# Three Essays on Predictability and Seasonality in the Cross-Section of Stock Returns 

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## Abstract

This thesis examines predictability and seasonality in the cross-section of stock returns.

The first chapter, titled "Infrequent Rebalancing, Return Autocorrelation, and Seasonality," shows that a model of infrequent rebalancing can explain specific predictability patterns in the time series and cross-section of stock returns. First, infrequent rebalancing produces return autocorrelations that are consistent with empirical evidence from intraday returns and new evidence from daily returns. Autocorrelations can switch sign and become positive at the rebalancing horizon. Second, the cross-sectional variance in expected returns is larger when more traders rebalance. This effect generates seasonality in the cross-section of stock returns, which can help explain available empirical evidence.

The second chapter, titled "Seasonalities in Anomalies," investigates return seasonalities in a set of well-known anomalies in the cross-section of U.S. stocks returns. A January seasonality goes beyond a size effect and strongly affects most anomalies, which can even switch sign in January. Both tax-loss selling and firm size are important in explaining the turn-of-the-year pattern. Return seasonality exists outside of January, with respect to the month of the quarter. Small stocks earn abnormally high average returns on the last day of each quarter, which significantly affects size, idiosyncratic volatility, and illiquidity portfolios. The results have implications for the interpretation and analysis of many anomalies, such as asset growth and momentum.

The third chapter, titled "The Cross-Section of Intraday and Overnight Returns," uses a thirtyyear sample of U.S. stock returns to document substantial cross-sectional variation in returns over the trading day and overnight. Market closures have a large impact on returns. Small and illiquid stocks earn high average returns in the last thirty minutes of trading. In contrast, large and liquid stocks perform poorly at this time. I find support for institutional and information asymmetry theories. But these theories do not fully explain the cross-sectional evidence. Portfolios based on other characteristics, such as beta and idiosyncratic volatility, earn their return gradually throughout the trading day-contrary to the market and a benchmark based on random portfolios. These portfolios also tend to incur large negative returns overnight,
consistent with mispricing at the open.

Key words: Return Predictability; Return Seasonality; Asset Pricing Anomalies; Intraday Returns; Liquidity; Infrequent Rebalancing

## Résumé

Cette thèse étudie la prévisibilité et la saisonnalité dans les données en coupe transversale de rendements d'actions financières.

Le premier chapitre, intitulé "Infrequent Rebalancing, Return Autocorrelation, and Seasonality," montre qu'un modèle de rééquilibrage peu fréquent explique des effets de prévisibilité dans les séries temporelles et la coupe transversale des rendements d'actions. Premièrement, le rééquilibrage peu fréquent produit des autocorrélations de rendements qui sont compatibles avec des résultats empiriques intra journaliers et de nouveaux résultats provenant des rendements quotidiens. Les autocorrélations peuvent changer de signe et devenir positives à l'horizon de rééquilibrage. Deuxièmement, la variance transversale des rendements espérés est plus élevée lorsque plus d'agents rééquilibrent leurs portefeuilles. Cet effet génère une saisonnalité dans la coupe transversale des rendements qui aide à expliquer les données empiriques disponibles.

Le deuxième chapitre, intitulé "Seasonalities in Anomalies," étudie les saisonnalités dans les rendements d'anomalies documentées dans la coupe transversale des rendements d'actions américaines. Un effet de saisonnalité en janvier n'est que partiellement expliqué par la capitalisation boursière et affecte fortement la plupart des anomalies. Le rendement moyen des anomalies peut même changer de signe en janvier. Le gain fiscal potentiel et la capitalisation boursière sont des variables importantes pour expliquer les rendements à la fin et au début de l'année. Une saisonnalité des rendements existe en dehors de janvier en fonction du mois du trimestre. Les actions d'entreprises de petite taille gagnent des rendements moyens anormalement élevés le dernier jour de chaque trimestre. Cet effet affecte de manière significative les portefeuilles basés sur la capitalisation boursière, la volatilité idiosyncratique et la liquidité. Les résultats ont des implications pour l'interprétation et l'analyse de nombreuses anomalies, telles que celles basées sur la croissance des actifs et l'élan des rendements.

Le troisième chapitre, intitulé "The Cross-Section of Intraday and Overnight Returns," utilise un échantillon de trente ans de rendements d'actions américaines pour documenter une variation transversale substantielle des rendements au cours de la journée. Les actions peu
liquides et de petites entreprises génèrent en moyenne des rendements élevés au cours des trente dernières minutes de négociation boursière. En revanche, les actions de grandes entreprises ont tendance à se déprécier en fin de journée. Les théories basées sur les chocs de liquidité et l'information asymétrique sont en partie validées mais n'expliquent pas l'effet de fin de journée pour les actions peu liquides. Les portefeuilles basés sur d'autres caractéristiques, telles que le beta et la volatilité idiosyncratique, gagnent leur rendement progressivement tout au long de la session boursière, contrairement au marché et à un indice de comparaison basé sur des portefeuilles aléatoires. Néanmoins, ces portefeuilles perdent de la valeur entre la fermeture du marché et l'ouverture le jour suivant. Ce résultat empirique est conforme avec une distorsion des prix à l'ouverture.

Mots-Clés: Prévisibilité des Rendements; Saisonnalité des Rendements; Anomalies Financières; Rendements Intra Journaliers; Liquidité; Rééquilibrage Infréquent

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## Introduction

Why do some stocks perform better than others? This apparently simple question has turned out to be a conundrum for research in finance. Over the past decades, many stock characteristics have been shown to predict the cross-section of stock returns but are not explained by standard finance theory. Even though these "anomalies" are the focus of a large literature, there is little consensus about their sources.

The global objective of this thesis is to contribute to research on this fundamental question by studying predictability and seasonality in the cross-section of stock returns. The three essays in this thesis build on each others to contribute to the literature on cross-sectional variation in stock returns, market efficiency, and liquidity.

The first chapter starts from the stylized fact that some stocks tend to perform systematically better than others at specific times of the day. While puzzling at first sight, I show that this observation can be explained by a theoretical model in which traders readjust their portfolios infrequently.

The second chapter provides an empirical overview of seasonality effects in stock returns at the monthly frequency. This empirical exercise is broadly motivated by the rebalancing model developed in the first chapter. Although the rebalancing model sheds light on intraday patterns, evidence of infrequent rebalancing exists at other frequencies. I show that significant cross-sectional variation in stock returns at the monthly frequency is linked to rebalancing effects such as tax-loss selling at the end of the year.

The third chapter expands the stylized fact that motivated the first chapter to provide a detailed analysis of the cross-section of intraday and overnight returns. I show that market closures generate significant cross-sectional variation in average stock returns and find partial support for theories of institutional effects and asymmetric information.

# 1 Infrequent Rebalancing, Return Autocorrelation, and Seasonality 


#### Abstract

A model of infrequent rebalancing can explain specific predictability patterns in the time series and cross-section of stock returns. First, infrequent rebalancing produces return autocorrelations that are consistent with empirical evidence from intraday returns and new evidence from daily returns. Autocorrelations can switch sign and become positive at the rebalancing horizon. Second, the cross-sectional variance in expected returns is larger when more traders rebalance. This effect generates seasonality in the cross-section of stock returns, which can help explain available empirical evidence. ${ }^{1}$


### 1.1 Introduction

Heston, Korajczyk, and Sadka (2010) document a striking pattern of periodicity in intraday returns. Reproducing their main finding, Figure 1.1 shows that the average estimate from a cross-sectional regression of current half-hour returns on lagged half-hour returns spikes at intervals of one trading day for several days. The estimate can be interpreted as the return on a momentum strategy-a high or low return on a stock in a given half-hour interval today can help predict the return on the stock at the same time tomorrow and over the next several days.

Changes in trading volume display a periodicity pattern that is similar to that for returns (Heston, Korajczyk, and Sadka (2010)), which suggests that investor trading is a natural candidate to explain the evidence. Motivated by this observation, in this paper I highlight the role of infrequent rebalancing for return and volume periodicity patterns at different frequencies.

The literature on slow-moving capital documents that many market participants are active only intermittently (Duffie (2010)). In particular, there is evidence of systematic trading and

[^0]

Figure 1.1. Cross-sectional regressions of intraday returns. The 9:30 to 16:00 trading day is divided into 13 separate half-hour intervals. For every half-hour interval $t$ and lag $l$, the following cross-sectional regression is estimated using NYSE stocks: $r_{i, t}=\alpha_{l, t}+\gamma_{l, t} r_{i, t-l}+u_{i, t}$, where $r_{i, t}$ is the simple return of stock $i$ in interval $t$ and $r_{i, t-l}$ is the simple return of stock $i$ in interval $t-l$. The cross-sectional regressions are run for each lag $l=1, \ldots, 65$ (past five trading days) using every half-hour return from January 2001 through December 2005 ( $T=16,261$ intervals). The figure plots the time-series averages of $\gamma_{l, t}$ scaled so that the units are percentages. The data are reproduced from Heston, Korajczyk, and Sadka (2010). Section 1.2 provides additional details about these regressions.
infrequent rebalancing at the intraday, daily, and monthly frequencies. ${ }^{2}$ To account for this evidence, I study a dynamic model in which a subset of agents trade only infrequently. ${ }^{3}$ I show that the model can help explain return autocorrelation and seasonality patterns at different frequencies.

In the model, infrequent rebalancing generates specific return autocorrelation patterns. After traders absorb a liquidity shock in an asset, they hold an excess position in the asset relative to its normal weight in their portfolio. At a rebalancing date, traders with an excess position in the asset unload part of their position in the market. This unloading is equivalent to another liquidity shock. Infrequent rebalancing can then result in positive return autocorrelation by propagating liquidity shocks across periods. This effect also modifies the dynamics of trading volume. A large liquidity shock results in high volume during both the current period and the rebalancing period.

[^1]Unless liquidity shocks are highly persistent, autocorrelations are negative at any horizon in the economy without infrequent traders. More importantly, all autocorrelations have the same sign. With infrequent rebalancing, autocorrelations can switch sign around traders' rebalancing horizon and become positive. Momentum at the rebalancing date is key in matching the empirical evidence. Similarly, change in trading volume is negatively autocorrelated at any horizon without infrequent rebalancing.

The infrequent rebalancing mechanism stressed by the theory can explain the empirical evidence shown in Figure 1.1. Assuming that a fraction of agents trade only once a day, the model can reproduce the periodicity documented by Heston, Korajczyk, and Sadka (2010). In the model, systematic trading generates predictable patterns in returns despite being perfectly anticipated. The model can also explain other recent evidence on intraday index returns. Gao et al. (2014) find that the first half-hour return on the SPDR S\&P 500 ETF predicts the last half-hour return. This result is in line with a fraction of agents adjusting their portfolios at the open and close of the market. ${ }^{4}$

Empirically, I provide new evidence on the impact of infrequent rebalancing on daily U.S. stock returns from 1983 to $2012 .{ }^{5}$ Cross-sectional regressions in the spirit of Jegadeesh (1990) reveal patterns in return autocorrelations that are consistent with a significant fraction of investors rebalancing at a weekly frequency. The model fits the short-term autocorrelation pattern. Neglected stocks do not drive the result since high turnover stocks display more pronounced patterns than low turnover stocks. This is in line with the theory, which suggests that infrequent rebalancing is distinct from thin trading. Daily volume change autocorrelations are broadly consistent with the theoretical predictions.

Empirical evidence from intraday and monthly returns displays persistent seasonality patterns that go beyond autocorrelation effects. In particular, Heston and Sadka (2008) document a persistent seasonality pattern in the cross-section of monthly U.S. stock returns. Contrary to the intraday evidence in Figure 1.1, Heston and Sadka's periodicity pattern does not show any decay with the horizon. More recently, Keloharju, Linnainmaa, and Nyberg (2016) provide substantial evidence on the pervasiveness of return seasonalities across asset classes and markets.

The well-known intraday U-shaped pattern in trading volume suggests that many market participants concentrate their trading at specific hours (Admati and Pfleiderer (1988)). Similarly, the fraction of agents who adjust their portfolios is likely not constant over a trading week

[^2]or year (Dellavigna and Pollet (2009), Hong and Yu (2009)). To account for this evidence, I extend the model to allow for variation in the proportion of infrequent traders across calendar periods. I show that this extended model can generate persistent seasonality patterns in line with the empirical evidence from intraday and monthly returns. ${ }^{6}$

In this extension, price impact varies across calendar periods. Traders require a larger risk premium to hold an asset when they expect the price impact to be higher next period. More precisely, variation in the proportion of infrequent traders across calendar periods generates seasonality in the market risk premium. If assets have different exposures to the market, then this mechanism amplifies the cross-sectional variance in expected returns in the period during which more traders rebalance. This effect generates seasonality in the cross-section of stock returns.

Crucially, infrequent rebalancing does not add an extra risk factor but rather generates seasonality in the factor risk premium. This is consistent with the evidence that seasonality strategies have low correlation across and within asset classes (Keloharju, Linnainmaa, and Nyberg (2016)). The seasonality strategies can have a low correlation in the context of the model if markets exhibit some degree of segmentation and, as a result, their risk factors are not perfectly correlated. Additionally, the seasonality strategies within an asset class (for instance, among small and large stocks) can have a low correlation if small and large stocks load on different factors. My equilibrium model features only one risk factor, but it is intuitive that infrequent rebalancing could generate seasonality in multiple risk factors.

Keloharju, Linnainmaa, and Nyberg (2016) argue that return seasonalities are not a distinct class of anomalies. However, one must explain why risk premia are seasonal to begin with. Seasonality in trading activity seems important in explaining seasonality in returns. This paper shows that trading by investors with heterogeneous rebalancing horizons can generate autocorrelation effects and persistent seasonality patterns consistent with empirical evidence at different frequencies.

Several papers examine the impact of infrequent rebalancing on asset prices. Duffie (2010) surveys the literature on slow-moving capital and studies the conditional price response to a large liquidity shock. He does not discuss unconditional return properties and trading volume. Bacchetta and Van Wincoop (2010) study the role of infrequent portfolio adjustments for the forward discount puzzle. Their setup, however, is tailored to the foreign exchange market. In particular, liquidity shocks do not matter for predictability in their economy, while they play a key role in mine. Chien, Cole, and Lustig (2012) show that intermittent rebalancing increases

[^3]the volatility of the market price of risk in a standard incomplete markets economy. Rinne and Suominen (2012) also investigate short-term return reversals, but they focus on liquidity and do not obtain the key prediction emphasized in this paper, namely, that infrequent rebalancing generates shifts in return autocorrelations. In contemporaneous research, Hendershott et al. (2014) test a modified version of Duffie's model to shed light on deviations from efficient prices at different frequencies. Their analysis uses impulse response functions and does not overlap my approach and results. None of these papers examines return seasonality.

More broadly, this paper relates to the literature on heterogeneous investment horizons and trading frequencies. For instance, Corsi (2009) motivates a cascade model of realized volatility with heterogeneity in market participants' trading frequencies. Beber, Driessen, and Tuijp (2012) use heterogeneous investment horizons to study the pricing of liquidity risk. More recently, Kamara et al. (2016) empirically highlight the role of differences in investors' rebalancing horizons in determining risk premia.

The paper is organized as follows. Section 1.2 decomposes the cross-sectional regressions used in Figure 1.1 and in the rest of the paper. Section 1.3 introduces a dynamic model with infrequent rebalancing. Section 1.4 studies return autocorrelation, Section 1.5 studies return seasonality, and Section 1.6 examines trading volume. Section 1.7 concludes. All the proofs are in Appendix A.1. Appendix A. 2 details the model's calibration.

### 1.2 Patterns in the Cross-Section of Stock Returns

Heston, Korajczyk, and Sadka (2010) estimate the following regression to obtain Figure 1.1:

$$
\begin{equation*}
r_{i, t}=\alpha_{l, t}+\gamma_{l, t} r_{i, t-l}+u_{i, t} \tag{1.1}
\end{equation*}
$$

where $r_{i, t}$ is the return on stock $i$ in half-hour interval $t$. Heston and Sadka (2008) estimate the same regression on monthly returns. The regression coefficients are first estimated crosssectionally at each date and then averaged over time (Fama and Macbeth (1973)). The crosssectional regression methodology avoids several shortcomings of time-series estimates of autocorrelation (Jegadeesh (1990), Lehmann (1990)). As explained below, however, the crosssectional regression estimates are not exactly equivalent to autocorrelations.

To better understand the empirical evidence, one can decompose the average cross-sectional regression coefficient. Let $\bar{r}_{t}=\frac{1}{N} \sum_{i=1}^{N} r_{i, t}$. The slope coefficient estimate is given by

$$
\begin{equation*}
\hat{\gamma}_{l, t}=\frac{1}{\frac{1}{N} \sum_{i=1}^{N}\left(r_{i, t-l}-\bar{r}_{t-l}\right)^{2}} \underbrace{\sum_{i=1}^{N} r_{i, t} \frac{1}{N}\left(r_{i, t-l}-\bar{r}_{t-l}\right)}_{\equiv \pi_{t}(l)} \tag{1.2}
\end{equation*}
$$

The above estimate closely relates to the profit of a relative strength strategy, denoted by $\pi_{t}(l)$. This zero-investment strategy is long past winners and short past losers based on their return in period $t-l$. Define the calendar function $c(t)$, which gives the calendar period for each date $t$ (for instance, the day of the week). The expected return on the strategy in calendar period $c(t)$ is

$$
\begin{align*}
\mathbb{E}\left[\pi_{t}(l) \mid c(t)\right]= & \frac{1}{N} \sum_{i=1}^{N} \operatorname{Cov}\left[r_{i, t}, r_{i, t-l} \mid c(t)\right]-\operatorname{Cov}\left[\bar{r}_{t}, \bar{r}_{t-l} \mid c(t)\right] \\
& +\frac{1}{N} \sum_{i=1}^{N}\left(\mu_{i, c(t)}-\mu_{c(t)}\right)\left(\mu_{i, c(t-l)}-\mu_{c(t-l)}\right) \tag{1.3}
\end{align*}
$$

where $\mu_{i, c(t)} \equiv \mathbb{E}\left[r_{i, t} \mid c(t)\right]$ and $\mu_{c(t)} \equiv \mathbb{E}\left[\bar{r}_{t} \mid c(t)\right]$. As a result, the average $\gamma_{l, t}$ coefficient in equation (1.2) reflects three components: return autocorrelation, return cross-autocorrelation, and cross-sectional variation in average returns (Lo and MacKinlay (1990)). ${ }^{7}$

In this paper, I explore how infrequent rebalancing can help explain the empirical evidence obtained from regression (1.1) at different frequencies. First, infrequent rebalancing generates specific return autocorrelation patterns linked to the rebalancing horizon of traders (first component in equation (1.3)). Second, infrequent rebalancing can generate persistent seasonality patterns. Indeed, the last component in equation (1.3) does not decay with the lag. Persistent seasonality patterns in the average $\gamma_{l, t}$ can therefore arise when expected returns vary across calendar periods. I show that infrequent rebalancing can generate such variation. The next section presents a model that formalizes this intuition.

### 1.3 A Dynamic Model with Infrequent Rebalancing

To better understand the impact of investors' trading on return and volume predictability patterns, I study a model in which some traders readjust their portfolio infrequently in an otherwise standard economy. The setup of the model builds on that of Duffie (2010). In particular, I extend the model to multiple assets to study the evidence from cross-sectional regressions.

In addition, as suggested by extant empirical evidence on trading volume, the fraction of agents who adjust their portfolios is likely not constant over a trading day, week, or year. In this respect, Heston, Korajczyk, and Sadka (2010) find that their pattern is strongest in the first and last half-hour of trading. Following this evidence, I further extend the model to allow

[^4]for a fixed but nonconstant proportion of infrequent traders across periods. Theoretically, Admati and Pfleiderer (1988) demonstrate that traders may optimally cluster their orders at given periods.

### 1.3.1 The Economy

Time is discrete and goes from zero to infinity. At each date, $N$ risky assets pay dividends. The $N \times 1$ vector of dividends follows a simple autoregressive process,

$$
\begin{equation*}
D_{t+1}=a_{D} D_{t}+\epsilon_{t+1}^{D} \tag{1.4}
\end{equation*}
$$

where $0 \leq a_{D} \leq 1$ represents common dividend persistence. I assume that $\epsilon_{t+1}^{D} \sim \mathscr{N}\left(0, \Sigma_{D}\right)$, where $\Sigma_{D}$ denotes the $N \times N$ variance-covariance matrix of dividend shocks. The mean dividend does not matter for return autocorrelation and seasonality and is assumed to be zero. In addition, a risk-free asset with gross return $R>1$ is available in perfectly elastic supply.

Two types of agents with exponential utility over terminal wealth trade in the economy. Frequent traders are present in the market at every date. A frequent trader of age $j$ maximizes the value of her terminal wealth in $h-j$ periods. At the end of her trading cycle, the agent starts investing again with a horizon $h$. I assume a constant fraction of frequent traders across investment horizons. Given this assumption, at each date the following groups of frequent traders are active in the market: a fraction $\frac{1}{h}$ of frequent traders with horizon $h$, a fraction $\frac{1}{h}$ of frequent traders with horizon $h-1$, and so on.

Allowing for frequent traders with a long horizon is a natural extension to evaluate the robustness of multiperiod return predictability patterns. Furthermore, investment horizons can have large effects on asset prices, as illustrated by Albagli (2015). Let $h-j$ be the remaining horizon of a frequent trader $(0 \leq j \leq h-1)$. Her optimization problem is then given by

$$
\begin{array}{ll} 
& \max _{X_{t, j}^{F}} \mathbb{E}_{t}\left[-e^{-\gamma_{F} W_{t+h-j}^{F}}\right],  \tag{1.5}\\
\text { s.t. } & W_{t+1}^{F}=\left(X_{t, j}^{F}\right)^{\prime}\left(P_{t+1}+D_{t+1}-R P_{t}\right)+R W_{t}^{F},
\end{array}
$$

where $X_{t}^{F}$ is the vector of asset demands, $P_{t}$ is the vector of asset prices, and $W_{t}^{F}$ is the initial wealth. The expectation is taken with respect to an information set that is common to all traders and includes the current and past levels of all state variables (defined below), as well as the current calendar period.

Infrequent traders-the second group of agents-trade to maximize the value of their terminal wealth and then leave the market for a period of length $k$. The inattention period $k$ is taken as exogenous. Bacchetta and Van Wincoop (2010), Duffie (2010), and Chien, Cole, and Lustig
(2012) make a similar assumption. The tractability offered by this assumption allows one to draw clear predictions from the model. Solving for endogenous participation or inattention in general equilibrium settings is challenging. ${ }^{8}$ It is unlikely that a fixed fraction of infrequent traders participate in the market each period; some investors may enter into the market when they perceive that profit opportunities outweigh their participation cost, which is a state-dependent trading rule, as opposed to the time-dependent rule implied by exogenous $k$. In a partial equilibrium setting, Abel, Eberly, and Panageas (2007) find that a constant rebalancing interval is optimal when agents are subject to observation costs. In further research, Abel, Eberly, and Panageas (2013) show that in the presence of both information costs and transaction costs, a time-dependent rule survives if the fixed component of transaction costs is small enough. Sections 1.4 and 1.5 show that the model's implications are consistent with the empirical evidence, and thus a simple approximation of investors' trading policies may help shed light on asset return properties.

The infrequent traders who are rebalancing at date $t$ select their vector of asset demands $X_{t}^{I}$ to maximize their expected utility according to

$$
\begin{array}{ll} 
& \max _{X_{t}^{I}} \mathbb{E}_{t}\left[-e^{-\gamma_{I} W_{t+k+1}^{I}}\right] \\
\text { s.t. } & W_{t+k+1}^{I}=\left(X_{t}^{I}+\theta_{t}\right)^{\prime}\left(P_{t+k+1}+\sum_{j=1}^{k+1} R^{k+1-j} D_{t+j}-R^{k+1} P_{t}\right)+R^{k+1} W_{t}^{I}, \tag{1.6}
\end{array}
$$

where $W_{t}^{I}$ is initial wealth. Infrequent traders adjust their portfolio and do not trade for the rest of their investment horizon. The dividends paid while the agent is out of the market are reinvested at the risk-free rate.

The model requires an additional element to generate trade. Here, liquidity traders supply inelastic quantities of assets every period. Equivalently, a fraction of frequent traders could receive state-contingent endowment shocks as in the setup of Biais, Bossaerts, and Spatt (2010). Liquidity traders' supplies are given by the zero-mean $N \times 1$ process

$$
\begin{equation*}
\theta_{t+1}=a_{\theta} \theta_{t}+\epsilon_{t+1}^{\theta} \tag{1.7}
\end{equation*}
$$

where $0 \leq a_{\theta} \leq 1$ represents liquidity trading persistence. I assume that $\epsilon_{t+1}^{\theta} \sim \mathscr{N}\left(0, \Sigma_{\theta}\right)$, where $\Sigma_{\theta}$ denotes the $N \times N$ variance-covariance matrix of liquidity shocks.

The autocorrelation effect highlighted in Section 1.4 requires that a shock affecting traders'

[^5]positions reverses over time. The model allows this shock to be asset-specific or common to many assets. Importantly, the infrequent rebalancing mechanism does not require any persistence in the shock to generate specific return predictability patterns. To focus on the simplest possible setting, I use an autoregressive process of order one. This assumption also makes the setup comparable to previous literature.

### 1.3.2 Equilibrium

Infrequent and frequent traders are present in the economy in proportion $q$ and $1-q$, respectively. I consider two cases. First, the mass of rebalancing infrequent traders at each date is constant over time. Second, the mass of rebalancing infrequent traders varies with the calendar period and is equal to $q_{c(t)}$, where $c(t)$ indicates the calendar period at date $t$. With $C$ calendar periods, $\sum_{j=1}^{C} q_{j}=q$. In this general case, market-clearing requires

$$
\begin{equation*}
q_{c(t)} X_{t}^{I}+\frac{1-q}{h} \sum_{j=0}^{h-1} X_{j, t}^{F}=\bar{S}+\theta_{t}-\sum_{i=1}^{k} q_{c(t-i)} X_{t-i}^{I}, \tag{1.8}
\end{equation*}
$$

where $\bar{S}$ is the $N \times 1$ vector of share supplies. ${ }^{9}$ The lagged demands of infrequent traders reduce the number of shares available in the market today.

The following three conditions define a linear rational expectations equilibrium (REE): (i) prices and demands are linear functions of the state variables, (ii) agents optimize problems (1.5) and (1.6), and (iii) markets clear according to (1.8).

I first limit attention to the case in which the mass of rebalancing infrequent traders is identical every period. This provides a benchmark model that focuses on return autocorrelation. I study the general model with a varying mass of infrequent traders in Section 1.5.

### 1.3.3 Constant Proportion of Infrequent Traders

An identical proportion of infrequent traders readjust their portfolio every period, hence $q_{c(t)}=\frac{q}{k+1}$.

Proposition 1. In a linear stationary REE, if it exists, the vector of asset prices is given by

$$
\begin{equation*}
P_{t}=\bar{P}+P_{\theta} \theta_{t}+\frac{a_{D}}{R-a_{D}} D_{t}+\sum_{i=1}^{k} P_{X_{i}} X_{t-i}^{I}, \tag{1.9}
\end{equation*}
$$

[^6]where the coefficient matrices are solutions to a system of nonlinear equations given in Appendix A.1.

The lagged demands of infrequent traders are state variables in equilibrium. The matrices $P_{X_{i}}$ determine how lagged demands affect current prices. The matrix $P_{\theta}$ reflects the price impact of liquidity shocks. The price vector includes the present value of expected future dividends discounted at the risk-free rate. Indeed, $\mathbb{E}_{t}\left[\sum_{j=1}^{\infty} R^{-j} D_{t+j}\right]=\frac{a_{D}}{R-a_{D}} D_{t}$.

Polar cases of the economy help gain intuition, since the equilibrium coefficients have to be solved numerically. ${ }^{10}$ When $q=0$ (or $k=0$ ), only frequent traders are active in the market and thus lagged demands are not state variables anymore. The price vector is then given by

$$
\begin{equation*}
P_{t}=\bar{P}+P_{\theta} \theta_{t}+\frac{a_{D}}{R-a_{D}} D_{t} \tag{1.10}
\end{equation*}
$$

The price vector (1.10) has the same form whether $h=1$ or $h>1$, but an analytical solution for $P_{\theta}$ is only available when $h=1$ because of the nonlinear hedging demands (see Spiegel (1998)). I refer to this economy as the frictionless economy. The following corollary solves for the equilibrium coefficients when the economy contains only infrequent traders with inattention period $k$ (infrequent rebalancing economy).

Corollary 1. Let $q=1$. In a linear stationary REE, the lagged demands' coefficient matrices in equation (1.9) are given by

$$
\begin{equation*}
P_{X_{1}}=P_{X_{2}}=\ldots=P_{X_{k}}=-\frac{1}{k+1}\left(\frac{R^{k+1}-a_{\theta}^{k+1}}{R^{k+1}-a_{\theta}^{k}}\right) P_{\theta} \tag{1.11}
\end{equation*}
$$

where $P_{\theta}$ solves a quadratic matrix equation given in Appendix A.1.

When $q=1$, equation (1.11) shows that $P_{X_{i}}$ and $P_{\theta}$ are proportional to each other. Since agents only trade on liquidity shocks, lagged demands directly reflect past liquidity shocks. To gain intuition, assume for example that liquidity traders sell a large quantity of the asset. The price drops to give agents an incentive to hold the additional asset supply. The traders who accommodate the liquidity shock now hold the asset in excess of their steady-state optimal position. As a result, these traders want to liquidate their abnormal holdings when they rebalance their portfolio in $k+1$ periods. At that future date, this desire to rebalance puts a downward pressure on the price that is proportional to the initial liquidity shock (traders cannot unload their positions in equilibrium since they trade only with liquidity traders in this polar case). This mechanism has specific implications for return autocorrelation, which I explain in Section 1.4.1.

[^7]
### 1.3.4 Equilibrium Multiplicity and Existence

The infrequent rebalancing economy solves the same problem as the frictionless economy with adjusted fundamental parameters. Thus, the results of Watanabe (2008) for the frictionless economy apply. In particular, he shows that if liquidity and dividend shock volatilities and correlations are the same for all assets, then only four "symmetric" equilibria exist (i.e., equilibria in which the price and demand coefficients are equal across assets): a "low volatility" equilibrium coexists with three "high volatility" equilibria. This multiplicity stems from the infinite horizon of the economy and the finite lives of agents. The low volatility equilibrium is the unique equilibrium of the finite-horizon frictionless economy (Banerjee (2011)). Moreover, as agents lives' goes to infinity in the frictionless economy (with intermediate consumption), a unique linear equilibrium always exists (Albagli (2015)). Albagli's analysis further suggests that the low volatility equilibrium converges to this unique equilibrium. I show in the Internet Appendix that the low volatility equilibrium is the only "stable" equilibrium when $q=0$ or $q=1 .{ }^{11}$

When $0<q<1$, I find multiple equilibria in all my numerical calibrations. Assuming that fundamental parameters are the same for all assets, I always find four symmetric equilibria that converge to the analytical polar cases as $q \rightarrow 0$ or $q \rightarrow 1$. For the previous reasons, I focus my analysis on the low volatility equilibrium. Importantly, the paper's main results also hold in the high volatility equilibria. This is because my analysis does not rely on comparative statics, for which different equilibria typically give opposite results (see, for instance, Banerjee (2011)).

With respect to existence, the effect of fundamental parameters is intuitive in both polar economies: more volatile and persistent sources of risk shrink the existence region. However, increasing the persistence of liquidity trading $a_{\theta}$ may widen the existence region when $q=1$, as explained in the Internet Appendix. The exact equilibrium existence conditions in the polar economies are given in the Internet Appendix. When $0<q<1$, numerical experiments indicate that small $q$ helps obtain an equilibrium. High volatility leads to nonexistence. More precisely, a risk-averse agent with a finite horizon requires a price discount to absorb a liquidity shock. This price discount increases price volatility. Increased volatility leads the agent to require an even larger discount. An equilibrium fails to exist if the loop does not converge. Since $a_{\theta}$ may have an opposite effect on the existence region when $q=0$ and $q=1, a_{\theta}$ can have an ambiguous effect on the existence region when $0<q<1$. Increasing $h$ helps find an equilibrium, in line with the results of Albagli (2015).

[^8]
### 1.4 Return Autocorrelation

This section examines return autocorrelation in a dynamic equilibrium model in which some traders adjust their portfolios infrequently.

### 1.4.1 Theory

Let $Q_{t+1}=P_{t+1}+D_{t+1}-R P_{t}$ denote the vector of (dollar) excess returns between time $t$ and $t+1$. In the frictionless economy, excess returns between time $t+s-1$ and $t+s$ are given by

$$
\begin{equation*}
Q_{t+s}=P_{\theta} \epsilon_{t+s}^{\theta}+\frac{R}{R-a_{D}} \epsilon_{t+s}^{D}+\left(a_{\theta}-R\right) P_{\theta} \theta_{t+s-1} . \tag{1.12}
\end{equation*}
$$

A dividend shock affects prices but does not modify expected returns (Wang (1994)). Return autocovariances are then given by

$$
\begin{equation*}
\operatorname{Cov}\left[Q_{t+s}, Q_{t}\right]=\left(a_{\theta} R-1\right) a_{\theta}^{s-1} \frac{R-a_{\theta}}{1-a_{\theta}^{2}} P_{\theta} \Sigma_{\theta} P_{\theta}^{\prime}, \quad a_{\theta}<1, s \geq 1 . \tag{1.13}
\end{equation*}
$$

Dividend persistence does not affect the sign of excess return autocovariances. ${ }^{12}$ Since the price vector (1.10) takes the same form when $h>1$, equation (1.13) also shows that long horizons affect neither the sign of the autocovariances nor their rate of decay. The frictionless model requires $a_{\theta} R<1$ to produce short-term return reversal, which is widely documented by previous research (see, for instance, Jegadeesh (1990)) and confirmed by the empirical analysis on daily returns in Section 1.4.4. ${ }^{13}$

When $0<a_{\theta} R<1$, the frictionless model predicts that all return autocovariances are negative at any horizon and decay exponentially. The negative autocorrelation of price changes stems from the reversal of transitory order flows and the risk aversion of frequent traders (Grossman and Miller (1988)). Makarov and Rytchkov (2012) demonstrate that a version of equation (1.13) holds for the more general case of asymmetrically informed traders. They show that asymmetric information alone cannot generate price momentum in the standard stationary setting in which liquidity trading follows a first-order autoregressive process. This implication contrasts with the finite-horizon model of Cespa and Vives (2012), in which autocorrelations are positive if information quality increases sufficiently across periods and liquidity trading is persistent enough.

[^9]In a stationary setup, liquidity shocks determine autocovariance dynamics because of the market-clearing condition. When $q=0$ (and $h=1$ ), the market-clearing condition is $\gamma_{F} \Sigma\left(\theta_{t}+\bar{S}\right)=$ $\mathbb{E}_{t}\left[Q_{t+1}\right]$, where $\Sigma \equiv \operatorname{Var}_{t}\left[P_{t+1}+D_{t+1}\right]$ is a constant matrix. This implies that $\operatorname{Cov}\left[Q_{t+1}, Q_{t}\right]=$ $\gamma_{F} \Sigma \operatorname{Cov}\left[\theta_{t}, Q_{t}\right]$. Since $\operatorname{Cov}\left[\epsilon_{t}^{D}, \epsilon_{t}^{\theta}\right]=0$, signals about future dividends are not informative about future liquidity shocks and cannot help generate positive return autocorrelation alone. ${ }^{14}$

According to the model, infrequent rebalancing can have a large impact on return autocorrelation. Figure 1.2 displays the first 10 autocorrelations generated by the model for different degrees of infrequent rebalancing and different degrees of liquidity trading persistence. The patterns are robust to variation in the other parameters. The calibration is detailed Appendix A. 2 and assumes that infrequent traders readjust their portfolios every five periods. To focus solely on the patterns generated by infrequent rebalancing, I scale the autocorrelations so that their absolute values sum up to one for the first 10 lags.

The left column shows autocorrelations in the frictionless economy. These autocorrelations are always negative and decay proportionally to the persistence of liquidity trading. As shown in the middle and right columns, infrequent rebalancing shifts the autocorrelations around the rebalancing horizon. In particular, autocorrelations can switch sign and become positive regardless of the persistence of liquidity trading. Even in a similar nonstationary setting, returns reverse when liquidity trading is transient. In the model of Cespa and Vives (2012), return autocorrelations are always negative when $a_{\theta}=0$, in spite of the nonstationary variance dynamics associated with the gradual revelation of information.

To understand the underlying mechanism, consider the single-asset case and assume that a large liquidity shock takes place at date $t$. The price drops so that agents who are present in the market accommodate the shock, and hence, $Q_{t}$ is low. Infrequent traders partially absorb the liquidity shock, and $X_{t}^{I}$ is larger than its steady-state level. At time $t+k+1$, infrequent traders come back to the market. Since liquidity trading is transient, these traders now hold an abnormal position in the asset relative to the current asset supply. They therefore liquidate part of their excess holdings. The resulting order flow is equivalent to a liquidity shock: the price drops and $Q_{t+k+1}$ is low. This effect increases $\operatorname{Cov}\left[Q_{t+k+1}, Q_{t}\right]$. Infrequent rebalancing is akin to serially correlated liquidity shocks, which is why autocorrelations can become positive despite the result of Makarov and Rytchkov (2012). A liquidity shock today transmits to the future date when agents rebalance their holdings.

More formally, consider a single-asset economy with $k=1$ and $a_{\theta}=0$. In this case, all auto-

[^10]

Figure 1.2. Autocorrelations (scaled) for different degrees of liquidity trading persistence $\left(a_{\theta}\right)$ and different degrees of infrequent rebalancing (q). The figure plots the scaled first element of the matrix $\operatorname{Corr}\left[Q_{t+s}, Q_{t}\right]$ for $s=1, \ldots, 10$. The autocorrelations are scaled so that their absolute values sum to one over the first 10 lags. The calibration is shown in Table A. 1 (left column).
covariances beyond the first lag are zero in the frictionless economy. This provides a clean benchmark. The next proposition formalizes the intuition developed previously.

Proposition 2. Let $a_{\theta}=0, k=1$, and $h=1$. In the single-asset economy with $0<q<1$, if $P_{\theta}<0$ and $P_{X}>0$, then $\operatorname{Cov}\left[Q_{t}, Q_{t+1}\right]<0$ and $\operatorname{Cov}\left[Q_{t}, Q_{t+2}\right]>0$.

