_b(m) + \frac{1}{2}\sigma^2(1 - \theta)^2E''_b(m) \\
E_b(\bar{m}) &= (1 - \alpha)E_a(\bar{m}) \\
E'(m) &= 1 \\
E''(m) &= 0.
\end{align*}
\]

So the expression for the value of equity before conversion is given by

\[
E_b(m) = [(1 - \alpha)V_a(\bar{m}) + \frac{e^{y_2 (\bar{m} + y_1 m) - e^{y_1 (\bar{m} + y_1 m)}}}{y_1 e^{y_1 m_a + y_2 m} - y_2 e^{y_2 m_a + y_1 m}} A(m)]
\]

where \( y_1 \) and \( y_2 \) are given in (1.11), \( A(m) \) is the expected probability of conversion given in (1.20), and the dividend pay-out threshold satisfies

\[
y_2^2 e^{y_2 (\bar{m} - m_a)} - y_1^2 e^{y_1 (\bar{m} - m_a)} = y_1 y_2 (y_2 - y_1)(1 - \alpha)E_a(\bar{m}).
\]
The value of CoCo bonds solves the following ODE
\[
\begin{align*}
  r CC(m) &= c + (1 - \theta)(\mu - c)CC'(m) + \frac{1}{2}\sigma^2(1 - \theta)^2 CC''(m) \\
  CC(\tilde{m}) &= \alpha E_a(\tilde{m}) \\
  CC'(m^*_b) &= 0.
\end{align*}
\]
CoCo bond holders get their coupon payment until the level of cash reserves hit \( \tilde{m} \) at which point CoCo bonds convert to equity and CoCo bond holders get a fraction \( \alpha \) of the bank. Since every dollar in addition to \( m^* \) is distributed to shareholders, at dividend boundary threshold the marginal value of CoCo bonds is zero. Solving the ODE gives the closed form solution for the value of CoCo bonds
\[
CC(m) = \frac{c}{r} [1 - A(m)] + \alpha V_2(\tilde{m}) A(m).
\]
So the value of the CoCo bond consists of two parts: the value of a risk-free debt until the time of conversion and the value to debt holders equal to a fraction \( \alpha \) of the whole bank at conversion.

The expressions for the value of equity and CoCo bonds stay the same in the cases of committed equity holders and regulators except that dividend threshold \( m^*_b \) and conversion threshold \( \tilde{m} \) are to be replaced by the appropriate dividend thresholds and conversion thresholds in each case.

In the social welfare case since no CoCo bonds are issued, the value of equity is equal to the total value of the bank.

### A.1.4 CoCo bonds - social welfare

To find the social value of SIFI at time zero we solve the problem starting from the social value after conversion which is given by the following ODE with its boundary conditions
\[
\begin{align*}
  r W_a(m) &= \theta \mu + (1 - \theta)\mu W'_a(m) + \frac{1}{2}\sigma^2(1 - \theta)^2 W''_a(m) \\
  W_a(0) &= -\gamma \\
  W'_a(m_{W,a}) &= 1 \\
  W''_a(m_{W,a}) &= 0.
\end{align*}
\]
So the closed form expression for the social value of the bank after conversion is given by
\[
W_a(m) = \frac{(1 + (\gamma + \frac{\theta \mu}{\gamma})z_2e^{z_2m_{W,b}})e^{z_1m} - (1 + (\gamma + \frac{\theta \mu}{\gamma})z_1e^{z_1m_{W,b}})e^{z_2m}}{z_1e^{z_1m_{W,b}} - z_2e^{z_2m_{W,b}}}.
\]
where \( z_1 \) and \( z_2 \) are given in (1.6), and \( m^*_{W,b} \) is the socially optimal dividend threshold after conversion which is determined by the super contact condition

\[
W_b''(m^*_{W,b}) = z_1^2 e^{-z_2 m^*_{W,b}} - z_2^2 e^{-z_1 m^*_{W,b}} - (y + \frac{\theta \mu}{r}) z_2 (z_2 - z_1) = 0.
\]

