A general framework for routing problems with stochastic demands

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Outline

1. Introduction
2. Stochastic Information
3. Formulation
4. Methodology
5. Numerical Experiments
6. Conclusion
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Introduction

Setup and concepts

- Logistic setting:
  - depots, supply points, demand points
  - non-stationary stochastic demands over a planning horizon
  - distribution or collection context
Setup and concepts

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- depots, supply points, demand points
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**Decisions:**
- visits
- routing
- inventory management
Introduction

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- Undesirable events:
  - stock-outs
  - overflows
  - breakdowns
  - route failures

The objective:
- minimize cost
- satisfying all constraints
- avoiding the occurrence of undesirable events
Introduction

Setup and concepts

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  - overflows
  - breakdowns
  - route failures

- The objective:
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Routing

Figure 1: Tour example

depot supply point demand point

trip 1 trip 2 trip 3
Motivation and Contribution

- Generality of the approach: VRP, IRP, others
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- Uses recourse actions to recover from undesirable events
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- Relies on dynamic probabilistic information to integrate the cost of undesirable events
-Uses recourse actions to recover from undesirable events
- Integrates demand forecasting
- Modeling framework corroborated by practical application
- High quality meta-heuristic solution approach
- Intuitive evaluation of various solution aspects by simulation
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2 Stochastic Information

3 Formulation

4 Methodology

5 Numerical Experiments

6 Conclusion
The demand of point \( i \in \mathcal{P} \) in period \( t \in \mathcal{T} \) decomposes trivially as:

\[
\rho_{it} = \mathbb{E}(\rho_{it}) + \varepsilon_{it}
\]  

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**Definition 1**

The error terms are modeled as $\varepsilon_{it} \sim D(\varpi)$, where $D(\varpi)$ may be any theoretical or empirical distribution.
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The error terms are modeled as $\varepsilon_{it} \sim D(\varpi)$, where $D(\varpi)$ may be any theoretical or empirical distribution.

**Definition 2**

A forecasting model provides the expected demands $\mathbb{E}(\rho_{it})$ for all $i \in P$, $t \in T$ and the error distribution $D(\varpi)$. 
Demand point states and probabilities

- Notation:
  - $\Lambda_{i0}$: inventory after delivery of demand point $i$ in period 0
  - $\omega_i$: inventory capacity of demand point $i$
  - $\sigma_{it}$: state of demand point $i$ in period $t$
    - $\sigma_{it} = 0$: normal
    - $\sigma_{it} = 1$: stock-out
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- **Delivery types:**
  - *regular delivery*: performed by a vehicle
  - *emergency delivery*: recourse action performed in a state of stock-out when no vehicle visits the point
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- **Relevant costs:**
  - **stock-out cost** $\chi$: paid in a state of stock-out
  - **emergency delivery cost** $\zeta$: paid in a state of stock-out when no vehicle visits the point
Figure 2: State probability tree

\[ \begin{align*}
\sigma_{i0} &= 0 \\
& \quad \vdots \\
\sigma_{i1} &= 1 \\
\sigma_{i2} &= 0 \\
& \quad \vdots \\
\sigma_{i3} &= 0 \\
& \quad \vdots \\
\sigma_{i4} &= 0 \\
& \quad \vdots \\
\end{align*} \]
Proposition 1

Under an OU policy in a distribution context, the stock-out probabilities can be pre-computed for any distribution $D(\varpi)$ using simulation.
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Corollary 1
Under an OU policy in a collection context, the overflow probabilities can be pre-computed for any distribution $D(\varpi)$ using simulation.
Maximum Level (ML) inventory policy

- Discretized ML policy:

  Figure 3: Level discretization for a demand point

[Diagram showing level discretization with discrete levels 1, 2, and 3]
Maximum Level (ML) inventory policy

- Discretized ML policy:

**Figure 3:** Level discretization for a demand point

```
  Discrete level 3
  Discrete level 2
  Discrete level 1
```

**Proposition 3**

Under a discretized ML policy, the relevant probabilities can be pre-computed, and the complexity is linear with the number of discrete levels.
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Objective function

Figure 4: Composition of the objective function

- Expected inventory holding cost
- Demand point visit cost
- Routing cost
- Workload balancing
- Expected stock-out and emergency delivery cost
- Expected route failure cost
Objective Function: Stochastic components

- **Expected Stock-Out and Emergency Delivery Cost (ESOEDC) component:**

  \[
  \text{ESOEDC} = \sum_{t \in T^+} \sum_{i \in P} \left( P \left( \sigma_{it} = 1 \mid \Lambda_{im} \right) \left( \chi + \zeta - \zeta \sum_{k \in K} y_{ikt} \right) \right), \tag{2}
  \]

  where

  - \( T^+ \): planning horizon plus following day
  - \( P \): set of demand points
  - \( K \): set of vehicles
  - \( \sigma_{it} = 1 \): state of stock-out of point \( i \) in period \( t \)
  - \( \Lambda_{im} \): inventory after delivery of point \( i \) in period \( m \)
  - \( m \): period of the previous delivery to point \( i \)
  - \( \chi \): stock-out cost
  - \( \zeta \): emergency delivery cost
  - \( y_{ikt} = 1 \) if point \( i \) is visited by vehicle \( k \) in period \( t \), 0 otherwise
Objective Function: Stochastic components