The conditions $P_{\theta}<0$ and $P_{X}>0$ are intuitive and hold in the polar economies. First, a liquidity shock should have a negative price impact. Second, a positive lagged demand should increase the price of the asset since it restricts the current asset supply. Under these conditions, infrequent traders absorb part of the liquidity shocks and therefore provide liquidity when $0<q<1$. ${ }^{15}$

Proposition 2 formally shows that infrequent rebalancing generates positive return autocorrelation when liquidity trading is transient and that autocorrelations can switch sign. As

[^11]indicated by Figure 1.2, a similar effect applies when $k>1$. In summary, with infrequent rebalancing, return autocorrelations are subject to shifts linked to traders' rebalancing horizon and can switch sign. Without infrequent rebalancing, all return autocorrelations have the same sign and decay exponentially. ${ }^{16}$

Positive return autocorrelation can be obtained by mechanically adjusting the liquidity trading process (1.7). Assuming that $\theta_{t}=\epsilon_{t}^{\theta}+\beta \epsilon_{t-k}^{\theta}$ leads to a price function of the form $P_{t}=P_{\theta} \epsilon_{t}^{\theta}+$ $\sum_{i=1}^{k} P_{\theta, i} \epsilon_{t-i}^{\theta}$. Economically, this specification of liquidity trading can be broadly interpreted as a form of order-splitting strategy. If $\beta>0$, this setup produces positive autocorrelation between the excess return today and the excess return in $k$ periods. This result illustrates that infrequent rebalancing propagates liquidity shocks across periods.

### 1.4.2 Heterogeneous Rebalancing Horizons

In the Internet Appendix, I extend the benchmark model to allow for infrequent traders with heterogeneous rebalancing horizons. More precisely, I consider an economy with two groups of infrequent traders (in addition to frequent traders). Group $i$ has mass $q_{i}$ and inattention period $k_{i}$. Though analytical solutions are again not available, the rebalancing mechanism seems robust to having multiple groups of infrequent traders. In particular, the autocorrelation pattern is subject to shifts at both rebalancing horizons, $k_{1}+1$ and $k_{2}+1$, that is, both autocorrelations can switch sign. This suggests that the model can simultaneously explain predictability patterns at different frequencies.

### 1.4.3 Empirical Evidence: Intraday Returns

Figure 1.2 suggests that a model in which a fraction of traders adjust their portfolio only once a day can help explain the predictability pattern documented by Heston, Korajczyk, and Sadka (2010) and reproduced in Figure 1.1. The multi-asset setting allows for an exact replication of the regressions using simulated returns from a calibrated version of the model. ${ }^{17}$ Since the current model relies only on the autocorrelation component of equation (1.3), the regression estimates are almost identical to autocorrelations in the model. For clarity, I report autocorrelations. This paper does not aim to provide an exact quantitative match to the data.

[^12]The parameters are therefore chosen to broadly match the patterns observed in the data while keeping the calibration as simple and transparent as possible. Appendix A. 2 details the calibrations used in the paper.

Figure 1.3 plots the autocorrelations obtained from the model. The results are in line with the empirical evidence-as expected, the regression coefficient spikes at horizons that are multiples of one trading day (since a trading day is composed of 13 half-hour intervals, traders' inattention period is set to $k=12$ ). Infrequent rebalancing produces a persistent pattern of return predictability despite being perfectly anticipated by frequent traders.


Figure 1.3. Autocorrelations predicted by the model for intraday returns. The calibration is shown in Table A.1.

In Figure 1.3, the proportion of infrequent traders must be set to a high level (i.e., $q=0.99$ ) for the pattern to persist over several days. A small fraction of frequent traders is consistent with the calibrations in related papers. ${ }^{18}$ The model also abstracts from transaction costs, which limit the arbitrage activity of frequent traders and could therefore partially explain the persistence of the pattern in the data (Heston, Korajczyk, and Sadka (2010)). The decay in the coefficients is consistent with a repeated shock explanation. But the persistence of the pattern at higher lags points towards cross-sectional variation in average returns that differs across calendar periods (see Section 1.2). Section 1.5 investigates this effect, which generates persistent seasonality patterns.

Heston, Korajczyk, and Sadka (2010) report that changes in trading volume exhibit similar periodic patterns. The model also predicts this relationship. A large liquidity shock results in high volume during the current period. One day later, infrequent traders reduce their

[^13]abnormal positions and generate high volume again. I examine trading volume in Section 1.6.

### 1.4.4 Empirical Evidence: Daily Returns

This section examines whether daily returns exhibit predictability consistent with infrequent rebalancing. I use daily returns on NYSE and Amex common stocks from CRSP over the period January 1983 to December 2012. The data are cleaned as follows: the CRSP share code is equal to 10 or 11 , penny stocks (average price less than one dollar) are eliminated, returns above $400 \%$ are winsorized, and each stock is required to have at least 250 days of data. This procedure leaves an average of 2,000 stocks each period in the data set. I focus on the last 30 years of data because structural shifts in investors' rebalancing frequencies are likely to be an issue over longer samples.

Intuitively, conjecture that some traders rebalance at a weekly frequency (i.e., every five consecutive trading days). This is consistent with Rakowski and Wang (2009), who find a day-of-the-week effect in mutual fund flows, or with investment products being rebalanced on specific days. To test this conjecture, I use the methodology of Jegadeesh (1990) and estimate a multiple cross-sectional regression of current returns on lagged returns at each date.

As explained in Section 1.2, cross-sectional variation in average returns across calendar periods can generate persistent seasonality patterns that are picked up by the regression coefficients. This is likely to be a concern here since prior research documents that average stock returns are not equal across days of the week (French (1980), Gibbons and Hess (1981)). The infrequent rebalancing model developed in Section 1.3.3 provides a repeated shock explanation for return predictability, although variation in unconditional expected returns across days of the week could arise from variation in the degree of infrequent trading throughout the week (see Section 1.5). To focus on the repeated shock mechanism, I estimate the following crosssectional regression at each date:

$$
\begin{equation*}
r_{i, t}=\alpha_{t}+\gamma_{1, t} r_{i, t-1}+\ldots+\gamma_{L, t} r_{i, t-L}+\gamma_{\mu, t} \mu_{i, t}+u_{i, t} \tag{1.14}
\end{equation*}
$$

where $\mu_{i t}$ is the average same-weekday (the same weekday as day $t$ ) return on stock $i$ over the previous year (excluding the past $L$ returns). Here, $\mu_{i t}$ controls for variation in expected returns across days of the week, which is similar to a day-of-the-week fixed effect (Keloharju, Linnainmaa, and Nyberg (2016)). A multiple regression provides a cleaner picture of autocorrelation patterns than a univariate regression.

The upper panel of Figure 1.4 plots the time-series averages of the cross-sectional regression estimates with $l=20$ and their associated Newey and West (1987) $t$-statistics. The results are not sensitive to the precise number of lags.


Figure 1.4. Cross-sectional multiple regressions of daily returns. The following crosssectional regression is estimated for each day $t$ : $r_{i, t}=\alpha_{t}+\gamma_{1, t} r_{i, t-1}+\ldots+\gamma_{20, t} r_{i, t-20}+\gamma_{\mu, t} \mu_{i, t}+$ $u_{i, t}$, where $r_{i, t}$ is the simple return of stock $i$ on day $t$ and $\mu_{i t}$ is the average same-weekday (the same weekday as day $t$ ) return on stock $i$ over the previous year excluding the past 20 returns. The sample consists of NYSE/Amex common stock returns over the period 1983 to 2012. The left-hand charts plot the time-series averages of $\gamma_{l, t}(l=1, \ldots, 20)$. The right-hand charts plot $t$-statistics computed using a Newey-West correction with 20 lags. Black lines indicate significance bounds at the $5 \%$ level. Panel A: all stocks. Panel B: the third of stocks with the highest average turnover over the past 250 days as of date $t-20$.

At short horizons, the coefficients are all negative and significant. The first estimate is large in absolute value because of bid-ask bounce ( -0.09 , truncated in the figure). The decaying pattern in slope coefficients is consistent with the $q=0$ model. But the fifth and tenth estimates appear abnormally high relative to the other estimates. More formally, the frictionless model predicts that all autocorrelations decay exponentially. This implies the following null hypothesis:

Hypothesis 1. $\left|\gamma_{5}\right| \geq\left|\gamma_{6}\right|$.

This hypothesis is rejected at the $1 \%$ level with a $t$-statistic of 2.93 , which is inconsistent with the frictionless model but is in line with infrequent rebalancing every five trading days as illustrated in Figure 1.2. Note, however, that hypothesis 1 can only invalidate the frictionless model-it does not constitute direct evidence of infrequent rebalancing. Still, infrequent rebalancing offers a plausible explanation that seems difficult to obtain with other theories. Furthermore, variation in average returns across days of the week does not generate the results, although the average estimated $\gamma_{\mu, t}$ is strongly significant. Using simple regressions or demeaning returns in the cross-section before estimating $\gamma_{\mu, t}$ does not affect this result.

To evaluate the role of trading volume, I split stocks into three portfolios at each date based on their average turnover over the past 250 days. The cross-sectional regression (1.14) is then estimated on the third of stocks that are in the high turnover portfolio at date $t-20$. Panel B of Figure 1.4 shows that the shift at lag five is markedly stronger for high turnover stocks. Hypothesis 1 is rejected at the $1 \%$ level with a $t$-statistic of 3.61 . Neglected stocks do not drive the results; the shift at lag five is weak for low turnover stocks. Moreover, the regression coefficients tend to be lower in absolute value for high turnover stocks, indicating smaller reversal for these stocks.

The model can match the predictability patterns in daily returns. As for intraday returns, I compare the regression estimates to the partial autocorrelations predicted by the model since they are almost identical. ${ }^{19}$ The left-hand side of Figure 1.5 reports the model's partial autocorrelations. The model seems to fit the short-term dependence in stock returns in Figure 1.4. Infrequent rebalancing generates a shift in the autocorrelation pattern at the rebalancing horizon.

The turnover results in Figure 1.4 are also potentially consistent with the model. A decrease in the persistence of liquidity trading $a_{\theta}$ increases turnover and decreases return autocorrelation (in absolute value). Nevertheless, $a_{\theta}$ has an ambiguous role on equilibrium price coefficients with infrequent rebalancing (see the Internet Appendix for details). Numerically, I find that when $a_{\theta}$ is large, the pattern becomes more pronounced as $a_{\theta}$ decreases, consistent with the

[^14]

Figure 1.5. Partial autocorrelations predicted by the model (with 20 lags) for daily returns with different degrees of liquidity trading persistence $\left(\mathbf{a}_{\theta}\right)$. The calibration is shown in Table A.l.
evidence. As an illustrative example, the left-hand side of Figure 1.5 shows that the infrequent rebalancing pattern is more pronounced for lower $a_{\theta}$. In particular, the autocorrelation becomes positive.

The previous results are robust to using midquote returns, controlling for firm size, and using subsamples. The Internet Appendix reports the detailed results. In addition, the results do not appear to be driven by a quarterly measure of institutional ownership after controlling for turnover. The coefficients are insignificant, however, over an older sample that runs from 1963 to 1993.

### 1.4.5 Additional Empirical Evidence

The model can potentially shed light on additional recent evidence from intraday returns. Gao et al. (2014) find that the first half-hour return on the SPDR S\&P 500 ETF predicts the last half-hour return of the trading day. The infrequent rebalancing model is consistent with this evidence assuming that some infrequent traders adjust their portfolios at the open and close of the market. This assumption is economically intuitive. The $U$-shaped pattern in trading volume across the trading day suggests that many market participants concentrate their trading at market open and close. Increasing the fraction of traders adjusting their portfolios in a given calendar period increases trading volume and strengthens the autocorrelation pattern in this period. Thus, the model can provide a simple explanation for the results of Gao et al. (2014). Furthermore, these results come from time-series regressions and therefore reflect autocorrelations only.

### 1.5 Return Seasonality

Return autocorrelation cannot explain the persistence of the coefficients for intraday returns in Figure 1.1. The same observation holds for monthly returns. Following Heston and Sadka (2008), Figure 1.6 plots the estimates of regression (1.1) obtained with monthly returns. The average coefficient spikes every twelfth lag and does not decay. According to the decomposition of Section 1.2, these results provide strong evidence that the cross-sectional variance in average returns varies across half-hour intervals of a trading day and months of the year. Indeed, the last term of equation (1.3) is

$$
\begin{equation*}
\frac{1}{N} \sum_{i=1}^{N}\left(\mu_{i, c(t)}-\mu_{c(t)}\right)\left(\mu_{i, c(t-l)}-\mu_{c(t-l)}\right) \tag{1.15}
\end{equation*}
$$

Hence, persistent seasonality patterns can arise whenever the cross-sectional variance in average returns varies across calendar periods. The benchmark infrequent rebalancing model of Section 1.3.3 focuses on the autocovariance component and abstracts from cross-sectional variation in expected returns. Next, I show that variation in the proportion of infrequent traders across calendar periods can generate persistent seasonality patterns.


Figure 1.6. Cross-sectional regressions of monthly returns. The following cross-sectional regression is estimated for each month $t: r_{i, t}=\alpha_{l, t}+\gamma_{l, t} r_{i, t-l}+u_{i, t}$ for $l=1, \ldots, 240$, where $r_{i, t}$ is the simple return of stock $i$ in month $t$. The sample consists of U.S. common stock returns over the period 1964 to 2013 for the dependent variable. The right-hand side series starts in 1944. Stocks with a price lower than $\$ 1$ are excluded from the regressions. The figure plots the time-series averages of $\gamma_{l, t}$.

In the general setup of Section 1.3 in which a mass $q_{c(t)}$ of infrequent traders rebalance in calendar period $c(t)$, the following proposition holds.

Proposition 3. In a linear stationary rational expectations equilibrium, if it exists, the vector of
asset prices is given by

$$
\begin{equation*}
P_{t}=\bar{P}_{c(t)}+\frac{a_{D}}{R-a_{D}} D_{t}+P_{\theta, c(t)} \theta_{t}+\sum_{i=1}^{k} P_{X_{i}, c(t)} X_{t-i}^{I} \tag{1.16}
\end{equation*}
$$

where the coefficient matrices are solutions to a system of nonlinear equations given in Appendix A.1.

The main insights developed using the simpler model of Section 1.3.3 hold, but here the equilibrium price coefficients vary with the calendar period $c(t)$ at date $t$. Expected returns now differ across calendar periods. ${ }^{20}$

To convey the main intuition in the simplest possible way, I focus on the case with two different calendar periods and let $k=1$. The mass of frequent traders is fixed and equals $1-q$, where $q=$ $q_{1}+q_{2}$. Further, let $h=1$ for ease of exposition and recall that $Q_{t+1}=P_{t+1}+D_{t+1}-R P_{t}$ denotes the vector of (dollar) excess returns. Using the market-clearing condition (1.8), the expected return in a given calendar period is

$$
\begin{equation*}
\mathbb{E}\left[Q_{t+1} \mid c(t)\right]=\frac{\gamma_{F}}{(1-q)} \operatorname{Var}\left[Q_{t+1} \mid c(t)\right]\left(\bar{S}-q_{c(t)} \mathbb{E}\left[X_{t}^{I} \mid c(t)\right]-q_{c(t-1)} \mathbb{E}\left[X_{t-1}^{I} \mid c(t)\right]\right) \tag{1.17}
\end{equation*}
$$

where I used the fact that $\operatorname{Var}_{t}\left[Q_{t+1}\right]=P_{\theta, c(t+1)} \Sigma_{\theta} P_{\theta, c(t+1)}^{\prime}+\left(\frac{R}{R-a_{D}}\right)^{2} \Sigma_{D}$ is constant for a given calendar period. The term in parentheses in equation (1.17) is independent of the calendar period. Thus, when $q_{1} \neq q_{2}$, differences in expected returns across calendar periods are generated solely by differences in conditional variances across calendar periods. When $q_{1}>q_{2}$, a larger mass of rational traders is present in the market in period 1 , which reduces the price impact of liquidity shocks. This implies that $\left|P_{\theta, i, 2}\right|>\left|P_{\theta, i, 1}\right|$ for asset $i$, and hence, expected returns are larger in period 1 than in period 2. In summary, traders require a higher premium to hold an asset when they anticipate the price impact to be higher next period. The next proposition formalizes this reasoning using the same intuitive conditions as Proposition 2.

Proposition 4. Consider a single-asset economy with two calendar periods, and assume that $k=1$ and $h=1$. Infrequent traders rebalance their portfolios only in the first calendar period. If $P_{\theta, c}<0$ and $P_{X, c}>0(c=1,2)$, then the expected excess return on the asset is larger in the first calendar period than in the second calendar period.

The previous result is specific to the infrequent rebalancing setup. As a point of comparison, consider a frictionless economy $(q=0)$ in which the mass of traders-or equivalently, the risk

[^15]aversion—varies deterministically from one calendar period to the next. In this economy, the opposite result holds.

Proposition 5. Consider a single-asset economy with two calendar periods and only frequent traders with $h=1$. The expected excess return on the asset is largest in the period when fewer traders are in the market.

A smaller mass of traders requires a larger expected return to absorb liquidity shocks. This effect dominates the price impact effect described above. In the infrequent rebalancing economy, the average asset supply that frequent traders must absorb is the same in both calendar periods, as shown in equation (1.17).

Expected returns are larger in the period in which more traders rebalance. ${ }^{21}$ This effect also leads to a larger spread in expected returns between assets in the rebalancing period. Intuitively, assets with large loadings on the risk factor are disproportionately affected relative to assets with small loadings-the extreme case being a riskless asset, which is not affected. To see this, note that a conditional form of the CAPM holds. The expected excess return on asset $i$ in a given calendar period can be written as

$$
\begin{equation*}
\mathbb{E}\left[Q_{i, t+1} \mid c(t)\right]=\frac{\operatorname{Cov}\left[Q_{i, t+1}, Q_{m, t+1} \mid c(t)\right]}{\operatorname{Var}\left[Q_{m, t+1} \mid c(t)\right]} \mathbb{E}\left[Q_{m, t+1} \mid c(t)\right], \tag{1.18}
\end{equation*}
$$

where $Q_{m, t+1}$ is the market excess return. ${ }^{22}$ Variation in the degree of infrequent rebalancing generates seasonality in the market risk premium. If assets have different exposure to market risk, then the model generates seasonality in the cross-section of asset returns. ${ }^{23}$

As an example, consider two assets that are identical except for their liquidity shock volatilities. Panel A of Figure 1.7 plots the expected excess return for each asset in both calendar periods as a function of the first asset's liquidity shock volatility. Since $q_{1}>q_{2}$ in this example, the cross-sectional variation in expected returns is larger in calendar period 1 than in calendar period 2. This effect comes from anticipated price impact-the conditional variance is more sensitive to variation in the mass of traders for the riskier asset than for the safer asset. In addition, expected returns are larger in the period in which more traders rebalance, in line with Proposition 4 (not shown in the figure since returns are normalized).

The above mechanism generates persistent return seasonality. Panel B of Figure 1.7 plots the average coefficients in regression (1.1) estimated from simulated returns with different

[^16]Panel A. Expected returns


Panel B. Average cross-sectional regression estimates (scaled)

$$
q_{1}=q_{2}=0 \quad q_{1}=q_{2} \quad q_{1} \neq q_{2}
$$





Figure 1.7. Return seasonality. Panel A shows the expected excess return for each stock in each calendar period as a function of the first stock's liquidity shocks volatility ( $\sigma_{\theta, 1}$ ). The expected returns are normalized to one for $\sigma_{\theta, 1}=\sigma_{\theta, 2}=0.5$. Panel B shows cross-sectional regression estimates (scaled) from $Q_{i, t}=\alpha_{l, t}+\gamma_{l, t} Q_{i, t-l}+u_{i, t}$ based on averages of 1000 simulations of a 20 -stock economy over $T=500$ periods. The calibration assumes $q_{1}=0.65$, $q_{2}=0.05, k=1, a_{\theta}=0, a_{D}=0, \sigma_{D}=0.2, \rho_{D}=0.3, R=1.05, h=2$, and $\bar{S}=10$. In Panel B, the stocks have either $\sigma_{\theta}=0.5$ or $\sigma_{\theta}=1.5$ in equal proportions.
proportions of infrequent traders. Return autocorrelation mainly determines the coefficients at lower lags-with infrequent rebalancing, the repeated shock mechanism produces a large positive autocorrelation in the second period (middle and right charts). At higher lags, the coefficients are positive because of cross-sectional variation in mean returns. When $q_{1} \neq q_{2}$, these coefficients shift from period to period since the cross-sectional variance in mean returns differs across calendar periods. Variation in the degree of infrequent rebalancing can thus potentially explain the evidence presented by Heston, Korajczyk, and Sadka (2010) and other persistent seasonality patterns in cross-sectional regression estimates. At low lags the
cross-sectional regressions pick up a repeated shock mechanism, while at high lags they only reflect cross-sectional variation in mean returns.

Crucially, infrequent rebalancing does not add an extra risk factor but rather generates seasonality in the factor risk premium. This is consistent with evidence that seasonality strategies have low correlation across and within asset classes (Keloharju, Linnainmaa, and Nyberg (2016)). The seasonality strategies can have a low correlation in the context of the model if markets exhibit some degree of segmentation and, as a result, their risk factors are not perfectly correlated.

To be more specific about monthly return seasonality in small and large stocks, some evidence of segmentation is provided by the rebalancing of traders in January (Ritter (1988)). The "January effect" plays an important role for monthly seasonality in U.S. stock returns. Excluding January lowers the magnitude of the monthly return seasonality strategy substantially: an average value-weighted return of $3 \%$ in January versus $0.57 \%$ outside of January over the period 1964 to 2014 (Bogousslavsky (2015)). In this case, infrequent rebalancing comes mainly from individual investors because of tax reasons. Since these investors tend to trade in small stocks, this creates a wedge between small and large stocks.

Additionally, the seasonality strategies can have a low correlation if the assets load on different factors. Market risk is the single risk factor in the model. With additional sources of risk, it is intuitive that variation in the proportion of rebalancing traders could generate seasonality in multiple risk premia. Return seasonalities could then persist even after sorting assets on specific characteristics or factors.

Consistent with the findings of Keloharju, Linnainmaa, and Nyberg (2016), a seasonality strategy is exposed to systematic risk in the model. These authors argue that return seasonalities are not a distinct class of anomalies. However, one must explain why risk premia are seasonal to begin with. Infrequent rebalancing provides a simple and intuitive channel to explain such seasonality.

### 1.5.1 Alternative Explanations

Alternative explanations based on seasonality in liquidity trading are possible. In the Internet Appendix, I show that a model with seasonality in mean liquidity trading can also generate persistent seasonality patterns. Buying or selling pressures on some stocks at the open and close could generate the seasonality in mean liquidity trading and explain the persistence of the pattern in Figure 1.1. This model cannot, however, explain the decaying pattern in the coefficients (from lag 13 to 26 and so on), the empirical daily return evidence in Section 1.4.4, and any predictability evidence based on time-series regressions (Section 1.4), which are all consistent with an autocorrelation effect from infrequent rebalancing.

Moreover, the seasonal mean model does not generate any calendar pattern in return volatility. In this model, it is the price of risk that varies with the calendar period. This model may therefore better apply to seasonality at a lower frequency, such as the January effect. Indeed, volatility does not appear to be larger in January. The shifts in mean liquidity trading could arise from tax-loss selling and rebalancing in January (Ritter (1988)).

### 1.6 Trading Volume

With only one group of agents, the dynamics of trading volume depend solely on the dynamics of liquidity trading. As a result, changes in trading volume $\left(\Delta V_{t}\right)$ are negatively autocorrelated.

Proposition 6. When $q=0$ or $q=1$, and $0<a_{\theta}<1$, changes in trading volume are negatively autocorrelated, that is, $\operatorname{Corr}\left[\Delta V_{t}, \Delta V_{t+s}\right]<0, s \geq 1$.

In the model, multiple groups of agents trade, which alters the dynamics of trading volume. Infrequent traders who rebalance their portfolios can trade with frequent traders. A large liquidity shock today reverberates in $k+1$ periods when traders readjust their portfolios. These rebalancing trades increase the autocorrelation between changes in trading volume, and hence, Corr $\left[\Delta V_{t}, \Delta V_{t+k+1}\right]$ can be positive. Proposition 6 shows that this is impossible in the frictionless economy.

Figure 1.8 plots $\operatorname{Corr}\left[\Delta V_{t}, \Delta V_{t+s}\right]$ when $q=0$ and $q=0.6$ using the daily frequency calibration (the Internet Appendix explains how to compute volume autocorrelations when $0<q<1$ ). In both cases, Corr [ $\left.\Delta V_{t}, \Delta V_{t+1}\right]$ is large and negative. Autocorrelations are negligible beyond the first lag when $q=0$. When $0<q<1$, the autocorrelations are still small but many times larger than in the frictionless economy. Patterns linked to infrequent rebalancing appear at the rebalancing horizon (fifth lag).

The setup can potentially explain why Heston, Korajczyk, and Sadka (2010) find that half-hour volume periodicity does not fully account for return periodicity. When $q=1$, liquidity trading determines trading volume (Proposition 6), but infrequent rebalancing still generates a return periodicity pattern. Therefore, the volume pattern cannot explain the return pattern in this polar case. When $q<1$, volume is still determined in part by liquidity trading and therefore cannot fully explain the return periodicity.

To test the model's predictions, I estimate the following regression on daily returns:

$$
\begin{equation*}
v_{i, t}=\alpha_{l, t}+\gamma_{l, t} v_{i, t-l}+\gamma_{v, t} v_{i, t}+u_{i, t} \tag{1.19}
\end{equation*}
$$

where $v_{i, t}=\ln \left(\frac{\text { Turnover }_{i, t}}{\text { Turnover }_{i, t-1}}\right)$ and $v_{i, t}$ is the average same-weekday (the same weekday as day $t$ ) change in turnover over the past year. To estimate regression (1.19), I exclude all stocks that



Figure 1.8. Volume change autocorrelations predicted by the model with different degrees of infrequent rebalancing (q). The calibration is shown in Table A.1.
have zero volume on one day from the sample. This procedure leaves an average of roughly 950 observations per period. The results of Section 1.4.4 are unaffected. Figure 1.9 plots the average $\gamma_{l, t}$ coefficients and their $t$-statistics.



Figure 1.9. Cross-sectional regressions of daily turnover. The following cross-sectional regression is estimated for each day $t: v_{i, t}=\alpha_{l, t}+\gamma_{l, t} \nu_{i, t-l}+\gamma_{v, t} v_{i, t}+u_{i, t}$ for $l=1, \ldots, 20$, where $v_{i, t}$ is the log turnover of stock $i$ on day $t$ and $v_{i t}$ is the average same-weekday (the same weekday as day $t$ ) turnover on stock $i$ over the previous year. The sample consists of NYSE/Amex common stock turnover series over 1983 to 2012. The left-hand chart plots the time-series averages of $\gamma_{l, t}$. The right-hand chart plots $t$-statistics computed using a Newey-West correction with 20 lags. Black lines indicate significance bounds at the level of $5 \%$.

The first coefficient (truncated in the figure) is large and negative ( -0.39 ). The regression reveals shifts in the autocorrelation at the fifth and tenth lags that are qualitatively consistent
with an infrequent rebalancing mechanism. Similar to the daily return evidence, the average of the coefficient $\gamma_{v, t}$ is positive and highly significant but does not explain the shifts in the other coefficients.

Nevertheless, the model overestimates the magnitude of the fifth coefficient and does not produce a large positive tenth lag coefficient. Moreover, the fourth lag coefficient does not exhibit any shift, which seems to indicate that traders either do not anticipate the repeated liquidity shocks on average or are not able to reliably trade on them.

As in the analysis of Section 1.5 for returns, infrequent rebalancing can also generate persistent seasonality patterns for changes in trading volume. Trading volume is higher when more infrequent traders rebalance. At the same time, differences in trading volume across assets are also more pronounced. The cross-sectional variance in average changes in trading volume is then higher when more traders rebalance, which can generate persistent seasonality patterns (Section 1.2). I leave a detailed investigation of these effects for future research.

### 1.7 Conclusion

This paper studies a dynamic equilibrium model in which some investors readjust their portfolio infrequently. I show that trading by investors with heterogeneous rebalancing horizons can generate return autocorrelation and seasonality consistent with empirical evidence at different frequencies.

In the model, return autocorrelations exhibit specific patterns linked to the rebalancing horizon of traders, consistent with empirical evidence from intraday returns and new evidence from daily returns. Despite being perfectly anticipated, the lagged demands of infrequent traders affect return dynamics. The model also makes specific predictions concerning changes in trading volume, for which I find support in the data.

Variation in the proportion of infrequent traders across calendar periods can generate return seasonality in line with empirical evidence from intraday and monthly returns. Infrequent rebalancing does not add an extra risk factor but rather generates seasonality in the factor risk premium. As a result, the spread in expected returns between assets with different exposures to the factor increases when more traders readjust their portfolios. This paper provides a first step in explaining why risk premia can be seasonal.

## 2 Seasonalities in Anomalies

This chapter investigates return seasonalities in a set of well-known anomalies in the crosssection of U.S. stock returns. A January seasonality goes beyond a size effect and strongly affects most anomalies, which can even switch sign in January. Both tax-loss selling and firm size are important in explaining the turn-of-the-year pattern. Return seasonality exists outside of January, with respect to the month of the quarter. Small stocks earn abnormally high average returns on the last day of each quarter, which significantly affects size, idiosyncratic volatility, and illiquidity portfolios. The results have implications for the interpretation and analysis of many anomalies, such as asset growth and momentum.

### 2.1 Introduction

Seasonalities have an important impact on the cross-section of stock returns. For example, a large literature studies the tendency of small stocks to earn abnormal returns relative to large stocks in January-i.e., the "January effect." ${ }^{1}$ This paper investigates seasonalities in "anomalies" portfolios built from U.S. stock returns. An anomaly is defined broadly as any factor that affects the cross-section of stock returns beyond the market factor (Fama and French, 2008). I consider a set of well-known anomalies based on accounting, price, return, and volume data.

First, I examine the January seasonality in anomalies. I find that the January seasonality goes beyond a size effect. Part of the evidence is consistent with previous research. But other results based on recently documented anomalies are new. In addition, many results are scattered in the literature and based on old data series. I aim to provide a fresh assessment of the January

[^17]
## Chapter 2. Seasonalities in Anomalies

seasonality as well as regroup evidence about its impact on stock returns.
Second, I show that strong seasonalities exist outside of January depending on the month of the quarter. One potential channel is that institutions may have incentives to manipulate or window dress their portfolios at the end of quarters, which may affect stock prices. ${ }^{2}$ The seasonalities are, however, especially strong for beginning-of-quarter months. The results have implications for the interpretation of several anomalies and, more generally, for studies of the cross-section of stock returns.

The January seasonality in anomaly return persists when controlling for the Fama-French three factors. Despite showing a marked January seasonality, the SMB and HML factors do not explain the January seasonality in anomalies. Moreover, the exposures of long-short anomaly portfolios to the Fama-French factors do not appear to vary between January and other months. On the other hand, month-of-the-quarter Fama-French alphas are much smaller than average returns. ${ }^{3}$ The SMB factor displays a beginning-of-the-quarter seasonality that reduces the beginning-of-the-quarter alpha of several anomalies portfolios. Even though the beginning-of-the-quarter seasonality disappears when controlling for this factor, this result does not explain why the factor is seasonal in the first place.

The previous results are robust to restricting the analysis on large caps portfolios with valueweighting or using price screens. Here, the January seasonality in anomalies cannot simply be understood as a January effect for small stocks. The results are also consistent over different subsamples.

I discuss explanations for the January effect put forward by previous research. The literature has not yet settled on an explanation for this seasonality. For instance, the tax-loss selling hypothesis—one of the leading explanations for the January effect—does not explain the long term seasonality patterns in January and the lack of price pressure in December for stocks with high potential tax-loss.

I investigate daily patterns around the turn of the year and the turn of each quarter. Small stocks earn an abnormally high average return on the last day of the quarter, which significantly affects size, idiosyncratic volatility, and illiquidity portfolios. A similar effect occurs on the last trading day of the year, which is puzzling from the point of view of tax-loss selling (Roll, 1983) but may be consistent with portfolio pumping by equity funds (Carhart et al., 2002).

Tax-loss selling as proxied by a measure of capital loss overhang and size are both important in

[^18]explaining the turn-of-the-year pattern. A marked seasonality exists in the long-short tax-loss selling potential portfolio built from large capitalization stocks. Similarly, a seasonality also exists in the long-short size portfolio built from stocks with low tax-loss selling potential. The pattern in daily returns persists after jointly controlling for these variables. Moreover, several anomalies characteristics are also significant in explaining year-end daily returns. Earnings announcements do not appear to explain the seasonalities.

An asset pricing theory that attempts to explain an anomaly should also offer an explanation as to why the anomaly exhibits seasonalities. The anomalies studied in the paper reflect a wide range of sorting variables; hence, the seasonalities seem to show pervasive features of the cross-section of stock returns. Furthermore, such seasonalities are important to take into account when constructing and backtesting strategies. These seasonalities challenge the economic interpretation of well-known strategies based on asset growth, size, momentum, illiquidity, and idiosyncratic volatility. For instance, removing low-priced stocks appears to be necessary to obtain positive illiquidity and size premia outside of January. But these premia vary within the quarter.

A number of recent papers examine the determinants and role of anomalies in the crosssection of stock returns. Most notably, studies explore the role of size (Fama and French, 2008), investor sentiment (Stambaugh, Yu, and Yuan, 2012), financial distress (Avramov et al., 2013), institutions (Edelen, Ince, and Kadlec, 2014) and shorting fees (Drechsler and Drechsler, 2014). Also related, McLean and Pontiff (2016) study the returns of many anomalies and find a post-publication decline in their returns. None of these papers discusses seasonalities in anomalies.

Keloharju, Linnainmaa, and Nyberg (2014) also present evidence of seasonality in anomalies. In particular, they show that seasonal variation in the average returns of several anomalies strongly dominates the unconditional cross-sectional variation in average returns. They examine return seasonality in different markets and focuses on the economic magnitude of seasonality strategies. Relative to their paper, this paper examines anomalies in detail and how the results challenge their economic interpretation. In addition, I specifically examine January and month-of-the-quarter effects.

### 2.2 Seasonalities in Anomalies

This section provides evidence that several well-known anomalies exhibit marked January and month-of-the-quarter seasonalities. The anomalies used in the paper are described in Table B. 1 in the Appendix. These anomalies reflect a broad range of sorting variables, such as past returns, accounting data, market capitalization, and trading volume. I use daily and monthly returns from CRSP on all common stocks (share code 10 or 11) on NYSE, Amex, and

NASDAQ from January 1964 to December 2014. ${ }^{4}$ I take accounting data from Compustat to compute book equity, gross profitability, asset growth, accruals, and net stock issues. All these accounting variables are computed once a year at the end of June using data for the previous fiscal year.

At the beginning of each month, I form decile portfolios based on the value of the variable at the end of the previous month. ${ }^{5}$ Long-short portfolios are always long (short) the stocks with the highest (lowest) value of the sorting variable in the formation period. For instance, the long-short size portfolio is long large stocks and short small stocks. To help limit the influence of microstructure noise, I focus on return-weighted and value-weighted portfolios (Asparouhova, Bessembinder, and Kalcheva, 2013). Return-weighting weights each stock by its gross return in the previous period. The weights are therefore positively correlated with any mechanical noise in previous period returns, such as bid-ask bounce. This positive correlation corrects for noise-induced reversals as a source of return. In addition, return-weighting does not discard the information in small stocks returns to the same extent as value-weighting. Equal-weighted portfolios show in general even stronger seasonalities.

Section 2.2.1 examines the average January returns on the anomalies portfolios. Section 2.2.2 studies non-January average returns. More specifically, I examine average returns for beginning-of-quarter, middle-of-quarter, and end-of-quarter months.

### 2.2.1 January

Table 2.1 displays the average return-weighted and value-weighted monthly returns of longshort decile portfolios in January and non-January months. All the portfolios exhibit a marked January seasonality. The magnitudes are economically large. Furthermore, the difference between the average January and non-January returns is statistically significant for all returnweighted portfolios and for all but two value-weighted portfolios.

The literature abounds on the "January effect," which is generally known as the tendency for small stocks to earn abnormal returns relative to large stocks in January. Table 2.1 directly illustrates this result. As first documented by Keim (1983), the size premium displays a strong January seasonality (see also Blume and Stambaugh, 1983). Strikingly, the size effect is small and insignificant outside of January.

To compute book-to-market, I divide a firm's book equity by its market capitalization six

[^19]Table 2.1
Average January (Jan) and non-January months (non-Jan) returns in percent of returnweighted and value-weighted long-short decile portfolios formed on different characteristics. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014 (the accruals portfolios start in July 1971). Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. Financial firms are excluded from book-to-market, gross profitability, asset growth, accruals, and net stock issues portfolios. NASDAQ stocks are excluded from the turnover and illiquidity portfolios. The characteristics are defined in Table B.1. Standard $t$-statistics are shown in parentheses. The diff columns report the difference between January and non-January average returns where ${ }^{*}$, ${ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | return-weighted |  |  | value-weighted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Jan | non-Jan | diff | Jan | non-Jan | diff |
| Market cap. | $\begin{gathered} -6.36 \\ (-7.36) \end{gathered}$ | $\begin{gathered} 0.32 \\ (1.87) \end{gathered}$ | $\begin{gathered} -6.68^{* * *} \\ (-7.58) \end{gathered}$ | $\begin{gathered} -5.56 \\ (-6.36) \end{gathered}$ | $\begin{gathered} 0.19 \\ (1.06) \end{gathered}$ | $\begin{gathered} -5.74^{* * *} \\ (-6.45) \end{gathered}$ |
| Book-to-market | $\begin{gathered} 3.58 \\ (5.44) \end{gathered}$ | $\begin{gathered} 0.72 \\ (4.24) \end{gathered}$ | $\begin{gathered} 2.87^{* * *} \\ (4.22) \end{gathered}$ | $\begin{gathered} 3.29 \\ (3.72) \end{gathered}$ | $\begin{gathered} 0.23 \\ (1.29) \end{gathered}$ | $\begin{gathered} 3.06^{* * *} \\ (3.38) \end{gathered}$ |
| Gross profitability | $\begin{gathered} -1.19 \\ (-2.27) \end{gathered}$ | $\begin{gathered} 1.03 \\ (8.18) \end{gathered}$ | $\begin{gathered} -2.22^{* * *} \\ (-4.12) \end{gathered}$ | $\begin{gathered} -0.88 \\ (-1.66) \end{gathered}$ | $\begin{gathered} 0.50 \\ (3.54) \end{gathered}$ | $\begin{gathered} -1.38^{* *} \\ (-2.51) \end{gathered}$ |
| Asset growth | $\begin{gathered} -3.95 \\ (-7.87) \end{gathered}$ | $\begin{gathered} -0.45 \\ (-3.96) \end{gathered}$ | $\begin{gathered} -3.50^{* * *} \\ (-6.80) \end{gathered}$ | $\begin{gathered} -2.21 \\ (-3.54) \end{gathered}$ | $\begin{gathered} -0.23 \\ (-1.58) \end{gathered}$ | $\begin{gathered} -1.98^{* * *} \\ (-3.09) \end{gathered}$ |
| Accruals | $\begin{gathered} -1.20 \\ (-3.11) \end{gathered}$ | $\begin{gathered} -0.33 \\ (-3.99) \end{gathered}$ | $\begin{aligned} & -0.87^{* *} \\ & (-2.20) \end{aligned}$ | $\begin{gathered} -0.56 \\ (-0.91) \end{gathered}$ | $\begin{gathered} -0.28 \\ (-1.87) \end{gathered}$ | $\begin{gathered} -0.28 \\ (-0.45) \end{gathered}$ |
| Net stock issues | $\begin{gathered} 0.15 \\ (0.28) \end{gathered}$ | $\begin{gathered} -1.16 \\ (-9.24) \end{gathered}$ | $\begin{aligned} & 1.31^{* *} \\ & (2.31) \end{aligned}$ | $\begin{gathered} -0.17 \\ (-0.34) \end{gathered}$ | $\begin{gathered} -0.54 \\ (-4.49) \end{gathered}$ | $\begin{gathered} 0.37 \\ (0.70) \end{gathered}$ |
| $\Delta$ turnover | $\begin{gathered} 3.25 \\ (5.71) \end{gathered}$ | $\begin{gathered} 0.89 \\ (9.46) \end{gathered}$ | $\begin{gathered} 2.36^{* * *} \\ (4.09) \end{gathered}$ | $\begin{gathered} 1.72 \\ (2.84) \end{gathered}$ | $\begin{gathered} 0.45 \\ (3.73) \end{gathered}$ | $\begin{aligned} & 1.27^{* *} \\ & (2.05) \end{aligned}$ |
| Illiquidity | $\begin{gathered} 6.10 \\ (6.75) \end{gathered}$ | $\begin{gathered} -0.16 \\ (-0.97) \end{gathered}$ | $\begin{gathered} 6.26^{* * *} \\ (6.82) \end{gathered}$ | $\begin{gathered} 4.46 \\ (5.44) \end{gathered}$ | $\begin{gathered} 0.16 \\ (0.96) \end{gathered}$ | $\begin{gathered} 4.31^{* * *} \\ (5.16) \end{gathered}$ |
| Idiosyncratic vol. | $\begin{gathered} 5.58 \\ (5.91) \end{gathered}$ | $\begin{gathered} -1.03 \\ (-4.47) \end{gathered}$ | $\begin{gathered} 6.61^{* * *} \\ (6.81) \end{gathered}$ | $\begin{gathered} 2.69 \\ (2.53) \end{gathered}$ | $\begin{gathered} -0.81 \\ (-2.92) \end{gathered}$ | $\begin{gathered} 3.50^{* * *} \\ (3.18) \end{gathered}$ |
| Momentum | $\begin{gathered} -3.47 \\ (-3.40) \end{gathered}$ | $\begin{gathered} 1.72 \\ (8.71) \end{gathered}$ | $\begin{gathered} -5.19 * * * \\ (-4.98) \end{gathered}$ | $\begin{gathered} -1.81 \\ (-1.59) \end{gathered}$ | $\begin{gathered} 0.99 \\ (3.95) \end{gathered}$ | $\begin{aligned} & -2.80^{* *} \\ & (-2.41) \end{aligned}$ |
| 12-month effect | $\begin{gathered} 3.28 \\ (6.20) \end{gathered}$ | $\begin{gathered} 0.48 \\ (5.67) \end{gathered}$ | $\begin{gathered} 2.81^{* * *} \\ (5.23) \end{gathered}$ | $\begin{gathered} 3.00 \\ (4.17) \end{gathered}$ | $\begin{gathered} 0.57 \\ (4.02) \end{gathered}$ | $\begin{gathered} 2.42^{* * *} \\ (3.30) \end{gathered}$ |

months ago. The book-to-market strategy is five times more profitable in January than in other months with return-weighting. The difference is even larger with value-weighting. Loughran (1997) documents a January seasonality in the book-to-market effect, which is robust to controlling for firm size. He further argues that the book-to-market effect is insignificant for large firms outside of January.

