Testing turbulence theories in the ocean: insights from state-of-the-art observations

Andrea Cimatoribus

andrea.cimatoribus@epfl.ch

Hans van Haren (NIOZ), Louis Gostiaux (EC Lyon), Ulrich Lemmin (EPFL), Damien Bouffard (EPFL/EAWAG), Andrew Barry (EPFL)

École polytechnique fédérale de Lausanne, Switzerland

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Contents

- A description of temperature fluctuations in the deep ocean
  - inferences on turbulence mechanisms from statistics
  - turbulent flux estimates
- Internal wave kinetic energy spectrum in Lake Geneva
  - linear vs. nonlinear spectra

A statistical perspective on turbulence

Statistics of scalar quantities (temperature, salinity,...):

- understanding intermittency (time and space dispersion of turbulence events);
- hints on the mechanisms leading to mixing;
- identification of different regimes at different scales.

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Well-studied topics (laboratory, numerics):

- passive scalars in isotropic turbulent flows (Warhaft, 2000);
- active scalars in convective turbulence (Zhou and Xia, 2002);
- scalars in *stably stratified* turbulent flows?

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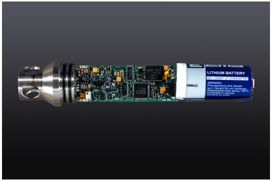
In the field:

- We cannot control what we observe in the field
  - e.g. control parameters are variable / undefined
- Statistics can help extracting information from "noisy" data

Sensors: NIOZ-HST (high-speed thermistors)



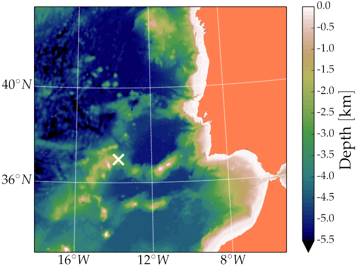
Main features:

- precision better than 5×10^{-4} K;
- response time 0.25 s;
- sampling frequency ≤ 2 Hz;
- long endurance (up to two years).

van Haren et al. (2009)

Data

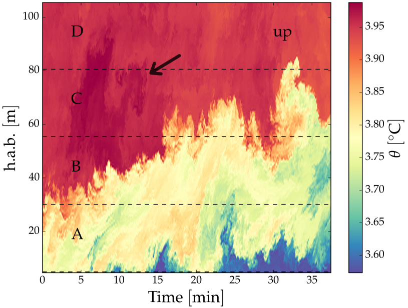
Latitude	36° 58.885′ N
Longitude	13° 45.523′ W
Max. depth	2205 m
Min. height above seafloor	5 m
Seafloor slope	9.4°
Number of sensors	144
Vertical spacing	0.7 m
Depth range	100.1 m
Deployment	13 Apr 2013
Recovery	12 Ago 2013
Sampling rate	1 Hz



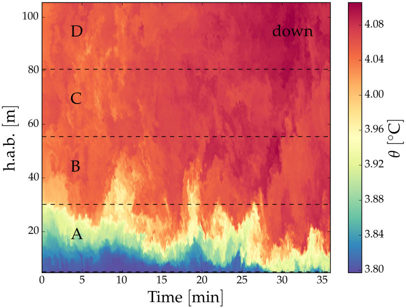
(Supercritical slope, $\gamma_{crit} \approx 5.7°$ for M_2 tide)

Data

Cooling phase (upslope)



Warming phase (downslope)



Taylor's hypothesis

Transform data to the spatial domain using Taylor's hypothesis (frozen turbulence)

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$$\Delta x = v\Delta t$$

Time series (time rate 1 s) → Spatial series (horizontal spatial resolution 0.2 m)

Using time-dependent velocity from ADCP data (only mean flow information)

Taylor's hypothesis

Transform data to the spatial domain using Taylor's hypothesis (frozen turbulence)

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Time series (time rate 1 s) → Spatial series (horizontal spatial resolution 0.2 m)

Using time-dependent velocity from ADCP data (only mean flow information)

All results are averages over the 4 months of data for each segment of the mooring, for each tidal phase

Methods I: generalised structure functions (GSF)

GSFs provide a way to characterise intermittency of the turbulent flow:

$$\gamma_q \equiv \gamma_q(r) = \left\langle |\Delta_r \theta|^q \right\rangle$$

In the inertial range:

$$\gamma_q \sim r^{\zeta(q)},$$

with $\zeta(q) = q/3$ according to the classical (non-intermittent) theory of Kolmogorov-Obukhov-Corrsin, and for r within the inertial range.

In presence of intermittency $\lim_{q\to\infty} \zeta(q) = \zeta_\infty$ (in practice for $q > 10$):

- Grid turbulence, shear driven $\to \zeta_\infty \approx 1.4$
- Convective turbulence, buoyancy driven $\to \zeta_\infty \approx 0.8$

Zhou and Xia (2002)

Methods II: flux estimates

Estimate the flux of a scalar quantity (temperature) with the method suggested by Winters and D'Asaro (1996):

- enable to resolve the fluxes vertically;
- clear definition of "background" stratification;
- no assumptions on the flow;
- estimate of the irreversible flux.

Flux as a function of the local temperature:

$$\phi_\theta(\theta_j) = -\kappa \left(\frac{\mathrm{d}z^T}{\mathrm{d}\theta} \right) (\theta_j) \left\langle |\nabla \theta|^2 \right\rangle (\theta_j)$$

θ_j: potential temperature at j-th sensor, κ: molecular diffusivity, $\mathrm{d}\theta/\mathrm{d}z^T$: background temperature gradient.

Biased low due to limits in resolution (for gradient estimation), but compensation is possible.