- **Expected Route Failure Cost (ERFC) component:**

  \[
  \text{ERFC} = \sum_{k \in \mathcal{K}} \sum_{\mathcal{S} \in \mathcal{S}_k} \sum_{n=1}^{N_{\mathcal{S}} - 1} C_{\mathcal{S}} \mathbb{P}(n \Omega_k < \Xi_{\mathcal{S}} \leq (n + 1) \Omega_k),
  \]

  where
  - \( \mathcal{K} \): set of vehicles
  - \( \mathcal{S}_k \): set of supply point delimited trips for vehicle \( k \)
  - \( N_{\mathcal{S}} \): number of demand points in trip \( \mathcal{S} \)
  - \( C_{\mathcal{S}} \): route failure cost for trip \( \mathcal{S} \)
  - \( \Xi_{\mathcal{S}} \): volume delivered in trip \( \mathcal{S} \)
  - \( \Omega_k \): capacity of vehicle \( k \)
Objective function: Tractability

Proposition 4

The route failure probabilities cannot be pre-computed.

- Because the volume to deliver in a given trip is a decision variable and not known in advance.

Assumption 1

Restrict the error terms as $\varepsilon \sim D(\varpi)$, where $D(\varpi)$ may be any theoretical or empirical distribution.

While we cannot pre-compute the probabilities themselves, we can derive their ECDFs. The number of ECDFs to derive is bounded by the number of demand points times the number of periods in the planning horizon.
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A reaction policy defines how the subsequent decisions are changed in response to an emergency delivery.
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**Proposition 5**

In the absence of inventory holding costs, the objective function always overestimates the real cost.
Constraints

- Routing aspect:
  - multiple depots
  - supply point visits
  - open tours
  - multi-period tours
  - periodicities and service frequency
  - etc...
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- **Inventory related:**
  - track inventory
  - implement the inventory policy
  - **forbid stock-Outs in the expected sense**
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- Vehicle capacity related
### Constraints

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- **Vehicle capacity related**
- **Duration and time window related**
- **Etc...**
Applications

- Stochastic demand problems:
  - vehicle routing problem
  - waste collection inventory routing
  - supermarket delivery routing
  - fuel delivery routing
  - home health care routing
  - maritime inventory routing
  - etc...

- Probability-based routing problems:
  - facility maintenance
  - epidemic prevention
  - etc...
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- State-of-the-art meta-heuristic framework
- Operators compete in modifying the current solution
- At each iteration, draw a destroy and a repair operator randomly
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- At each iteration, draw a destroy and a repair operator randomly
- The destroy (repair) operator $i \in \mathcal{O}$ is drawn with probability:
  \[ P(i) = \frac{\omega_i}{\sum_{j \in \mathcal{O}} \omega_j} \] (4)
- The weights $\omega_i$ are periodically updated by an adaptive layer that tracks operator performance
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- The weights \( \omega_i \) are periodically updated by an adaptive layer that tracks operator performance
- Rich operator pools reflecting the problem structure
- Simulated annealing solution guiding principle
Numerical Experiments

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Benchmarks: Archetti et al. (2007) Instances

- First classical IRP testbed
- 160 instances in total
- 5 to 50 customers
- 3 or 6 periods in the planning horizon
- Single vehicle
- Low and high inventory holding costs
- Optimal solutions (branch-and-cut) by Archetti et al. (2007)
## Numerical Experiments