The size seasonality is well-known in the literature. Less is known, however, about the behavior of other asset pricing anomalies in January. I investigate strategies based on the following accounting data: gross profitability (Novy-Marx, 2013), asset growth (see, for instance, Cooper, Gulen, and Schill, 2008), accruals (Sloan, 1996), and net stock issues (Daniel and Titman, 2006). ${ }^{6}$

The gross profitability strategy's average return is negative in January and positive in all other months. Though the strategy yields large and strongly significant average returns outside of January, the previous results suggest that a convincing explanation should be able to account for the different January behavior. Similarly, the asset growth strategies are seasonal and earn more than $40 \%$ of their average annual return in January. For both of these anomalies, the authors do not mention the seasonality.

The accruals anomaly exhibits a clear seasonality with return-weighted portfolios. For valueweighted portfolios, the January average return is large but not statistically significant. The average decile returns are not monotonic for this strategy. In January, they are U-shaped. Outside of January, they are driven by the poor performance of the highest decile portfolio (i.e., stocks with high accruals).

Previous research shows that firms with high net stock issues tend to have low future returns (Daniel and Titman, 2006, Pontiff and Woodgate, 2008). The net stock issues strategy is also seasonal. The strategy gives small and insignificant average returns in January, while the average return is large outside of January. The difference is especially marked for returnweighted portfolios.

Strategies based on volume data seem to perform differently in January. The "change in turnover" strategy ( $\Delta$ turnover) sorts stocks on their change in turnover in the previous month relative to the past six-month average turnover. This strategy is highly profitable in all months of the year. The January profit is, however, much larger than in other months. The seasonality vanishes once one skips the last month in the portfolio formation: High December $\Delta$ turnover stocks tend to have high average returns in January, but this is not the case for high November $\Delta$ turnover stocks.

[^20]Table 2.1 also examines the return on a strategy based on the average price impact measure ILLIQ (Amihud, 2002) in the previous year like in Acharya and Pedersen (2005); see Table B.1. The illiquidity strategy exhibits a strong January seasonality. Surprisingly, this strategy has the wrong sign outside of January for return-weighted returns. Stocks with low price impact—as measured by ILLIQ—in the previous year tend to underperform high price impact stocks. Looking at each decile return separately gives no evidence of a robust return pattern outside of January. Since ILLIQ is measured over one year, the seasonality does not reflect a short-term reversal effect specific to December and January. These results highlight the importance of controlling for January-related effects when testing liquidity variables (see Eleswarapu and Reinganum, 1993, Datar, Y. Naik, and Radcliffe, 1998).

Ang et al. (2006) find a negative return on a high-minus-low strategy formed on past month idiosyncratic volatility-a puzzling result that contradicts standard theories. As shown in Table 2.1, the strategy's average January and non-January returns strongly differ. Average returns are sizable in both periods but have opposite signs. ${ }^{7}$ The non-January average return is not monotonic across deciles (not reported). The highest-decile portfolio completely drives the negative average return; that is, stocks with the highest idiosyncratic volatility in the last month perform extremely poorly in the current month. The previous results are consistent with the analysis of Peterson and Smedema (2011).

Momentum strategies tend to perform poorly in January (Jegadeesh and Titman, 2001). For completeness, I compute the average monthly return on a momentum strategy that sorts stocks based on the past six months skipping the last month and with a one-month holding period. Table 2.1 reports the results, which are in line with previous research. Furthermore, it is also well-known that strategies based on long-term reversal exhibit a strong January seasonality (De Bondt and Thaler, 1987).

Finally, I examine a seasonality strategy based on past returns (Heston and Sadka, 2008). At the end of month $t-1$, the 12-month strategy allocates stocks into ten portfolios based on their average return in month $t-k 12$, for $k=6,7,8,9,10$. This strategy is profitable when the same stocks tend to perform well in the same months every year. Unsurprisingly, the 12-month strategy exhibits a strong January seasonality. Contrary to other months, the average return pattern across deciles is not monotonic but U-shaped in January.

The January seasonality appears robust to value-weighting. Standing out are the accruals and net issues portfolios, for which the value-weighted average January and non-January returns do not differ significantly. In addition, value-weighting tends to lower the long-short portfolios

[^21]
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average returns. The book-to-market and asset growth strategies give small and insignificant average returns outside of January when using value-weighted portfolios. ${ }^{8}$

### 2.2.2 Beginning and End-of-Quarter Effects

This section shows that the average return of anomalies portfolios varies with the month of the quarter. Returns in beginning-of-quarter and end-of-quarter months can differ from returns in mid-quarter months for several reasons. Previous research has shown that institutions may have incentives to manipulate or window dress their portfolios at the end of quarters (Carhart et al., 2002, Ben-David et al., 2013). In this respect, Sias (2007) finds that a momentum strategy has a higher return on quarter-ending months than on non-quarter-ending months. In addition, most firms announce their earnings for the previous quarter in the first month of the next quarter. Since most firms end their fiscal year in December, the months of April, July, and October contain almost half of all earnings announcement over the period 1973-2004 (Lamont and Frazzini, 2007, Table I). Important quarterly macroeconomic data may also be released in the month right after the end of the relevant quarter. This is the case for the advanced estimate of U.S. gross domestic product.

Table 2.2 displays the average return of the long-short anomalies portfolios separately for middle-quarter, beginning-of-quarter, and end-of-quarter months. The beginning-of-quarter results exclude the month of January. Specific seasonalities emerge for both return-weighted and value-weighted portfolios.

A size effect is economically large and statistically significant only in beginning-of-quarter months. Since the market capitalization strategy is long large caps and short small caps, this result contrasts with the average January return in Table 2.1, where small caps tend to strongly outperform large caps. Therefore, two different seasonalities appear to drive the size premium: a January seasonality and a beginning-of-quarter seasonality.

Book-to-market average returns differ between return-weighted and value-weighted portfolios. Value-weighted returns are small and not statistically significant. While there is no clear seasonality in the return-weighted gross profitability portfolio, the value-weighted portfolio average return is small and insignificant in end-of-quarter months. On the contrary, the asset growth value-weighted portfolio average return is only large and significant in end-of-quarter months. The accruals, net stock issues, $\Delta$ turnover, illiquidity, and idiosyncratic volatility average returns tend to display a seasonality in beginning-of-quarter months. The results are especially marked for value-weighted portfolios. For instance, the accruals and net stock

[^22]Table 2.2
Average returns in percent of long-short return-weighted and value-weighted decile portfolios formed on different characteristics shown separately for middle-of-quarter, beginning-of-quarter (excluding January), and end-of-quarter months. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014 (the accruals portfolios start in July 1971). Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. Financial firms are excluded from book-to-market, gross profitability, asset growth, accruals, and net stock issues portfolios. NASDAQ stocks are excluded from the turnover and illiquidity portfolios. The characteristics are defined in Table B.1. The $t$-statistics are in parentheses, and ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | return-weighted |  |  | value-weighted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mid | beg | end | mid | beg | end |
| Market cap. | $\begin{gathered} 0.15 \\ (0.52) \end{gathered}$ | $\begin{gathered} 0.93 * * * \\ (2.75) \end{gathered}$ | $\begin{gathered} 0.03 \\ (0.11) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.04) \end{gathered}$ | $\begin{gathered} 0.95 * * * \\ (2.64) \end{gathered}$ | $\begin{gathered} -0.21 \\ (-0.81) \end{gathered}$ |
| Book-to-market | $\begin{aligned} & 0.57^{*} \\ & (1.93) \end{aligned}$ | $\begin{gathered} 1.08^{* * *} \\ (3.12) \end{gathered}$ | $\begin{aligned} & 0.59^{* *} \\ & (2.39) \end{aligned}$ | $\begin{gathered} 0.13 \\ (0.41) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.62) \end{gathered}$ | $\begin{gathered} 0.34 \\ (1.23) \end{gathered}$ |
| Gross profitability | $\begin{gathered} 0.96^{* * *} \\ (4.10) \end{gathered}$ | $\begin{gathered} 1.17^{* * *} \\ (5.35) \end{gathered}$ | $\begin{gathered} 0.99^{* * *} \\ (5.09) \end{gathered}$ | $\begin{gathered} 0.70^{* * *} \\ (3.38) \end{gathered}$ | $\begin{aligned} & 0.53^{*} \\ & (1.82) \end{aligned}$ | $\begin{gathered} 0.27 \\ (1.10) \end{gathered}$ |
| Asset growth | $\begin{gathered} -0.59 * * * \\ (-2.66) \end{gathered}$ | $\begin{aligned} & -0.34^{*} \\ & (-1.75) \end{aligned}$ | $\begin{aligned} & -0.40^{* *} \\ & (-2.36) \end{aligned}$ | $\begin{gathered} -0.12 \\ (-0.48) \end{gathered}$ | $\begin{gathered} -0.14 \\ (-0.53) \end{gathered}$ | $\begin{aligned} & -0.41^{*} \\ & (-1.66) \end{aligned}$ |
| Accruals | $\begin{gathered} -0.38^{* * *} \\ (-2.62) \end{gathered}$ | $\begin{gathered} -0.30^{*} \\ (-1.76) \end{gathered}$ | $\begin{aligned} & -0.32^{* *} \\ & (-2.50) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.16) \end{gathered}$ | $\begin{aligned} & -0.67^{* *} \\ & (-2.08) \end{aligned}$ | $\begin{gathered} -0.29 \\ (-1.21) \end{gathered}$ |
| Net stock issues | $\begin{gathered} -0.93^{* * *} \\ (-3.94) \end{gathered}$ | $\begin{gathered} -1.68^{* * *} \\ (-7.70) \end{gathered}$ | $\begin{gathered} -1.00^{* * *} \\ (-5.30) \end{gathered}$ | $\begin{gathered} -0.22 \\ (-1.17) \end{gathered}$ | $\begin{gathered} -1.22^{* * *} \\ (-5.56) \end{gathered}$ | $\begin{gathered} -0.36 \\ (-1.67) \end{gathered}$ |
| $\Delta$ turnover | $\begin{gathered} 1.05^{* * *} \\ (6.20) \end{gathered}$ | $\begin{gathered} 0.78^{* * *} \\ (4.30) \end{gathered}$ | $\begin{gathered} 0.81^{* * *} \\ (5.79) \end{gathered}$ | $\begin{gathered} 0.22 \\ (1.20) \end{gathered}$ | $\begin{gathered} 0.81^{* * *} \\ (3.37) \end{gathered}$ | $\begin{aligned} & 0.41^{* *} \\ & (1.97) \end{aligned}$ |
| Illiquidity | $\begin{gathered} -0.11 \\ (-0.36) \end{gathered}$ | $\begin{gathered} -0.93^{* * *} \\ (-2.97) \end{gathered}$ | $\begin{gathered} 0.37 \\ (1.52) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.83) \end{gathered}$ | $\begin{aligned} & -0.80^{* *} \\ & (-2.52) \end{aligned}$ | $\begin{gathered} 0.80^{* * *} \\ (3.32) \end{gathered}$ |
| Idiosyncratic vol. | $\begin{gathered} -0.55 \\ (-1.30) \end{gathered}$ | $\begin{gathered} -1.66^{* * *} \\ (-3.72) \end{gathered}$ | $\begin{gathered} -1.04^{* * *} \\ (-3.17) \end{gathered}$ | $\begin{gathered} -0.25 \\ (-0.52) \end{gathered}$ | $\begin{gathered} -1.55 * * * \\ (-2.89) \end{gathered}$ | $\begin{aligned} & -0.82^{*} \\ & (-1.92) \end{aligned}$ |
| Momentum | $\begin{gathered} 1.37 * * * \\ (4.69) \end{gathered}$ | $\begin{gathered} 1.20^{* * *} \\ (2.69) \end{gathered}$ | $\begin{gathered} 2.45 * * * \\ (8.03) \end{gathered}$ | $\begin{aligned} & 0.73^{*} \\ & (1.77) \end{aligned}$ | $\begin{gathered} 0.07 \\ (0.13) \end{gathered}$ | $\begin{gathered} 1.95 * * * \\ (5.12) \end{gathered}$ |
| 12-month | $\begin{gathered} 0.36^{* * *} \\ (2.84) \end{gathered}$ | $\begin{gathered} 0.65^{* * *} \\ (3.34) \end{gathered}$ | $\begin{gathered} 0.47^{* * *} \\ (3.65) \end{gathered}$ | $\begin{gathered} 0.29 \\ (1.26) \end{gathered}$ | $\begin{gathered} 0.93^{* * *} \\ (3.02) \end{gathered}$ | $\begin{gathered} 0.59^{* * *} \\ (2.74) \end{gathered}$ |

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issues value-weighted portfolios earn negative average returns more than twice as large at the beginning of quarters than in other months. The illiquidity portfolio average return in beginning-of-quarter months goes against economic intuition since low ILLIQ stocks strongly outperform high ILLIQ stocks. Like the size portfolio, the illiquidity and idiosyncratic volatility portfolios average beginning-of-quarter return have opposite sign than in January. Therefore, the seasonalities in these anomalies at the beginning of quarters and in January appear to be distinct.

Consistent with Sias (2007), the momentum average return is stronger in end-of-quarter months. In addition, the value-weighted average return is close to zero at the beginning of quarters. The 12 -month strategy, which is a seasonality strategy, is more profitable at the beginning and end of quarters. The average value-weighted return is small and insignificant in middle-of-quarter months.

Overall, strong seasonalities arise at the beginning of quarters for many anomalies. The average return of value-weighted long-short portfolios in middle-quarter months is economically small and statistically insignificant for nine out of eleven anomalies. On the contrary, the average return is large and significant in beginning-of-quarter months eight times out of eleven. Besides, the zero momentum return in beginning-of-quarter months also reflects a seasonality. The beginning-of-quarter seasonality does not, however, necessarily reflect an attenuated January seasonality since some anomalies have opposite sign than in January. Last, return-weighted and value-weighted portfolios can exhibit marked differences for some of the anomalies. ${ }^{9}$

### 2.2.3 Discussion

Many anomalies exhibit strong seasonalities. January-related effects are weaker in recent years but remain marked for most of the anomalies. Almost all of the anomalies under consideration have a beginning-of-quarter seasonality. Here, the seasonalities are often stronger with valueweighting. These seasonalities should be taken into account when backtesting strategies or performing asset pricing tests on recent data. For instance, if the return on a strategy is seasonal and the seasonality has decreased in recent years, then the average return on the strategy is misleading.

Taking a specific case, the strong seasonality in ILLIQ may lead to biased inferences. Many papers use ILLIQ as a measure of illiquidity but do not discuss the impact of seasonality on their results (see, for instance, Acharya and Pedersen, 2005). The average long-short ILLIQ

[^23]return is small and insignificant outside of January with return-weighted and value-weighted returns. Going one step further, the average ILLIQ return is large but with an opposite sign at the beginning and end of quarters. Hence, it is not clear that the return on the long-short ILLIQ portfolio reflects an illiquidity premium. As shown below, removing low-priced stocks appears to be necessary to obtain positive illiquidity and size premia outside of January. But these premia are only large and significant in the last month of quarters.

### 2.2.4 Fama-French Factors

I do not adjust returns for exposure to the Fama-French factors (Fama and French, 1993). The economic interpretation of these factors is subject to debate. For instance, as illustrated by the previous analysis, the long-short size portfolio does not appear to consistently reflect a real size premium. Nevertheless, since these factors are widely used, this section discusses their impact on the analysis.

Fama and French's SMB and HML factors also display a marked seasonality during the sample period 1964-2014. The SMB factor has a mean of $1.94 \%$ ( $t$-statistic: 4.04) in January and $0.10 \%$ (0.76) in other months. The HML factor has a mean of $1.37 \%$ (2.70) in January and $0.27 \%$ (2.31) in other months. ${ }^{10}$ This is not surprising in light of the previous results about size and book-to-market portfolios.

Since the factors are seasonal, they may explain the seasonalities in anomalies. I estimate the following time-series regression for each anomaly:

$$
\begin{align*}
r_{i, t}^{\mathrm{LS}}= & \alpha+\alpha_{\mathrm{Jan}} 1_{\mathrm{Jan}}+\alpha_{\mathrm{Beg}} 1_{\mathrm{Beg}}+\alpha_{\mathrm{End}} 1_{\mathrm{End}} \\
& +\beta_{m}\left(r_{m, t}-r_{f, t}\right)+\beta_{\mathrm{SMB}} \mathrm{SMB}_{t}+\beta_{\mathrm{HML} \mathrm{HML}_{t}+\epsilon_{t}} \tag{2.1}
\end{align*}
$$

where $r_{i, t}^{\mathrm{LS}}$ is the long-short decile portfolio return on anomaly $i$, and $r_{m, t}-r_{f, t}$ is the excess market return. The regression includes January, beginning-of-quarter (excluding January), and end-of-quarter dummies.

If the factors control for the seasonalities, then none of the dummies should be significant. Table 2.3 presents the results for equal and value-weighted long-short portfolios. The January dummy is significant for all return-weighted portfolios and for most value-weighted portfolios. Even when the January dummy is insignificant, its magnitude is often large relative to the mid-quarter alpha. Despite being seasonal, the factors do not explain the January seasonality in anomalies.

Beginning and end-of-quarter dummies are insignificant most of the time. Still, the end-of-

[^24]Table 2.3
Estimates of time series regressions controlling for Fama-French three factors. Monthly returns of long-short portfolios as described in Table C. 2 are regressed on the Fama-French three factors and January, beginning-of-quarter (excluding January), and end-of-quarter dummies. The table reports the intercept and dummies expressed in percent. The $t$-statistics are shown in parentheses and computed using White heteroskedasticity robust standard errors. The symbols ${ }^{*}$, ${ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | return-weighted |  |  |  | value-weighted |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\alpha$ | $\alpha_{\text {Jan }}$ | $\alpha_{\text {Beg }}$ | $\alpha_{\text {End }}$ | $\alpha$ | $\alpha_{\text {Jan }}$ | $\alpha_{\text {Beg }}$ | $\alpha_{\text {End }}$ |
| MC | $\begin{gathered} 0.41^{* * *} \\ (2.66) \end{gathered}$ | $\begin{gathered} -4.13^{* * *} \\ (-9.14) \end{gathered}$ | $\begin{gathered} -0.13 \\ (-0.59) \end{gathered}$ | $\begin{gathered} 0.26 \\ (1.06) \end{gathered}$ | $\begin{gathered} 0.34^{* * *} \\ (2.62) \end{gathered}$ | $\begin{gathered} -2.85^{* * *} \\ (-6.96) \end{gathered}$ | $\begin{gathered} -0.02 \\ (-0.09) \end{gathered}$ | $\begin{gathered} 0.19 \\ (0.94) \end{gathered}$ |
| BM | $\begin{aligned} & 0.38^{* *} \\ & (2.28) \end{aligned}$ | $\begin{gathered} 1.62^{* * *} \\ (3.69) \end{gathered}$ | $\begin{gathered} 0.45 \\ (1.64) \end{gathered}$ | $\begin{gathered} -0.08 \\ (-0.36) \end{gathered}$ | $\begin{gathered} -0.27 \\ (-1.44) \end{gathered}$ | $\begin{aligned} & 0.83^{*} \\ & (1.84) \end{aligned}$ | $\begin{gathered} 0.26 \\ (0.82) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.06) \end{gathered}$ |
| GP | $\begin{gathered} 0.94^{* * *} \\ (4.04) \end{gathered}$ | $\begin{gathered} -2.22^{* * *} \\ (-3.71) \end{gathered}$ | $\begin{gathered} 0.16 \\ (0.53) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.12) \end{gathered}$ | $\begin{gathered} 0.82^{* * *} \\ (4.11) \end{gathered}$ | $\begin{aligned} & -0.97^{*} \\ & (-1.66) \end{aligned}$ | $\begin{gathered} -0.19 \\ (-0.55) \end{gathered}$ | $\begin{gathered} -0.39 \\ (-1.28) \end{gathered}$ |
| AG | $\begin{gathered} -0.53^{* * *} \\ (-2.76) \end{gathered}$ | $\begin{gathered} -2.56^{* * *} \\ (-5.16) \end{gathered}$ | $\begin{gathered} 0.04 \\ (0.18) \end{gathered}$ | $\begin{gathered} 0.31 \\ (1.18) \end{gathered}$ | $\begin{gathered} -0.00 \\ (-0.00) \end{gathered}$ | $\begin{gathered} -1.16^{* *} \\ (-2.18) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.04) \end{gathered}$ | $\begin{gathered} -0.23 \\ (-0.77) \end{gathered}$ |
| AC | $\begin{gathered} -0.36^{* *} \\ (-2.51) \end{gathered}$ | $\begin{aligned} & -0.65^{*} \\ & (-1.65) \end{aligned}$ | $\begin{gathered} 0.03 \\ (0.15) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.50) \end{gathered}$ | $\begin{gathered} 0.00 \\ (0.00) \end{gathered}$ | $\begin{gathered} -0.87 \\ (-1.36) \end{gathered}$ | $\begin{gathered} -0.50 \\ (-1.36) \end{gathered}$ | $\begin{gathered} -0.38 \\ (-1.20) \end{gathered}$ |
| NSI | $\begin{gathered} -0.94^{* * *} \\ (-5.42) \end{gathered}$ | $\begin{aligned} & 1.15^{* *} \\ & (2.49) \end{aligned}$ | $\begin{aligned} & -0.49^{* *} \\ & (-2.06) \end{aligned}$ | $\begin{gathered} -0.11 \\ (-0.47) \end{gathered}$ | $\begin{gathered} -0.24 \\ (-1.44) \end{gathered}$ | $\begin{gathered} -0.10 \\ (-0.21) \end{gathered}$ | $\begin{gathered} -0.74^{* * *} \\ (-2.80) \end{gathered}$ | $\begin{gathered} -0.20 \\ (-0.81) \end{gathered}$ |
| $\Delta \mathrm{T}$ | $\begin{gathered} 1.04^{* * *} \\ (6.08) \end{gathered}$ | $\begin{gathered} 2.08^{* * *} \\ (3.98) \end{gathered}$ | $\begin{gathered} -0.21 \\ (-0.90) \end{gathered}$ | $\begin{gathered} -0.26 \\ (-1.15) \end{gathered}$ | $\begin{gathered} 0.21 \\ (1.09) \end{gathered}$ | $\begin{gathered} 1.46^{* * *} \\ (2.60) \end{gathered}$ | $\begin{aligned} & 0.55^{*} \\ & \text { (1.84) } \end{aligned}$ | $\begin{gathered} 0.20 \\ (0.70) \end{gathered}$ |
| IL | $\begin{gathered} -0.37^{* *} \\ (-2.33) \end{gathered}$ | $\begin{gathered} 3.74^{* * *} \\ (8.94) \end{gathered}$ | $\begin{gathered} 0.07 \\ (0.34) \end{gathered}$ | $\begin{gathered} 0.08 \\ (0.36) \end{gathered}$ | $\begin{gathered} -0.08 \\ (-0.62) \end{gathered}$ | $\begin{gathered} 1.67^{* * *} \\ (4.75) \end{gathered}$ | $\begin{gathered} -0.15 \\ (-0.84) \end{gathered}$ | $\begin{gathered} 0.17 \\ (0.97) \end{gathered}$ |
| IV | $\begin{gathered} -0.91^{* * *} \\ (-4.08) \end{gathered}$ | $\begin{gathered} 4.29^{* * *} \\ (7.99) \end{gathered}$ | $\begin{gathered} -0.22 \\ (-0.64) \end{gathered}$ | $\begin{aligned} & -0.78^{* *} \\ & (-2.58) \end{aligned}$ | $\begin{aligned} & -0.65^{* *} \\ & (-2.27) \end{aligned}$ | $\begin{gathered} 0.77 \\ (1.18) \end{gathered}$ | $\begin{gathered} -0.22 \\ (-0.50) \end{gathered}$ | $\begin{aligned} & -0.92^{* *} \\ & (-2.42) \end{aligned}$ |
| MOM | $\begin{gathered} 1.53^{* * *} \\ (5.24) \end{gathered}$ | $\begin{gathered} -4.22^{* * *} \\ (-4.14) \end{gathered}$ | $\begin{gathered} -0.22 \\ (-0.42) \end{gathered}$ | $\begin{gathered} 1.12^{* * *} \\ (2.65) \end{gathered}$ | $\begin{gathered} 0.90^{* *} \\ (2.24) \end{gathered}$ | $\begin{gathered} -2.13 \\ (-1.75) \end{gathered}$ | $\begin{gathered} -0.61 \\ (-0.92) \end{gathered}$ | $\begin{aligned} & 1.22^{* *} \\ & (2.22) \end{aligned}$ |
| 12m | $\begin{gathered} 0.35^{* * *} \\ (2.76) \end{gathered}$ | $\begin{gathered} 2.88^{* * *} \\ (5.33) \end{gathered}$ | $\begin{gathered} 0.29 \\ (1.23) \end{gathered}$ | $\begin{gathered} 0.10 \\ (0.56) \end{gathered}$ | $\begin{gathered} 0.33 \\ (1.49) \end{gathered}$ | $\begin{gathered} 2.82^{* * *} \\ (3.69) \end{gathered}$ | $\begin{aligned} & 0.73^{*} \\ & (1.91) \end{aligned}$ | $\begin{gathered} 0.29 \\ (0.92) \end{gathered}$ |

quarter dummy stands out for the momentum and idiosyncratic volatility strategies. Consistent with the results of Sias (2007), past winners outperform past losers at the end of quarters. In addition, high idiosyncratic volatility stocks yield a large negative alpha at the end of quarters. I come back to this effect in Section 2.3.2.

The month-of-the-quarter evidence from the regressions is much weaker than the evidence from the portfolio sorts. First, in several cases the differences between mid-quarter and beginning-of-quarter average monthly returns are not significant in Table 2.2. Second, the SMB factor displays a beginning-of-quarter seasonality that reduces the beginning-of-quarter alphas for many anomalies. Even though the beginning-of-quarter seasonality disappears when controlling for this factor, this procedure does not explain why the factor is seasonal in the first place. Stated differently, this evidence does not explain why the market capitalization strategy in Table 2.2 displays such a marked beginning-of-quarter seasonality.

To complement the analysis, I estimate separately time-series regressions for January returns and non-January returns (not reported). Overall, the Fama-French factor loadings remain similar in both periods. In other words, the long-short anomalies portfolios sensitivities to the Fama-French factors do not appear to vary between January and other months.

### 2.2.5 Robustness Checks

It is well-known that firm size plays a key role for the January effect. In Table 2.4 I try to assess whether the January and month-of-the-quarter seasonalities in anomalies are purely a size effect. Before sorting on the characteristic under study, I require the stocks to be larger than the median NYSE market capitalization in the formation period. The portfolios are then value-weighted.

The strategies' returns tend to be smaller in the value-weighted portfolios restricted to large caps. But the main results are qualitatively similar. Anomalies tend to exhibit specific January patterns in large caps portfolios. For instance, the book-to-market strategy gives a markedly larger average return in January. Excluding stocks with low market capitalization gives a positive and significant size premium except at the beginning of quarters, where the size premium has the "wrong" sign. Beginning-of-quarter months continue to show strong seasonalities relative to other months. This result is marked for size, net stock issues, $\Delta$ turnover, illiquidity, idiosyncratic volatility, and 12-month strategies. The difference in average returns between mid-quarter months and other months is often striking. Though some of these results may be spurious due to noise, the evidence indicates that a wide range of well-known anomalies display strong seasonalities even in value-weighted portfolios restricted to large caps.

In Section B. 1 of the Appendix, I provide additional robustness checks with a larger price screen and subsamples. Overall, the seasonalities in many anomalies are not simple artifacts

## Chapter 2. Seasonalities in Anomalies

Table 2.4
Average returns in percent of long-short value-weighted decile portfolios formed on different characteristics with size screen. Before sorting on the characteristic, stocks are restricted to be larger than the median NYSE market capitalization. The average return is shown separately for middle-quarter, January, beginning-of-quarter (excluding January), and end-of-quarter months. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014 (the accruals portfolios start in July 1971). Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. Financial firms are excluded from book-to-market, gross profitability, asset growth, accruals, and net stock issues portfolios. NASDAQ stocks are excluded from the turnover and illiquidity portfolios. The characteristics are defined in Table B.1. The symbols *, **, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | mid | Jan | beg | end |
| :--- | :---: | :---: | :---: | :---: |
|  |  |  |  |  |
| MC | $-0.39^{*}(-1.82)$ | $-1.47^{* *}(-2.51)$ | $0.58^{* *}(2.03)$ | $-0.52^{* * *}(-2.60)$ |
| BM | $-0.15(-0.48)$ | $2.10^{* * *}(2.75)$ | $-0.02(-0.04)$ | $0.22(0.75)$ |
| GP | $0.61^{* * *}(2.78)$ | $-0.67(-1.15)$ | $0.48(1.55)$ | $0.09(0.33)$ |
| AG | $0.02(0.08)$ | $-1.04^{*}(-1.67)$ | $-0.18(-0.66)$ | $-0.31(-1.22)$ |
| AC | $0.07(0.32)$ | $-0.41(-0.74)$ | $-0.56^{*}(-1.75)$ | $-0.24(-0.99)$ |
| NSI | $-0.09(-0.49)$ | $-0.17(-0.31)$ | $-0.87^{* * *}(-4.08)$ | $-0.30(-1.39)$ |
| $\Delta T$ | $0.05(0.30)$ | $0.99^{*}(1.95)$ | $0.68^{* * *}(2.86)$ | $0.39^{* *}(2.14)$ |
| IL | $0.47^{* *}(2.46)$ | $0.77(1.49)$ | $-0.58^{* *}(-2.35)$ | $0.59^{* * *}(3.19)$ |
| IV | $0.11(0.27)$ | $1.56^{*}(1.76)$ | $-0.82^{*}(-1.78)$ | $-0.57(-1.45)$ |
| MOM | $0.40(0.98)$ | $-1.26(-1.15)$ | $-0.49(-1.07)$ | $1.49^{* * *}(4.20)$ |
| I2m | $0.30(1.31)$ | $1.34^{*}(1.95)$ | $0.85^{* * *}(2.88)$ | $0.52^{* *}(2.45)$ |

of small market capitalization or low-priced stocks and hold over multiple subsamples.

### 2.3 Analysis

Section 2.3.1 briefly reviews current explanations for the January and month-of-the-quarter seasonalities. Section 2.3 .2 studies daily returns around the turns of years and quarters. Section 2.3.3 studies the role of tax-loss selling for the January seasonality in anomalies. Section 2.3.4 reports additional results on price pressure, long-term seasonality, and the role of earnings announcements.

### 2.3.1 Potential Explanations

Previous studies put forward several explanations for the January effect among which the most prominent are tax-loss selling and institutional "window dressing."

According to the tax-loss selling explanation, taxable investors tend to sell their losing stocks in December to realize capital losses. Realized losses offset realized gains and therefore reduce the amount of taxes owed. Since taxes are calculated over a calendar year, investors have an incentive to realize capital losses before the turn of the year. Tax-loss selling puts temporary price pressure on the stocks in December, which bounce back to their equilibrium values in January. ${ }^{11}$

Window dressing means that portfolio managers sell their low-performing stocks before yearend since they do not want poor-performing investments to appear on their reports (Haugen and Lakonishok, 1988). Like tax-loss selling, window dressing cannot explain why small stocks that are past winners also earn large January returns (Reinganum, 1983). The window dressing and tax-loss selling hypotheses make similar predictions but rely on different types of investors since only individual taxable investors should be subject to the latter. ${ }^{12}$

Ritter (1988) suggests yet another explanation, which complements tax-loss selling. Namely, individual investors reinvest the proceeds from tax sales and year-end cash infusions in January. These individual investors mainly invest in small stocks, generating a size effect in January even for prior winners. Furthermore, Ritter (1988) hypothesizes that these investors tend to buy back the stocks they previously sold for tax reasons. Grinblatt and Keloharju (2004) provide consistent evidence for the Finnish stock market. The "rebalancing" hypothesis predicts that

[^25]
## Chapter 2. Seasonalities in Anomalies

all small stocks outperform in January, including prior winners. In addition, small stocks that are prior losers should exhibit the strongest effect since they are the best candidates for tax-loss selling.

As mentioned in the introduction, institutions may want to manipulate or window dress their portfolios at the end of quarters. Such trades are likely to affect stock prices, especially for illiquid stocks. Moreover, Kang et al. (2015) argue that tax-loss selling may occur at the end of quarters following recessions.

### 2.3.2 Evidence from Daily Returns

## Turn-of-the-Year

Figure 2.1 shows the average daily return of value-weighted size portfolios around the turn-of-the-year. The large average return of small stocks on the last trading day of December, already pointed out by Roll (1983), seems difficult to reconcile with the tax-loss selling and window dressing explanations. In addition, the average daily returns preceding the turn-of-the-year also tend to be positive.

Figure 2.1. Average daily return of value-weighted size portfolios around the turn-of-theyear. Jan indicates the first trading day of January. Market capitalization is measured each year at the end of June. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014. Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded.


The seasonality is similarly unilateral for illiquidity, idiosyncratic volatility, and momentum portfolios. For other anomalies, both legs tend to display a seasonality but often to a different extent, which leads to the January seasonality reported in Table 2.1.

## Turn-of-the-Quarter

Small stocks earn large average returns on the last trading day of each quarter. Figure 2.2 displays the average daily return of value-weighted long-short size portfolio for each quarterturn outside of the year-turn. ${ }^{13}$ Small stocks (the short leg) drive the pattern in each case. The average returns are especially large on the last trading day of the first and second quarters and are highly significant with $t$-statistics of -5.15 and -3.90 . For the third quarter, the average daily return is smaller but remains significant at the level of $5 \%$ with a $t$-statistic of -2.36 . I did not find evidence of a similar effect at the turn-of-the-month within the calendar quarter.

Figure 2.2. Average daily return of value-weighted long-short size portfolios around the turn-of-the-quarter. $Q$ indicates the first trading day of the quarter. Market capitalization is measured each year at the beginning of June. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014. Breakpoints are based on NYSE deciles. Stocks with a price smaller than \$1 at the formation date are excluded.


This small stock effect on the last trading day of the quarter may help explain why small stocks tend to earn large average returns on the day before the turn-of-the-year (Figure 2.1). If the same underlying force is at work in both cases, then the high average year-end return should not be taken as evidence against tax-loss selling.

The pattern may reflect the evidence of portfolio pumping by equity funds on the last day of each quarter documented by Carhart et al. (2002). One would expect portfolio pumping to be implemented using small stocks since those tend to be less liquid and therefore easier to manipulate. Furthermore, Figure 2.2 indicates that return reversal occurs to some extent on the first day of the quarter.

[^26]I now investigate whether turn-of-the-quarter effects drive the seasonalities in long-short anomalies portfolios shown in Table 2.2. Most anomalies do not show any clear pattern at the turn-of-the-quarter, but net stock issues, illiquidity, and idiosyncratic volatility stand out. Figure 2.3 shows the average daily return of the value-weighted long-short illiquidity and idiosyncratic volatility portfolios for each quarter-turn outside of the year-turn. Like the size portfolio, the average daily return on the last trading day of the quarter is large and significant. ${ }^{14}$ The only exception is the IV portfolio in the third quarter.

Figure 2.3 can therefore explain why the illiquidity portfolio earns a large return at quarterend (Table 2.2). The return on the last trading day generates a large part of the apparent positive illiquidity premium at the end of quarters. These results do not explain, however, the difference in average returns between beginning-of-quarter months and middle-of-quarter months. Similarly, Figure 2.3 indicates that excluding the last day of the quarter strengthens the idiosyncratic volatility puzzle. The evidence in Section 2.2.4 confirms this effect: Controlling for the SMB factor increases the absolute alpha of the IV strategy at the end of quarters. In summary, the small stocks turn-of-the-quarter effect has a significant impact on several asset pricing anomalies.