Before conversion the social value of the SIFI solves the following ODE

\[
\begin{align*}
  r W_b(m) &= c(1 - \theta) + \theta \mu + (1 - \theta)(\mu - c) W'_b(m) + \frac{1}{2} \sigma^2 (1 - \theta)^2 W''_b(m) \\
  W_b(\bar{m}_W) &= W_d(\bar{m}_W) \\
  W'_b(m^*_{W,b}) &= 1 \\
  W''_b(m^*_{W,b}) &= 0.
\end{align*}
\]  

(A.9)

Solving this ODE gives the following closed form solution for the social value of the bank before conversion

\[
W_b(m) = \frac{c(1 - \theta) + \mu \theta}{r} [1 - A_W(m)] + [W_d(\bar{m}_W) + \frac{e^{y_2 \bar{m}_W + y_1 m} - e^{y_1 \bar{m}_W + y_2 m}}{y_1 e^{y_1 m^*_{W,b} + y_2 m_W} - y_2 e^{y_2 m^*_{W,b} + y_1 m_W}} A_W(m),
\]

where \( y_1 \) and \( y_2 \) are given in (1.11) and \( A_W(m) \) which is the discounted expected probability of conversion in this case is given by

\[
A_W(m) = \frac{y_1 e^{y_1 m^*_{W,b} + y_2 m} - y_2 e^{y_2 m^*_{W,b} + y_1 m}}{y_1 e^{y_1 m^*_{W,b} + y_2 m_W} - y_2 e^{y_2 m^*_{W,b} + y_1 \bar{m}_W}}.
\]

To maximize SIFI’s social value, and given the dividend threshold that solves the super contact condition on the social value of the bank, an optimal conversion threshold and an optimal amount of CoCo bond to be issued should be decided on at time zero when initial cash injection is \( m_0 = m^*_{W,b} \)

\[
\max_{c,m_W} W_b(m^*_{W,b}) - m^*_{W,b}.
\]
Figure A.1: Optimal capital structure and pay-out policy - CoCo bond

Panel A ($\mu$)

Equity holders

Regulators

0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14

Conversion threshold

Optimal coupon payment

$\mu = 12\%$

$\mu = 15\%$

$\mu = 18\%$

Conversion threshold

Dividend threshold

Equity holders

Regulators

0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14

0.0

0.1

0.2

0.3

0.4

0.5

0.6

0.7

Conversion threshold

Dividend threshold

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Panel B ($\sigma$)

![Graphs showing the relationship between conversion threshold and optimal coupon payment for different values of \(\sigma\).]
Panel C ($\theta$)

\[ \theta = 25\% \]

Equity holders
Regulators

\[ \theta = 35\% \]

Equity holders
Regulators

\[ \theta = 40\% \]

Equity holders
Regulators
The figure shows how the optimal dividend threshold and the optimal coupon payment of the CoCo bond change with conversion threshold. Solid lines and dashed lines represent shareholders and regulators respectively. Panels (A), (B), (C), and (D) show the results for different levels of profitability, volatility, tax rate, and external cost of failure respectively.
A.2 Appendix to Chapter 2

A.2.1 Values of bank’s securities - straight debt

Given the system of equation in (2.6), and the boundary conditions discussed in Section 2.3.1, I can solve for the closed form solution of equity value in the bad state of the world:

\[
E_B(w) = \begin{cases} 
\frac{\pi_{BG}(E_G(W^*_G)-W^*_G-\gamma_{E,G})}{\pi_{BG}+\pi_{GB}} + \frac{\pi_{BG}(1-\theta)(\mu-c)}{(r+\pi_{GB}+\pi_{GB})^2} + \alpha_3 e^{x_1 w} + \alpha_4 e^{x_2 w}, & \forall w \in (0, W) \\
\beta_1 e^{x_1 w} + \beta_2 e^{x_2 w} - \frac{\pi_{BG}(\alpha_1 e^{x_1 w} + \alpha_2 e^{x_2 w})}{\pi_{BG}+\pi_{GB}}, & \forall w \in (W^*_G, W^*_B) \\
E_B(W^*_G) + w - W^*_B, & \forall w > W^*_B
\end{cases}
\]