### Benchmarking: Archetti et al. (2007) instances

**Table 1: Results on Archetti et al. (2007) Instances**

| $|T|$ | n  | High Inventory Holding Cost | Low Inventory Holding Cost |
|----|----|------------------------------|-----------------------------|
|    |    | Runtime(s.) | Best Gap(%) | Avg Gap(%) | Worst Gap(%) | Runtime(s.) | Best Gap(%) | Avg Gap(%) | Worst Gap(%) |
| 3  | 5  | 69.08        | 0.00        | 0.00       | 0.00         | 85.69       | 0.00        | 0.00       | 0.00         |
| 3  | 10 | 183.94       | 0.00        | 0.00       | 0.00         | 156.36      | 0.00        | 0.00       | 0.00         |
| 3  | 15 | 317.93       | 0.00        | 0.00       | 0.00         | 274.05      | 0.00        | 0.00       | 0.00         |
| 3  | 20 | 440.02       | 0.00        | 0.00       | 0.01         | 444.68      | 0.00        | 0.00       | 0.00         |
| 3  | 25 | 523.42       | 0.00        | 0.08       | 0.25         | 501.78      | 0.01        | 0.20       | 0.66         |
| 3  | 30 | 835.21       | 0.01        | 0.15       | 0.32         | 649.09      | 0.00        | 0.41       | 0.98         |
| 3  | 35 | 866.06       | 0.00        | 0.15       | 0.36         | 731.21      | 0.00        | 0.46       | 1.68         |
| 3  | 40 | 896.91       | 0.02        | 0.18       | 0.44         | 976.83      | 0.16        | 0.47       | 0.97         |
| 3  | 45 | 1124.57      | 0.05        | 0.42       | 0.91         | 1074.19     | 0.00        | 1.05       | 2.53         |
| 3  | 50 | 1424.27      | 0.06        | 0.35       | 0.79         | 1223.56     | 0.13        | 1.19       | 2.15         |
| 6  | 5  | 105.86       | 0.00        | 0.00       | 0.00         | 73.28       | 0.00        | 0.00       | 0.00         |
| 6  | 10 | 184.48       | 0.00        | 0.01       | 0.08         | 181.93      | 0.00        | 0.00       | 0.00         |
| 6  | 15 | 333.82       | 0.01        | 0.09       | 0.15         | 272.03      | 0.00        | 0.03       | 0.16         |
| 6  | 20 | 394.39       | 0.00        | 0.17       | 0.41         | 420.28      | 0.05        | 0.34       | 0.82         |
| 6  | 25 | 636.27       | 0.12        | 0.34       | 0.82         | 546.85      | 0.09        | 0.67       | 1.60         |
| 6  | 30 | 725.63       | 0.10        | 0.47       | 0.93         | 733.12      | 0.44        | 1.43       | 2.63         |
| Average | 566.37 | 0.02 | 0.15 | 0.34 | 521.56 | 0.05 | 0.39 | 0.89 |
Waste collection inventory routing problem

- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016
- Planning horizon of 7 days
- Up to 2 heterogeneous vehicles
- Up to 53 containers (41 on average)
- 2 dumps located far apart from each other
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- **Overflow cost: 100 CHF**
- **Simulation of undesirable events on the final solution**
Waste collection IRP: Geography

Figure 5: Geneva service area
Waste collection IRP: Policies

- **Probabilistic objective:**
  - *routing cost*
  - *expected overflow and emergency collection cost*
  - *expected route failure cost*
  - we vary the emergency collection cost (100 CHF, 50 CHF, 25 CHF) and the route failure cost multiplier (1.00, 0.50, 0.25, 0.00)
Waste collection IRP: Policies

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  - we vary the emergency collection cost (100 CHF, 50 CHF, 25 CHF) and the route failure cost multiplier (1.00, 0.50, 0.25, 0.00)

- **Deterministic objective:**
  - *routing cost only*
  - reduced container effective capacity
  - reduced truck effective capacity
  - we vary the container and truck effective capacities (1.00, 0.90, 0.75, 0.60)
Numerical Experiments

Waste collection IRP: Routing costs

Figure 6: Comparison of routing costs for probabilistic and deterministic policies
Waste collection IRP: Overflows and route failures

Figure 7: Comparison of undesirable events at different simulated percentiles

(a) Overflows

(b) Route Failures

Policies
Waste collection IRP: Realized costs

Figure 8: Comparison of realized costs at different simulated percentiles

Objective Complete ECC=100, RFCM=1 Routing-only
## Waste collection IRP: Simulation and Tractability

### Table 2: Impact of ECDFs on computation time

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<th>ALNS version</th>
<th>Bins</th>
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<th>RFCM</th>
<th>Cost (CHF)</th>
<th>Runtime (s.)</th>
<th>ECDF calls (millions)</th>
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<td>672.73</td>
</tr>
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May 17–19, 2017  
33 / 42
Figure 9: Objective function overestimation for two reaction policy extremes
Figure 10: Heuristic bounds, single visit, gap = 17%
Waste collection IRP: Bounds

Figure 11: Heuristic bounds with re-optimization, single visit, gap = 7%
Facility maintenance

- 24 instances derived from the waste collection instances
- Planning horizon of 7 days
- Up to 2 vehicles
- Up to 50 containers (41 on average)
Facility maintenance

- 24 instances derived from the waste collection instances
- Planning horizon of 7 days
- Up to 2 vehicles
- Up to 50 containers (41 on average)
- **Simulation of breakdowns on the final solution**
Facility maintenance: Breakdown probability

Figure 12: Facility cumulative breakdown probability
Facility maintenance: Routing cost and breakdowns

Figure 13: Verification of modeling approach

(a) Routing cost

(b) Breakdowns
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- Computationally tractable
- Corroborated by practical applications
- Much superior to classical deterministic approaches
- High quality efficient and stable solution methodology
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Next steps:
- further work on the facility maintenance problem
- further stability tests
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Future work:
- further work on bounds
- comparison to alternative approaches
- generation of additional sets of realistic instances
Thank you.
Questions?