### 2.3.3 Anomalies and Tax-Loss Selling

What is the role of tax-loss selling for the January seasonality in anomalies? To obtain a stock's potential for tax-loss selling, I follow Grinblatt and Han (2005) to compute a measure of capital loss overhang for each stock. Each week $t$, a stock's capital loss overhang (CLO) is given by

$$
\begin{equation*}
\mathrm{CLO}_{t}=\frac{R_{t}-P_{t}}{P_{t}} \tag{2.2}
\end{equation*}
$$

where $P_{t}$ is the price at the end of the week, and $R_{t}$ is a reference price. The reference price equals

$$
\begin{equation*}
R_{t}=\sum_{n=1}^{260} k\left(V_{t-n} \prod_{\tau=1}^{n-1}\left(1-V_{t-n+\tau}\right)\right) P_{t-n} \tag{2.3}
\end{equation*}
$$

where $V_{t}$ is a stock's turnover ratio in week $t$ (sum of daily trading volume over shares outstanding), and $k$ is a normalization constant that makes the price weights sum up to one. The reference price $R_{t}$ gives a larger weight to prices set during weeks with high turnover. A weight for a given week represents the probability that an investor bought the stock during this week and did not sell it afterward. Like Grinblatt and Han (2005), I take an horizon of five years to

[^27]Figure 2.3. Average daily return of value-weighted long-short illiquidity and idiosyncratic volatility portfolios around the turn-of-the-quarter. $Q$ indicates the first trading day of the quarter. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014. Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded.

compute the reference price (see also Kang et al., 2015). A high capital loss overhang makes a stock a good candidate for tax-loss selling since an investor is then likely to have bought the stock at a price higher than the current price.

## Sequential Sorts

I use sequential sorts to investigate whether capital loss overhang (i.e., tax-loss selling potential) can explain the turn-of-the-year seasonality in small stocks. ${ }^{15}$ Table 2.5 presents the average returns of portfolios from quintile sequential sorts on CLO and then on the characteristics used to build the anomalies in the paper. For comparison, Table 2.6 presents similar sorts using market capitalization as the first sorting variable. All the portfolios are formed at the beginning of December and are value-weighted.

If tax-loss selling is the main driver of the January seasonality in anomalies, then the seasonality should not exist among stocks in the low CLO quintile. In general, the seasonality is markedly stronger for the portfolios formed within the high capital loss quintile. But several anomalies continue to display a strong seasonality even when built using stocks that have a low potential for tax-loss selling. A similar remark applies when market capitalization is the first sorting variable, which confirms the robustness checks of Section 2.2.5.

Anomalies portfolios built using stocks in the low CLO quintile seem less subject to seasonalities than anomalies portfolios built using stocks in the high size quintile. This evidence potentially indicates that CLO identifies the seasonalities more precisely than size. Neither size nor CLO, however, subsumes the other: Within the large size quintile, sorting stocks on CLO gives an average cumulative return in the ten days after the turn-of-the-year of $1.21 \%$ with a $t$-statistic of 2.46 . Similarly, sorting on size within the low CLO quintile gives an average cumulative return of $-2.23 \%$ with a $t$-statistic of -4.22 . Strikingly, the average return of small stocks on the last trading day of December remains large among low CLO stocks with a $t$-statistic of -8.25 . Hence, tax-loss selling as proxied by CLO does not solely generate the end-of-the-year effect in small stocks. This result is consistent with the end-of-the-year effect in small stocks reflecting a general end-of-the-quarter effect (Figure 2.2).

The average return of many anomalies switches sign between January and non-January months in Table 2.1. Table 2.5 shows that controlling for tax-loss selling does not help explain this finding.

[^28]Table 2.5
Average returns in percent of long-short value-weighted quintile portfolios around the turn-of-the-year split by capital loss overhang. At the beginning of December, stocks are allocated into capital loss overhang quintiles. Within each quintile, stocks are sorted again into quintiles based on different characteristics. T-10:T-2 indicates the average cumulative return from ten to two days before the turn-of-the-year. T-1 indicates the average return on the last trading day of December. $\mathrm{T}+1: \mathrm{T}+10$ indicates the average cumulative return from the first to the tenth trading day of January. Sample: NYSE, Amex, and NASDAQ stocks from December 1968 to December 2014. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. The characteristics are defined in Table B.1. The $t$-statistics are in parentheses, and ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | low capital loss overhang |  |  | high capital loss overhang |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | T-10:T-2 | T-1 | $\mathrm{T}+1: \mathrm{T}+10$ | T-10:T-2 | T-1 | $\mathrm{T}+1: \mathrm{T}+10$ |
| MC | $\begin{gathered} 0.16 \\ (0.53) \end{gathered}$ | $\begin{gathered} -0.91 * * * \\ (-8.25) \end{gathered}$ | $\begin{gathered} -2.23^{* * *} \\ (-4.22) \end{gathered}$ | $\begin{gathered} 1.96^{* * *} \\ (3.48) \end{gathered}$ | $\begin{gathered} -1.49^{* * *} \\ (-5.88) \end{gathered}$ | $\begin{gathered} -7.37^{* * *} \\ (-8.53) \end{gathered}$ |
| BM | $\begin{gathered} 0.31 \\ (0.78) \end{gathered}$ | $\begin{aligned} & 0.20^{* *} \\ & (2.29) \end{aligned}$ | $\begin{aligned} & 0.93^{*} \\ & (1.86) \end{aligned}$ | $\begin{gathered} -0.56 \\ (-1.01) \end{gathered}$ | $\begin{gathered} 0.83^{* * *} \\ (3.72) \end{gathered}$ | $\begin{gathered} 4.04^{* * *} \\ (4.88) \end{gathered}$ |
| GP | $\begin{gathered} -0.06 \\ (-0.18) \end{gathered}$ | $\begin{gathered} 0.10 \\ (-1.09) \end{gathered}$ | $\begin{aligned} & -0.88^{* *} \\ & (-2.12) \end{aligned}$ | $\begin{gathered} 0.40 \\ (0.67) \end{gathered}$ | $\begin{aligned} & -0.45^{* *} \\ & (-2.39) \end{aligned}$ | $\begin{gathered} -1.13 \\ (-1.62) \end{gathered}$ |
| AG | $\begin{gathered} 0.18 \\ (0.58) \end{gathered}$ | $\begin{gathered} -0.16^{*} \\ (-1.85) \end{gathered}$ | $\begin{gathered} -1.55^{* * *} \\ (-3.73) \end{gathered}$ | $\begin{gathered} -0.21 \\ (-0.40) \end{gathered}$ | $\begin{aligned} & -0.47^{* *} \\ & (-2.02) \end{aligned}$ | $\begin{gathered} -3.33^{* * *} \\ (-4.27) \end{gathered}$ |
| AC | $\begin{gathered} 0.27 \\ (0.80) \end{gathered}$ | $\begin{gathered} 0.11 \\ (1.19) \end{gathered}$ | $\begin{gathered} -0.41 \\ (-0.87) \end{gathered}$ | $\begin{gathered} -0.49 \\ (-0.86) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.10) \end{gathered}$ | $\begin{aligned} & -2.07^{* *} \\ & (-2.51) \end{aligned}$ |
| NSI | $\begin{gathered} 0.26 \\ (0.86) \end{gathered}$ | $\begin{aligned} & 0.14^{*} \\ & (1.91) \end{aligned}$ | $\begin{gathered} 0.59 \\ (1.57) \end{gathered}$ | $\begin{gathered} -0.44 \\ (-0.89) \end{gathered}$ | $\begin{gathered} 0.68^{* * *} \\ (3.49) \end{gathered}$ | $\begin{gathered} -0.10 \\ (-0.13) \end{gathered}$ |
| $\Delta \mathrm{T}$ | $\begin{gathered} 0.14 \\ (0.39) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.76) \end{gathered}$ | $\begin{gathered} 0.02 \\ (0.06) \end{gathered}$ | $\begin{gathered} 0.75 \\ (0.83) \end{gathered}$ | $\begin{gathered} 0.89 * * * \\ (4.42) \end{gathered}$ | $\begin{aligned} & 1.83^{* *} \\ & (2.49) \end{aligned}$ |
| IL | $\begin{gathered} 0.28 \\ (0.79) \end{gathered}$ | $\begin{gathered} 0.68^{* * *} \\ (6.61) \end{gathered}$ | $\begin{gathered} 2.19 * * * \\ (3.91) \end{gathered}$ | $\begin{gathered} -0.89 \\ (-1.45) \end{gathered}$ | $\begin{gathered} 1.51^{* * *} \\ (6.24) \end{gathered}$ | $\begin{gathered} 5.80^{* * *} \\ (4.42) \end{gathered}$ |
| IV | $\begin{gathered} 0.45 \\ (1.11) \end{gathered}$ | $\begin{gathered} 0.56^{* * *} \\ (3.28) \end{gathered}$ | $\begin{aligned} & 1.81^{* *} \\ & (2.35) \end{aligned}$ | $\begin{gathered} -0.67 \\ (-1.16) \end{gathered}$ | $\begin{gathered} 1.75^{* * *} \\ (6.64) \end{gathered}$ | $\begin{gathered} 5.13^{* * *} \\ (4.48) \end{gathered}$ |
| MOM | $\begin{aligned} & 0.82^{*} \\ & (1.90) \end{aligned}$ | $\begin{gathered} 0.18 \\ (1.60) \end{gathered}$ | $\begin{gathered} 0.07 \\ (0.14) \end{gathered}$ | $\begin{gathered} -0.29 \\ (-0.49) \end{gathered}$ | $\begin{gathered} -0.63^{* * *} \\ (-3.51) \end{gathered}$ | $\begin{aligned} & -2.38^{* *} \\ & (-2.39) \end{aligned}$ |
| 12m | $\begin{aligned} & 0.84^{* *} \\ & (2.05) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.53) \end{gathered}$ | $\begin{gathered} 1.88^{* * *} \\ (2.93) \end{gathered}$ | $\begin{gathered} -0.46 \\ (-0.69) \end{gathered}$ | $\begin{aligned} & 0.51^{* *} \\ & (2.57) \end{aligned}$ | $\begin{gathered} 3.54^{* * *} \\ (3.60) \end{gathered}$ |

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Table 2.6
Average returns in percent of long-short value-weighted quintile portfolios around the turn-of-the-year split by size. At the beginning of December, stocks are allocated into size quintiles. Within each quintile, stocks are sorted again into quintiles based on different characteristics. T-10:T-2 indicates the average cumulative return from ten to two days before the turn-of-the-year. T-1 indicates the average return on the last trading day of December. $\mathrm{T}+1: \mathrm{T}+10$ indicates the average cumulative return from the first to the tenth trading day of January. Sample: NYSE, Amex, and NASDAQ stocks from December 1968 to December 2014. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. The characteristics are defined in Table B.1. The $t$-statistics are in parentheses, and *, **, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.


## Regression Analysis

Regression analysis can give a better view of how market capitalization (MC), tax-loss potential (CLO), and other characteristics jointly interact. I estimate the following pooled OLS regression on daily returns in a window of ten days before and after the turn-of-the-year:

$$
\begin{align*}
r_{i, t}= & c_{0}+c_{1} 1_{T-1}+c_{2} 1_{T+1: T+10}+c_{3} X_{i, t}+c_{4} \ln \left(\mathrm{MC}_{i, t}\right)+c_{5} \mathrm{CLO}_{i, t} \\
& +c_{6} 1_{T-1} X_{i, t}+c_{7} 1_{T-1} \ln \left(\mathrm{MC}_{i, t}\right)+c_{8} 1_{T-1} \mathrm{CLO}_{i, t} \\
& +c_{9} 1_{T+1: T+10} X_{i, t}+c_{10} 1_{T+1: T+10} \ln \left(\mathrm{MC}_{i, t}\right)+c_{11} 1_{T+1: T+10} \mathrm{CLO}_{i, t}+\epsilon_{i, t}, \tag{2.4}
\end{align*}
$$

where $1_{T-1}$ and $1_{T+1: T+10}$ are dummies for the last trading day of the year and the first ten trading days of the year. The characteristic under consideration is denoted by $X$. Importantly, $X$ is measured at the beginning of December and is fixed over a given turn-of-the-year. Similarly, CLO and MC are measured in the middle of December and are not updated over the turn-of-the-year. Hence, there is only cross-sectional variation in those variables within a given turn-of-the-year. All the variables except returns are winsorized at the $0.5 \%$ and $99.5 \%$ fractiles over the sample. The $t$-values are computed using standard errors clustered by day.

Panel (a) of Table 2.7 reports the results. The $1_{T-1}$ and $1_{T+1: T+10}$ coefficients are large and strongly significant in all regressions. Hence, both size and potential for tax-loss selling cannot explain those dummies away, even when interacted with them. These interaction terms have the expected signs and are all large and significant. To get an idea of the economic magnitude of the coefficients, Panel (b) reports various percentiles for the characteristics averaged across securities. All else equal, a stock in the $90^{\text {th }}$ CLO (size) percentile earns an extra return of $0.32 \%(0.53 \%)$ per day during the first ten trading days of the year relative to a stock in the $10^{\text {th }}$ CLO (size) percentile. Therefore, in line with Tables 2.5 and 2.6, both size and CLO matter for returns around the turn-of-the-year. The interacted coefficients for the last trading day of the year are also especially large. Both small stocks and stocks with a high potential for tax-loss selling earn substantial returns on this specific day.

Table 2.7 also shows that several anomalies characteristics have statistically significant interaction dummies around the turn-of-the-year. But these effects are in general economically small relative to the contribution of size and tax-loss potential; the only exceptions being the impacts of book-to-market and idiosyncratic volatility on the last trading of the year. I did not find any significant term for the asset growth, accruals, and illiquidity characteristics.

There is little evidence of price pressure before the turn-of-the-year. Price pressure on stocks subject to tax-loss selling would imply a negative coefficient on CLO, but this coefficient is small and insignificant. Similarly, the regression constant, which represents a fixed return in the ten days before the turn-of-the-year, is negligible in all specifications. Section 2.3.4 further discusses price pressure.

Table 2.7
Tax-loss selling (pooled regression). Panel (a): Pooled OLS regression estimates with $t$-values in parentheses. Daily returns ten days before and after the turn-of-the-year are regressed on different variables. MC is market capitalization measured at the middle of December. CLO is the capital loss overhang measured at the middle of December. BM is book-to-market. GP is gross profitability. NSI is net stock issues. MOM is six-month return momentum. IV is idiosyncratic volatility. These characteristics are defined in Table B. 1 and measured at the beginning of December. $1_{T-1}$ is a dummy for the day before the turn-of-the-year. $1_{T+1: T+10}$ is a dummy for the ten days after the turn-of-the-year. Sample: NYSE, Amex, and NASDAQ stocks from January 1968 to December 2014. Financial firms are excluded from the regressions with accounting variables. Stocks with a price smaller than $\$ 1$ at the middle of December are excluded. Non-return variables are winsorized at the $0.5 \%$ and $99.5 \%$ fractiles over the full sample. The $t$-values are computed using standard errors clustered by day. ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level. Panel (b): Average of descriptive statistics across securities for different variables during the sample.

|  | Panel (a) |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $X$ | $\ln (\mathrm{BM})$ | GP | NSI | MOM | IV |
| constant | -0.0007 | -0.0004 | -0.0004 | -0.0008 | -0.0003 |
|  | $(-0.67)$ | $(-0.35)$ | $(-0.36)$ | $(-0.73)$ | $(-0.18)$ |
| $1_{T-1}$ | $0.0353^{* * *}$ | $0.0344^{* * *}$ | $0.0338^{* * *}$ | $0.0335^{* * *}$ | $0.0204^{* * *}$ |
|  | $(9.32)$ | $(8.58)$ | $(8.64)$ | $(8.95)$ | $(3.86)$ |
| $1_{T+1: T+10}$ | $0.0116^{* * *}$ | $0.0125^{* * *}$ | $0.0115^{* * *}$ | $0.0119^{* * *}$ | $0.0091^{* * *}$ |
|  | $(6.80)$ | $(6.98)$ | $(6.38)$ | $(6.75)$ | $(2.91)$ |
| $X$ | 0.0002 | -0.0001 | -0.0008 | $0.0013^{*}$ | -0.0044 |
|  | $(0.77)$ | $(-0.37)$ | $(-1.33)$ | $(1.80)$ | $(-0.27)$ |
| $\ln (\mathrm{MC})$ | $0.0002^{* *}$ | $0.0002^{*}$ | $0.0002^{*}$ | $0.0002^{* *}$ | 0.0002 |
|  | $(2.38)$ | $(1.92)$ | $(1.90)$ | $(2.15)$ | $(1.41)$ |
| CLO | -0.0003 | -0.0003 | -0.0003 | -0.0000 | -0.0003 |
|  | $(-0.57)$ | $(-0.65)$ | $(-0.64)$ | $(-0.23)$ | $(-0.79)$ |
| $1_{T-1} X$ | $-0.0017^{* * *}$ | -0.0005 | $0.0061^{* * *}$ | -0.0040 | $0.1906^{* * *}$ |
|  | $(-2.65)$ | $(-0.65)$ | $(2.93)$ | $(-1.57)$ | $(4.26)$ |
| $1_{T-1} \ln (\mathrm{MC})$ | $-0.0026^{* * *}$ | $-0.0024^{* * *}$ | $-0.0024^{* * *}$ | $-0.0023^{* * *}$ | $-0.0017^{* * *}$ |
|  | $(-7.42)$ | $(-6.53)$ | $(-6.50)$ | $(-6.69)$ | $(-3.86)$ |
| $1_{T-1} \mathrm{CLO}$ | $0.0085^{* * *}$ | $0.0081^{* * *}$ | $0.0082^{* * *}$ | $0.0076^{* * *}$ | $0.0068^{* * *}$ |
|  | $(5.45)$ | $(5.30)$ | $(5.32)$ | $(5.36)$ | $(4.86)$ |
| $1_{T+1: T+10} X$ | 0.0000 | $-0.0015^{* * *}$ | $0.0023^{* *}$ | 0.0000 | 0.0423 |
|  | $(0.20)$ | $(-3.69)$ | $(2.58)$ | $(0.03)$ | $(1.53)$ |
| $1_{T+1: T+10} \ln (\mathrm{MC})$ | $-0.0009^{* * *}$ | $-0.0010^{* * *}$ | $-0.0009^{* * *}$ | $-0.0010^{* * *}$ | $-0.0008^{* * *}$ |
|  | $(-6.39)$ | $(-6.00)$ | $(-5.82)$ | $(-6.08)$ | $(-3.59)$ |
| $1_{T+1: T+10} \mathrm{CLO}$ | $0.0040^{* * *}$ | $0.0040^{* * *}$ | $0.0040^{* * *}$ | $0.0039^{* * *}$ | 0.0035 |
|  | $(4.84)$ | $(4.98)$ | $(5.01)$ | $(5.21)$ | $(5.22)$ |
| Obs. | $1 ’ 924^{\prime} 510$ | $1^{\prime} 973^{\prime} 597$ | $1^{\prime} 967^{\prime} 870$ | $2 \prime 380^{\prime} 811$ | $2 \prime 386^{\prime} 713$ |

(Table 2.7 continued.)
Panel (b)

|  | \#securities | mean | std | $\min$ | $10 \%$ | $50 \%$ | $90 \%$ | $\max$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\ln (\mathrm{MC})$ | $11^{\prime} 624$ | 11.628 | 1.868 | 6.712 | 9.311 | 11.467 | 14.160 | 17.507 |
| CLO | $11^{\prime} 624$ | 0.293 | 0.533 | -0.496 | -0.081 | 0.151 | 0.787 | 6.697 |
| $\ln (\mathrm{BM})$ | $8^{\prime} 984$ | -0.596 | 0.816 | -3.925 | -1.651 | -0.520 | 0.335 | 1.798 |
| GP | '' $^{\prime} 47$ | 0.370 | 0.280 | -0.739 | 0.101 | 0.344 | 0.719 | 1.442 |
| NSI | '' $^{\prime} 38$ | 0.052 | 0.110 | -0.618 | -0.012 | 0.019 | 0.151 | 1.091 |

### 2.3.4 Additional Results

## Price Pressure and Long-Term Seasonality

At the beginning of each January, I allocate stocks into sixteen portfolios based on their average return over the past year (excluding the last month) $r_{-2: 12 m}$ and their long-term seasonality return $r_{-6: 10 y .}{ }^{16}$ More precisely, stocks with a negative return over the past year are split into two equal-sized groups: losers (L) and extreme losers (EL). Stocks with a positive return are split in a similar way between winners (W) and extreme winners (EW). The stocks are then independently allocated into the portfolios based on $r_{-6: 10 y}$.

Panel (a) of Table 2.8 shows the average January return of the portfolios. ${ }^{17}$ Independently of the previous year return, stocks with a high average return in January six to ten years ago strongly outperform stocks with a low past-January return. The return pattern is monotonically increasing in past average January return for each past-year return group.

Conditional on $r_{-6: 10 y}$, past-year losers outperform past-year winners in January, consistent with tax-loss selling and window dressing. The past-year return does not fully explain what happens at the turn of the year. First, past-year losers should outperform past-year winners in January. Among the eight portfolios with a negative return in the past year, half of them perform worse in January than the $W$ and $E W$ portfolios that were winners six to ten years ago. Second, past-year losers that belong to the $r_{-6: 10 y}$ winners group perform better than the first three quartiles portfolios of extreme losers, which seems inconsistent with tax-loss selling.

[^29]Using long-term seasonality returns over two to five years ( $r_{-2: 5 y}$ ) or eleven to fifteen years ( $r_{-11: 15 y}$ ) gives similar results.

Panel (b) of Table 2.8 presents the average returns of the January-built portfolios in February. There is no significant evidence of reversal for the W-P4 and EW-P4 portfolios relative to the W-P1 and EW-P1 portfolios. The large January average returns of these portfolios do not appear to reverse. The results are similar in March (not reported). The long-short EW-EL portfolios seem to reverse partly. Three out of the four portfolios have positive and significant average returns in February. The February average returns are markedly larger than their March average returns, which do no exhibit any specific pattern.

Table 2.8 also shows in Panel (c) the average December return for the portfolios formed in January (these portfolios do not depend on any December data since $r_{-2: 12 m}$ excludes the last month). It is difficult to find any strong evidence of price pressure. The spreads between extreme winners and losers are positive but insignificant and, more importantly, much lower than the January spreads. This result is not consistent with tax-loss selling. According to tax-loss selling, momentum profits should be especially large in December (Grinblatt and Moskowitz, 2004). Overall, these results are difficult to reconcile with a price pressure story. The absence of pattern in December relative to January is puzzling. Market liquidity could be higher in December than in January, but then some reversal should occur in February for the long-short portfolios.

## Earnings Announcements

Following Lamont and Frazzini (2007), a firm is predicted to have an earning announcement next month if it has an announcement in the same month one year ago. The firm is also required to have at least four announcements in the previous year. Over the sample period January 1974 to December 2013, the long expected announcement stocks and short no announcement stocks value-weighted portfolio earns an average monthly return of $0.61 \%$ with a $t$-statistic of 5.39, consistent with the results of Lamont and Frazzini (2007). The return of this portfolio is, however, zero in January $(-0.03 \%$ with a $t$-statistic of -0.10$)$. The equal-weighted long-short portfolio earns a smaller monthly average return of $0.22 \%$ with a $t$-statistic of 3.33. In this case, the January average return is $-0.95 \%$ ( $t$-statistic -3.28 ). Hence, the earnings announcement premium displays a strong January seasonality over the sample period. The average premium with value-weighting is also more than two-third larger in beginning and end-of-quarter months than mid-quarter months (not reported).

Intuitively, earnings announcements may partly explain seasonalities in anomalies portfolios since, as mentioned previously, a majority of announcements take place in beginning-ofquarter months. To answer this question, I examine long-short anomalies portfolios separately

Table 2.8
Tax-loss selling (portfolios). Average monthly return in percent of equal-weighted portfolios formed at the beginning of January. Stocks are first allocated into four groups based on their average return over the past year excluding the last month ( $r_{-2: 12 m}$ ): extreme losers (EL), losers (L), winners (W), and extreme winners (EW). Stocks are then independently allocated into sixteen portfolios based on their average return in the same month as the current month six to ten years ago ( $r_{-6: 10 y}$ ). The sample includes NYSE, Amex, and Nasdaq common stocks from January 1964 to December 2013. Stocks with a price smaller than $\$ 1$ at the beginning of the holding period are excluded. The $t$-statistics for the long-short portfolios are in parentheses, and ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | (a) January |  |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $r_{-6: 10 y}$ |  |  |  |  |
| $r_{-2: 12 m}<0$ | EL | P1 (L) | P 2 | P 3 | $\mathrm{P} 4(\mathrm{~W})$ | $\mathrm{P} 4-\mathrm{P} 1$ |  |
|  | L | 2.12 | 4.95 | 5.38 | 7.15 | $2.72^{* * *}(4.77)$ |  |
| $r_{-2: 12 m} \geq 0$ | W | 1.60 | 2.87 | 2.93 | 6.03 | $3.47^{* * *}(5.10)$ |  |
|  | EW | 2.23 | 2.53 | 3.24 | 4.92 | $3.32^{* * *}(5.83)$ |  |
|  | EW-EL | $-2.19^{* * *}$ | $-2.68^{* * *}$ | $-2.14^{* * *}$ | $-2.55^{* * *}$ | $2.36^{* * *}(4.97)$ |  |
|  |  | $(-2.66)$ | $(-3.33)$ | $(-2.77)$ | $(-2.80)$ |  |  |
|  |  |  |  |  |  |  |  |

(b) February

|  |  | $r_{-6: 10 y}$ |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| $r_{-2: 12 m}<0$ | EL | 0.36 | -0.34 | 0.76 | 0.91 | $0.19(0.42)$ |
|  | L | 0.66 | 0.95 | 1.15 | 1.79 | $0.97^{* *}(2.11)$ |
| $r_{-2: 12 m}>0$ | W | 1.20 | 1.13 | 1.32 | 1.49 | $0.29(0.78)$ |
|  | EW | 2.02 | 1.41 | 2.01 | 1.76 | $-0.26(-0.55)$ |
|  | EW-EL | $1.61^{* * *}$ | $1.83^{* * *}$ | $1.42^{* *}$ | 0.86 |  |
|  |  | $(3.06)$ | $(3.88)$ | $(2.20)$ | $(1.55)$ |  |

(c) December

|  |  | $r_{-6: 10 y}$ |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{P} 1(\mathrm{~L})$ | P 2 | P 3 | $\mathrm{P} 4(\mathrm{~W})$ | $\mathrm{P} 4-\mathrm{P} 1$ |
| $r_{-2: 12 m}<0$ | EL | 2.09 | 1.93 | 1.77 | 2.33 | $0.16(0.33)$ |
|  | L | 2.09 | 1.97 | 2.15 | 1.71 | $-0.44(-1.34)$ |
| $r_{-2: 12 m}>0$ | W | 2.26 | 2.33 | 2.39 | 2.58 | $0.32(0.99)$ |
|  | EW | 2.29 | 2.69 | 2.74 | 3.03 | $0.74^{*}(1.81)$ |
|  | EW-EL | 0.29 | 0.96 | $1.16^{*}$ | 0.70 |  |
|  |  | $(0.42)$ | $(1.54)$ | $(1.88)$ | $(0.92)$ |  |

## Chapter 2. Seasonalities in Anomalies

within the groups of stocks with and without expected announcements. In a nutshell, I do not find any evidence of a systematic pattern in anomalies linked to expected announcements.

### 2.4 Conclusion

Well-known anomalies exhibit strong January and month-of-the-quarter seasonalities. These seasonalities are in general robust to controlling for size and tax-loss selling potential. In addition, small stocks earn an abnormally high average return on the last day of the quarter, which significantly affects size, idiosyncratic volatility, and illiquidity portfolios. As a result, taking into account such seasonalities is important when studying the cross-section of stock returns. These seasonalities challenge the economic interpretation of many anomalies.

## 3 The Cross-Section of Intraday and Overnight Returns

Using a thirty-year sample of U.S. stock returns, I document substantial cross-sectional variation in returns over the trading day and overnight. Market closures have a large impact on returns. Small and illiquid stocks earn high average returns in the last thirty minutes of trading. In contrast, large and liquid stocks perform poorly at this time. I find support for institutional and information asymmetry theories. But these theories do not fully explain the cross-sectional evidence. Portfolios based on other characteristics, such as beta and idiosyncratic volatility, earn their return gradually throughout the trading day-contrary to the market and a benchmark based on random portfolios. These portfolios also tend to incur large negative returns overnight, consistent with mispricing at the open.

### 3.1 Introduction

This paper provides new evidence on the determinants of cross-sectional variation in expected stock returns by examining returns over the trading day and overnight. First, an examination of intraday and overnight returns gives insights on what factors affect the cross-section of stock returns. If returns on two portfolios exhibit markedly different intraday patterns, then an understanding of this difference sheds light on what drives stock returns. Second, an analysis of intraday return patterns around market closures has important implications for liquidity and market efficiency. ${ }^{1}$

To follow this intuition, I compute intraday half-hour returns and overnight returns on all U.S. common stocks from January 1986 to December 2015. The overnight return is the return outside of regular trading hours and is therefore defined by the change from the closing price

[^30]on a given day to the opening price on the next day. I am not aware of any related paper using such an extensive data set to examine intraday average returns. ${ }^{2}$

Research in finance has reported many variables that predict the cross-section of stock returns and are not explained by standard finance theory. These "anomalies" are the focus of a large literature, but there is little consensus about their sources. I show that anomaly portfolios exhibit strikingly different intraday return patterns. Substantial differences in intraday average returns exist both within and across the portfolios.

Anomalies fall into three groups: Anomalies that accrue in a specific period during the day (size, illiquidity, and momentum), referred to as "period-specific" anomalies; anomalies that accrue gradually over the trading day (betting-against-beta, gross profitability, idiosyncratic volatility, and net stock issues), referred to as "gradual" anomalies; and anomalies that display no clear pattern (accruals and book-to-market). The results are robust across subsamples and days of the week and remain after applying a volume filter to limit the impact of nonsynchronous trading. Furthermore, microstructure effects are unlikely to explain the findings: Portfolios are value-weighted and returns computed from quote midpoints.

In contrast, the market portfolio earns close to zero returns over most of the trading day. To further benchmark the results, I simulate thousands of random strategies using monthly returns, select the profitable ones, and examine their intraday return patterns. The average random strategy earns the majority of its profits overnight. Profitable random strategies are highly unlikely to accrue intraday in a consistent manner over multiple subsamples and days of the week, contrary to the period-specific anomalies. Similarly, none of the random strategies is able to reproduce the consistently positive and statistically significant intraday average returns of the gradual anomalies. These comparisons suggest that intraday return patterns of anomalies have economic content that can help to understand cross-sectional variation in stock returns.

Market closures have a large impact on stock returns. I document the novel finding that a large fraction of size and illiquidity premia is realized in the last half hour of trading. Returns on these strategies accrue like noise outside of the opening and closing hours. Both legs contribute to the end-of-the-day return: Small and illiquid stocks perform well at the end of the day while, on the contrary, large and liquid stocks tend to perform poorly.

[^31]High end-of-the-day returns are difficult to reconcile with standard theories of size and illiquidity. For instance, if size proxies for distress risk, then it is unclear why small stocks would earn most of their returns at the end of the day. I test theories based on closure effects, institutional effects, and coordination effects. ${ }^{3}$ Closures should affect stock returns because it is more complicated to hedge risk when the stock market is closed. Institutional effects can affect intraday return patterns. Examples include mutual funds trading around the close to limit tracking error and overnight margin constraints imposed on day traders. Coordination can lead informed traders and liquidity traders to concentrate their trades at specific periods of the day, which in turn has implications for intraday return properties.

End-of-the-day returns of small stocks do not reverse in the next overnight period. This rules out a simple price pressure story in which small stocks are subject to buying pressure at the end of the day. Moreover, I do not find evidence of common liquidity shocks at the close for small stocks.

Evidence from double sorts and panel regressions show that illiquidity-as proxied by the measure of Amihud (2002)—dominates size in explaining the end-of-the-day return. Illiquidity remains a statistically significant explanatory variable of positive end-of-the-day returns even in panel regressions that include stock fixed effects.

Given that illiquidity dominates size in explaining the end-of-the-day return, it is natural to look for an explanation based on liquidity. High end-of-the-day returns can be rationalized by a model in which liquidity deteriorates at the end of the day. Risk-averse market makers (with positive inventories) require a higher compensation for risk to absorb supply shocks at this time of the day. Previous research documents that liquidity deteriorates at the end of the day: Effective spreads are U-shaped over the day (McInish and Wood (1992)), quoted depths are reverse U-shaped (Lee, Mucklow, and Ready (1993)), and the price impact of transitory shocks increases at the close (Madhavan, Richardson, and Roomans (1997), Cushing and Madhavan (2000)).

The previous framework nests both a pure liquidity shocks theory and an information asymmetry theory. First, the price impact of supply shocks may increase because supply shocks are more volatile around the close, for instance, due to institutional effects around the close. Second, the price impact of supply shocks may increase because there is more informed trading around the close. This framework can explain the cross-sectional evidence if small and illiquid stocks are subject to more volatile liquidity shocks or more information asymmetry than large stocks.

To help disentangle liquidity shock effects from asymmetric information effects, I examine return patterns at the end of quarters. There is evidence consistent with portfolio pumping

[^32]at the end of quarters, which disproportionately affects illiquid stocks (Carhart et al. (2002)). Portfolio pumping is, however, insufficient to explain the end-of-the-day returns of illiquid stocks.

To test the information-based explanation, I use earnings announcements to proxy for a change in the degree of asymmetric information. I conjecture that the degree of information asymmetry is higher in the days preceding an announcement than in the days following one. I find a positive end-of-the-day effect for all stocks on the days preceding an announcement. On the contrary, a negative end-of-the-day effect exists for all stocks on the days following an announcement. This marked asymmetry is consistent with the asymmetric information theory. There is, however, only limited evidence that illiquid stocks are more affected than other stocks. Asymmetric information is therefore not supported as the primary driver of the end-of-the-day effect in illiquid stocks.

In summary, I find evidence consistent with institutional effects and information asymmetry in generating high end-of-the-day returns. But these theories fail to explain why small and illiquid stocks are disproportionately affected relative to large stocks. It remains unclear whether differences in liquidity can explain the cross-sectional difference in intraday average returns between small and large stocks.

Gradual anomalies (i.e., betting-against-beta, gross profitability, net stock issues, and idiosyncratic volatility), earn consistently positive and statistically significant returns over most of the trading day but tend to incur large negative returns overnight. Negative overnight returns are difficult to explain with a risk-based theory. The evidence rejects overnight liquidity risk and is difficult to reconcile with asymmetric information theories. Furthermore, noise at the open does not drive the negative overnight returns: The evidence is robust to using volume-weighted average prices in the first half hour of trading.

The short leg of the gradual anomalies drives their negative overnight returns. Hence, an explanation based on time-varying mispricing over the day may better accommodate the evidence than a risk-based explanation. Mispricing increases at the open-for instance, due to systematic retail buying pressure at this time (Berkman et al. (2012)). Overall, the results emphasize the role of market closures for the cross-section of stock returns.

Puzzling patterns in intraday and overnight stock returns have been documented by previous research. Heston, Korajczyk, and Sadka (2010) provide evidence that some stocks tend to perform systematically better than others during specific half hours of the trading day. Lou, Polk, and Skouras (2016) show that momentum profits accrue solely overnight for U.S. stocks over 1993 to 2013. They also report the intraday return and the overnight return of several other anomalies but focus their analysis on momentum and do not decompose the intraday return as I do.

My paper contributes more broadly to studies of intraday and overnight returns: Overnight returns on aggregate portfolios are large, but their magnitude is sensitive to the definition of the opening price. Overnight returns are lower when they include the first five minutes of trading or are computed using volume-weighted average prices. ${ }^{4}$

In addition, my research relates to a few recent papers that attempt to distinguish among competing explanations of anomalies by examining variables such as investor sentiment (Stambaugh, Yu, and Yuan (2012)) or out-of-sample and post-publication returns (McLean and Pontiff (2016)).

The paper is organized as follows. Section 3.2 discusses theoretical determinants of intraday and overnight returns. Section 3.3 presents the data and methodology. Section 3.4 explores the cross-section of intraday and overnight returns. Section 3.5 examines end-of-the-day return patterns. Section 3.6 examines gradual intraday return patterns. Section 3.7 provides robustness checks and Section 3.8 concludes.

### 3.2 Theories of Intraday and Overnight Average Returns

Studies that examine average returns over trading and non-trading periods go back to French (1980). French tests a calendar time hypothesis and a trading time hypothesis by comparing returns on different days of the week. The calendar time hypothesis predicts that the Monday average return is three times the average return on the other days of the week. The trading time hypothesis predicts that the Monday average return is the same as for the other days of the week. French (1980) strongly rejects both hypotheses in light of the large negative Monday average return over his sample.

The benchmark considered by French (1980) with the calendar (trading) time hypothesis is that returns accrue evenly over the (trading) day. If agents require a risk premium to hold an asset, the premium required over a half hour in the morning should not differ substantially from the premium required over a half hour in the afternoon. While this hypothesis is a natural benchmark, there are theoretical reasons to expect intraday average returns not to be constant over the day.

Hong and Wang (2000) solve an equilibrium model with periodic market closures. ${ }^{5}$ They

[^33]model a competitive setup with informed and uninformed traders. Both groups hedge returns from a private investment opportunity, but informed traders receive a private signal about mean dividend growth.

When agents are homogeneous, there is no trade and market closures do not matter. When agents are heterogeneous, the interaction of two effects can generate a rich set of dynamics in average returns. First, investors cannot use the stock as a hedge overnight. This makes the stock more risky to hold overnight, and investors want to reduce their hedging demand in the stock before the market closes. As a result, the stock price decreases over the day. Second, the level of information asymmetry tends to decrease over the trading day since uninformed investors cannot learn from the stock price overnight. Indeed, information asymmetry decreases as more information is incorporated into prices through trading. Uninformed investors then require a lower discount to hold the stock. This effect makes the stock price increase over the day.

In line with the hedging channel modeled by Hong and Wang (2000), Gerety and Mulherin (1992) find evidence that high expected overnight volatility leads to high trading volume at the close and at the next day's open. This evidence is consistent with traders that unload their positions before the close and reopen them on the following day. Gerety and Mulherin (1992) do not explore implications for average returns. Risk-averse liquidity providers require a price discount to absorb temporary order imbalances (Grossman and Miller (1988)). Previous research documents evidence consistent with liquidity provision being compensated at the open (Stoll and Whaley (1990)) but has not investigated liquidity provision at the close.

The model of Hong and Wang (2000) is a competitive setup in which everyone trades continuously. The mix of traders active in the market may, however, vary over the day. Admati and Pfleiderer (1988) develop a model in which informed investors can time their information production. Trading volume is highest when transaction costs are lowest. ${ }^{6}$ Since the asset price follows a martingale, the model is silent about average returns. Intuitively, one may expect uninformed investors to require a larger premium to hold stocks during periods with more informed trading.

Note that the first type of models (Hong and Wang (2000)) predicts intraday patterns because investors cannot trade during a closure. The second type of models (Admati and Pfleiderer (1988)) predicts intraday patterns at the open and at the close only to the extent that these represent natural focal points for investors. Thus, the magnitude of the effects should not depend on the length of the closure.