(A.10)

where \(y_1 > 0 > y_2\) are the roots of the characteristic equation

\[
r + \pi_{GB} + \pi_{BG} = (1-\theta)(\mu-C)y + \frac{1}{2} \sigma^2 (1-\theta)^2 y^2,
\]

(A.11)

\(z_1 > 0 > z_2\) are the roots of the characteristic equation

\[
r = (1-\theta)(\mu-C)z + \frac{1}{2} \sigma^2 (1-\theta)^2 z^2,
\]

(A.12)

and \(x_1 > 0 > x_2\) are the roots of the characteristic equation

\[
r + \pi_{BG} = (1-\theta)(\mu-C)x + \frac{1}{2} \sigma^2 (1-\theta)^2 x^2.
\]

(A.13)

Similarly given the equation in (2.5), and the boundary conditions discussed, the closed form expression for the value of equity in the good state of the world is as follows:

\[
E_G(w) = \begin{cases} 
E_G(W^*_G) + w - W^*_G - \gamma_{E,G} & \forall w \in (0, W) \\
\beta_1 e^{y_1 w} + \beta_2 e^{y_2 w} - \frac{\pi_{BG}(\alpha_1 e^{y_1 w} + \alpha_2 e^{y_2 w})}{\pi_{BG}+\pi_{GB}}, & \forall w \in [W^*_G, W^*_B) \\
E_G(W^*_G) + w - W^*_G, & \forall w > W^*_G
\end{cases}
\]

(A.14)

where \(y_1, y_2, z_1,\) and \(z_2\) are given in (A.11), and (A.12). \(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6,\) and \(\beta_1,\) and \(\beta_2\) are solved for by using the relevant boundary conditions in Section 2.3.1.
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The value of straight debt in the bad state is given by:

\[
SD_B(w) = \begin{cases} 
\frac{C_{SD} + \pi_{BG}SD_G(W^*_G)}{r + \pi_{BG}} + \alpha_9 e^{x_1w} + \alpha_{10} e^{x_2w}, & \forall w \in (0, W] \\
\frac{C_{SD} + \beta_3 e^{y_1w} + \beta_4 e^{y_2w}}{r + \pi_{BG}} - \frac{\pi_{BG}(\alpha_9 e^{x_1w} + \alpha_{10} e^{x_2w})}{r + \pi_{BG}}, & \forall w \in (W, W^*_G) \\
\frac{C_{SD} + \pi_{BG}SD_G(W^*_G)}{r + \pi_{BG}} + \alpha_{11} e^{x_1w} + \alpha_{12} e^{x_2w}, & \forall w \in (W^*_G, W^*_B) \\
SD_B(W^*_B), & \forall w > W^*_B 
\end{cases}
\] (A.15)

where \(y_1, y_2, z_1, z_2, \) and \(x_1, \) and \(x_2\) are given in (A.11), (A.12), and (A.13).

Finally, the closed form solution to the value of straight debt in the good state of the world is:

\[
SD_G(w) = \begin{cases} 
SD_G(W^*_G), & \forall w \in (0, W] \\
\frac{C_{SD} + \beta_3 e^{y_1w} + \beta_4 e^{y_2w}}{r + \pi_{BG}} + \frac{\pi_{BG}(\alpha_9 e^{x_1w} + \alpha_{10} e^{x_2w})}{r + \pi_{BG}}, & \forall w \in (W, W^*_G) \\
SD_G(W^*_G), & \forall w > W^*_G 
\end{cases}
\] (A.16)

where \(y_1, \) and \(y_2,\) and \(z_1, \) and \(z_2\) are given in (A.11), and (A.12). \(\alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \) and \(\alpha_{12}, \) and \(\beta_3, \) and \(\beta_4\) can be solved for by using the boundary conditions of the value of straight debt that are discussed in Section 2.3.1.

