Flexarray: Random Phased Array Layouts for Analytical Spatial Filtering

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Abstract — We propose a method for designing phased-arrays according to a given target beamshape. Building on the **Flexibeam** framework, antenna locations are *sampled* from a probabilistic density function. We prove that the achieved beamshapes **converge uniformly** to the target beamshapes as the number of antennas increases. We illustrate the technique with examples.

1. Beamforming in a Probabilistic Setup

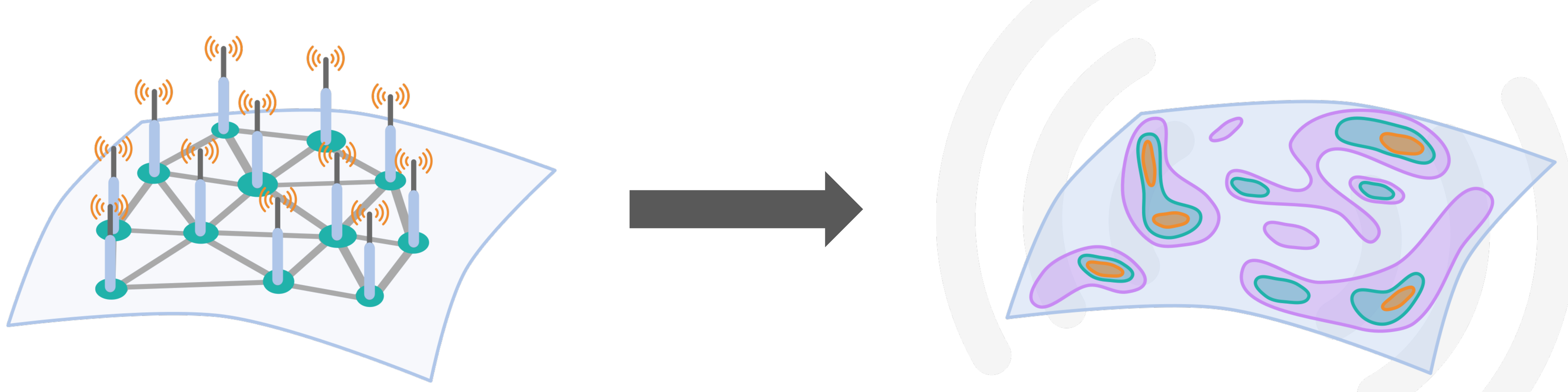
As in the **Flexibeam [1]** framework, consider a continuous array with *target beamshape:*

$$\hat{\omega}(\theta) = \int_0^{\infty}\int_0^{2\pi} \omega(p,\phi)\, e^{j2\pi p\cos(\theta-\phi)}\, p\, d\phi\, dp.$$

The **beamforming function** is defined as

$$\omega(p,\phi) = \int_0^{\infty}\int_0^{2\pi} \hat{\omega}_e(r,\theta)\, e^{-j2\pi r p\cos(\theta-\phi)}\, r\, d\theta\, dr,$$

where $\hat{\omega}_e(r,\theta)$ is called the *extended filter*.



For **circularly symmetric** extended filters

$$\hat{\omega}_e(\boldsymbol{r}) = \hat{g}(\|\boldsymbol{r}-\boldsymbol{r}_0\|),$$

we can define a *beamforming density function*

$$f_b(p,\phi) = \frac{|g(p)|}{\|g\|_1}, \quad \forall p \in \mathbb{R}_+,$$

so that:

$$\hat{\omega}(\boldsymbol{r}) = \alpha\, \mathbb{E}_{\boldsymbol{P}}\left[\sigma_g(\|\boldsymbol{P}\|)e^{-j2\pi\langle \boldsymbol{r}_0,\boldsymbol{P}\rangle}\, e^{j2\pi\langle \boldsymbol{r},\boldsymbol{P}\rangle}\right], \quad \forall \boldsymbol{r} \in \mathbb{S}^1,$$

where $\sigma_g(\|\boldsymbol{P}\|) = \mathrm{sign}(g(\|\boldsymbol{P}\|)$ and $g(p)$ is the *Hankel transform of order zero* of $\hat{g}(r)$.