Institutional features may cause temporary price pressure at specific times of the day. For

[^34]instance, S\&P500 futures and options settle based on the opening prices of the constituents, which generates large liquidity shocks at the open (Barclay, Hendershott, and Jones (2008)). Relatedly, Berkman et al. (2012) argue that buying by attention-constrained investors drives up the opening price of stocks with large fluctuations in the previous day (i.e., stocks who caught investors' attention). Closing prices may also be subject to pressures induced by institutions. For instance, share in open-end mutual funds trade at the net asset value (NAV), which is computed once a day based on closing prices. Hence mutual fund managers may concentrate their trades towards the end of the day, when there is less uncertainty about net daily flows. In line with this idea, Goetzmann and Massa (2003) show that, for a sample of index funds, daily net flows are correlated with afternoon index returns but not with morning returns.

Last, nonsynchronous trading can generate spurious time-of-the-day patterns in average returns. Consider the extreme example of a stock that is traded only at the end of the day. If the stock's average return over the period is positive, then one observes a high end-of-the-day return and zero returns over the rest of the day for this stock. This pattern is mechanical and must be controlled for when studying intraday returns.

Overall, any theory that sets out to explain the cross-section of average returns has to be able to accommodate the intraday patterns observed in the data. It remains an open question to which extent cross-sectional variation in average intraday stock returns can shed light on sources of cross-sectional variation in returns at lower frequencies.

### 3.3 Data and Methodology

The data used in this paper come from several databases. Institute for the Study of Securities Market (ISSM) and Trade and Quote (TAQ) data are used to compute intraday half-hour returns and volumes for each trading day from January 1, 1986, to December 31, 2015. ISSM data is available back to January 1, 1983, but I begin the sample on January 1, 1986, three months after the NYSE started opening at 9:30 a.m. (The month of August 1987 is excluded from the analysis because of missing data.) TAQ data is used starting from January 1,1993 , and is stamped to the millisecond (daily TAQ) from 2004 onwards. At the beginning of each quarter, I select all NYSE, Amex, and NASDAQ common stocks with a price higher than $\$ 5$ and a market capitalization larger than 100 million. Before 1993, I use only NYSE and Amex stocks.

I compute intraday returns based on quote midpoints at the beginning of each half-hour interval during regular trading hours (9:30 a.m. to 4:00 p.m.). Intervals of thirty minutes limit the influence of microstructure effects but still capture a rich set of dynamics. The last half-hour return (3:30 p.m. to 4:00 p.m.) is computed using the last quote available during trading hours. ${ }^{7}$

[^35]Inaccurate quotes at the open generate spurious reversals in midquote returns. For instance, an abnormally high ask price at the open biases the quote midpoint upward and results in a high overnight return, but this return is immediately reversed in the first half hour when quotes are updated. This problem is marked for small stocks in the recent part of the sample. The Appendix provides a specific example and additional details. To limit the scope of this issue, I consider quotes starting at 9:45 a.m.; hence, the first return interval goes from 9:45 a.m. to 10:00 a.m.

In addition to standard error filters (e.g., Chordia, Roll, and Subrahmanyam (2001)), quotes with a spread lower than zero or greater than $\$ 5$ are excluded. The ISSM data is filtered as in Hausman, Lo, and MacKinlay (1992). I also delete any observation for which the spread is larger than 30 times the median spread during the day for a given stock. Finally, I screen the returns to discard obvious reporting mistakes-for instance, extreme price moves that reverse and are not accompanied by any trading volume.

Overnight returns are computed following Lou, Polk, and Skouras (2016); namely,

$$
\begin{equation*}
r_{\text {overnight }, t}=\frac{1+r_{\text {close-to-close }, t}}{1+r_{\text {intraday }, t}}-1 \tag{3.1}
\end{equation*}
$$

where $r_{\text {close-to-close }, t}$ is the daily midquote return and $r_{\text {intraday }, t}$ is the intraday return computed using the midquote at 9:45 a.m. as described above. As a result, the overnight return includes the first 15 minutes of trading. To compute daily midquote returns, quote midpoints at the close are adjusted for stock splits and dividends using CRSP factor to adjust prices (FACPR) and CRSP dividend amount (DIVAMT). If the absolute difference between the daily midquote return and the daily CRSP return is larger than $25 \%$, the daily CRSP return is used instead of the midquote return.

If a stock has no intraday data on a given day, the CRSP daily return, if it exists, is allocated to the overnight return. If a return is missing in the CRSP daily file and intraday trade data exists, I discard the data for this stock on this day. ${ }^{8}$ In the analysis below, I focus on stocks that have non-zero volume in at least $80 \%$ of the traded days in the previous month.

The main analysis uses returns computed from quote midpoints. Quotes may be updated when there is no trade, which limits the selection bias associated with the occurrence of a trade. I provide robustness checks using trade prices as well as volume-weighted average prices (VWAP) in the first half hour of trading. Trade-based returns are computed using the

[^36]first available transaction price in each half-hour interval and the last available price of the day. A return is set to zero if there are no transactions during the interval. To remove abnormal data, I exclude transactions at prices that are greater than the ask plus the spread and lower than the bid minus the spread (Barndorff-Nielsen et al. (2009)). Bid and ask quotes are matched to trades with a five-second lag before 1999 and no lag afterwards. To compute VWAP in the first half hour, a stock is required to have a share volume greater than 1,000 in the first half hour on at least $95 \%$ of the days in which the stock is traded in a given quarter.

To compute excess returns, daily risk-free returns obtained from Kenneth French's data library are subtracted from overnight returns. As pointed out by Heston, Korajczyk, and Sadka (2010), the risk-free rate should not be earned intraday because transactions are settled at the end of the trading day.

Accounting data is taken from Compustat to compute accruals, book equity, gross profitability, and net stock issues. The accounting variables are computed once a year at the end of June using data for the previous fiscal year. Table C. 2 in the Appendix provides additional details about the construction of each variable. Earnings announcement dates are obtained from Compustat.

### 3.4 Intraday and Overnight Average Returns

This section provides new evidence on the determinants of cross-sectional variation in average stock returns by examining returns over the trading day and overnight.

### 3.4.1 Evidence from Anomaly Portfolios

To analyze the cross-section of stock returns, I start by forming portfolios every month based on well-known characteristics. The anomalies literature documents a large number of characteristics associated with abnormal returns relative to the market. My analysis uses sorting variables based on accounting data, market capitalization, past returns, and trading volume. These variables are described in Table C. 2 in the Appendix. The anomalies that I study are similar to the anomalies considered in Fama and French (2016), to which I add an illiquidity variable.

At the beginning of each month, decile portfolios are formed using NYSE breakpoints based on the values of the sorting variable under consideration at the end of the previous month. The long-short portfolio is long the highest decile portfolio and short the lowest decile portfolio (or vice versa depending on the sorting variable). To exclude highly illiquid stocks and attenuate microstructure effects, each stock is required to have at least ten days with non-zero volume in the previous month and a price greater than $\$ 10$ at the end of the previous month to be
included. Value-weighted portfolios returns are used to limit the influence of microstructure noise (Blume and Stambaugh (1983)). Importantly, there is no intraday rebalancing: Portfolio returns are those of a buy-and-hold portfolio rebalanced at the beginning of each month. ${ }^{9}$

The day is split into $k=1, \ldots, K$ periods, where 1 indicates the overnight period and $K$ indicates the last half hour of trading. Let $r_{t}$ denote the return of a portfolio in interval $t$. The following regression is estimated:

$$
\begin{equation*}
\frac{r_{t}}{\hat{\sigma}_{t}}=\sum_{k=1}^{K} \frac{1_{t, k}}{\hat{\sigma}_{k}} \mu_{k}+\epsilon_{t} \tag{3.2}
\end{equation*}
$$

where $\hat{\sigma}_{k}$ denotes the standard deviation of returns in period $k, 1_{t, k}$ is a dummy variable that takes the value one if interval $t$ belongs to period $k$ and zero otherwise, and $\hat{\sigma}_{t}=\sum_{k=1}^{K} 1_{t, k} \hat{\sigma}_{k}$. Estimating equation (3.2) is equivalent to computing average returns and standard deviations separately for each period of the day. This is important to control for heteroskedasticity given that return volatility is not constant over the day. In addition, standard errors are adjusted for heteroskedasticity and autocorrelation using a Newey and West (1987) correction with 14 lags (1 day). Similarly, to compute alpha in a given period, I estimate

$$
\begin{equation*}
\frac{r_{t}}{\hat{\sigma}_{t}}=\sum_{k=1}^{K} \frac{1_{t, k}}{\hat{\sigma}_{k}} \alpha_{k}+\sum_{k=1}^{K} \frac{1_{t, k}}{\hat{\sigma}_{k}} r_{m, t}^{e} \beta_{k}+\epsilon_{t} \tag{3.3}
\end{equation*}
$$

where $r_{m, t}^{e}$ is the market (excess) return in interval $t$. Alpha in a given half hour is estimated using returns in the same half hour. This methodology recognizes that beta may vary over the day. Theoretically, such variation can occur if, for instance, the proportion of traders active in the market is not constant across the day (Bogousslavsky (2016)). The results are robust to including lagged market returns in equation (3.3).

Table 3.1 reports average returns, market alphas, and several other statistics for each portfolio over the full sample. Average returns and alphas are estimated using equation (3.3). Table 3.1 shows that marked differences in intraday average returns exist both within and across anomalies. This variation is the building block of my analysis. Indeed, I aim to show that useful information about cross-sectional variation in stock returns can be extracted from intraday returns.

[^37]3.4. Intraday and Overnight Average Returns
Table 3.1
Intraday and overnight return properties of long-short decile portfolios. This table reports average returns ( $\bar{r}$ ) and alpha in basis points $(\alpha)$, volatility in percent $(\sigma)$, skewness (skew), and minimum return (min) in percent. At the end of each month, stocks are split into decile portfolios based on the NYSE breakpoints of the characteristics defined in Table C.2. Portfolios are value-weighted and held for one month. A stock is required to have a price greater than $\$ 10$ at the end of the previous month and at least $80 \%$ of traded days with non-zero volume in the previous month to be included. Financial firms are excluded from portfolios based on accounting variables. NASDAQ stocks are excluded from the illiquidity portfolio. Stock returns are computed using quote midpoints. Before computing skewness, returns are winsorized at $0.1 \%$ separately for each half hour and the overnight period. The first interval starts at 9:45 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. The sample is composed of NYSE, Amex, and NASDAQ common stocks from January 1, 1986, to December 31, 2015. NASDAQ stocks are included since 1993. $t$-statistics are shown in parentheses and based on Newey and West (1987) standard errors with 14 lags. *, **, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | OV | 9:45 | 10:00 | 10:30 | 11:00 | 11:30 | 12:00 | 12:30 | 1:00 | 1:30 | 2:00 | 2:30 | 3:00 | 3:30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Accruals |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\bar{r}$ | 0.44 | 0.86*** | 0.35 | 0.05 | -0.48*** | 0.01 | 0.24 | 0.24 | 0.03 | -0.27* | 0.08 | -0.05 | -0.28 | -0.28 |
|  | (0.88) | (3.46) | (1.35) | (0.23) | (-2.61) | (0.06) | (1.51) | (1.56) | (0.20) | (-1.81) | (0.50) | (-0.33) | (-1.61) | (-1.35) |
| $\alpha$ | 0.34 | 0.95*** | 0.38 | 0.04 | -0.46 ** | 0.01 | 0.25 | 0.19 | 0.01 | -0.24 | 0.10 | -0.12 | -0.32* | -0.29 |
|  | (0.69) | (3.76) | (1.49) | (0.18) | (-2.52) | (0.07) | (1.61) | (1.28) | (0.09) | (-1.63) | (0.65) | (-0.73) | (-1.88) | (-1.38) |
| $\sigma$ | 0.43 | 0.22 | 0.23 | 0.18 | 0.16 | 0.15 | 0.14 | 0.13 | 0.13 | 0.13 | 0.13 | 0.14 | 0.15 | 0.18 |
| skew | 0.04 | 0.19 | 0.33 | 0.03 | -0.29 | -0.05 | 0.11 | 0.18 | 0.13 | -0.21 | 0.19 | -0.01 | -0.23 | 0.34 |
| min | -4.04 | -1.68 | -1.59 | -1.95 | -1.31 | -2.26 | -1.65 | -1.03 | -1.12 | -2.89 | -1.11 | -1.44 | -1.14 | -1.91 |
| Beta |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\bar{r}$ | -6.97*** | $1.82^{* * *}$ | 1.74*** | 0.25 | 0.59 | 0.61* | 0.41 | -0.20 | 0.23 | 0.46 | 0.85*** | -0.48 | -0.01 | 0.05 |
|  | (-6.28) | (4.34) | (3.33) | (0.58) | (1.65) | (1.90) | (1.42) | (-0.68) | (0.77) | (1.50) | (2.59) | (-1.40) | (-0.01) | (0.11) |
| $\alpha$ | -2.86*** | $1.05 * * *$ | 0.91 *** | 0.45 * | $0.30$ | $0.59^{* * *}$ | $0.26$ | $0.28$ | $0.53^{* * *}$ | $0.15$ | $0.51^{* * *}$ | $0.25$ | 0.57*** | $0.04$ |
| $\sigma$ | 0.97 | 0.36 | 0.45 | 0.37 | 0.31 | 0.28 | 0.25 | 0.25 | 0.26 | 0.26 | 0.29 | 0.30 | 0.32 | 0.36 |
| skew | -0.34 | 0.20 | 0.03 | -0.08 | 0.18 | -0.06 | 0.21 | -0.05 | 0.07 | -0.31 | -0.48 | -0.46 | -0.37 | -0.11 |
| min | -14.62 | -4.75 | -4.04 | -3.48 | -4.67 | -2.23 | -3.42 | -8.08 | -7.64 | -5.40 | -3.00 | -3.36 | -5.11 | -4.56 |
| Book-to-market |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\bar{r}$ | -3.38*** | 0.83*** | $0.58{ }^{*}$ | -0.38 | 0.01 | 0.10 | 0.11 | -0.08 | 0.39** | 0.03 | 0.16 | -0.18 | 0.01 | 1.00** |
|  | (-6.30) | (3.17) | (1.96) | (-1.54) | (0.04) | (0.53) | (0.64) | (-0.50) | (2.18) | (0.20) | (0.89) | (-0.95) | (0.03) | (4.12) |
| $\alpha$ | -2.81*** | 0.83 ** | 0.32 | -0.32 | -0.07 | 0.04 | 0.05 | 0.15 | 0.48 *** | -0.06 | 0.11 | -0.00 | 0.28 | 0.89*** |
|  | (-5.77) | (2.49) | (1.15) | (-1.41) | (-0.36) | (0.21) | (0.28) | (0.96) | (2.93) | (-0.35) | (0.65) | (-0.02) | (1.46) | (4.05) |
| $\sigma$ | 0.47 | 0.23 | 0.26 | 0.21 | 0.18 | 0.17 | 0.15 | 0.14 | 0.15 | 0.15 | 0.16 | 0.17 | 0.18 | 0.21 |
| skew | -0.10 | 0.12 | 0.04 | -0.28 | -0.16 | 0.04 | 0.18 | -0.24 | 0.06 | -0.21 | -0.30 | 0.28 | 0.16 | 0.79 |
| min | -3.62 | -2.33 | -3.52 | -2.30 | -2.27 | -1.56 | -1.59 | -2.20 | -5.11 | -2.40 | -1.84 | -2.63 | -2.21 | -2.00 |

Chapter 3．The Cross－Section of Intraday and Overnight Returns

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Anomalies fall into three groups: Anomalies that accrue in a specific period during the day (size, illiquidity, and momentum), referred to as "period-specific" anomalies; anomalies that accrue gradually over the trading day (betting-against-beta, gross profitability, idiosyncratic volatility, and net stock issues), referred to as "gradual" anomalies; and anomalies that display no clear pattern (accruals and book-to-market). These patterns are robust across subsamples (Figure 3.1), though statistical significance tends to be lower because of the smaller number of observations. ${ }^{10}$

Harris (1986), Smirlock and Starks (1986), and Jain and Joh (1988) all document a strong day-of-the-week effect in intraday index returns. In particular, returns tend to be markedly negative over the first hours of trading on Mondays. This evidence follows from the "weekend effect," i.e., returns tend to be particularly low on Mondays for the U.S. stock market (see, for instance, French (1980)). While the weekend effect does not appear in recent data, Birru (2016) finds day-of-the-week effects for anomalies in a sample that goes from 1995 to 2013. For each anomaly, Figure 3.2 plots the statistical significance of intraday average returns separately for each day of the week. The period-specific and gradual patterns are robust across days of the week. The size portfolio is, however, subject to a day-of-the-week effect. This observation is discussed in greater detail below.

In Table 3.1, market alphas display similar intraday patterns as average returns. This is not surprising since the average market return is small throughout most of the trading day (see below). Furthermore, anomaly betas are small and often close to zero. For most anomalies, I find that betas are relatively stable across the trading day and leave a detailed investigation of intraday exposures for future research. Betting-against-beta is the only anomaly for which intraday returns and alphas show non-negligible differences. Given that nonsynchronous trading can bias beta, it is reassuring that the results are similar for average returns and alphas. ${ }^{11}$

Anomalies differ on other dimensions than average returns. While several anomalies exhibit a marked U-shaped pattern in volatility across the trading day, other anomalies exhibit a L-shaped pattern. Intraday patterns in skewness and minimum return also differ considerably across anomalies.

[^38]Figure 3.1. Intraday and overnight $\boldsymbol{t}$-statistics of market alphas of long-short portfolios across subsamples. The first interval starts at 9:45 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. Portfolio construction is detailed in the caption of Table 3.1. Dashed red lines indicate significance at the level of $10 \%$. $t$-statistics are based on Newey and West (1987) standard errors with 14 lags.


Figure 3.2. Intraday and overnight $\boldsymbol{t}$-statistics of market alphas of long-short portfolios across days of the week. The first interval starts at 9:45 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. Portfolio construction is detailed in the caption of Table 3.1. Dashed red lines indicate significance at the level of $10 \%$. $t$-statistics are based on heteroskedasticity-adjusted standard errors.

(Figure 3.2 continued.)





### 3.4.2 Nonsynchronous Trading and Thin Trading

Nonsynchronous trading is an important issue to consider when studying returns over short horizons. Nonsynchronous trading smoothes portfolio returns, which generates positive portfolio return autocorrelation (e.g., Fisher (1966)) and lowers a portfolio's volatility below its true economic volatility. The use of midquote returns, which are not necessarily associated with trades, and the filters described in Section 3.3 should limit the problem. Still, quotes may not be revised actively, especially during the old part of the sample.

To assess the impact of nonsynchronous trading and thin trading, I apply the following volume filter: Each year, a stock is required to have trades in the first, second, second to last, and last half hours of the trading day on at least $90 \%$ of the days for which the stock has a valid CRSP daily return. ${ }^{12}$ In addition to excluding stocks that trade particularly infrequently, this restriction ensures that the overnight and opening half-hour returns are associated with actual transactions. In 1986, this criterion excludes $85 \%$ of the stocks for which I have ISSM data. In 2016, this criterion excludes $20 \%$ of the stocks for which I have TAQ data.

Table 3.2 reports intraday and overnight alphas of anomalies portfolios with the volume filter. The patterns are robust. Alphas tend, however, to be slightly smaller over the trading day, and a few differences arise for overnight and first-hour returns. For instance, both size and illiquidity now earn positive and statistically significant overnight alpha. In summary, the patterns in Table 3.1 do not appear to be driven by nonsynchronous trading.

[^39]Table 3.2
Intraday and overnight alphas $(\alpha)$ in basis points of long-short portfolios with volume filter. Each year, a stock is required to have trades in the first, second, second to last, and last half-hours of the trading day on at least $90 \%$ of the days for which it has a valid CRSP daily return. At the end of each month, stocks are split into decile portfolios based on the NYSE breakpoints of the characteristics defined in Table C.2. Portfolios are value-weighted and held for one month. A stock is required to have a price greater than $\$ 10$ at the end of the previous month and at least $80 \%$ of traded days with non-zero volume in the previous month to be included. Financial firms are excluded from portfolios based on accounting variables. NASDAQ stocks are excluded from the illiquidity portfolio. Stock returns are computed using quote midpoints. The first interval starts at 9:45 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. The sample is composed of NYSE, Amex, and NASDAQ common stocks from January 1, 1986, to December 31, 2015. NASDAQ stocks are included since 1993. $t$-statistics are shown in parentheses and based on Newey and West (1987) standard errors with 14 lags. ${ }^{*}$, **, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.


### 3.4.3 Benchmarking the Results

The previous findings show that there is marked cross-sectional variation in intraday and overnight return patterns. In particular, patterns in intraday average returns differ across anomalies. To which benchmark should these patterns be compared?

The market portfolio is a natural benchmark. Panel (a) of Figure 3.3 reports the $t$-statistics of average intraday market returns, where the market return is computed as the value-weighted return of all stocks in the sample and is rebalanced on a monthly basis. The market portfolio tends to earn high returns overnight and displays no clear pattern over the trading day. Over the full sample, the hypothesis that the market portfolio's intraday half-hour returns (starting from 10 a.m.) are jointly equal to zero cannot be rejected at the level of $1 \%$. After 1990, the hypothesis cannot be rejected at the level of $5 \%$.

It is puzzling that the portfolio proxying for the aggregate risk in the economy tends to earn close to zero returns over most of the trading day. This puzzle is further deepened when excluding FOMC announcement days (not reported). ${ }^{13}$

As a second benchmark, I compute returns on "random" portfolios. At the beginning of each year, stocks are allocated randomly into decile portfolios. I impose the same filters as for the anomaly portfolios. Two of the decile portfolios are selected randomly to compute monthly value-weighted returns on a long-short decile portfolio over the following year. The long and short legs are determined ex post to obtain a positive average monthly return over the sample period (1986-2015). This procedure is repeated 10,000 times.

Two remarks are in order. First, I assume annual rebalancing to simplify the computations. Second, given that there is no persistence in the sorts over a period greater than a year, the unconditional persistence in the composition of the random portfolios may not match the persistence in the composition of the anomaly portfolios. ${ }^{14}$ While the first point is unlikely to be a concern, the second point may make the random strategies not fully comparable to the anomalies. Still, these random strategies provide a neat benchmark to evaluate intraday and overnight return patterns.

Among all random strategies, 1,065 earn average monthly returns that are statistically signifi-

[^40]Figure 3.3. Intraday benchmarks: market portfolio and random portfolios. Panel (a) reports the $t$-statistics of the average intraday market returns. The market return is computed as the value-weighted return of all stocks in the sample and is rebalanced on a monthly basis. Random portfolios: At the beginning of each year, stocks with a price larger than $\$ 10$ and at least ten days of nonzero volume over the previous month are allocated randomly into decile portfolios. Two of the decile portfolios are selected randomly to compute monthly value-weighted returns on a long-short decile portfolio over the following year. The long and short legs are determined ex post to obtain a positive average monthly return over the full sample period (1986-2015). This procedure is repeated 10,000 times. The 1,065 strategies that have an average monthly return significant at the level of $10 \%$ are labeled as significant strategies. Panel (b) reports the first quartile, median, and third quartile of alpha's $t$-statistics across all significant strategies in each interval of the day. Panel (c) reports an histogram of the number of significant strategies with a given number of positive and significant (at the level of $10 \%$ ) intraday half-hour alphas. The same statistic is indicated for accruals (AC), beta (BE), book-to-market (BM), gross profitability (GP), idiosyncratic volatility (IV), illiquidity (IL), momentum (MO), net stock issues (NI), and size (SI).
(a) Market return ( $t$-statistic)

(b) Quartiles of random portfolios ( $t$-statistic)


cant at the level of $10 \%$. In what follows, I refer to these strategies as "significant strategies." The best significant strategy has a $t$-statistic of 3.98. Unsurprisingly, market returns do not explain the simulated strategies' returns, and average returns and alphas are highly similar. For each significant strategy, I compute intraday half-hour and overnight alphas with associated $t$-statistics.

Panel (b) of Figure 3.3 plots the first quartile, median, and third quartile of $t$-statistics across all significant strategies in each interval. These statistics shed light on the average alpha profile over the day of a significant strategy. The overnight period drives the profitability of most random strategies. In fact, less than $20 \%$ of significant strategies have a negative overnight alpha. Overall, the average random portfolio does not appear to earn returns gradually over the day, which is confirmed by a joint test below.

To benchmark period-specific anomalies, I evaluate whether a significant strategy can earn statistically significant alpha in a given period across all subsamples. The probability for a random strategy is close to zero for all periods except overnight ( $1.22 \%$ ) and, to a lesser extent, in the last half hour ( $0.47 \%$ ). Among the 10,000 original strategies, only one strategy earns significant alpha in a given period in all subsamples and across all days of the weeks. Like momentum, this strategy has a positive overnight alpha. This evidence suggests that concentrated patterns similar to that of the period-specific anomalies are not replicated by simple random strategies.

To benchmark the gradual anomalies, the histogram in Panel (c) of Figure 3.3 reports the number of random strategies that have a given number of positive and significant half-hour alphas (at the level of $10 \%$ ). Not a single random strategy has more than six statistically significant intraday intervals and only two attain this threshold. The random strategies do not appear to earn positive and statistically significant returns consistently across the trading day. This result contrasts with the gradual anomalies identified above, for which the same statistic is indicated in the histogram.

The comparison between these two benchmarks and the anomaly portfolios suggests that stock characteristics matter for intraday patterns. Portfolios of stocks formed on different characteristics can exhibit strikingly different intraday return patterns. These patterns differ from the two natural benchmarks that are the market portfolio and random portfolios. The next step is to understand why such cross-sectional differences in average returns exist at the intraday level and whether these differences can explain the cross-section of stock returns at lower frequencies.

### 3.5 Size and Illiquidity

Strikingly, the bulk of size and illiquidity average returns (alpha) is earned in the last half hour of trading. The end-of-the-day spike in size return in Table 3.1 translates to roughly $0.60 \%$ on a monthly basis. This result is statistically significant across all subsamples and days of the week (Figures 3.1 and 3.2), robust to excluding all January observations, and not limited to extreme deciles. Overnight returns show no marked relation to firm size. But last half-hour returns increase monotonically with size, while first half-hour returns decrease monotonically with size (not reported). The last half hour return is also robust to excluding NASDAQ stocks or forming a (size) portfolio using only NASDAQ stocks (not reported).

Small stocks earn a positive and statistically significant average excess return over the sample period. Hence, if small stocks trade mostly around the close or, equivalently, their quotes are updated mostly around the close, then positive returns should be concentrated at this time. As shown in Section 3.4.2, the high end-of-the-day return of size and illiquidity is not a mechanical side effect of nonsynchronous trading.

As a simple robustness check against data mining concerns, I examine size returns on days with anticipated early closures of the exchanges. The NYSE and NASDAQ may close early on July $3^{\text {rd }}$, July $5{ }^{\text {th }}$, the day after Thanksgiving, and Christmas Eve. Over my sample, I identify 51 days with early closures. The average return in the last half hour (12:30-1:00 p.m.) is not statistically significant but remains largest among all intraday average returns ( 4.28 bp ).

To the best of my knowledge, this evidence has not been highlighted before. Using transaction data on NYSE stocks over December 1981 to January 1983, Harris (1986) documents that prices rise on the last trade of the day. This rise is in large part due to the tendency of the last transaction to be at the ask (Harris (1989)). This effect cannot be at play in my sample of midquote returns. Moreover, the cross-sectional difference between large and small stocks is not emphasized by Harris (1989).

Figure 3.4 shows that small and large stocks display radically different patterns in average returns across days of the week and over the trading day. Both legs contribute to the end-of-the-day effect. Furthermore, small stocks perform poorly at the beginning of the week, while large stocks display the opposite pattern.

The previous result is difficult to reconcile with standard theories of size and illiquidity. For instance, if size proxies for distress risk, then it is not clear why small stocks should earn such large returns at the end of the day. I explore next whether theories of intraday and overnight returns can explain the previous evidence. More precisely, I test theories based on closure effects, institutional effects, and coordination (information asymmetry).

## Chapter 3. The Cross-Section of Intraday and Overnight Returns

Figure 3.4. Intraday and overnight market alphas of small and large stocks portfolios across days of the week. The first interval starts at 9:45 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. Portfolio construction is detailed in the caption of Table 3.1. Dashed red lines indicate significance at the level of $10 \%$. $t$-statistics are based on heteroskedasticity-adjusted standard errors.




### 3.5.1 Closure Effects

The large average return of small stocks (the long leg) over the last half hour of trading does not appear to be consistent with overnight liquidity risk being compensated. Prices should go down for liquidity providers to hold risky stocks overnight (see Section 3.2). Moreover, evidence from extreme negative returns and skewness does not support a crash risk story (Table 3.1). Therefore, the hedging story detailed in Section 3.2 cannot explain the high end-of-the-day average return of small stocks. This explanation may help explain the low end-of-the-day average return of large stocks, but it remains unclear why the opposite pattern is observed for small stocks.

### 3.5.2 Institutional Effects: Price Pressure

Another potential explanation is that exogenous buy imbalances-for instance due to institutional effects as described in Section 3.2—may cause an increase in the price of small and illiquid stocks at the end of the day. In this case, one would expect some reversal over the following overnight and morning periods. However, autocorrelations provide no evidence of reversal between the end-of-the-day return and the following overnight return (not reported). In addition, Figure 3.4 shows that high average end-of-the-day returns tend not to be reversed on the following day for most days of the week. In summary, there is no evidence of large price pressure effects—correlated across small stocks to show up in portfolio returns-at the end of the day. ${ }^{15}$

Following Llorente et al. (2002), I estimate the following regression for the small stocks portfolio:

$$
\begin{equation*}
r_{\mathrm{OV}, t+1}=a+b r_{3: 30, t}+c r_{3: 30, t} \operatorname{turn}_{3: 30, t}+\epsilon_{t+1} \tag{3.4}
\end{equation*}
$$

where turn ${ }_{3: 30, t}$ is the logarithm of turnover (trading volume over shares outstanding) between 3:30 and 4:00 p.m. on day $t$ minus its average over the past 250 days. Here, the turnover of the portfolio is the value-weighted turnover of the stocks in the portfolio. We expect the coefficient $c$ to be negative if there are common liquidity shocks at the end of the day. There is no such evidence for the small stocks portfolio except in the last part of the sample, in which $c$ is negative and statistically significant at the level of $10 \%$. It remains possible, however, that the shocks reverse over longer horizons.

[^41]
### 3.5.3 Liquidity at the Close

Table 3.1 shows that size and illiquidity exhibit a similar intraday pattern. To evaluate which characteristic dominates the other, Table 3.3 reports the last half hour average return of doublesorted long-short portfolios. Stocks are first sorted into illiquidity (size) quintiles and then, within each quintile, stocks are sorted again into size (illiquidity) quintile portfolios. As can be seen, illiquidity dominates size in generating a positive and statistically significant average return in the last half hour of trading.

Given that illiquidity dominates size in explaining the end-of-the-day return, it is natural to look for an explanation based on liquidity. A high return at the end of the day is consistent with a model in which risk-averse market makers with positive inventories have to absorb supply shocks at the end of the day. ${ }^{16}$ In this model, the price impact of supply shocks increases at the end of the day, which leads market makers to require higher returns to hold stocks over this period.

This framework nests both a pure liquidity shocks theory and an information asymmetry theory. First, the price impact of supply shocks may increase because supply shocks are more volatile around the close, for instance, due to institutional effects around the close. Second, the price impact of supply shocks may increase because there is more informed trading around the close. This framework can explain the cross-sectional evidence if small and illiquid stocks are subject to more volatile liquidity shocks or more information asymmetry than large stocks.

There is evidence that liquidity deteriorates around the close. Effective spreads are U-shaped over the day (McInish and Wood (1992)) while quoted depths are reverse U-shaped (Lee, Mucklow, and Ready (1993)). Furthermore, Madhavan, Richardson, and Roomans (1997) find that temporary price impact increases over the day. Cushing and Madhavan (2000) find that the return sensitivity to order flow is higher in the last half hour of trading than during the rest of the day. More precisely, they document a common factor in stock returns at the end of the day and link their finding to institutional trading at this time. ${ }^{17}$ Still, recent anecdotal evidence suggests that liquidity is higher at the close (see the references in Footnote 1).

The liquidity of small and large stocks may possibly diverge at this time of the day. As a simple test, I compute Amihud's illiquidity coefficient separately for each interval of the day. For both small and large stocks, the illiquidity coefficient is actually lowest at the end of the day (not reported). This result does not support a liquidity explanation. Liquidity risk may be higher for small stocks at the end of the day, but this channel is not supported by the limited evidence

[^42]Table 3.3
Average end-of-the-day return of double-sorted size-illiquidity portfolios. At the end of each month, stocks are sorted sequentially into size/illiquidity (ILLIQ) long-short quintile portfolios. The table reports the average return in the last half hour of trading ( $\bar{r}_{3: 30}$ ) in basis points. Portfolios are value-weighted and held for one month. A stock is required to have a price greater than $\$ 10$ at the end of the previous month and at least $80 \%$ of traded days with non-zero volume in the previous month to be included. Stock returns are computed using quote midpoints. The first interval starts at 9:45 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. The sample is composed of NYSE and Amex common stocks from January 1, 1986, to December 31, 2015. $t$-statistics are shown in parentheses and based on Newey and West (1987) standard errors with 14 lags. ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | Size/ILLIQ quintile |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{r}_{3: 30}[\mathrm{bp}]$ | 1 | 2 | 3 | 4 | 5 |  |
| Size (given ILLIQ) | $0.87^{* * *}$ | -0.18 | -0.19 | -0.06 | 0.30 |  |
|  | $(4.77)$ | $(-1.08)$ | $(-1.11)$ | $(-0.10)$ | $(1.28)$ |  |
|  |  |  |  |  |  |  |
| ILLIQ (given Size) | $0.75^{* * *}$ | 0.31 | $0.65^{* *}$ | $1.08^{* * *}$ | $1.59^{* * *}$ |  |
|  | $(3.26)$ | $(1.65)$ | $(3.63)$ | $(6.74)$ | $(8.91)$ |  |

of return reversal in portfolios of small and illiquid stocks. Clearly, more work remains to be done to assess intraday liquidity patterns in the cross-section.

## Institutional Effects

To disentangle liquidity shock effects from asymmetric information effects, I examine return patterns at the end of quarters. Liquidity effects may be more pronounced at the end of quarters. As shown by Carhart et al. (2002), portfolio pumping by fund managers often takes place on the last day of each quarter. This aggressive trading can affect the cross-section of stock returns and, in particular, size and illiquidity portfolios. ${ }^{18}$ I estimate a panel regression with end-of-the-quarter indicator variables as follows.

$$
\begin{equation*}
r_{i, t}^{3: 30}=\alpha_{i}+\gamma_{\mathrm{IL}} \mathrm{ILLIQ}_{i, t}+\gamma_{\mathrm{EoQ}} 1_{\mathrm{EoQ}, t}+\gamma_{\mathrm{EoQ}, I L} 1_{\mathrm{EoQ}, t} \mathrm{ILLIQ}_{i, t}+u_{i, t} \tag{3.5}
\end{equation*}
$$

where $\gamma_{\mathrm{EoQ}} 1_{\mathrm{EoQ}, t}$ takes the value one on the last day of a quarter.
The results are reported in Table 3.4. In line with prior evidence, illiquid stocks are subject to a significant end-of-the-quarter effect. The coefficient on ILLIQ remains, however, positive and

[^43]statistically significant at the level of $10 \%$, even though stock fixed effects are included in the regression. Excluding all January observations does not affect this result.

In summary, explanations based on pure liquidity shocks (institutional effects) have some merit in explaining high returns at the end of the day. But, in view of the lack of reversal observed in small stocks returns (Section 3.5.2), they do not appear to explain why marked cross-sectional differences are observed between small and large stocks.

## Information Asymmetry

This section examines whether an increase in the degree of information asymmetry for small stocks can explain the end-of-the-day pattern. A shift in the degree of information asymmetry is potentially consistent with strategic models in which informed traders select when to trade (Section 3.2). According to this explanation, the end-of-the-day pattern in small stocks may result from small stocks being subject to a higher degree of information asymmetry than large stocks.

To test the information-based explanation, I use earnings announcements to proxy for a change in the degree of asymmetric information. One may expect the degree of information asymmetry to be higher on the days preceding an earnings announcement than on the days following one. The model of Kim and Verrecchia (1994) predicts the opposite pattern: Some traders have better information processing ability than others and, as a result, information asymmetry increases following an announcement. I expect, however, such effects to last for less than a day, especially over my sample which mostly spans recent years.

If an earnings announcement is made on a given day after the close of trading, I allocate it to the next trading day. ${ }^{19}$ The following panel regression is estimated:

$$
\begin{equation*}
r_{i, t}^{h}=\alpha_{t}+\gamma_{\mathrm{IL}} \mathrm{ILLIQ}_{i, t}+\sum_{k=-3}^{3} \gamma_{\mathrm{EA}-k} 1_{\mathrm{EA}-\mathrm{k}, i, t}+\sum_{k=-3}^{3} \gamma_{\mathrm{EA}-k, \mathrm{IL}} 1_{\mathrm{EA}-\mathrm{k}, i, t} \mathrm{ILLIQ}_{i, t}+u_{i, t} \tag{3.6}
\end{equation*}
$$

where $1_{\mathrm{EA}-\mathrm{k}, i, t}$ is an indicator variable that takes the value one if firm $i$ has an earnings announcement on date $t+k$. I focus on a range of three days before and after an earnings announcement date. According to the asymmetric information theory, the coefficients $\gamma_{\text {EA }-k}$ should be larger in the days preceding the announcement. Furthermore, the coefficients $\gamma_{\mathrm{EA}-k \text {, IL }}$ should be positive in the days preceding the announcement. The coefficient $\gamma_{\mathrm{IL}}$ should be statistically insignificant if earnings announcement account for the end-of-the-day effect.

[^44]Table 3.4
End of quarters and intraday returns. The following panel regression is estimated: $r_{i, t}^{h}=$ $\alpha_{i}+\gamma_{\mathrm{IL}} \mathrm{ILLIQ}_{i, t}+\gamma_{\mathrm{EoQ}} 1_{\mathrm{EoQ}, t}+\gamma_{\mathrm{EoQ}, I L} 1_{\mathrm{EoQ}, t} \mathrm{ILLIQ}_{i, t}+u_{i, t}$, where $r_{i, t}^{h}$ is stock $i$ 's return in interval $h$ on day $t$ (in basis points), ILLIQ $_{i, t}$ is the logarithm of stock $i$ 's Amihud (2002) illiquidity coefficient estimated over the previous year, and $1_{\mathrm{EA}-\mathrm{k}, i, t}$ is an indicator variable that takes the value one if firm $i$ has an earnings announcement on date $t$. The regression includes stock fixed effects. A stock is required to have a price greater than $\$ 10$ at the end of the previous month and at least $80 \%$ of traded days with non-zero volume in the previous month to be included. Stock returns are computed using quote midpoints. All the variables are winsorized at $0.05 \%$. ILLIQ is normalized by its standard deviation. The sample is composed of NYSE and Amex common stocks from January 1, 1986, to December 31, 2015. $t$-statistics are shown in parentheses and based on standard errors that are double-clustered by date and firm. *, **, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

| dependent variable | $r_{i, t}^{3: 30}[\mathrm{bp}]$ |
| :--- | :---: |
|  | $0.34^{*}$ |
| ILLIQ | $(1.80)$ |
|  | 6.40 |
| $\mathrm{l}_{\mathrm{EoQ}}$ | $(1.47)$ |
|  | $2.81^{* * *}$ |
| $\mathrm{l}_{\mathrm{EoQ}}$ * ILLIQ | $(3.43)$ |
|  | Yes |
| Stock fixed effects |  |
|  | $6,186,818$ |

Table 3.5 reports the results. The left column shows that there is a positive end-of-the-day effect for all stocks on the days preceding an announcement. On the contrary, there is a negative end-of-the-day effect for all stocks on the day following an announcement. This striking asymmetry is consistent with the asymmetric information theory.