A.2.2 Values of bank’s securities - CoCo debt

When the bank commits to a bail-in plan the value of equity in the good state of the world satisfies the following equation

\[
rE_G(w) = (1 - \theta)(\mu - C)E'_G(w) \\
+ \frac{1}{2}(1 - \theta)^2 \sigma^2 E''_G(w) + \pi_{BG}(E_B(w) - E_G(w)).
\] (A.17)

The value of equity in the bad state of the world satisfies the following system of equations:

\[
rE_B(w) = \begin{cases} 
(1 - \theta)(\mu - C) E'_B(w) + \frac{(1 - \theta)^2 \sigma^2}{2} E''_B(w) + \pi_{BG}(E_G(W^*_G) + w - W^*_G - \gamma_E - E_B(w)), & \forall w \in (0, W] \\
(1 - \theta)(\mu - C) E'_B(w) + \frac{(1 - \theta)^2 \sigma^2}{2} E''_B(w) + \pi_{BG}(E_G(w) - E_B(w)), & \forall w \in (W, W^*_G) \\
(1 - \theta)(\mu - C) E'_B(w) + \frac{(1 - \theta)^2 \sigma^2}{2} E''_B(w) + \pi_{BG}(E_G(W^*_G) + w - W^*_G - E_B(w)), & \forall w \in (W^*_G, W^*_B) 
\end{cases}
\] (A.18)
These equations each include a term (the last term on the right hand side) that reflects the effects of the time varying financing frictions on the equity value depending on the level of cash buffer. (A.17) and (A.18) are solved subject to the following boundary conditions

\[
\begin{align*}
E_i(W_i^*) &= 1 \\
E_i'(W_i^*) &= 0 \\
E_G(W) &= 1 \\
E_B(0) &= (1 - \alpha)(E_B(W_B^*) + CD_B(W_B^*) - W_B^* - \gamma_C)
\end{align*}
\]

and the following continuity and smoothness conditions at \(W_G^*\) and \(W_B^*\):

\[
\begin{align*}
\lim_{w \downarrow W_G^*} E_B(w) &= \lim_{w \uparrow W_G^*} E_B(w) \\
\lim_{w \downarrow W_G^*} E_B'(w) &= \lim_{w \uparrow W_G^*} E_B'(w), \\
\lim_{w \downarrow W_G^*} E_B(w) &= \lim_{w \uparrow W_G^*} E_B(w), \\
\lim_{w \downarrow W_G^*} E_B'(w) &= \lim_{w \uparrow W_G^*} E_B'(w).
\end{align*}
\]

The value of CoCo debt in the good state of the world satisfies the following ODE:

\[
r CD_G(w) = C_C D + (1 - \theta)(\mu - C)CD_G'(w) + \frac{1}{2}(1 - \theta)^2 \sigma^2 CD_G''(w) + \pi_B G CD_B(C D_G(W_G^*) - C D_B(w)), \quad w \in (0, W_G^*]. \tag{A.19}
\]

The value of CoCo debt in the bad state of the world satisfies the following system of equations:

\[
r CD_B(w) = \begin{cases} 
C_C D + (1 - \theta)(\mu - C) \left( CD_B'(w) + \frac{(1 - \theta) \sigma^2}{2} CD_B''(w) + \pi_B G (CD_G(W_G^*) - C D_B(w)) \right), & w \in [0, W_B^*] \\
C_C D + (1 - \theta)(\mu - C) \left( CD_B'(w) + \frac{(1 - \theta) \sigma^2}{2} CD_B''(w) + \pi_B G (CD_G(W_G^*) - C D_B(w)) \right), & w \in [W_G^*, W_B^*] \tag{A.20}
\end{cases}
\]
Appendix A. Appendices