2. Flexarray

Flexarray works in three steps:

1. Compute the *beamforming density function*,
2. Sample random locations

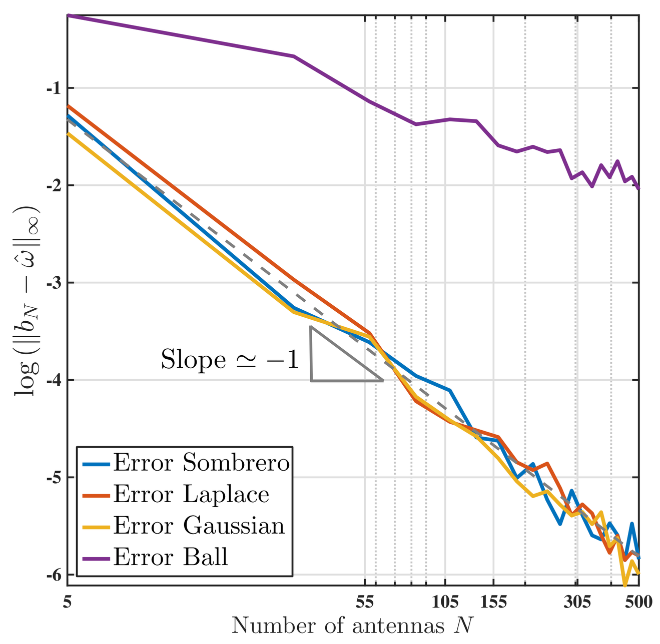
$$\{(p_i,\phi_i)\}_{i=1,\dots,N} \overset{i.i.d.}{\sim} f_b$$

3. Apply *beamforming weights*

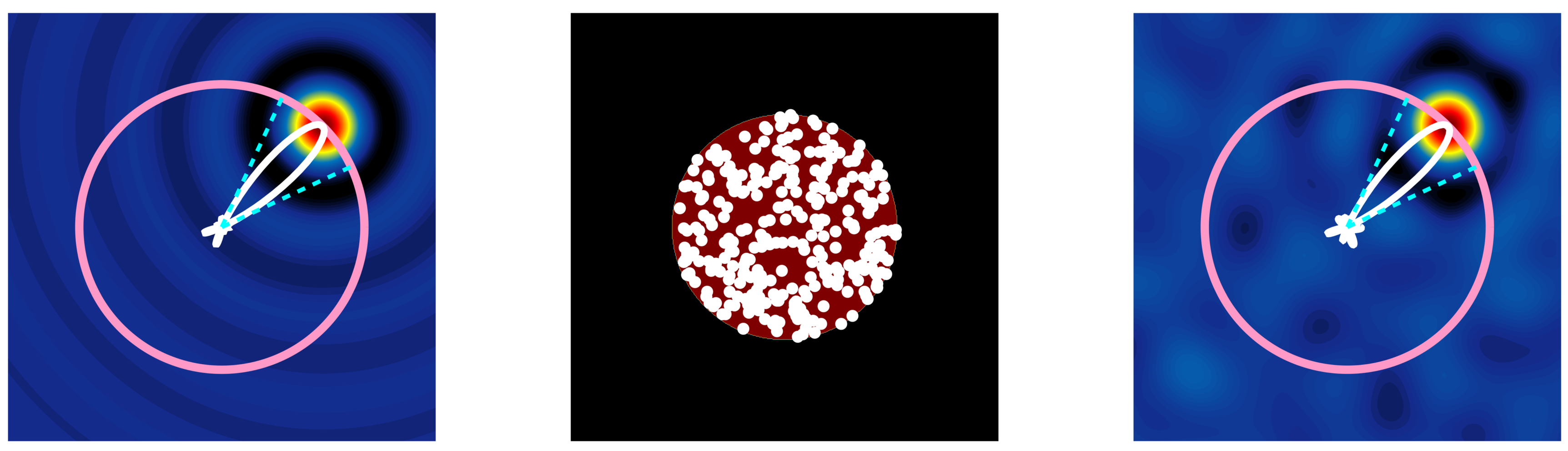
$$w_i = \frac{\alpha}{N}\sigma_g(p_i)e^{-j2\pi p_i\cos(\phi_i-\theta_0)},$$