Moreover, as shown in the middle and right columns of Table 3.5, this asymmetry does not exist during the intervals 2:30-3:00 p.m. and 3:00-3:30 p.m. There is, however, only limited evidence that illiquid stocks are more affected than other stocks. The coefficient on ILLIQ remains large and positive, which shows that earnings announcement do not explain the end-of-the-day effect in illiquid stocks. There is an announcement day effect since average returns tend to be positive and statistically significant on announcement days for all shown intervals. The previous results are robust to excluding the ISSM data.

Overall, the evidence does not support asymmetric information as the primary driver of the end-of-the-day effect in illiquid stocks. However, the evidence in Table 3.5 is consistent with a role of asymmetric information for end-of-the-day returns. Table 3.5 does not directly show
that there is more asymmetric information at the end of the day, but the intraday return pattern seems hard to reconcile with alternative explanations.

### 3.5.4 Summary

I find evidence consistent with institutional and information asymmetry theories in generating high end-of-the-day returns. However, these theories fail to explain why small and illiquid stocks are disproportionately affected relative to large stocks. Importantly, it remains unclear whether liquidity improves or deteriorates at the end of the trading day and whether variations in liquidity can explain the striking cross-sectional difference in intraday average returns between small and large stocks.

### 3.6 Gradual Anomalies

Gradual anomalies-i.e., betting-against-beta, gross profitability, idiosyncratic volatility, and net stock issues-earn consistently positive and statistically significant average returns over the trading day. This evidence is robust across subsamples and days of the week.

These anomalies realize, however, large negative returns overnight and in the last half hour of trading. Such returns are difficult to reconcile with risk-based explanations. Since the overnight returns of gradual anomalies are negatively skewed, overnight crash risk does not seem to explain the low overnight returns. ${ }^{20}$

Another potential explanation for the large overnight returns is that the quote midpoints of the stocks in these portfolios tend to be associated with low depth at 9:45 a.m. Hence, even small trades could easily bias the quotes, which would reverse shortly afterwards. In this respect, Section 3.7.1 shows that the definition of the opening price has a large impact on the magnitude of overnight returns of aggregate portfolios. Such reversal at the open is not economically meaningful for understanding of anomalies over longer horizons.

To test this explanation, I compute overnight returns using volume-weighted average prices (VWAP) in the first half hour of trading. As detailed in Section 3.3, I use only stocks that have a sufficient number of shares traded over this interval. The results-reported in the Internet Appendix-show that overnight alphas remain large and negative for all gradual anomalies except gross profitability. ${ }^{21}$

Mispricing theories generally predict an asymmetry between an anomaly long leg return and

[^45]Table 3.5
Earnings announcements and intraday returns. The following panel regression is estimated: $r_{i, t}^{h}=\alpha_{t}+\gamma_{\mathrm{IL}} \mathrm{ILLIQ}_{i, t}+\sum_{k=-3}^{3} \gamma_{\mathrm{EA}-k} 1_{\mathrm{EA}-\mathrm{k}, i, t}+\sum_{k=-3}^{3} \gamma_{\mathrm{EA}-k, \mathrm{IL}} 1_{\mathrm{EA}-\mathrm{k}, i, t} \mathrm{ILLIQ}_{i, t}+u_{i, t}$, where $r_{i, t}^{h}$ is stock $i$ 's return in interval $h$ on day $t$ (in basis points), ILLIQ $_{i, t}$ is the logarithm of stock $i$ 's Amihud (2002) illiquidity coefficient estimated over the previous year, and $1_{\mathrm{EA}-\mathrm{k}, i, t}$ is an indicator variable that takes the value one if firm $i$ has an earnings announcement on date $t+k$. The regression includes day fixed effects. A stock is required to have a price greater than $\$ 10$ at the end of the previous month and at least $80 \%$ of traded days with non-zero volume in the previous month to be included. Stock returns are computed using quote midpoints. All the variables are winsorized at $0.05 \%$. ILLIQ is normalized by its standard deviation. The sample is composed of NYSE and Amex common stocks from January 1, 1986, to December 31, 2015. $t$-statistics are shown in parentheses and based on standard errors that are double-clustered by date and firm. ${ }^{*},{ }^{* *}$, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

| dependent variable | $r_{i, t}^{3: 30}[\mathrm{bp}]$ | $r_{i, t}^{3: 00}[\mathrm{bp}]$ | $r_{i, t}^{2: 30}[\mathrm{bp}]$ |
| :--- | :---: | :---: | :---: |
|  |  |  |  |
| ILLIQ | $0.43^{* * *}$ | 0.04 | -0.05 |
|  | $(4.30)$ | $(0.54)$ | $(-0.80$ |
| EA-3 | 1.18 | 0.60 | 0.49 |
|  | $(1.45)$ | $(0.89)$ | $(0.81)$ |
| EA-2 | 1.35 | 0.17 | 0.51 |
|  | $(1.59)$ | $(0.24)$ | $(0.76)$ |
| EA-1 | 1.50 | 0.42 | $-1.32^{*}$ |
|  | $(1.55)$ | $(0.56)$ | $(-1.82)$ |
| EA | $3.62^{* * *}$ | $2.38^{* * *}$ | $3.00^{* * *}$ |
|  | $(3.63)$ | $(2.74)$ | $(3.66)$ |
| EA+1 | $-1.77^{* *}$ | 0.62 | -0.53 |
|  | $(-2.12)$ | $(0.86)$ | $(-0.79)$ |
| EA+2 | -0.49 | 0.33 | -0.19 |
|  | $(-0.60)$ | $(0.48)$ | $(-0.30)$ |
| EA+3 | $-1.63^{* *}$ | 0.14 | 0.03 |
|  | $(-2.03)$ | $(0.20)$ | $(0.05)$ |
| EA-3 * ILLIQ | 0.26 | 0.14 | 0.06 |
|  | $(1.30)$ | $(0.84)$ | $(0.40)$ |
| EA-2 * ILLIQ | 0.13 | 0.00 | 0.09 |
|  | $(0.64)$ | $(0.02)$ | $(0.56)$ |
| EA-1 * ILLIQ | -0.10 | 0.08 | $-0.39^{* *}$ |
| EA * ILLIQ | $(-0.42)$ | $(0.44)$ | $(-2.24)$ |
| EA+1 * ILLIQ | $0.46^{*}$ | $0.39^{*}$ | $0.63^{* * *}$ |
| EA+2 * ILLIQ | $(1.92)$ | $(1.82)$ | $(3.08)$ |
| EA+3 * ILLIQ | $-0.48^{* *}$ | 0.16 | -0.05 |
| Obs. | $(-2.39)$ | $(0.91)$ | $(-0.32)$ |
|  | -0.07 | 0.12 | -0.02 |
|  | $(-0.33)$ | $(0.71)$ | $(-0.10)$ |
|  | -0.30 | 0.04 | 0.06 |
|  | $(-1.52)$ | $(0.24)$ | $(0.44)$ |
|  | $6,186,889$ | $6,186,889$ | $6,186,889$ |
|  |  |  |  |

short leg return because buying stocks is easier than shorting them (e.g., Stambaugh, Yu, and Yuan (2012)). Table 3.6 reports average returns on the long and short legs of the four gradual anomalies. While both legs contribute to the anomalies' intraday profits, the short leg drives the low overnight return and-to a lesser extent-the low return at the end of the day. According to mispricing theories, this evidence is consistent with mispricing that worsens at the open and in the last half hour of trading as the short leg becomes more overvalued.

There is evidence that mispricing can worsen at the open. Neal (1996) documents that the degree of mispricing associated with stock index arbitrage is highest at the open. Bid-ask spreads tend to be especially high at the open (McInish and Wood (1992)), which may hinder arbitrage. Furthermore, systematic buying pressure by retail investors at the open may increase mispricing, as suggested by the analysis of Berkman et al. (2012).

Intuitively, mispricing may also increase around the close. One potential explanation is that arbitrageurs tend to close their short positions at the end of the day; for example, they may not want to carry short positions overnight. ${ }^{22}$ This theory predicts a low return on the short leg of anomalies portfolios in the last half hour of trading. This explanation could be tested using intraday data on short sales.

Even though the negative overnight returns are most consistent with mispricing, the gradual returns over the trading day may still represent a compensation for risk, mispricing that gradually resolves over the day, or a combination of both. The evidence suggests that mispricing matters around market closures. But it is an open question why portfolios of stocks formed on certain characteristics earn their return gradually over the day. This pattern stands in sharp contrast with the market portfolio, which earns the bulk of its return overnight.

### 3.7 Robustness

This section examines the robustness of overnight returns to the measure of the opening price (Section 3.7.1) and the robustness of the results to the use of trade-based returns (Section 3.7.2).

### 3.7.1 Do Stocks Earn High Overnight Returns?

As shown in the main analysis, anomaly overnight returns are robust to the choice of the opening price. Overnight returns on long-only portfolios are, however, more sensitive to this choice.

Stocks are allocated into micro, small, and large value-weighted portfolios based on the $20^{\text {th }}$

[^46]Table 3.6
Intraday and overnight alphas in basis points of long $\left(\alpha_{L}\right)$ and short $\left(\alpha_{S}\right)$ portfolios. At the end of each month, stocks are split into decile portfolios based on the NYSE breakpoints of the characteristics defined in Table C.2. Portfolios are value-weighted and held for one month. A stock is required to have a price greater than $\$ 10$ at the end of the previous month and at least ten days with non-zero volume in the previous month to be included. Financial firms are excluded from portfolios based on accounting variables. NASDAQ stocks are excluded from the illiquidity portfolios. Stock returns are computed using quote midpoints. The first interval starts at 9:35 a.m. 10:00 indicates the half-hour interval that starts at 10:00 a.m. and ends before 10:30 a.m. OV indicates the overnight return. The sample is composed of NYSE, Amex, and NASDAQ common stocks from January 1,1986 , to December 31, 2015. NASDAQ stocks are included since 1993. $t$-statistics are shown in parentheses and based on Newey and West (1987) standard errors with 14 lags. *, ,*, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | OV | 9:45 | 10:00 | 10:30 | 11:00 | 11:30 | 12:00 | 12:30 | 1:00 | 1:30 | 2:00 | 2:30 | 3:00 | 3:30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Beta |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{L}$ | $\begin{gathered} -0.84^{* * *} \\ (-2.79) \end{gathered}$ | $\begin{gathered} 0.48^{* * *} \\ (3.32) \end{gathered}$ | $\begin{gathered} 0.46^{* * *} \\ (2.97) \end{gathered}$ | $\begin{gathered} -0.03 \\ (-0.23) \end{gathered}$ | $\begin{gathered} 0.11 \\ (1.09) \end{gathered}$ | $\begin{aligned} & 0.19^{*} \\ & (1.96) \end{aligned}$ | $\begin{gathered} 0.01 \\ (0.13) \end{gathered}$ | $\begin{gathered} 0.13 \\ (1.50) \end{gathered}$ | $\begin{aligned} & 0.19^{* *} \\ & (2.36) \end{aligned}$ | $\begin{gathered} -0.03 \\ (-0.30) \end{gathered}$ | $\begin{aligned} & 0.19 * * \\ & (1.99) \end{aligned}$ | $\begin{gathered} -0.03 \\ (-0.29) \end{gathered}$ | $\begin{gathered} 0.15 \\ (1.47) \end{gathered}$ | $\begin{gathered} 0.37^{* * *} \\ (3.06) \end{gathered}$ |
| $\alpha_{S}$ | 2.02*** | -0.57** | -0.44* | -0.48 ** | -0.19 | -0.40** | -0.25* | -0.16 | -0.33** | -0.18 | -0.32** | -0.28* | -0.43** | 0.34* |
|  | (3.93) | (-2.54) | (-1.82) | (-2.34) | (-1.11) | (-2.56) | (-1.77) | (-1.20) | (-2.51) | (-1.23) | (-2.21) | (-1.88) | (-2.54) | (1.73) |
| Gross Profitability |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{L}$ | -0.01 | -0.14 | -0.02 | 0.33** | 0.22* | 0.19* | 0.19* | 0.07 | 0.20** | 0.03 | 0.03 | 0.36*** | 0.25** | -0.09 |
|  | (-0.03) | (-0.94) | (-0.12) | (2.38) | (1.79) | (1.71) | (1.87) | (0.71) | (2.20) | (0.33) | (0.26) | (3.59) | (2.32) | (-0.73) |
| $\alpha_{S}$ | 0.26 | -0.18 | -0.50*** | -0.28* | -0.26** | -0.26** | -0.12 | -0.12 | -0.04 | -0.15 | 0.05 | -0.14 | 0.15 | 1.04*** |
|  | (0.85) | (-1.18) | (-2.88) | (-1.93) | (-2.08) | (-2.17) | (-1.05) | (-1.15) | (-0.39) | (-1.48) | (0.48) | (-1.36) | (1.33) | (7.70) |
| Idiosyncratic Volatility |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{L}$ | -1.18*** | 0.37*** | 0.67*** | 0.29*** | 0.34*** | 0.03 | 0.03 | -0.00 | 0.17** | 0.05 | 0.07 | 0.17* | 0.34*** | -0.26** |
|  | (-4.63) | (3.05) | (4.84) | (2.59) | (3.55) | (0.39) | (0.39) | (-0.06) | (2.25) | (0.67) | (0.79) | (1.87) | (3.60) | (-2.27) |
| $\alpha_{S}$ | 3.45*** | -0.99*** | -1.58*** | -0.88*** | -0.76*** | -0.28* | -0.29** | -0.22 | -0.22 | -0.22 | -0.10 | -0.54*** | -0.74*** | 0.99*** |
|  | (6.77) | (-4.03) | $(-6.08)$ | $(-4.21)$ | $(-4.28)$ | $(-1.76)$ | $(-2.02)$ | $(-1.62)$ | $(-1.58)$ | $(-1.47)$ | (-0.61) | $(-3.37)$ | $(-4.28)$ | (4.52) |
| Net Stock Issues |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\alpha_{L}$ | 0.33 | 0.14 | 0.27* | 0.33** |  | 0.29*** | -0.13 | 0.26*** | -0.06 | 0.08 | 0.06 | 0.27** | -0.30*** | -0.94*** |
|  | (1.04) | (0.91) | (1.66) | (2.46) | (0.49) | (2.66) | (-1.39) | (2.64) | (-0.68) | (0.81) | (0.65) | (2.52) | (-2.63) | (-6.69) |
| $\alpha_{S}$ | 1.64*** | -0.52*** | -0.95*** | -0.64*** | -0.24** | -0.24** | -0.09 | -0.09 | 0.04 | -0.25** | -0.11 | -0.22** | -0.01 | 0.77*** |
|  | (5.22) | (-3.12) | $(-5.60)$ | (-4.60) | (-1.96) | (-2.15) | (-0.89) | $(-0.87)$ | (0.45) | $(-2.50)$ | (-1.06) | (-1.99) | (-0.05) | (5.18) |

Table 3.7
Intraday (IN) and overnight (OV) average returns in basis points of aggregate portfolios for different measures of the opening price. The table also reports the correlation (corr) between overnight and intraday returns. Stocks are allocated into micro, small, and large value-weighted portfolios based on the $20^{\text {th }}$ and $50^{\text {th }}$ percentiles of NYSE market capitalization each year at the end of June. Opening prices are computed using the first trade of the day (trade), the first quote of the day after 9:35 (quote), and the volume-weighted average price in the first half-hour of trading (VWAP). Each stock is required to have a share volume greater than 1,000 in the first half-hour of trading on at least $95 \%$ of the days in a given quarter (using days for which the stock has a valid CRSP daily return). The sample is composed of NYSE, Amex, and NASDAQ common stocks. NASDAQ stocks are included since 1993. A stock is required to have a price greater than $\$ 5$ and a market capitalization greater than $\$ 100$ million at the end of the previous quarter to be included. Standard $t$-statistics are shown in parentheses. *, **, and ${ }^{* * *}$ denote significance at the $10 \%, 5 \%$, and $1 \%$ level.

|  | $1986-1992$ |  |  | $1993-2004$ |  |  |  | $2005-2015$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | OV | IN | corr | OV | IN | corr | OV | IN | corr |  |
| Large |  |  |  |  |  |  |  |  |  |  |
| trade | $3.30^{* *}$ | $3.43^{*}$ | 0.06 | $5.83^{* * *}$ | -1.07 | -0.02 | $2.90^{* *}$ | 0.68 | 0.05 |  |
|  | $(2.18)$ | $(1.66)$ |  | $(5.74)$ | $(-0.62)$ |  | $(2.20)$ | $(0.36)$ |  |  |
| quote | $5.03^{* * *}$ | 1.69 | -0.02 | $5.47^{* * *}$ | -0.76 | -0.02 | $2.82^{* *}$ | 0.74 | 0.05 |  |
|  | $(3.19)$ | $(0.84)$ |  | $(5.22)$ | $(-0.45)$ |  | $(2.12)$ | $(0.40)$ |  |  |
| VWAP | 0.69 | $6.12^{* * *}$ | -0.06 | $3.25^{* * *}$ | 1.59 | 0.03 | 2.02 | 1.45 | 0.13 |  |
|  | $(0.42)$ | $(2.89)$ |  | $(3.11)$ | $(0.94)$ |  | $(1.55)$ | $(0.81)$ |  |  |
| Small |  |  |  |  |  |  |  |  |  |  |
| trade | $5.08^{* * *}$ | 1.08 | 0.12 | $8.75^{* * *}$ | $-3.26^{*}$ | 0.12 | $2.81^{* *}$ | 1.61 | 0.08 |  |
|  | $(3.89)$ | $(0.54)$ |  | $(10.17)$ | $(-1.77)$ |  | $(2.07)$ | $(0.69)$ |  |  |
| quote | $6.47^{* * *}$ | -0.30 | 0.12 | $7.71^{* * *}$ | -2.26 | 0.08 | 2.30 | 2.11 | 0.03 |  |
|  | $(4.49)$ | $(-0.15)$ |  | $(7.79)$ | $(-1.27)$ |  | $(1.55)$ | $(0.93)$ |  |  |
| VWAP | 1.65 | $4.27^{* *}$ | 0.06 | $4.78^{* * *}$ | 0.40 | 0.07 | 0.21 | $3.94^{*}$ | 0.12 |  |
|  | $(1.06)$ | $(2.09)$ |  | $(4.41)$ | $(0.22)$ |  | $(0.14)$ | $(1.79)$ |  |  |
| Micro |  |  |  |  |  |  |  |  |  |  |
| trade | $8.26^{* * *}$ | -2.62 | 0.06 | $11.99^{* * *}$ | -3.59 | 0.06 | $4.70^{* * *}$ | 1.00 | 0.10 |  |
|  | $(4.94)$ | $(-1.06)$ |  | $(10.76)$ | $(-1.55)$ |  | $(3.34)$ | $(0.38)$ |  |  |
| quote | $8.98^{* * *}$ | -3.33 | 0.06 | $11.51^{* * *}$ | -3.25 | 0.03 | $4.53^{* * *}$ | 1.13 | 0.07 |  |
|  | $(4.97)$ | $(-1.38)$ |  | $(9.01)$ | $(-1.46)$ |  | $(3.05)$ | $(0.44)$ |  |  |
| VWAP | 3.12 | 2.83 | -0.00 | $8.66^{* * *}$ | -1.99 | 0.02 | 0.83 | 3.28 | 0.13 |  |
|  | $(1.58)$ | $(1.18)$ |  | $(5.74)$ | $(-0.82)$ |  | $(0.55)$ | $(1.32)$ |  |  |

and $50^{\text {th }}$ percentiles of NYSE market capitalization each year at the end of June. Table 3.7 reports intraday and overnight average returns for each of the portfolios using three measures of the opening price: quote midpoints, trade prices, and VWAP as described in Section 3.3. To make an exact comparison, each portfolio in Table 3.7 has the same composition as the VWAP portfolio. The results for the micro portfolio over the 1986-1992 period should be taken with a grain of salt since this portfolio holds on average only 50 stocks during this period.

The choice of the opening price matters. Overnight returns are lowest using VWAP. The differences are particularly marked in the first part of the sample and for the small and micro stocks portfolios. Midquote and trade prices yield negative intraday returns for the large stocks portfolio between 1993 and 2004, while VWAP yield positive intraday returns.

Cliff, Cooper, and Gulen (2008) claim that the U.S. equity premium over 1993 to 2006 is entirely earned overnight. Table 3.7 shows that this statement depends on the definition of the opening price. A substantial fraction of overnight returns computed from trades and quote midpoints can be explained by short-term price movements in the first half hour of trading. This evidence indicates abnormally high prices at the open that revert over the following half hour of trading.

### 3.7.2 Trade-Based Returns

Intraday and overnight average trade-based returns on anomalies are reported in the Internet Appendix. In general, returns computed from trade prices give similar results than returns computed from quote midpoints. As hinted by the evidence in Section 3.7.1, most differences occur in the old part of the sample and are due to the inclusion of the first fifteen minutes of trading when computing trade-based returns. Intraday returns on anomalies do not differ much. The results in the main analysis are therefore robust. In fact, return patterns around market closures tend to be more pronounced with trade-based returns.

### 3.8 Conclusion

Portfolios of stocks formed on different characteristics exhibit strikingly different intraday return patterns. This evidence is novel and helps understand the economic drivers behind cross-sectional variation in stock returns. Portfolios of stocks formed on different characteristics exhibit strikingly different intraday return patterns. These patterns are robust and differ from two natural benchmarks: the market portfolio and portfolios based on random strategies.

Market closures have a large impact on stock returns. Small and illiquid stocks earn large returns in the last half hour of trading. In contrast, large stocks tend to perform poorly at the end of the day. Therefore, any explanation that rejects the role of market closures should be able to provide an alternative as to why a large fraction of size returns accrue in the last half hour of trading

High end-of-the-day returns are difficult to explain with standard theories of size and illiquidity. I find evidence consistent with information asymmetry and institutional effects resulting in high end-of-the-day returns across stocks. However, the empirical evidence is inconclusive relative to why small and illiquid stocks behave differently than large stocks. In this respect, more work is needed to understand liquidity around the close and whether variations in
liquidity can explain cross-sectional differences in average intraday returns. Such work is also important from a trade execution point of view.

Overall, this paper provides new evidence on the determinants of cross-sectional variation in stock returns. But it remains an open question to which extent cross-sectional variation in average intraday stock returns can explain cross-sectional variation in lower frequency returns. As pointed out by Chordia, Roll, and Subrahmanyam (2005) in a study of market efficiency, it is puzzling how extant empirical evidence goes from apparent weak-form efficiency at very short horizons to predictability at long horizons. This paper provides an intermediate step that can help bridge this gap. It appears crucial to consider separately the different periods of the day. For instance, specific patterns around the close may be masked by noise over the rest of the trading day.

## Conclusion

This thesis contributes to research on cross-sectional variation in stock returns, market efficiency, and liquidity.

The first chapter studies a dynamic equilibrium model in which some investors readjust their portfolio infrequently. I show that trading by investors with heterogeneous rebalancing horizons can generate return autocorrelation and seasonality consistent with empirical evidence at different frequencies. This chapter provides a first step in explaining why risk premia can be seasonal.

The second chapter documents that well-known anomalies exhibit strong January and month-of-the-quarter seasonalities. These seasonalities are in general robust to controlling for size and tax-loss selling potential. In addition, small stocks earn an abnormally high average return on the last day of the quarter, which significantly affects size, idiosyncratic volatility, and illiquidity portfolios. This chapter shows that taking into account such seasonalities is important when studying the cross-section of stock returns.

The third chapter contributes to extant literature by documenting substantial cross-sectional variation in average stock returns over the trading day and overnight. This evidence is novel and helps understand the economic drivers behind cross-sectional variation in stock returns. Portfolios of stocks formed on different characteristics exhibit strikingly different intraday return patterns. These patterns are robust and differ from natural benchmarks. Market closures have a large impact on stock returns. I find evidence consistent with information asymmetry and liquidity shocks resulting in high end-of-the-day returns across stocks.

To come back to the question raised in the introduction-why do some stocks perform better than others?--this thesis finds that investor rebalancing and institutional effects are important drivers of cross-sectional variation in stock returns. As discussed below, much more work remains to be done in this area.

## Outline for Further Research

A study of how the intraday return patterns taken as stylized facts in the first chapter of this thesis evolve over time would be of high interest. With the rise of high-frequency trading, such predictability patterns may be expected to become weaker as they are arbitraged away. At the same time, index investing has grown tremendously in recent years. This increase could lead to more pronounced time-of-the-day effects. Indeed, many institutional investors rely on closing prices as benchmarks. For example, open-end mutual funds use closing prices to set the net asset value at which their shares can be bought and sold each day. Similarly, leveraged exchange-traded funds tend to trade heavily towards the end of the day. This increase in passive investing appears to have led to a dramatic increase in trading volume over the last thirty minutes of trading in recent years.

As a result, it is unclear whether intraday return predictability patterns have become stronger or weaker over time. Further work in this area is necessary to understand trends in market efficiency and the role of price pressures in explaining excess return volatility. In particular, more work is needed to estimate the consequences of trading concentration at the close for the average retail investor. A deep understanding of liquidity around the close and at other times of the day is also important from a trade execution point of view. The length and granularity of the data set constructed in the third chapter of this thesis can potentially shed light on these important questions.

Relative to standard asset pricing research, more work is needed to understand the properties of intraday and overnight returns. I have found preliminary evidence that the canonical asset pricing model-the market model—explains better overnight returns than intraday returns (see also Bollerslev, Li, and Todorov (2016)). An analysis of why such a difference exists can potentially help develop asset pricing models with increased power to explain the cross-section of stock returns.

Overall, this thesis provides new evidence on the determinants of cross-sectional variation in stock returns. But it remains an open question to which extent cross-sectional variation in average intraday stock returns can explain cross-sectional variation in lower frequency returns. As pointed out by Chordia, Roll, and Subrahmanyam (2005) in a study of market efficiency, it is puzzling how extant empirical evidence goes from apparent weak-form efficiency at very short horizons to predictability at long horizons.

## A Appendix to Chapter 1

## A. 1 Proofs

To derive Proposition 1, I first conjecture that asset prices and infrequent traders' demands are linear in the state variables (defined below). Using this conjecture, I derive frequent traders' demands (Lemma 1) and infrequents traders' demands (Lemma 3). Finally, I verify the initial conjectures by plugging the demands into the market-clearing condition (Proposition 1). The dividend and liquidity trading mean vectors are given by $\bar{D}$ and $\bar{\theta}$ in the proofs and are set to $0_{N \times 1}$ in the analysis.

Derivation of the state variables process: I follow Duffie (2010) and focus on linear equilibria. Let the price and infrequent traders' demand vectors be given by

$$
\begin{align*}
& P_{t}=A Y_{t}, \quad \text { and } \\
& X_{t}^{I}=B Y_{t}, \tag{A.1}
\end{align*}
$$

where $A$ and $B$ are constant parameter matrices of dimensions $N \times 1+(2+k) N$, and $Y_{t}$ is the $(1+(2+k) N)$-dimensional vector of state variables $\left(\begin{array}{cccccc}1 & \theta_{t}^{\prime} & D_{t}^{\prime} & X_{t-1}^{I \prime} & \ldots & X_{t-k}^{I \prime}\end{array}\right)^{\prime}$. Let $I_{N}$ $\left(0_{N}\right)$ denote the identity (zero) matrix of dimension $N \times N$. One can write $Y_{t+1}=A_{Y} Y_{t}+B_{Y} \epsilon_{t+1}$, where $\epsilon_{t} \equiv\left(\epsilon_{t}^{\theta \prime} \quad \epsilon_{t}^{D \prime}\right)^{\prime} \sim \mathscr{N}\left(0_{2 N \times 1}, \Sigma_{Y}\right)$ is the vector of innovations with variance-covariance

## Appendix A. Appendix to Chapter 1

matrix $\Sigma_{Y}=\left[\begin{array}{cc}\Sigma_{\theta} & 0_{N} \\ 0_{N} & \Sigma_{D}\end{array}\right]$, and the matrices $A_{Y}$ and $B_{Y}$ are given by

$$
A_{Y}=\left[\right] \text { and } B_{Y}=\left[\begin{array}{cc}
0_{1 \times N} & 0_{1 \times N} \\
I_{N} & 0_{N} \\
0_{N} & I_{N} \\
0_{k N \times N} & 0_{k N \times N}
\end{array}\right] .
$$

The dynamics of $Y_{t}$ imply that

$$
\begin{equation*}
Y_{t+j}=A_{Y}^{j} Y_{t}+\sum_{i=1}^{j} A_{Y}^{j-i} B_{Y} \epsilon_{t+i}, \quad j \geq 1 \tag{A.2}
\end{equation*}
$$

To simplify notation, let $A_{Y}^{0}=I_{N}$. I also introduce the following matrices for convenience: $\varphi_{D}$, $\varphi_{\theta}$, and $\varphi_{x}$, which are defined such that $\theta_{t}=\varphi_{\theta} Y_{t}, D_{t}=\varphi_{D} Y_{t},\left(X_{t-1}^{I \prime} \ldots X_{t-k}^{I \prime}\right)^{\prime}=\varphi_{X} Y_{t}$, and $\varphi_{\bar{S}} Y_{t}=\bar{S}$.

Define $Q_{t+1} \equiv P_{t+1}+D_{t+1}-R P_{t}$, the vector of excess dollar returns. It follows that $Q_{t+1}=$ $A_{Q} Y_{t}+B_{Q} \epsilon_{t+1}$, where $A_{Q} \equiv\left(A+\varphi_{D}\right) A_{Y}-R A$ and $B_{Q} \equiv\left(A+\varphi_{D}\right) B_{Y}$. Finally, denote the cumulative payoff from $t$ (ex-dividend) to $t+k+1$ as $T_{t, t+k+1} \equiv P_{t+k+1}+\sum_{j=1}^{k+1} R^{k+1-j} D_{t+j}$.

Lemma 1. Given the initial conjectures (A.1), the demands of frequent traders with remaining horizon $h-j(0 \leq j<h)$ at date $t$ are given by

$$
\begin{equation*}
X_{t, j}^{F}=\frac{1}{\alpha_{j+1}} F_{j+1} Y_{t} \tag{A.3}
\end{equation*}
$$

where $\quad F_{j+1}=\left(B_{Q} \Xi_{j+1} B_{Q}^{\prime}\right)^{-1}\left(A_{Q}-B_{Q} \Xi_{j+1} B_{Y}^{\prime} U_{j+1}^{\prime} A_{Y}\right) \quad$ and $\quad \alpha_{j}=R \alpha_{j+1}$.
The coefficients are solved recursively starting from the conditions $\alpha_{h}=\gamma_{F}$ and $U_{h}=0_{1+2 N+k N}$. The matrices $\Xi_{j+1}$ and $U_{j+1}(0 \leq j<h)$ are defined below.

Proof of Lemma 1: The proof parallels the derivations of He and Wang (1995) in a nonstationary setup. Let $j$ be the age of the investor $(0 \leq j<h)$ and $J\left(W_{t}, Y_{t}, j\right)$ be the value function. The Bellman optimization problem for an investor aged $j$ at date $t$ is

$$
\begin{equation*}
J\left(W_{t}, Y_{t}, j\right)=\max _{X_{t, j}} \mathbb{E}_{t}\left[J\left(W_{t+1}, Y_{t+1}, j+1\right)\right] \tag{A.4}
\end{equation*}
$$

where $W_{t+1}=X_{t, j}^{\prime} Q_{t+1}+R W_{t}$ and $J\left(W_{t}, Y_{t}, h\right)=-e^{-\gamma_{F} W_{t}}$. Conjecture that $J\left(W_{t+1}, Y_{t+1}, j+1\right)=$ $-e^{-\alpha_{j+1} W_{t+1}-\frac{1}{2} Y_{t+1}^{\prime} U_{j+1} Y_{t+1}}$. It then follows that

$$
\begin{align*}
\mathbb{E}_{t}\left[J\left(W_{t+1}, Y_{t+1}, j+1\right)\right]= & -e^{-\alpha_{j+1}\left(R W_{t}+X_{t, j}^{\prime} A_{Q} Y_{t}\right)} \mathbb{E}_{t}\left[e^{-\alpha_{j+1} X_{t, j}^{\prime} B_{Q} \epsilon_{t+1}-\frac{1}{2} Y_{t+1}^{\prime} U_{j+1} Y_{t+1}}\right] \\
= & -e^{-\alpha_{j+1}\left(R W_{t}+X_{t, j}^{\prime} A_{Q} Y_{t}\right)-\frac{1}{2} Y_{t}^{\prime} A_{Y}^{\prime} U_{j+1} A_{Y} Y_{t}} \\
& \mathbb{E}_{t}\left[e^{\left(-\alpha_{j+1} X_{t, j}^{\prime} B_{Q}-Y_{t}^{\prime} A_{Y}^{\prime} U_{j+1} B_{Y}\right) \epsilon_{t+1}-\frac{1}{2} \epsilon_{t+1}^{\prime} B_{Y}^{\prime} U_{j+1} B_{Y} \epsilon_{t+1}}\right] . \tag{A.5}
\end{align*}
$$

Using the multivariate normality of $\epsilon_{t+1}$ (see, for instance, Vives (2010)) gives

$$
\begin{align*}
\mathbb{E}_{t}
\end{align*}\left[e^{\left(-\alpha_{j+1} X_{t, j}^{\prime} B_{Q}-Y_{t}^{\prime} A_{Y}^{\prime} U_{j+1} B_{Y}\right) \epsilon_{t+1}-\frac{1}{2} \epsilon_{t+1}^{\prime} B_{Y}^{\prime} U_{j+1} B_{Y} \epsilon_{t+1}}\right]=\left|I+\Sigma_{Y} B_{Y}^{\prime} U_{j+1} B_{Y}\right|^{-\frac{1}{2}} . ~ e^{\frac{1}{2}\left(-\alpha_{j+1} X_{t, j}^{\prime} B_{Q}-Y_{t}^{\prime} A_{Y}^{\prime} U_{j+1} B_{Y}\right)\left(I+\Sigma_{Y} B_{Y}^{\prime} U_{j+1} B_{Y}\right)^{-1} \Sigma_{Y}\left(-\alpha_{j+1} B_{Q}^{\prime} X_{t, j}-B_{Y}^{\prime} U_{j+1}^{\prime} A_{Y} Y_{t}\right)} .
$$

Define $\rho_{j+1} \equiv\left|I+\Sigma_{Y} B_{Y}^{\prime} U_{j+1} B_{Y}\right|^{-\frac{1}{2}}$ and $\Xi_{j+1} \equiv\left(\Sigma_{Y}^{-1}+B_{Y}^{\prime} U_{j+1} B_{Y}\right)^{-1}$. Using the previous results, the first-order condition is

$$
\begin{equation*}
A_{Q} Y_{t}-\alpha_{j+1} B_{Q} \Xi_{j+1} B_{Q}^{\prime} X_{t, j}-B_{Q} \Xi_{j+1} B_{Y}^{\prime} U_{j+1}^{\prime} A_{Y} Y_{t}=0, \tag{A.7}
\end{equation*}
$$

which gives (A.3). The second-order condition is satisfied if $-\alpha_{j+1} B_{Q} \Xi_{j+1} B_{Q}^{\prime}$ is negative definite. Last, I verify the conjecture for the value function. Plugging the optimal demand expression into the optimization problem gives

$$
\begin{equation*}
\mathbb{E}_{t}\left[J\left(W_{t+1}, Y_{t+1}, j+1\right)\right]=-\rho_{j+1} e^{-\alpha_{j+1} R W_{t}-\frac{1}{2} Y_{t}^{\prime} M_{j+1} Y_{t}} \tag{A.8}
\end{equation*}
$$

where $M_{j+1} \equiv A_{Y}^{\prime} U_{j+1} A_{Y}+F_{j+1}^{\prime} B_{Q} \Xi_{j+1} B_{Q}^{\prime} F_{j+1}-A_{Y}^{\prime} U_{j+1} B_{Y} \Xi_{j+1} B_{Y}^{\prime} U_{j+1}^{\prime} A_{Y}$. Matching terms with the conjectured value function yields $\alpha_{j}=R \alpha_{j+1}$ and $U_{j}=M_{j+1}-2 \ln \left(\rho_{j+1}\right) I_{11}$, where $I_{11}$ is a matrix whose first element is one and all others are zero. The terminal condition gives $\alpha_{h}=\gamma_{F}$ and $U_{h}=0_{1+2 N+k N}$. The coefficients can then be solved recursively.

The next lemma is needed to derive infrequent traders' demands.

Lemma 2. Given the initial conjectures (A.1), the equilibrium stationary $j$-period cumulative payoff variance, $\operatorname{Var}_{t}\left[T_{t, t+j}\right]$, is a constant matrix $\Sigma_{j}$ given by $($ for $j \geq 1)$

$$
\begin{equation*}
\sum_{i=1}^{j}\left(A A_{Y}^{j-i}+\frac{R^{j-i+1}-a_{D}^{j-i+1}}{R-a_{D}} \varphi_{D}\right) B_{Y} \Sigma_{Y} B_{Y}^{\prime}\left(A A_{Y}^{j-i}+\frac{R^{j-i+1}-a_{D}^{j-i+1}}{R-a_{D}} \varphi_{D}\right)^{\prime} \tag{A.9}
\end{equation*}
$$

## Appendix A. Appendix to Chapter 1

Proof of Lemma 2: Since $T_{t, t+j}=A Y_{t+j}+\sum_{i=1}^{j} R^{j-i} D_{t+i}$, it follows (using (A.2)) that

$$
\begin{equation*}
\operatorname{Var}_{t}\left[T_{t, t+j}\right]=\operatorname{Var}\left[A \sum_{i=1}^{j} A_{Y}^{j-i} B_{Y} \epsilon_{t+i}+\sum_{i=1}^{j} R^{j-i}\left(\sum_{s=1}^{i-1} a_{D}^{s} \epsilon_{t+i-s}^{D}+\epsilon_{t+i}^{D}\right)\right] \tag{A.10}
\end{equation*}
$$

To compute $\operatorname{Var}_{t}\left[T_{t, t+j}\right]$, note that

$$
\begin{equation*}
\sum_{i=1}^{j} R^{j-i}\left(\sum_{s=1}^{i-1} a_{D}^{s} \epsilon_{t+i-s}^{D}+\epsilon_{t+i}^{D}\right)=\sum_{i=1}^{j} g\left(R, a_{D}, j-i\right) \varphi_{D} B_{Y} \epsilon_{t+i} \tag{A.11}
\end{equation*}
$$

where the function $g\left(R, a_{D}, j-i\right)$ is defined recursively by $g\left(R, a_{D}, i\right)=g\left(R, a_{D}, i-1\right) R+a_{D}^{i}$, $i \geq 1$, and $g\left(R, a_{D}, 0\right)=1$. By induction, $g\left(R, a_{D}, i\right)=\frac{R^{i+1}-a_{D}^{i+1}}{R-a_{D}}$. Plugging this function into the conditional variance expression (A.10) gives

$$
\begin{equation*}
\Sigma_{j}=\operatorname{Var}\left[\sum_{i=1}^{j}\left(A A_{Y}^{j-i}+\frac{R^{j-i+1}-a_{D}^{j-i+1}}{R-a_{D}} \varphi_{D}\right) B_{Y} \epsilon_{t+i}\right] \tag{A.12}
\end{equation*}
$$

Since the vectors $\epsilon_{t+i}$ in (A.12) are independent of each other, the lemma follows.