(A.19) and (A.20) are subject to the following boundary conditions:

\[
\begin{align*}
CD_i'(W_i^*) &= 0 \\
CD_G(W) &= CD_G(W_G^*) \\
CD_B(0) &= \alpha (E_B(W_B^*) + CD_B(W_B^*) - W_B^* - \gamma_C)
\end{align*}
\]

and the following continuity and smoothness conditions at $W_G^*$ and $W$:

\[
\begin{align*}
\lim_{w \downarrow W_G^*} CD_B(w) &= \lim_{w \uparrow W_G^*} CD_B(w) \\
\lim_{w \downarrow W_G^*} CD_B'(w) &= \lim_{w \uparrow W_G^*} CD_B'(w), \\
\lim_{w \downarrow W} CD_B(w) &= \lim_{w \uparrow W} CD_B(w), \\
\lim_{w \downarrow W} CD_B'(w) &= \lim_{w \uparrow W} CD_B'(w),
\end{align*}
\]

The expressions for the values of equity in the bad and good states of the world are identical to (A.10), and (A.14) respectively. However, the boundary conditions that are used to solve the coefficients $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$, and $\beta_1$, and $\beta_2$ are different as discussed in Section 2.4.1.

The closed form expressions for the values of CoCo debt in both bad and good states are also given in (A.15), and (A.16) but the coefficients are solved using the boundary conditions relevant to the case of CoCo debt.
## Unregulated straight debt

Table A.1: Optimal bank capital structure and policy choices - unregulated straight debt

<table>
<thead>
<tr>
<th></th>
<th>Optimal debt $C^*_SD(%)$</th>
<th>Bank value</th>
<th>Reissuance threshold ($W$)</th>
<th>Target level in G ($W^*_G$)</th>
<th>Target level in B ($W^*_B$)</th>
<th>Debt ratio band in G (%)</th>
<th>Debt ratio band in B (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Base case environment</strong></td>
<td></td>
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<tr>
<td></td>
<td>8.24</td>
<td>3.102</td>
<td>0.069</td>
<td>0.238</td>
<td>0.285</td>
<td>[108.78, 114.03]</td>
<td>[107.41, 116.33]</td>
</tr>
<tr>
<td><strong>B. Higher cash flow volatility ($\sigma = 0.10$)</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>7.65</td>
<td>2.991</td>
<td>0.094</td>
<td>0.300</td>
<td>0.363</td>
<td>[102.25, 108.23]</td>
<td>[100.57, 111.21]</td>
</tr>
<tr>
<td><strong>C. Lower cash flow volatility ($\sigma = 0.06$)</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>8.95</td>
<td>3.222</td>
<td>0.045</td>
<td>0.175</td>
<td>0.208</td>
<td>[116.54, 120.90]</td>
<td>[115.50, 122.50]</td>
</tr>
<tr>
<td><strong>D. Higher reissuance costs ($\gamma_E = 0.02$)</strong></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>8.20</td>
<td>3.098</td>
<td>0.050</td>
<td>0.245</td>
<td>0.285</td>
<td>[108.27, 114.33]</td>
<td>[107.09, 116.00]</td>
</tr>
<tr>
<td><strong>E. Lower reissuance costs ($\gamma_E = 0.005$)</strong></td>
<td></td>
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<tr>
<td></td>
<td>8.28</td>
<td>3.106</td>
<td>0.086</td>
<td>0.232</td>
<td>0.285</td>
<td>[109.20, 113.72]</td>
<td>[107.63, 116.58]</td>
</tr>
<tr>
<td><strong>F. More frequent jumps to state B ($\pi_{GB} = 0.20$)</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>8.23</td>
<td>3.099</td>
<td>0.073</td>
<td>0.241</td>
<td>0.285</td>
<td>[108.60, 113.82]</td>
<td>[107.32, 116.24]</td>
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<tr>
<td><strong>G. Less frequent jumps to state B ($\pi_{GB} = 0.16$)</strong></td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>8.26</td>
<td>3.106</td>
<td>0.065</td>
<td>0.235</td>
<td>0.285</td>
<td>[108.98, 114.28]</td>
<td>[107.51, 116.45]</td>
</tr>
</tbody>
</table>

This table reports the optimal coupon payment, the optimal bank value net of capital injection, the equity reissuance threshold, the target levels of cash in the good and bad states, and the optimal debt ratio bands in the good and bad states under different parametric assumptions for the case of unregulated straight debt.


Bibliography


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