This leads to the *empirical beamshape:*

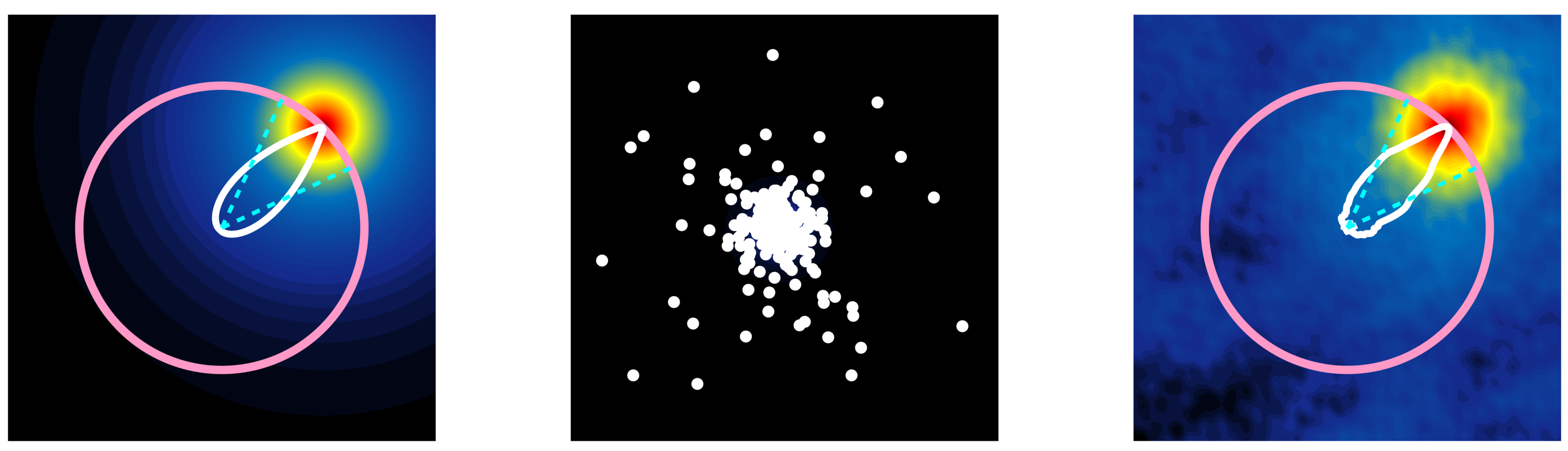
$$b_N(\theta) = \frac{\alpha}{N}\sum_{i=1}^{N}\sigma_g(p_i)e^{-j2\pi p_i\cos(\phi_i-\theta_0)}e^{j2\pi p_i\cos(\theta-\phi_i)}.$$



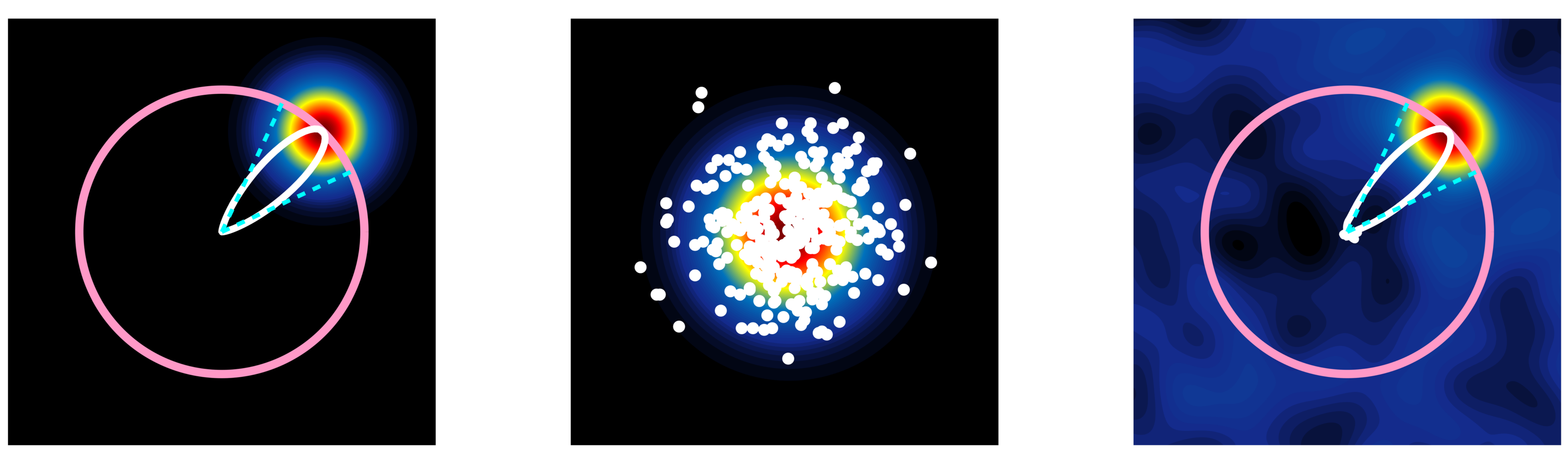
Convergence rate towards target beamshape is as ***1/N***



Sombrero beamshape



Laplace beamshape



Gaussian beamshape

3. References

1. P. Hurley and M. Simeoni, "Flexibeam: analytic spatial filtering by beamforming," in *International Conference on Acoustics, Speech and Signal Processing (ICASSP), IEEE*, March 2016.