Lemma 3. Given the initial conjectures (A.1), infrequent traders' demands are given by

$$
\begin{equation*}
X_{t}^{I}=\frac{1}{\gamma_{I}} \Sigma_{k+1}^{-1} \sum_{j=0}^{k} R^{k-j} A_{Q} A_{Y}^{j} Y_{t} \tag{A.13}
\end{equation*}
$$

where $\Sigma_{k+1} \equiv \operatorname{Var}_{t}\left[T_{t, t+k+1}\right]$ is the equilibrium stationary $(k+1)$-period payoff variance and is shown to be constant in Lemma 2.

Proof of Lemma 3: From the optimization problem (1.6) and given that prices are normally distributed under the conjecture (A.1), infrequent traders' demands are

$$
\begin{align*}
X_{t}^{I} & =\frac{1}{\gamma_{I}} \Sigma_{k+1}^{-1}\left(\mathbb{E}_{t}\left[P_{t+k+1}+\sum_{j=1}^{k+1} R^{k+1-j} D_{t+j}\right]-R^{k+1} P_{t}\right) \\
& =\frac{1}{\gamma_{I}} \Sigma_{k+1}^{-1} \sum_{j=0}^{k} R^{k-j} \mathbb{E}_{t}\left[Q_{t+j+1}\right] . \tag{A.14}
\end{align*}
$$

Using (A.2), (A.14) reduces to (A.13). The vector of demands is linear in the state variables, as conjectured.

Proof of Proposition 1: Replacing the demands (A.13) and (A.3) in the market-clearing condition (1.8) with $q_{c(t)}=\frac{q}{k+1}$ and rearranging terms yields the following system of fixed point
equations:

$$
\begin{array}{r}
\frac{q / \gamma_{I}}{k+1} \Sigma_{k+1}^{-1}\left(\sum_{j=0}^{k} R^{k-j} A_{Q} A_{Y}^{j}\right)+\frac{1-q}{h}\left(\sum_{j=0}^{h-1} \frac{1}{\alpha_{j+1}} F_{j+1}\right)-\varphi_{\theta}-\varphi_{\bar{S}}+\frac{q}{k+1} \varphi_{X}=0, \\
\frac{1}{\gamma_{I}} \Sigma_{k+1}^{-1}\left(\sum_{j=0}^{k} R^{k-j} A_{Q} A_{Y}^{j}\right)-B=0 . \tag{A.16}
\end{array}
$$

A linear REE exists if this system of equations admits a solution. Using the expressions for $A_{Q}$ and $A_{Y}$, the dividend coefficient matrix in (A.16) can be rewritten as

$$
\begin{equation*}
\left(a_{D}\left(P_{D}+I_{N}\right)-R P_{D}\right)\left(\sum_{j=1}^{k} R^{k-j} a_{D}^{j}+R^{k}\right)=0_{N} \tag{A.17}
\end{equation*}
$$

where the equality follows from the fact that agents do not trade on dividends (no-trade theorem). Hence, $P_{D}=\frac{a_{D}}{R-a_{D}} I_{N}$.

Proof of Corollary 1: For simplicity, let $\bar{\theta}=0, \bar{D}=0$, and $\bar{S}=0$. This implies that $\bar{P}=0$. When $q=1$, the market-clearing condition becomes

$$
\begin{equation*}
\frac{1}{k+1} X_{t}^{I}=\theta_{t}-\frac{1}{k+1} \sum_{i=1}^{k} X_{t-i}^{I} \tag{A.18}
\end{equation*}
$$

This gives the $B$ coefficients in (A.1). Plugging infrequent traders' demands (A.14) into the market-clearing condition yields

$$
\begin{equation*}
R^{k+1} P_{t}=\mathbb{E}_{t}\left[T_{t, t+k+1}\right]-\gamma_{I}(k+1) \Sigma_{k+1} \theta_{t}+\gamma_{I} \Sigma_{k+1} \sum_{i=1}^{k} X_{t-i}^{I} \tag{A.19}
\end{equation*}
$$

where $T_{t, t+k+1}=P_{\theta} \theta_{t+k+1}+\left(P_{D}+I_{N}\right) D_{t+k+1}+\sum_{i=1}^{k} R^{k+1-i} D_{t+i}+\sum_{i=1}^{k} P_{X_{i}} X_{t+k+1-i}^{I}$. Using (1.4) and matching terms for the dividends in (A.19) gives $P_{D}=\frac{a_{D}}{R-a_{D}} I_{N}$.

The $(t+k)$-demand of an infrequent trader equals her $(t-1)$-demand plus the additional liquidity trading that takes place between $t+k-1$ and $t+k$ :

$$
\left[\begin{array}{c}
X_{t+k}^{I}  \tag{A.20}\\
\vdots \\
X_{t+1}^{I}
\end{array}\right]=\left[\begin{array}{c}
X_{t-1}^{I} \\
\vdots \\
X_{t-k}^{I}
\end{array}\right]+(k+1)\left[\begin{array}{c}
\theta_{t+k}-\theta_{t+k-1} \\
\theta_{t+k-1}-\theta_{t+k-2} \\
\vdots \\
\theta_{t+1}-\theta_{t}
\end{array}\right]
$$

This equation follows from the market-clearing condition (A.18) and the fact that agents trade only every $k+1$ periods. Using this result and (1.7), it follows that $\mathbb{E}_{t}\left[X_{t+i}^{I}\right]=X_{t+i-(k+1)}^{I}-(k+$ 1) $a_{\theta}^{i-1}\left(1-a_{\theta}\right) \theta_{t}$. Finally, using the previous results and matching terms for the liquidity shocks

## Appendix A. Appendix to Chapter 1

and lagged demands in (A.19) gives

$$
\begin{align*}
& R^{k+1} P_{\theta}=a_{\theta}^{k+1} P_{\theta}-\gamma_{I}(k+1) \Sigma_{k+1}-(k+1)\left(1-a_{\theta}\right)\left(\sum_{i=1}^{k-1} a_{\theta}^{k-i} P_{X_{i}}+P_{X_{k}}\right), \text { and }  \tag{A.21}\\
& R^{k+1} P_{X_{i}}=P_{X_{i}}+\gamma_{I} \Sigma_{k+1}, \quad i=1, \ldots, k \tag{A.22}
\end{align*}
$$

Equation (1.11) follows from (A.21), (A.22), and ( $1-a_{\theta}$ ) $\left(\sum_{i=1}^{k-1} a_{\theta}^{k-i}+1\right)=1-a_{\theta}^{k}$.
To prove the second part of the corollary, I show that $\Sigma_{k+1}=\operatorname{Var}_{t}\left[T_{t, t+k+1}\right]$ defines a system of quadratic matrix equations that admits $2^{N}$ solutions under some parametric condition. Since $P_{X_{1}}=P_{X_{2}}=\ldots=P_{X_{k}} \equiv P_{X}$, it follows that $\sum_{i=1}^{k} P_{X_{i}} X_{t+k+1-i}^{I}=P_{X}\left(\sum_{i=1}^{k} X_{t-i}^{I}+(k+1)\left(\theta_{t+k}-\theta_{t}\right)\right)$. Plugging this formula into the expression for $T_{t, t+k+1}$ and using (1.11) to replace $P_{X}$ with $P_{\theta}$ gives

$$
\begin{equation*}
\Sigma_{k+1}=\operatorname{Var}_{t}\left[\frac{a_{D}}{R-a_{D}} D_{t+k+1}+\sum_{i=1}^{k+1} R^{k+1-i} D_{t+i}+P_{\theta} \epsilon_{t+k+1}^{\theta}-\frac{R^{k+1}\left(1-a_{\theta}\right)}{R^{k+1}-a_{\theta}^{k}} P_{\theta} \theta_{t+k}\right] \tag{A.23}
\end{equation*}
$$

Since dividends and liquidity shocks are uncorrelated, I can focus on both terms separately. Tedious computations show that

$$
\begin{equation*}
\operatorname{Var}_{t}\left[\frac{a_{D}}{R-a_{D}} D_{t+k+1}+\sum_{i=1}^{k+1} R^{k+1-i} D_{t+i}\right]=\left(\frac{R}{R-a_{D}}\right)^{2}\left(\sum_{i=0}^{k} R^{2 i}\right) \Sigma_{D} \tag{A.24}
\end{equation*}
$$

For liquidity shocks, tedious computations show that

$$
\begin{equation*}
\operatorname{Var}_{t}\left[P_{\theta} \epsilon_{t+k+1}^{\theta}-\frac{R^{k+1}\left(1-a_{\theta}\right)}{R^{k+1}-a_{\theta}^{k}} P_{\theta} \theta_{t+k}\right]=\left(1+\left(\frac{R^{k+1}\left(1-a_{\theta}^{k}\right)}{R^{k+1}-a_{\theta}^{k}}\right)^{2}\right) P_{\theta} \Sigma_{\theta} P_{\theta}^{\prime} \tag{A.25}
\end{equation*}
$$

This last expression implies that $\Sigma_{k+1}$ defines a quadratic matrix equation for $P_{\theta}$. Finally, using (A.21) and (A.22), simplify terms to get $\Sigma_{k+1}+\frac{\left(R^{k+1}-1\right)\left(R^{k+1}-a_{\theta}^{k+1}\right)}{\gamma_{I}(k+1)\left(R^{k+1}-a_{\theta}^{k}\right)} P_{\theta}=0$. Replacing $\Sigma_{k+1}$ with (A.24) and (A.25) gives the following quadratic matrix equation for $P_{\theta}$ :

$$
\begin{equation*}
\left(1+\left(\frac{R^{k+1}\left(1-a_{\theta}^{k}\right)}{R^{k+1}-a_{\theta}^{k}}\right)^{2}\right) P_{\theta} \Sigma_{\theta} P_{\theta}^{\prime}+\frac{\left(R^{k+1}-1\right)\left(R^{k+1}-a_{\theta}^{k+1}\right)}{\gamma_{I}(k+1)\left(R^{k+1}-a_{\theta}^{k}\right)} P_{\theta}+\left(\frac{R}{R-a_{D}}\right)^{2}\left(\sum_{i=0}^{k} R^{2 i}\right) \Sigma_{D}=0 \tag{A.26}
\end{equation*}
$$

It can be shown that this quadratic matrix equation admits $2^{N}$ solutions if $\frac{1}{4}\left(\frac{\left(R^{k+1}-1\right)\left(R^{k+1}-a_{\theta}^{k+1}\right)}{\gamma_{I}(k+1)\left(R^{k+1}-a_{\theta}^{k}\right)}\right)^{2} I_{N^{-}}$ $\left(\frac{R}{R-a_{D}}\right)^{2}\left(1+\sum_{i=1}^{k} R^{2 i}\right)\left(1+\left(\frac{R^{k+1}\left(1-a_{\theta}^{k}\right)}{R^{k+1}-a_{\theta}^{k}}\right)^{2}\right) \Sigma_{\theta}^{\frac{1}{2}} \Sigma_{D} \Sigma_{\theta}^{\frac{1}{2}}$ is positive definite (see the Internet Appendix
for details)
To prove Proposition 2, I use the following lemma.
Lemma 4. Let $k=1$ and $h=1$. In the single-asset economy with $0<q<1$, if $P_{\theta}<0$ and $P_{X}>0$, then infrequent traders absorb part of the liquidity shocks (i.e., $X_{\theta}>0$ ) in any equilibrium.

Proof of Lemma 4: Infrequent traders' demand at time $t$ is linear in the state variables and can be written as $X_{t}^{I}=\bar{X}_{I}+X_{\theta} \theta_{t}+X_{X} X_{t-1}^{I}$, where $\bar{X}_{I}, X_{\theta}$, and $X_{X}$ are constant parameters. When $k=1, h=1$, and $N=1$, the following four equations hold:

$$
\begin{align*}
& \frac{q}{2} X_{\theta}+(1-q) \gamma_{F}^{-1} \Sigma_{1}^{-1}\left(\left(a_{\theta}-R\right) P_{\theta}+P_{X} X_{\theta}\right)=1  \tag{A.27}\\
& \frac{q}{2} X_{X}+(1-q) \gamma_{F}^{-1} \Sigma_{1}^{-1} P_{X}\left(X_{X}-R\right)=-\frac{q}{2}  \tag{A.28}\\
& X_{\theta}=\gamma_{I}^{-1} \Sigma_{2}^{-1}\left(a_{\theta}\left(a_{\theta} P_{\theta}+P_{X} X_{\theta}\right)+P_{X} X_{X} X_{\theta}-R^{2} P_{\theta}\right), \quad \text { and }  \tag{A.29}\\
& X_{X}=\gamma_{I}^{-1} \Sigma_{2}^{-1}\left(P_{X} X_{X}^{2}-R^{2} P_{X}\right) \tag{A.30}
\end{align*}
$$

Equations (A.27) and (A.28) are obtained from the market-clearing condition. Equations (A.29) and (A.30) follow from the optimization problem of infrequent traders. Since I assume that $P_{\theta}<0$ and $P_{X}>0$, equation (A.28) implies that $X_{X}<R$. But then equation (A.30) requires $-R<X_{X}<0$.

Next, assume that $X_{\theta} \leq 0$. Equation (A.27) then implies that $\left(a_{\theta}-R\right) P_{\theta}+P_{X} X_{\theta}>0$, which is equivalent to $R P_{\theta}<a_{\theta} P_{\theta}+P_{X} X_{\theta}<0$. Moreover, equation (A.29) implies that $a_{\theta}\left(a_{\theta} P_{\theta}+P_{X} X_{\theta}\right)+$ $P_{X} X_{X} X_{\theta}-R^{2} P_{\theta}<0$. Combining the last two conditions gives $a_{\theta} R P_{\theta}+P_{X} X_{X} X_{\theta}-R^{2} P_{\theta}<0$. This is a contradiction since $P_{X} X_{X} X_{\theta}>0$ under our assumption and $a_{\theta} R P_{\theta}-R^{2} P_{\theta}>0$. Therefore, if $P_{\theta}<0$ and $P_{X}>0$, then $X_{\theta}>0$ in any equilibrium.

Proof of Proposition 2: Using the notation of Lemma 4, one has $X_{t}^{I}=\bar{X}_{I}+X_{\theta} \epsilon_{t}^{\theta}+X_{X} X_{t-1}^{I}$. It then follows that

$$
\begin{align*}
\operatorname{Cov}\left[Q_{t+1}, Q_{t}\right]= & P_{\theta}^{2} \operatorname{Cov}\left[\epsilon_{t+1}^{\theta}-R \epsilon_{t}^{\theta}, \epsilon_{t}^{\theta}-R \epsilon_{t-1}^{\theta}\right]+P_{X} P_{\theta} \operatorname{Cov}\left[X_{t}^{I}-R X_{t-1}^{I}, \epsilon_{t}^{\theta}-R \epsilon_{t-1}^{\theta}\right] \\
& +P_{X}^{2} \operatorname{Cov}\left[X_{t}^{I}-R X_{t-1}^{I}, X_{t-1}^{I}-R X_{t-2}^{I}\right]  \tag{A.31}\\
= & -R P_{\theta}^{2} \Sigma_{\theta}+P_{X} P_{\theta} X_{\theta}\left(1-R\left(X_{X}-R\right)\right) \Sigma_{\theta}+P_{X}^{2}\left(X_{X}-R\right)^{2} X_{X} \operatorname{Var}\left[X_{t}^{I}\right] \tag{A.32}
\end{align*}
$$

Since Lemma 4 implies that $X_{\theta}>0$ and $X_{X}<0$, each term is negative. Similarly,

$$
\begin{align*}
\operatorname{Cov}\left[Q_{t+2}, Q_{t}\right] & =P_{X}^{2} \operatorname{Cov}\left[X_{t+1}^{I}-R X_{t}^{I}, X_{t-1}^{I}-R X_{t-2}^{I}\right]+P_{X} P_{\theta} \operatorname{Cov}\left[X_{t+1}^{I}-R X_{t}^{I}, \epsilon_{t}^{\theta}-R e_{t-1}^{\theta}\right] \\
& =P_{X}^{2}\left(X_{X}-R\right)^{2} X_{X}^{2} \operatorname{Var}\left[X_{t}^{I}\right]+P_{X}^{2}\left(X_{X}-R\right) X_{X} X_{\theta}^{2} \Sigma_{\theta}+P_{X} P_{\theta}\left(X_{X}-R\right)\left(1-R X_{X}\right) X_{\theta} \Sigma_{\theta} \tag{A.33}
\end{align*}
$$

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Since $X_{\theta}>0$ and $X_{X}<0$, each term is positive
Proof of Proposition 3: Since the proof is quite similar to the proof of Proposition 1, I only provide the key steps. Conjecture that $P_{t}=A_{c(t)} Y_{t}$ and $X_{t}^{I}=B_{c(t)} Y_{t}$. The dynamics of the state variables and excess returns are then given by $Y_{t+1}=A_{Y, c(t)} Y_{t}+B_{Y} \epsilon_{t+1}$ and $Q_{t+1}=$ $A_{Q, c(t)} Y_{t}+B_{Q, c(t+1)} \epsilon_{t+1}$, where the matrices are defined as in Proposition 1 but vary depending on the calendar period $c(t)$. It follows that

$$
\begin{equation*}
Y_{t+j}=\left(\prod_{i=1}^{j} A_{Y, c(t+j-i)}\right) Y_{t}+\sum_{i=1}^{j-1}\left(\prod_{s=1}^{i} A_{Y, c(t+j-s)}\right) B_{Y} \epsilon_{t+j-i}+B_{Y} \epsilon_{t+j} \tag{A.34}
\end{equation*}
$$

Infrequent traders' demand can then be written as

$$
\begin{equation*}
X_{t}^{I}=\frac{1}{\gamma_{I}} \Sigma_{k+1, c(t)}^{-1}\left\{A_{c(t+k+1)}\left(\prod_{i=1}^{k+1} A_{Y, c(t+k+1-i)}\right)+\left(\sum_{i=1}^{k+1} R^{k+1-i} a_{D}^{i}\right) \varphi_{D}\right\} Y_{t} \tag{A.35}
\end{equation*}
$$

where $\Sigma_{k+1, c(t)}$ is a constant matrix. Demands are linear in the state variables, as conjectured. Consider next the problem of frequent traders. The value function of a frequent trader of age $j$ who trades in calendar period $c(t)$ is

$$
\begin{equation*}
J\left(W_{t}, Y_{t}, j, c(t)\right)=\max _{X_{t}} \mathbb{E}_{t}\left[J\left(W_{t+1}, Y_{t+1}, j+1, c(t+1)\right)\right] \tag{A.36}
\end{equation*}
$$

where $W_{t+1}=X_{t, j}^{\prime} Q_{t+1}+R W_{t}$ and $J\left(W_{t}, Y_{t}, h, c(t)\right)=-e^{-\gamma_{F} W_{t}}$. When the agent is one period older, the calendar period is $c(t+1)$. Conjecture that $J\left(W_{t+1}, Y_{t+1}, j+1, c(t+1)\right)=$ $-e^{-\alpha_{j+1} W_{t+1}-\frac{1}{2} Y_{t+1}^{\prime} U_{j+1, c(t+1)} Y_{t+1}}$.

Using standard arguments, it follows that

$$
\begin{equation*}
X_{t, j}=\frac{1}{\alpha_{j+1}} F_{j+1, c(t+1)} Y_{t} \tag{A.37}
\end{equation*}
$$

where $F_{j+1, c(t+1)}=\left(B_{Q, c(t+1)} \Xi_{j+1, c(t+1)} B_{Q, c(t+1)}^{\prime}\right)^{-1}\left(A_{Q, c(t)}-B_{Q, c(t+1)} \Xi_{j+1, c(t+1)} B_{Y}^{\prime} U_{j+1, c(t+1)}^{\prime} A_{Y, c(t)}\right)$. All the parameter matrices are defined recursively from $\alpha_{h}=\gamma_{F}$ and $U_{h, c(t)}=0_{1+2 N+k N}$. It
then follows that

$$
\begin{align*}
& \alpha_{j}=\alpha_{j+1} R,  \tag{A.38}\\
& U_{j, c(t)}=M_{j+1, c(t+1)}-2 \ln \rho_{j+1, c(t+1)} I_{11},  \tag{A.39}\\
& M_{j+1, c(t+1)}=A_{Y, c(t)}^{\prime} U_{j+1, c(t+1)} A_{Y, c(t)}+F_{j+1, c(t+1)}^{\prime} B_{Q, c(t+1)} \Xi_{j+1, c(t+1)} B_{Q, c(t+1)}^{\prime} F_{j+1, c(t+1)} \\
& \quad-A_{Y, c(t)}^{\prime} U_{j+1, c(t+1)} B_{Y} \Xi_{j+1, c(t+1)} B_{Y}^{\prime} U_{j+1, c(t+1)}^{\prime} A_{Y, c(t)},  \tag{A.40}\\
& \rho_{j+1, c(t+1)}=\left|I+\Sigma_{Y} B_{Y}^{\prime} U_{j+1, c(t+1)} B_{Y}\right|^{-\frac{1}{2}}, \quad \text { and }  \tag{A.41}\\
& \Xi_{j+1, c(t+1)}=\left(\Sigma_{Y}^{-1}+B_{Y}^{\prime} U_{j+1, c(t+1)} B_{Y}\right)^{-1} . \tag{A.42}
\end{align*}
$$

The market-clearing condition is $q_{c(t)} X_{t}^{I}+\frac{1-q}{h} \sum_{j=0}^{h-1} X_{t, j}^{F}=\left(\varphi_{\bar{S}}+\varphi_{\theta}-\sum_{i=1}^{k} q_{c(t-i)} \varphi_{X_{i}}\right) Y_{t}$, which verifies the conjecture that the price is linear in the state variables. Using equations (A.35) and (A.37), the market-clearing condition determines a system of fixed point equations for the $A_{c(t)}$ coefficients. The demand coefficients $B_{c(t)}$ can be solved using the fixed point system from equation (A.35):

$$
\begin{equation*}
\frac{1}{\gamma_{I}} \Sigma_{k+1, c(t)}^{-1}\left\{A_{c(t+k+1)}\left(\prod_{i=1}^{k+1} A_{Y, c(t+k+1-i)}\right)+\left(\sum_{i=1}^{k+1} R^{k+1-i} a_{D}^{i}\right) \varphi_{D}\right\}-B_{c(t)}=0 \tag{A.43}
\end{equation*}
$$

This concludes the proof.
Proof of Proposition 4: Assuming that $k=1$, one can write $X_{t}^{I}=\bar{X}_{I, c(t)}+X_{\theta, c(t)} \theta_{t}+X_{X, c(t)} X_{t-1}^{I}$. When $h=1$, market-clearing implies

$$
\begin{equation*}
q_{c(t)} X_{\theta, c(t)}+(1-q) \gamma_{F}^{-1} \Sigma_{c(t)}^{-1}\left(a_{\theta} P_{\theta, c(t+1)}-R P_{\theta, c(t)}+P_{X, c(t+1)} X_{\theta, c(t)}\right)=1 \tag{A.44}
\end{equation*}
$$

where $\Sigma_{c(t)}=P_{\theta, c(t+1)} \Sigma_{\theta} P_{\theta, c(t+1)}^{\prime}+\left(\frac{R}{R-a_{D}}\right)^{2} \Sigma_{D}$.
Consider the case with two calendar periods, and let $q_{2}=0$. Equation (A.44) implies

$$
\begin{align*}
& q_{1} X_{\theta, 1}+\left(1-q_{1}\right) \gamma_{F}^{-1} \Sigma_{1}^{-1}\left(a_{\theta} P_{\theta, 2}-R P_{\theta, 1}+P_{X, 2} X_{\theta, 1}\right)=1, \quad \text { and }  \tag{A.45}\\
& \left(1-q_{1}\right) \gamma_{F}^{-1} \Sigma_{2}^{-1}\left(a_{\theta} P_{\theta, 1}-R P_{\theta, 2}\right)=1 \tag{A.46}
\end{align*}
$$

For simplicity, assume that there is only one asset. Plugging (A.46) into (A.45) gives

$$
\begin{equation*}
q_{1} X_{\theta, 1}+\left(\frac{a_{\theta} P_{\theta, 2}-R P_{\theta, 1}+P_{X, 2} X_{\theta, 1}}{a_{\theta} P_{\theta, 1}-R P_{\theta, 2}}\right) \frac{\Sigma_{2}}{\Sigma_{1}}=1 \tag{A.47}
\end{equation*}
$$

Equation (A.46) implies that $a_{\theta} P_{\theta, 1}-R P_{\theta, 2}>0$. Using the methodology of Lemma 4, the conditions $P_{\theta, c(t)}<0$ and $P_{X, c(t)}>0$ imply that $X_{\theta, 1}>0$. In that case, if $P_{\theta, 1}<P_{\theta, 2}$, then $\frac{\Sigma_{2}}{\Sigma_{1}}>1$ and $\frac{a_{\theta} P_{\theta, 2}-R P_{\theta, 1}+P_{X, 2} X_{\theta, 1}}{a_{\theta} P_{\theta, 1}-R P_{\theta, 2}}>1$, which is impossible. As a result, $P_{\theta, 1}>P_{\theta, 2}$ in any equilibrium. Equiva-
lently, $\Sigma_{1}>\Sigma_{2}$. Equation (1.17) therefore implies that $\mathbb{E}\left[Q_{t+1} \mid c(t)=1\right]>\mathbb{E}\left[Q_{t+1} \mid c(t)=2\right]$.
Proof of Proposition 5: In this setup, risk aversion is inversely related to the mass of traders. Let the risk aversion of frequent traders vary with the calendar period and be denoted by $\gamma_{c(t)}$. In equilibrium, $a_{\theta} P_{\theta, c(t+1)}-R P_{\theta, c(t)}=\gamma_{c(t)}\left(P_{\theta, c(t+1)} \Sigma_{\theta} P_{\theta, c(t+1)}^{\prime}+\left(\frac{R}{R-a_{D}}\right)^{2} \Sigma_{D}\right)$. With two calendar periods and one asset, if $\gamma_{1}>\gamma_{2}$, then $P_{\theta, 1}<P_{\theta, 2}$ in any equilibrium (by contradiction). Since price impact is negative, this implies that $a_{\theta} P_{\theta, 2}-R P_{\theta, 1}>a_{\theta} P_{\theta, 1}-R P_{\theta, 2}$. Using the market-clearing condition, the expected excess return is given by $\mathbb{E}\left[Q_{t+1} \mid c(t)\right]=$ $\left(a_{\theta} P_{\theta, c(t+1)}-R P_{\theta, c(t)}\right) \bar{S}$. The proof follows from applying the previous result in the last equation.

Proof of Proposition 6: Consider an asset with liquidity shock volatility $\sigma_{\theta}$. When $q=0$ (or $q=1$ ), trading volume is given by $V_{t}=\left|\theta_{t}-\theta_{t-1}\right|$. To compute volume autocorrelation, note that if $X$ and $Y$ are jointly normal random variables with zero mean, variances $\sigma_{X}^{2}$ and $\sigma_{Y}^{2}$, and correlation $\rho$, then $\operatorname{Cov}[|X|,|Y|]=\frac{2}{\pi}\left(\rho \arcsin (\rho)+\sqrt{1-\rho^{2}}-1\right) \sigma_{X} \sigma_{Y}$.

Since $\theta_{t+s}-\theta_{t+s-1}=\mathrm{const}+\left(a_{\theta}-1\right) a_{\theta}^{s} \theta_{t-1}+\epsilon_{t+s}^{\theta}+\left(a_{\theta}-1\right) \sum_{i=0}^{s-1} a_{\theta}^{s-1-i} \epsilon_{t+i}^{\theta}, s \geq 1$, the autocovariance of $\Delta \theta_{t+s} \equiv \theta_{t+s}-\theta_{t+s-1}$ for a single asset is given by $\operatorname{Cov}\left[\Delta \theta_{t}, \Delta \theta_{t+s}\right]=-\left(\frac{1-a_{\theta}}{1+a_{\theta}}\right) a_{\theta}^{s-1} \sigma_{\theta}^{2}$, $s \geq 1$. It follows that $\operatorname{Corr}\left[\Delta \theta_{t+s}, \Delta \theta_{t}\right] \equiv \rho_{\Delta \theta}(s)=\frac{-\left(\frac{1-a_{\theta}}{1+a_{\theta}}\right) a_{\theta}^{s-1} \sigma_{\theta}^{2}}{\frac{1}{1+a_{\theta}} \sigma_{\theta}^{2}}=-\left(\frac{1-a_{\theta}}{2}\right) a_{\theta}^{s-1}, s \geq 1$. Thus, $\rho_{\Delta \theta}(s)<0$ and is an increasing concave function of $s$ for $0<a_{\theta}<1$.

Using the previous result,

$$
\begin{equation*}
\operatorname{Cov}\left[V_{t}, V_{t+s}\right]=\frac{2}{\pi}\left(\rho_{\Delta \theta}(s) \arcsin \left(\rho_{\Delta \theta}(s)\right)+\sqrt{1-\rho_{\Delta \theta}(s)^{2}}-1\right) \sigma_{\Delta \theta}^{2} \tag{A.48}
\end{equation*}
$$

Note that $\frac{d \operatorname{Cov}\left[V_{t}, V_{t+s}\right]}{d \rho_{\Delta \theta}(s)}=\frac{2}{\pi} \arcsin \left(\rho_{\Delta \theta}(s)\right) \sigma_{\Delta \theta}^{2}$. Using this fact and the properties of the arcsin function, it is direct to show that $\operatorname{Cov}\left[V_{t}, V_{t+s}\right]>0$ and is a decreasing convex function of $s(s \geq$ 1). Note that when $a_{\theta}=1, \operatorname{Cov}\left[V_{t}, V_{t+s}\right]=0, s \geq 1 . \operatorname{Since} \operatorname{Cov}\left[\Delta V_{t}, \Delta V_{t+s}\right]=2 \operatorname{Cov}\left[V_{t}, V_{t+s}\right]-$ $\operatorname{Cov}\left[V_{t}, V_{t+s-1}\right]-\operatorname{Cov}\left[V_{t}, V_{t+s+1}\right]$, it follows that $\operatorname{Cov}\left[\Delta V_{t}, \Delta V_{t+s}\right]<0$ by Jensen's inequality.

## A. 2 Calibration

Table A. 1 shows the calibration used to compare the model's predictions to the empirical analysis on intraday returns in Section 1.4.3 and daily returns in Section 1.4.4. This paper does not aim to provide an exact quantitative match to the data. The parameters are therefore chosen to broadly match the patterns observed in the data while keeping the calibration as simple and transparent as possible.

Trading frequencies: Section 1.4.3 discuss the calibration for intraday returns. For daily returns, I assume that $60 \%$ of the agents rebalance once a week ( $q=0.6, k=4$ ). The other agents trade

Table A. 1
Model calibration for daily and intraday returns.

| Parameter |  | Daily returns | Intraday returns |
| :--- | :---: | :---: | :---: |
| Proportion of infrequent traders | $q$ | 0.6 | 0.99 |
| Inattention period | $k$ | 4 | 12 |
| Risk aversion | $\gamma_{F}, \gamma_{I}$ | 1 | 1 |
| Risk-free rate | $R$ | $1.05 \frac{1}{250}$ | 1.0001 |
| Persistence of dividends | $a_{D}$ | 0 | 0 |
| Persistence of liquidity trading | $a_{\theta}$ | 0.8 | 0.7 |
| Volatility of dividend shocks | $\sigma_{D}$ | 0.04 | 0.01 |
| Volatility of liquidity shocks | $\sigma_{\theta}$ | 1 | 0.6 |
| Correlation of dividend shocks | $\rho_{D}$ | 0.3 | 0.3 |
| Correlation of liquidity shocks | $\rho_{\theta}$ | 0 | 0 |
| Number of assets | $N$ | 2 | 2 |
| Horizon of frequent traders | $h$ | 20 | 20 |

every period with a monthly horizon $(h=20)$.
Dividends: Dividend persistence does not affect excess return autocorrelation and is set to zero. Dividend shocks' volatility and correlation do not qualitatively affect the results.

Liquidity shocks: The persistence of liquidity shocks is the only parameter that can generate persistence in excess return autocorrelation in this setup. For daily returns, Figure 1.4 suggests a relatively high persistence. The persistence required by the model to broadly match the decaying autocorrelation pattern for the first lags in the data seems lower for intraday returns than for daily returns (Figures 1.1 and 1.4). This evidence is inconsistent with a single liquidity trading process driving both intraday and daily returns. For instance, a mix of low frequency and high frequency liquidity shocks would result in a more complicated process than an AR(1). Still, the AR(1) assumption represents a natural benchmark. Furthermore, the rebalancing mechanism does not require any persistence in liquidity shocks (Section 1.4.1). Liquidity shock volatility is hard to estimate. I set it to a lower value than the equivalent value estimated by Campbell, Grossman, and Wang (1993). Liquidity shocks' correlation is set to zero for simplicity.

## B Appendix to Chapter 2

## B. 1 Additional Robustness Checks

## B.1.1 Price Screen

Bhardwaj and Brooks (1992) assert that the January effect is mainly a low-price effect. I therefore exclude all stocks with a price lower than $\$ 10$ at the formation date (instead of $\$ 1$ in the paper). A majority of anomalies still exhibit a marked January seasonality (not reported). The average returns are often several times larger in January than in other months. The larger price screen reduces, however, the January seasonality for the accruals and net stock issues anomalies. The month-of-quarter results are mostly similar with a larger price screen (not reported). The two exceptions are illiquidity and size. The illiquidity strategy average return at the beginning of quarters is still negative but now insignificant, while the average return at the end of quarters remains positive and strongly significant. The size portfolio has a large negative average return—a positive size premium since large caps underperform small caps—at the end of quarters and a positive but statistically insignificant average return at the beginning of quarters.

## B.1.2 Subsamples

I split the sample in two subsamples. Table B. 2 shows that the January seasonality tends to be less pronounced across anomalies in the second subsample. This is especially true for valueweighted portfolios, though most of the anomalies still exhibit a marked seasonality. Contrary to the January effect, the month-of-the-quarter effects are in general more pronounced in the most recent sample (see Tables B. 3 and B.4). Statistical significance often declines relative to the full sample, which is not surprising given the smaller number of observations, but the magnitudes remain large.

Table B. 1
Description of the anomalies used in the paper. All the accounting variables are computed once a year at the end of June using data for the previous fiscal year.

$$
\text { Name } \quad \text { Sorting variable }
$$

| Market cap. (MC) | Market capitalization in the previous month |
| :---: | :---: |
| Book-to-market (BM) | Book equity over market value, where market value is the market capitalization of the firm six months ago (stockholders' equity is computed as in Novy-Marx (2013) and negative BE firms are excluded from the portfolios) |
| Gross profitability (GP) | Revenue minus cost of good sold, divided by total assets |
| Asset growth (AG) | Yearly growth rate of total assets |
| Accruals (AC) | Change in working capital (excluding cash) minus depreciation, scaled by average total assets over the previous two years |
| Net stock issues (NSI) | Growth rate of the split-adjusted shares outstanding at fiscal year end as in Fama and French (2008) |
| $\Delta$ turnover ( $\Delta \mathrm{T}$ ) | Change in turnover in the previous month relative to the past sixmonth average turnover (excluding the last month) |
| Illiquidity (IL) | Average ILLIQ in the previous year (the portfolios are formed once a year at the beginning of January); <br> $\operatorname{ILLIQ}_{i, t}=\frac{1}{D_{i, y}} \sum_{d=1}^{D_{i, y}} \frac{\left\|r_{i, y, d}\right\|}{\mathrm{DVOL}_{i, y, d}} 10^{6}$, where $D_{i, y}$ is the number of trading days and DVOL is the dollar volume (at least 100 trading days to be included) |
| Idiosyncratic volatility (IV) | Standard deviation of the residuals from regressing the stock's daily excess returns on the Fama-French three factors (at least 17 return observations in the month to be included) |
| Momentum (MOM) | Past six months return skipping the last month and with a onemonth holding period |
| 12-month effect (12m) | Average of the stock's return in the same calendar month six to ten years ago |

Table B. 2
Average January ( J ) and non-January months ( nJ ) returns in percent of high-minus-low return-weighted and value-weighted decile portfolios formed on different characteristics by subsample. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 2014 (the accruals portfolios start in July 1971). Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. Financial firms are excluded from book-to-market, gross profitability, asset growth, accruals, and net stock issues portfolios. NASDAQ stocks are excluded from the turnover and illiquidity portfolios. The characteristics are defined in Table B.1. The $t$-statistics are in parentheses.

|  |  | return-weighted |  | value-weighted |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1964-1988 | 1989-2014 | 1964-1988 | 1989-2014 |
| Market cap. | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{gathered} -8.06(-6.56) \\ 0.19(0.73) \end{gathered}$ | $\begin{gathered} -4.73(-4.12) \\ 0.44(2.03) \end{gathered}$ | $\begin{gathered} -7.25(-5.93) \\ 0.09(0.33) \end{gathered}$ | $\begin{gathered} -3.92(-3.32) \\ 0.28(1.21) \end{gathered}$ |
| Book-to-market | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{aligned} & 5.13(4.60) \\ & 0.60(2.87) \end{aligned}$ | $\begin{aligned} & 2.10(3.43) \\ & 0.83(3.13) \end{aligned}$ | $\begin{aligned} & 5.48(4.37) \\ & 0.22(0.93) \end{aligned}$ | $\begin{aligned} & 1.19(1.06) \\ & 0.24(0.90) \end{aligned}$ |
| Gross profit. | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{gathered} -0.78(-1.83) \\ 0.90(5.89) \end{gathered}$ | $\begin{gathered} -1.59(-1.68) \\ 1.16(5.82) \end{gathered}$ | $\begin{gathered} -1.59(-2.37) \\ 0.53(2.60) \end{gathered}$ | $\begin{gathered} -0.20(-0.24) \\ 0.46(2.40) \end{gathered}$ |
| Asset growth | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{aligned} & -4.28(-7.08) \\ & -0.34(-2.43) \end{aligned}$ | $\begin{aligned} & -3.95(-7.79) \\ & -0.45(-3.94) \end{aligned}$ | $\begin{aligned} & -3.23(-3.88) \\ & -0.22(-1.17) \end{aligned}$ | $\begin{aligned} & -2.21(-3.51) \\ & -0.23(-1.57) \end{aligned}$ |
| Accruals | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{aligned} & -1.48(-2.07) \\ & -0.33(-2.66) \end{aligned}$ | $\begin{aligned} & -1.20(-3.11) \\ & -0.33(-3.99) \end{aligned}$ | $\begin{aligned} & -0.23(-0.26) \\ & -0.45(-1.90) \end{aligned}$ | $\begin{aligned} & -0.56(-0.91) \\ & -0.28(-1.87) \end{aligned}$ |
| Net stock issues | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{aligned} & -1.08(-1.73) \\ & -1.13(-9.56) \end{aligned}$ | $\begin{gathered} 0.15(0.28) \\ -1.16(-9.24) \end{gathered}$ | $\begin{aligned} & -1.17(-1.88) \\ & -0.65(-4.25) \end{aligned}$ | $\begin{gathered} 0.17(-0.34) \\ -0.54(-4.49) \end{gathered}$ |
| $\Delta$ turnover | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{aligned} & 4.90(5.23) \\ & 0.64(5.03) \end{aligned}$ | $\begin{aligned} & 1.67(3.30) \\ & 1.14(8.25) \end{aligned}$ | $\begin{aligned} & 3.11 \text { (3.20) } \\ & 0.54(3.96) \end{aligned}$ | $\begin{aligned} & 0.37(0.58) \\ & 0.36(1.83) \end{aligned}$ |
| Illiquidity | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{gathered} 8.79(6.24) \\ -0.01(-0.05) \end{gathered}$ | $\begin{gathered} 3.52(3.86) \\ -0.30(-1.42) \end{gathered}$ | $\begin{aligned} & 6.74(5.04) \\ & 0.21(0.87) \end{aligned}$ | $\begin{aligned} & 2.27(2.95) \\ & 0.10(0.48) \end{aligned}$ |
| Idiosyncratic vol. | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{gathered} 6.67(4.90) \\ -1.09(-3.91) \end{gathered}$ | $\begin{gathered} 4.53(3.48) \\ -0.97(-2.66) \end{gathered}$ | $\begin{gathered} 3.40(2.13) \\ -0.81(-2.72) \end{gathered}$ | $\begin{gathered} 2.00(1.40) \\ -0.81(-1.74) \end{gathered}$ |
| Momentum | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{gathered} -4.32(-3.14) \\ 1.75(8.14) \end{gathered}$ | $\begin{gathered} -2.66(-1.75) \\ 1.69(5.16) \end{gathered}$ | $\begin{gathered} -1.39(-0.89) \\ 1.22(4.32) \end{gathered}$ | $\begin{gathered} -2.22(-1.32) \\ 0.77(1.88) \end{gathered}$ |
| 12-month effect | $\begin{gathered} \mathrm{J} \\ \mathrm{~nJ} \end{gathered}$ | $\begin{aligned} & 4.16(5.54) \\ & 0.25(2.19) \end{aligned}$ | $\begin{aligned} & 2.44(3.38) \\ & 0.70(5.66) \end{aligned}$ | $\begin{aligned} & 2.27(2.54) \\ & 0.27(1.51) \end{aligned}$ | $\begin{aligned} & 3.70(3.30) \\ & 0.87(3.93) \end{aligned}$ |

## Table B. 3

Quarter analysis (1964-1988). Average returns in percent of long-short return-weighted and value-weighted decile portfolios formed on different characteristics. The average return is shown separately for middle-quarter, beginning-of-quarter (excluding January), and end-of-quarter months. Sample: NYSE, Amex, and NASDAQ stocks from January 1964 to December 1988 (the accruals portfolios start in July 1971). Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. Financial firms are excluded from book-to-market, gross profitability, asset growth, accruals, and net stock issues portfolios. NASDAQ stocks are excluded from the turnover and illiquidity portfolios. The characteristics are defined in Table B.1. The $t$-statistics are in parentheses.

|  | return-weighted |  |  | value-weighted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mid | beg | end | mid | beg | end |
| MC | $\begin{gathered} 0.18 \\ (0.37) \end{gathered}$ | $\begin{gathered} 0.73 \\ (1.45) \end{gathered}$ | $\begin{gathered} -0.20 \\ (-0.48) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.19) \end{gathered}$ | $\begin{gathered} 0.72 \\ (1.40) \end{gathered}$ | $\begin{gathered} -0.39 \\ (-0.94) \end{gathered}$ |
| BM | $\begin{gathered} 0.25 \\ (0.70) \end{gathered}$ | $\begin{gathered} 0.94 \\ (2.17) \end{gathered}$ | $\begin{gathered} 0.71 \\ (2.20) \end{gathered}$ | $\begin{gathered} 0.03 \\ (0.07) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.54) \end{gathered}$ | $\begin{gathered} 0.40 \\ (1.06) \end{gathered}$ |
| GP | $\begin{gathered} 1.04 \\ (4.43) \end{gathered}$ | $\begin{gathered} 0.66 \\ (2.16) \end{gathered}$ | $\begin{gathered} 0.93 \\ (3.56) \end{gathered}$ | $\begin{gathered} 0.64 \\ (2.16) \end{gathered}$ | $\begin{gathered} 0.50 \\ (1.16) \end{gathered}$ | $\begin{gathered} 0.45 \\ (1.26) \end{gathered}$ |
| AG | $\begin{gathered} -0.15 \\ (-0.64) \end{gathered}$ | $\begin{gathered} -0.46 \\ (-1.77) \end{gathered}$ | $\begin{gathered} -0.43 \\ (-1.89) \end{gathered}$ | $\begin{gathered} 0.01 \\ (0.03) \end{gathered}$ | $\begin{gathered} 0.00 \\ (0.01) \end{gathered}$ | $\begin{gathered} -0.61 \\ (-2.08) \end{gathered}$ |
| AC | $\begin{gathered} -0.43 \\ (-2.24) \end{gathered}$ | $\begin{gathered} -0.32 \\ (-1.13) \end{gathered}$ | $\begin{gathered} -0.23 \\ (-1.26) \end{gathered}$ | $\begin{gathered} -0.16 \\ (-0.49) \end{gathered}$ | $\begin{gathered} -0.78 \\ (-1.45) \end{gathered}$ | $\begin{gathered} -0.49 \\ (-1.25) \end{gathered}$ |
| NSI | $\begin{gathered} -0.98 \\ (-5.19) \end{gathered}$ | $\begin{gathered} -1.28 \\ (-5.49) \end{gathered}$ | $\begin{gathered} -1.18 \\ (-5.86) \end{gathered}$ | $\begin{gathered} -0.27 \\ (-1.06) \end{gathered}$ | $\begin{gathered} -0.94 \\ (-3.25) \end{gathered}$ | $\begin{gathered} -0.82 \\ (-3.24) \end{gathered}$ |
| $\Delta \mathrm{T}$ | $\begin{gathered} 0.73 \\ (3.57) \end{gathered}$ | $\begin{gathered} 0.62 \\ (2.20) \end{gathered}$ | $\begin{gathered} 0.55 \\ (2.97) \end{gathered}$ | $\begin{gathered} 0.18 \\ (0.82) \end{gathered}$ | $\begin{gathered} 1.12 \\ (3.80) \end{gathered}$ | $\begin{gathered} 0.48 \\ (2.28) \end{gathered}$ |
| IL | $\begin{gathered} -0.07 \\ (-0.15) \end{gathered}$ | $\begin{gathered} -0.64 \\ (-1.35) \end{gathered}$ | $\begin{gathered} 0.52 \\ (1.43) \end{gathered}$ | $\begin{gathered} 0.14 \\ (0.32) \end{gathered}$ | $\begin{gathered} -0.56 \\ (-1.25) \end{gathered}$ | $\begin{gathered} 0.86 \\ (2.39) \end{gathered}$ |
| IV | $\begin{gathered} -0.69 \\ (-1.51) \end{gathered}$ | $\begin{gathered} -1.75 \\ (-2.95) \end{gathered}$ | $\begin{gathered} -1.00 \\ (-2.35) \end{gathered}$ | $\begin{gathered} -0.18 \\ (-0.37) \end{gathered}$ | $\begin{gathered} -1.78 \\ (-2.89) \end{gathered}$ | $\begin{gathered} -0.72 \\ (-1.57) \end{gathered}$ |
| MOM | $\begin{gathered} 1.48 \\ (4.94) \end{gathered}$ | $\begin{gathered} 1.74 \\ (4.08) \end{gathered}$ | $\begin{gathered} 2.01 \\ (5.08) \end{gathered}$ | $\begin{gathered} 1.16 \\ (2.65) \end{gathered}$ | $\begin{gathered} 1.18 \\ (2.02) \end{gathered}$ | $\begin{gathered} 1.31 \\ (2.76) \end{gathered}$ |
| 12 m | $\begin{gathered} 0.19 \\ (1.00) \end{gathered}$ | $\begin{gathered} 0.24 \\ (1.15) \end{gathered}$ | $\begin{gathered} 0.30 \\ (1.63) \end{gathered}$ | $\begin{gathered} 0.18 \\ (0.59) \end{gathered}$ | $\begin{gathered} 0.50 \\ (1.39) \end{gathered}$ | $\begin{gathered} 0.18 \\ (0.66) \end{gathered}$ |

Table B. 4
Quarter analysis (1989-2014). Average returns in percent of long-short return-weighted and value-weighted decile portfolios formed on different characteristics. The average return is shown separately for middle-quarter, beginning-of-quarter (excluding January), and end-of-quarter months. Sample: NYSE, Amex, and NASDAQ stocks from January 1989 to December 2014. Breakpoints are based on NYSE deciles. Stocks with a price smaller than $\$ 1$ at the formation date are excluded. Financial firms are excluded from book-to-market, gross profitability, asset growth, accruals, and net stock issues portfolios. NASDAQ stocks are excluded from the turnover and illiquidity portfolios. The characteristics are defined in Table B.1. The $t$-statistics are in parentheses.

|  | return-weighted |  |  | value-weighted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | mid | beg | end | mid | beg | end |
| MC | $\begin{gathered} 0.13 \\ (0.36) \end{gathered}$ | $\begin{gathered} 1.11 \\ (2.46) \end{gathered}$ | $\begin{gathered} 0.24 \\ (0.75) \end{gathered}$ | $\begin{gathered} -0.06 \\ (-0.17) \end{gathered}$ | $\begin{gathered} 1.17 \\ (2.31) \end{gathered}$ | $\begin{gathered} -0.05 \\ (-0.14) \end{gathered}$ |
| BM | $\begin{gathered} 0.89 \\ (1.87) \end{gathered}$ | $\begin{gathered} 1.21 \\ (2.25) \end{gathered}$ | $\begin{gathered} 0.47 \\ (1.28) \end{gathered}$ | $\begin{gathered} 0.22 \\ (0.50) \end{gathered}$ | $\begin{gathered} 0.23 \\ (0.38) \end{gathered}$ | $\begin{gathered} 0.28 \\ (0.70) \end{gathered}$ |
| GP | $\begin{gathered} 0.89 \\ (2.20) \end{gathered}$ | $\begin{gathered} 1.66 \\ (5.46) \end{gathered}$ | $\begin{gathered} 1.05 \\ (3.63) \end{gathered}$ | $\begin{gathered} 0.76 \\ (2.60) \end{gathered}$ | $\begin{gathered} 0.56 \\ (1.40) \end{gathered}$ | $\begin{gathered} 0.09 \\ (0.28) \end{gathered}$ |
| AG | $\begin{gathered} -0.59 \\ (-2.65) \end{gathered}$ | $\begin{gathered} -0.34 \\ (-1.75) \end{gathered}$ | $\begin{gathered} -0.40 \\ (-2.35) \end{gathered}$ | $\begin{gathered} -0.12 \\ (-0.47) \end{gathered}$ | $\begin{gathered} -0.14 \\ (-0.53) \end{gathered}$ | $\begin{gathered} -0.41 \\ (-1.66) \end{gathered}$ |
| AC | $\begin{gathered} -0.38 \\ (-2.62) \end{gathered}$ | $\begin{gathered} -0.30 \\ (-1.76) \end{gathered}$ | $\begin{gathered} -0.32 \\ (-2.50) \end{gathered}$ | $\begin{gathered} 0.03 \\ (0.16) \end{gathered}$ | $\begin{gathered} -0.67 \\ (-2.08) \end{gathered}$ | $\begin{gathered} -0.29 \\ (-1.21) \end{gathered}$ |
| NSI | $\begin{gathered} -0.93 \\ (-3.94) \end{gathered}$ | $\begin{gathered} -1.68 \\ (-7.70) \end{gathered}$ | $\begin{gathered} -1.00 \\ (-5.30) \end{gathered}$ | $\begin{gathered} -0.22 \\ (-1.17) \end{gathered}$ | $\begin{gathered} -1.22 \\ (-5.56) \end{gathered}$ | $\begin{gathered} -0.36 \\ (-1.67) \end{gathered}$ |
| $\Delta \mathrm{T}$ | $\begin{gathered} 1.36 \\ (5.12) \end{gathered}$ | $\begin{gathered} 0.93 \\ (4.06) \end{gathered}$ | $\begin{gathered} 1.07 \\ (5.11) \end{gathered}$ | $\begin{gathered} 0.26 \\ (0.88) \end{gathered}$ | $\begin{gathered} 0.51 \\ (1.36) \end{gathered}$ | $\begin{gathered} 0.35 \\ (0.98) \end{gathered}$ |
| IL | $\begin{gathered} -0.15 \\ (-0.40) \end{gathered}$ | $\begin{gathered} -1.21 \\ (-2.91) \end{gathered}$ | $\begin{gathered} 0.22 \\ (0.70) \end{gathered}$ | $\begin{gathered} 0.32 \\ (0.92) \end{gathered}$ | $\begin{gathered} -1.03 \\ (-2.28) \end{gathered}$ | $\begin{gathered} 0.74 \\ (2.30) \end{gathered}$ |
| IV | $\begin{gathered} -0.42 \\ (-0.59) \end{gathered}$ | $\begin{gathered} -1.57 \\ (-2.35) \end{gathered}$ | $\begin{gathered} -1.08 \\ (-2.16) \end{gathered}$ | $\begin{gathered} -0.32 \\ (-0.39) \end{gathered}$ | $\begin{gathered} -1.32 \\ (-1.51) \end{gathered}$ | $\begin{gathered} -0.91 \\ (-1.28) \end{gathered}$ |
| MOM | $\begin{gathered} 1.26 \\ (2.54) \end{gathered}$ | $\begin{gathered} 0.69 \\ (0.89) \end{gathered}$ | $\begin{gathered} 2.88 \\ (6.25) \end{gathered}$ | $\begin{gathered} 0.31 \\ (0.46) \end{gathered}$ | $\begin{gathered} -1.00 \\ (-1.15) \end{gathered}$ | $\begin{gathered} 2.57 \\ (4.37) \end{gathered}$ |
| 12 m | $\begin{gathered} 0.53 \\ (3.10) \end{gathered}$ | $\begin{gathered} 1.03 \\ (3.28) \end{gathered}$ | $\begin{gathered} 0.63 \\ (3.54) \end{gathered}$ | $\begin{gathered} 0.40 \\ (1.14) \end{gathered}$ | $\begin{gathered} 1.34 \\ (2.71) \end{gathered}$ | $\begin{gathered} 0.99 \\ (3.00) \end{gathered}$ |

## C Appendix to Chapter 3

## C. 1 Reversals in Midquote Returns

Spurious reversals plague midquote returns computed from TAQ. These reversals are especially prevalent across small stocks in the second part of the sample. Table C. 1 illustrates the problem for a randomly selected stock by showing the first and last available intraday quotes on several dates.

## Table C. 1

First and last available intraday quotes for symbol IT on several dates extracted from the TAQ database.

| Date | Time | Bid | Ask | Bid Size | Ask Size |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $2005-10-11$ | $15: 59: 50.0$ | 11.38 | 11.39 | 5 | 5 |
| $2005-10-12$ | $9: 30: 54.0$ | 11.03 | 16.03 | 1 | 1 |
|  | $9: 34: 57.0$ | 11.3 | 11.36 | 1 | 1 |
|  | $\vdots$ |  |  |  |  |
| $2005-10-12$ | $15: 59: 42.0$ | 11.3 | 11.31 | 2 | 23 |
| $2005-10-13$ | $9: 30: 31$ | 10.35 | 13.67 | 30 | 1 |
|  | $9: 30: 32$ | 10.35 | 14.38 | 30 | 1 |
|  | $9: 30: 33$ | 10.35 | 15.09 | 30 | 1 |
|  | $9: 32: 19$ | 11.24 | 11.25 | 2 | 1 |

As can be seen in the table, the best ask at the open can be biased. A high ask generates a large overnight return and a negative first half-hour return (i.e., spurious reversal). Furthermore, even the second and third quoted ask prices can be too high. The best bid is subject to similar problems. It takes a few minutes for the quotes to stabilize to what appears to be their normal level. Note that there is a nonzero trade size at both bid and ask quotes. The criterion of Berkman et al. (2012) of taking the first valid quote (i.e., with nonzero trade size on both bid
and ask) does not seem sufficient. Numerous similar examples can be found for stocks that display more frequent quote updates.

To deal with these spurious reversals, I use the following criteria. First, I only consider quotes after 9:45. This threshold is based on an empirical investigation of many spurious reversals. For all stocks that have quote updated on a regular basis, I find that quotes seem to have normalized by 9:45. Second, I always delete the first quote available during the day. It is often the case that this quote is biased. This restriction is important for stocks whose first available quote is released after 9:45. Third, I delete any observation for which the spread is larger than 30 times the median spread during the day. This restriction helps exclude outliers that may have passed the other filters. Finally, I screen the data to eliminate large outliers; in particular, large return reversals that are not accompanied by any trading volume.

Table C. 2
List of anomalies. All the accounting variables are computed once a year at the end of June using data for the previous fiscal year.

Name
Sorting variable

| Accruals | Change in working capital (excluding cash) minus depreciation, <br> scaled by average total assets over the previous two years (Sloan, <br> 1996). The strategy shorts stocks with high accruals. |
| :--- | :--- |
| Beta |  |
| Market beta for each stock estimated using daily returns over the |  |
| past year. The market return is the value-weighted return of all |  |
| stocks in the sample excluding stocks with a price below $\$ 5$ and |  |
| is rebalanced once a month. The strategy shorts stocks with high |  |
| beta. |  |$\quad$| Book equity over market value, where market value is the market |
| :--- |
| capitalization of the firm six months ago. Stockholders' equity |
| is computed as in Novy-Marx (2013) and negative BE firms are |
| excluded from the portfolios. |$\quad$| Revenue minus cost of good sold, divided by total assets (Novy- |
| :--- |
| Idiosyncratic volatility |
| Sarx, 2013). The strategy is long stocks with high gross prof. |

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Visiting Scholar, Columbia Business School, Spring 2016
M.Sc. in Finance, University of Lausanne (HEC), 2011

Exchange student, UCLA Anderson School of Management, Fall 2010
B.Sc. in Economics, University of Lausanne (HEC), 2008

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## Research Interests

Asset Pricing, Market Efficiency and Liquidity, Market Microstructure

## Publication

Infrequent Rebalancing, Return Autocorrelation, and Seasonality [link]
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## Working Papers

The Cross-Section of Intraday and Overnight Returns, 2016 [link]
Slow-Moving Capital and Execution Costs: Evidence from a Major Trading Glitch, 2016, with Pierre Collin-Dufresne and Mehmet Sağlam [link]
Seasonalities in Anomalies, 2015 [link]

## Teaching Experience

Ecole Polytechnique Fédérale de Lausanne (EPFL)
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University of Lausanne
Teaching Assistant, Prof. J. Lahaye, Market Microstructure, M.Sc. in Finance, 2011

## Honors, Awards, \& Grants

Swiss Finance Institute Advanced Doctoral Grant, 2016
AFA 2016 Travel Grant
NASDAQ OMX - CQA Prize (Runner-up), EFA Doctoral Tutorial, 2014
Prize "Wegelin et Co.," highest GPA, M.Sc. in Finance, University of Lausanne, 2011
Faculty Prize, highest GPA, B.Sc. in Economics, University of Lausanne, 2008
Gustave-Louis Chappuis Prize, University of Lausanne, 2007
HEC Lausanne Alumni Association Award, 2006

## Conference and Seminar Presentations

2017: Boston College; Bocconi; INSEAD; London School of Economics; Oxford; University of Maryland; Washington University in St. Louis; University of British Columbia; Emory; BoAML Global Quant Conference, London (scheduled); WFA (scheduled)
2016: SFI Research Days; UNIL-EPFL (brown bag); University of Zürich (brown bag); University of Geneva; SFI PhD Workshop
2013-2015: UNIL-EPFL (brown bag); CQA Fall Meeting, Chicago; EFA Doctoral Tutorial, Lugano; Geneva-China Workshop in International Finance and Macroeconomics, Geneva (discussant); Swiss Doctoral Workshop in Finance, Gerzensee

## Professional Experience

Pictet Asset Management, Geneva, 6-8/2009
Intern, team "Analysis, Projects \& Implementation"
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Computer Skills: Matlab, Python, Mathematica, SAS
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[^0]:    ${ }^{1}$ This chapter is the postprint version of the article published in The Journal of Finance. Permission to reproduce this article was obtained from John Wiley and Sons. BOGOUSSLAVSKY, V. (2016), Infrequent Rebalancing, Return Autocorrelation, and Seasonality. The Journal of Finance, 71: 2967-3006. doi:10.1111/jofi.12436

[^1]:    ${ }^{2}$ Heston, Korajczyk, and Sadka (2010) discuss why institutional fund flows and trading algorithms may lead to periodicity in trading volume and order imbalances. Rakowski and Wang (2009) find a day-of-the-week effect in mutual fund flows. Similarly, the rebalancing methodology documentation of several investment products suggests that weekly reviews may take place on specific days of the week. Ritter (1988) provides evidence that individual investors strongly rebalance their portfolios in January.
    ${ }^{3}$ The setup builds on the model of Duffie (2010) and relates to the finance literature on overlapping generations models. See Spiegel (1998), Watanabe (2008), Biais, Bossaerts, and Spatt (2010), Banerjee (2011), and Albagli (2015), among others.

[^2]:    ${ }^{4}$ As anecdotal evidence, The Wall Street Journal (September 10,2010) reports the story of a proprietary-trading firm that is mostly active at the open and close of the market ("The Traders Who Skip Most of the Day").
    ${ }^{5}$ Papers that are closest to this one include Jegadeesh (1990) on the profitability of monthly contrarian strategies and Lehmann (1990) on weekly return reversal in individual securities. Nagel (2012) provides a more recent analysis on the profitability of reversal strategies.

[^3]:    ${ }^{6}$ Investor inertia has been shown to affect asset properties at longer horizons. Lou (2012) shows that the high persistence in mutual fund flows can explain part of the medium- and long-term predictability in stock returns. Vayanos and Woolley (2013) provide a theory of momentum and reversal based on investment flows in a setup with rational agents.

[^4]:    ${ }^{7}$ Many papers investigate the source of momentum profits using a similar decomposition (see, for instance, Conrad and Kaul (1998) and Jegadeesh and Titman (2002)). Jegadeesh and Titman (1995) point out, however, that applying this decomposition empirically may not correctly distinguish between the autocovariance and cross-autocovariance components.

[^5]:    ${ }^{8}$ Orosel (1998) studies an overlapping generations economy with endogenous participation arising from a fixed cost of participation, but his setup does not include liquidity shocks. Taking another modeling approach, Peng and Xiong (2006) define an agent's attention to a particular stock as the precision of the signal he receives about the stock's future dividend. In this case, the agent is always active in the market but allocates his limited attention across different stocks.

[^6]:    ${ }^{9}$ If $\bar{S}=0_{N \times 1}$, then the unconditional expected excess return is zero for all assets. Thus, to study expected returns I assume that all assets are in positive supply. Some securities can be in zero net supply as long as they are correlated with securities in positive supply.

[^7]:    ${ }^{10}$ This is due to the heterogeneity in traders. Watanabe (2008), Biais, Bossaerts, and Spatt (2010), and Banerjee (2011) also resort to numerical solutions.

[^8]:    ${ }^{11}$ Multiple equilibria arise because agents have self-fulfilling beliefs about the volatility of future prices. Following Bacchetta and Van Wincoop (2006), stability requires an equilibrium to be robust to a small deviation in next period's belief regarding volatility. The Internet Appendix is available in the online version of this article on The Journal of Finance's website.

[^9]:    ${ }^{12}$ Campbell, Grossman, and Wang (1993) derive a similar equation in a single-asset setup with myopic agents and exogenously time-varying risk aversion instead of liquidity shocks. Following their paper and the related literature, I focus my analysis on dollar returns $Q_{t}$ to highlight the economic intuition. Percentage returns are not well defined with normally distributed prices and do not have analytical expressions. Numerical experiments indicate that the main qualitative results hold with percentage returns.
    ${ }^{13}$ The infrequent rebalancing model can generate short-term reversal even when $a_{\theta}>1 / R$. Still, all return autocorrelations become positive when $a_{\theta}$ approaches one.

[^10]:    ${ }^{14}$ The previous result holds in the model of Biais, Bossaerts, and Spatt (2010), which uses endowment shocks. Asymmetric information can increase return autocorrelation but cannot make it positive unless $a_{\theta} R>1$. In a stationary setup, Albuquerque and Miao (2014) obtain positive autocorrelation with a signal about future dividends. The main trading mechanism of their model, however, is the existence of a nontraded investment opportunity as in the model of Wang (1994). The hedging motive relies on a nonzero correlation between dividend shocks and private investment shocks, which is why the signal affects return autocorrelation.

[^11]:    ${ }^{15}$ See Lemma 4. These conditions always held in the four symmetric equilibria that I found numerically. Assuming that $k=1$ and $h=1$ is made for convenience and does not appear to affect the result.

[^12]:    ${ }^{16}$ The Internet Appendix presents a model in which liquidity trading occurs at low and high frequencies. That is, a fraction of liquidity traders trade infrequently. Autocorrelations are negative unless liquidity trading is highly persistent and cannot switch sign if the first autocorrelation is negative. The key difference is that infrequent traders provide liquidity (Lemma 4). Thus, when they liquidate their abnormal positions, they trade in the same direction as the initial liquidity shock that they absorbed.
    ${ }^{17}$ Solving the model for a large number of assets is numerically challenging with high $k$ and correlated assets. To ease the procedure, one can assume that the variance-covariance matrices of dividends and liquidity shocks commute and use an eigenvalue decomposition. The method only requires that one solve for ( $2 k+2$ ) eigenvalues independently of the number of assets.

[^13]:    ${ }^{18}$ Chien, Cole, and Lustig (2012) assume 5\% of active traders, $45 \%$ of intermittent traders, and $50 \%$ of nonparticipants in their economy. Bacchetta and Van Wincoop (2010) study a foreign exchange market setup populated only by infrequent traders. The results are robust to variation in the other parameters; for instance, liquidity shock volatility can be adjusted to calibrate the magnitudes of the coefficients.

[^14]:    ${ }^{19}$ The regression coefficients cannot be directly compared to partial autocorrelations. Nevertheless, adjusting the volatility of dividends or liquidity shocks can fit the magnitudes of the autocorrelations while preserving the shape of the autocorrelation pattern. The calibration is discussed in detail in Appendix A.2.

[^15]:    ${ }^{20}$ Let date $t$ be the beginning of a calendar period. The vector of expected returns in calendar period $j$ is then given by $\mathbb{E}\left[P_{t+1}+D_{t+1}-R P_{t} \mid c(t)=j\right]$. This definition ensures that increasing traders' risk aversion in a calendar period increases expected returns in the same calendar period.

[^16]:    ${ }^{21}$ In line with this result, Jain and Joh (1988) find that the average return on the S\&P 500 is largest in the first and last hour of the trading day (except on Mondays).
    ${ }^{22}$ The market return is computed using the expected number of shares available in the market. More precisely, $Q_{m, t+1}=\sum_{i=1}^{N} s_{i} Q_{i, t+1}$, where $s_{i}$ is the $i^{\text {th }}$ element of the vector $\bar{S}-\sum_{j=0}^{k} q_{c(t-j)} \mathbb{E}\left[X_{t-j}^{I} \mid c(t)\right]$.
    ${ }^{23}$ Assets' betas may also change. Numerically, I find that infrequent rebalancing increases the spread in betas, which strengthens the return seasonality. With many assets, however, this effect is small in the model.

[^17]:    ${ }^{1}$ Rozeff and Kinney (1976) document larger January returns relative to other months in an equal-weighted index of NYSE stocks. Keim (1983) shows that the large January returns stem from the abnormal performance of small stocks.

[^18]:    ${ }^{2}$ Musto (1997) shows evidence consistent with agency problems related to end-of-quarter portfolio disclosures for commercial paper. Carhart et al. (2002) document mutual funds manipulation at the end of quarters. Ben-David et al. (2013) provide similar evidence for hedge funds. For an example of anecdotal evidence, The Wall Street Journal (December 6, 2012) reports this story: "Fund Managers Lift Results With Timely Trading Sprees."
    ${ }^{3}$ Two noteworthy exceptions are the momentum and idiosyncratic volatility strategies; see Section 2.2.4.

[^19]:    ${ }^{4}$ To build some of the characteristics, I use returns starting from 1954. Returns are adjusted for a potential delisting bias (Shumway, 1997). A missing delisting return that is performance-related (code 500, 520-584) is set to $-30 \%$.
    ${ }^{5}$ The breakpoints for the portfolios are based on NYSE deciles. Stocks with a price smaller than $1 \$$ at the formation date are excluded from the portfolios. Financial firms (SIC code between 6000 and 6999) are excluded from all the portfolios based on accounting variables.

[^20]:    ${ }^{6}$ Due to lack of data I start the sample in July 1971 for the accruals portfolios and in July 1964 for the asset growth and net stock issues portfolios.

[^21]:    ${ }^{7}$ Contrary to Ang et al. (2006), the portfolios are built using NYSE breakpoints instead of CRSP breakpoints. Using value-weighted portfolios with CRSP breakpoints, I find that the average January return is $3.57 \%$ ( $t$-stat: 2.85), and the average non-January return is $-1.55 \%$ ( $t$-stat: -4.94 ), which gives a total average return of $-1.13 \%$ ( $t$-stat: -3.60) -in line with the findings of Ang et al. (2006). Bali and Cakici (2008) show that "the idiosyncratic volatility puzzle" is sensitive to the weighting scheme and the definition of the breakpoints.

[^22]:    ${ }^{8}$ Cooper, Gulen, and Schill (2008) perform a battery of robustness checks, such as sorting on size. They do not, however, control for a January seasonality. To provide a complete picture, I restrict the sample from July 1963 to June 2003 like their analysis. The non-January average value-weighted return becomes $-0.33 \%$ with a $t$-statistic of -1.87, which is more than seven times lower than in January.

[^23]:    ${ }^{9}$ The same remark holds with equal-weighting instead of value-weighting. Examining anomalies' returns separately for each month of the year, the results are consistent but noisier. Still, October solely drives the beginning-of-quarter seasonality in value-weighted size portfolios. Nevertheless, this seasonality is robust over multiple subsamples (Section B.1.2).

[^24]:    ${ }^{10}$ I take the risk-free return, market return, SMB, and HML from Kenneth French's website. I thank him for making these data available.

[^25]:    ${ }^{11}$ Reinganum (1983) finds supportive evidence; the January effect is more pronounced for stocks with poor performance over the previous year (see also Roll, 1983). Poterba and Weisbenner (2001) provide consistent evidence and explain in greater detail the relevant U.S. tax regime. In a recent study, Kang et al. (2015) highlight the role of interest rates for the January effect.
    ${ }^{12}$ Sias and Starks (1997) use institutional ownership to disentangle the two hypotheses.

[^26]:    ${ }^{13}$ Market capitalization is measured once a year at the beginning of June, so that the composition of the portfolios does not vary over the turn of any quarter.

[^27]:    ${ }^{14}$ The $t$-statistics are 4.61, 5.07, and 2.71 for the IL portfolio; and 4.61, 5.07, and 2.71 for the IV portfolio. In March/April, the average daily return of the IV portfolio on the last trading day of the quarter is not the largest in absolute value but is the only one to be statistically significant.

[^28]:    ${ }^{15}$ Using independent sorts gives similar results, but some of the portfolios hold less than 20 stocks at several dates.

[^29]:    ${ }^{16}$ The monthly long-term seasonality return $r_{-6: 10 y}$ is a stock's average return in the same month as the current month six to ten years ago. This return proxies for the long-term seasonal component of a stock return in a given month. The results are similar using Reinganum's measure of potential for tax-loss selling, where the stock price at the end of November is divided by its maximum price over January to November.
    ${ }^{17}$ The portfolios contain on average more than seventy stocks, but some of them have a couple of missing values since I require a minimum of ten stocks in each portfolio. For instance, few stocks may have negative returns following a stock market boom. Since this effect concerns around $1 \%$ of the portfolio-month observations and relative sorts give similar results, this issue is not likely to be a serious concern. This also explains why the average returns on some of the long-short portfolios are not exactly equal to the average long minus the average short return.

[^30]:    ${ }^{1}$ As an example of the importance of studying intraday return and volume patterns, the rise of passive investing appears to have led to a dramatic increase in trading volume over the last thirty minutes of trading in recent years. See, e.g., Robin Wiggleworth, "Machines and Markets: 5 Areas to Watch," Financial Times, March 17, 2017; and Dan Strumpf, "Stock-Market Traders Pile In at the Close," Wall Street Journal, May 27, 2015.

[^31]:    ${ }^{2}$ There is scant empirical evidence about intraday average returns. Wood, McInish, and Ord (1985), Harris (1986), and Jain and Joh (1988) document patterns in intraday average returns, but they rely on short samples dating from before 1984. Smirlock and Starks (1986) use a longer sample-twenty-one years of hourly returns-but only for the Dow Jones Industrial Average. On the contrary, patterns in return volatility and volume over the trading day are well-documented, robust, and appear in different markets; see, for instance, Wood, McInish, and Ord (1985), Amihud and Mendelson (1987), Jain and Joh (1988), Gerety and Mulherin (1994), and Andersen and Bollerslev (1997).

[^32]:    ${ }^{3}$ Theories of intraday and overnight returns are reviewed in Section 3.2.

[^33]:    ${ }^{4}$ Cliff, Cooper, and Gulen (2008), Kelly and Clark (2011) and Berkman et al. (2012) find that overnight returns account for a sizable fraction of the U.S. equity premium. Marked intraday and overnight patterns in average returns exist in other asset classes. Breedon and Ranaldo (2013) document time-of-day effects in currencies. Muravyev and Ni (2016) find that the variance risk premium for S\&P 500 and equity options is only negative overnight and is in fact mildly positive intraday.
    ${ }^{5}$ Slezak (1994) develops an equilibrium model with a single closure that is a pure information event: The variance of private news increases in the period after the closure, but the variance of liquidity trading remains the same.

[^34]:    ${ }^{6}$ See also Foster and Viswanathan (1990). Relatedly, Collin-Dufresne and Fos (2016) solve a strategic model in which informed investors have long-lived information and time their trades.

[^35]:    ${ }^{7}$ The results are robust to using the quote midpoint taken from the Center for Research in Security Prices (CRSP)

[^36]:    data or the closing price if no midpoint is reported. Before 2004, end-of-the-day CRSP midquotes tend to be higher than TAQ midquotes for all stocks. The main results of the paper are, however, qualitatively unaffected. After 2004, midquotes are close to identical.
    ${ }^{8}$ I use the TCLINK macro provided by WRDS to link TAQ symbol to CRSP PERMNO. In a few cases, there are more than one TAQ symbol associated with a given PERMNO on the same day. Among these overlapping observations, I keep the TAQ symbol with the most observations over the current quarter and discard the others.

[^37]:    ${ }^{9}$ I verify that no discernible difference exists between the average monthly portfolio return computed by compounding intraday half-hour and overnight returns and the average monthly value-weighted portfolio return computed using CRSP monthly returns.

[^38]:    ${ }^{10}$ The sample is split into three parts. The first part spans the ISSM data and goes from January 1,1986 , to December 31, 1992. The second part covers 1993 to 2004 included. Finally, the last part covers 2005 to 2015.
    ${ }^{11}$ Since I focus on portfolios, I do not adjust the regressions for nonsynchronous trading. On average over all stocks in the market, measured betas and alphas are equal to true alphas and betas (Scholes and Williams (1977)). Section 3.4.2 shows that nonsynchronous trading and thin trading do not appear to be a major concern for my results.

[^39]:    ${ }^{12}$ The ISSM data set misses volume data in 1987. I use as a benchmark the maximum number of days for which a stock has ISSM volume data in this year (210).

[^40]:    ${ }^{13}$ Lucca and Moench (2015) document that, from January 1994 to March 2011, about $80 \%$ of annual realized market excess returns accrue in the 24 hours before scheduled FOMC announcements. Most anomalies tend not to perform well on FOMC announcement days. But apart from beta and idiosyncratic volatility, which incur large negative overnight returns and continue to lose value over the day, intraday and overnight average returns on other anomalies are not significantly different at the level of $10 \%$. The short leg eliminates most of the exposure to the announcement. These results are reported in the Internet Appendix available at www.vincentbogousslavsky.com.
    ${ }^{14}$ The average rank correlation of the characteristics from one year to the next ranges from 0.04 for momentum to 0.97 for illiquidity. Net stock issues ( 0.40 ), beta ( 0.63 ), idiosyncratic volatility ( 0.65 ), and gross profitability ( 0.93 ) lie in between.

[^41]:    ${ }^{15}$ Evidence from panel regressions (detailed below) is inconclusive as well. A one basis point increase in the last half-hour return is associated with a 0.09 basis point decrease in the following overnight return.

[^42]:    ${ }^{16}$ Comerton-Forde et al. (2010) document that NYSE specialists had positive end-of-the-day inventories $94 \%$ of the time over 1994 to 2004.
    ${ }^{17}$ Strategic models such as Admati and Pfleiderer (1988) and Foster and Viswanathan (1990) predict that liquidity measures improve with higher trading volume. Hence, these models have trouble reconciling the evidence of U-shaped intraday patterns in volume and trading costs.

[^43]:    ${ }^{18}$ Bogousslavsky (2015) shows that small stocks earn large returns on the last day of each quarter that partly reverse on the following day.

[^44]:    ${ }^{19}$ Since Compustat reports only the date of the announcement, I use a simple volume test to determine whether the announcement is made after trading hours. The firm's turnover net of market turnover is compared between the reported day and the following trading day. The announcement is allocated to the day with the highest turnover.

[^45]:    ${ }^{20}$ Moreover, even though a series of positive returns over the day is in line with information asymmetry being gradually resolved with trading (Section 3.2), there is no reason to expect that stocks in the long leg of these portfolios are subject to more information asymmetry than stocks in the short leg.
    ${ }^{21}$ Overnight returns on long-only portfolios are, however, more sensitive to this choice as shown in Section 3.7.1.

[^46]:    ${ }^{22}$ The initial margin requirements of Regulation $T$ in the U.S. are typically applied at the end of the day; see for instance https://gdcdyn.interactivebrokers.com/en/index.php?f=marginnew\&p=overviewl.