Generalised structure functions

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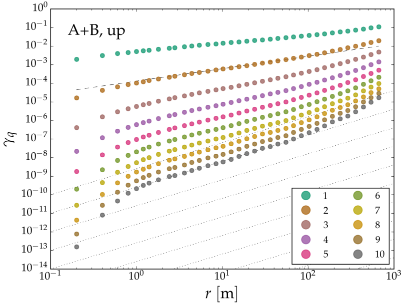

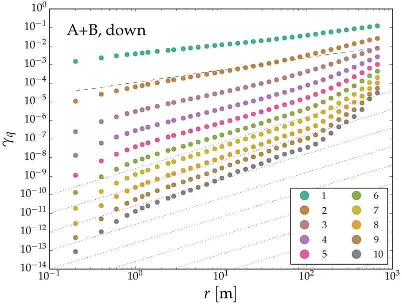

Dashed line: $\zeta(2) = 2/3$ slope. Dotted lines: "grid turbulence" slope.

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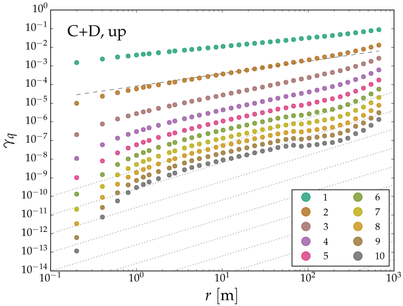

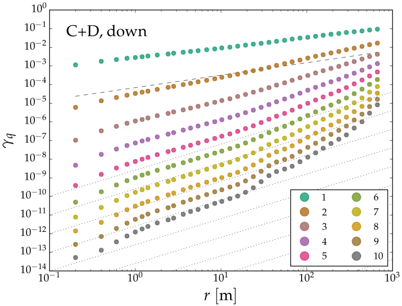

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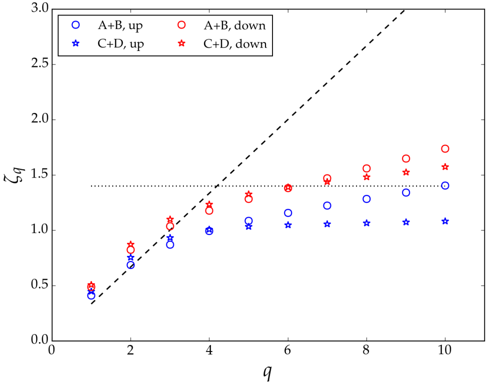
Scaling exponents and saturation of GSFs

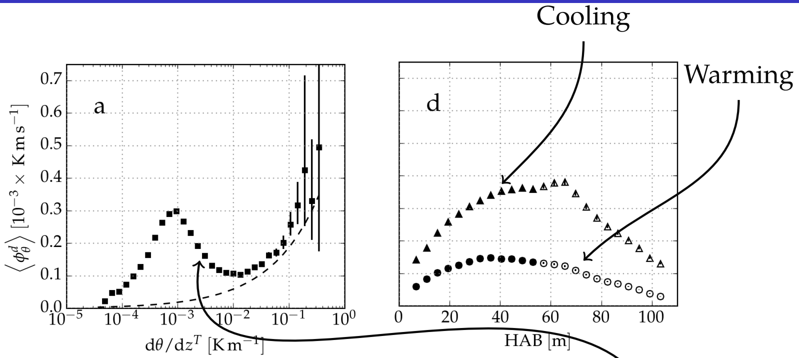


Scaling exponent within the turbulence scaling range.

Dashed line: $\zeta(q) = q/3$ slope. Dotted line: “grid turbulence” asymptote.

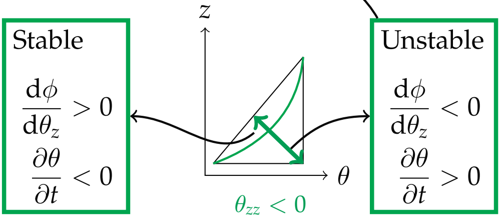
Much more on statistics in Cimatoribus and van Haren (2015)

Estimates of the flux



$$\frac{\partial \theta}{\partial t} = \frac{\partial \phi}{\partial z} = \frac{\mathrm{d}\phi}{\mathrm{d}\theta_z}\theta_{zz}$$

Phillips (1972), Posmentier (1977)



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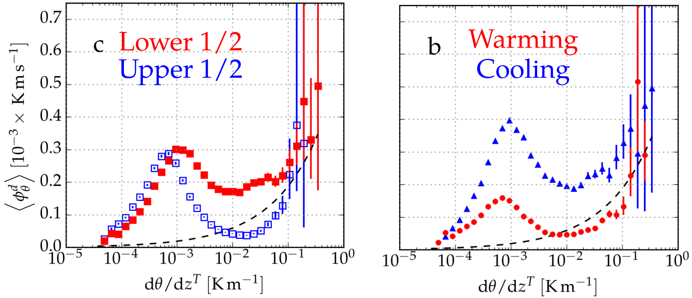

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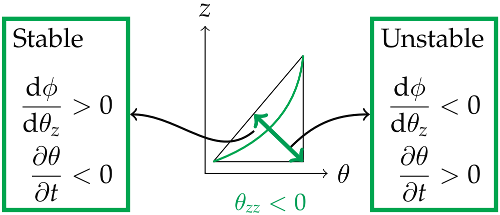

A framework for interpretation

A minimal analytical model:

- Based on Balmforth et al. (1998),
- steady states for kinetic energy density e and density gradient g.
- Mixing length (l) model (turbulent flux $\propto l$),
- horizontally homogeneous (1D vertical model).
- l constrained by the density gradient and by the height above the seafloor (h).
- Energy production = internal waves breaking at a particular scale λ (scaling break of γ_q).

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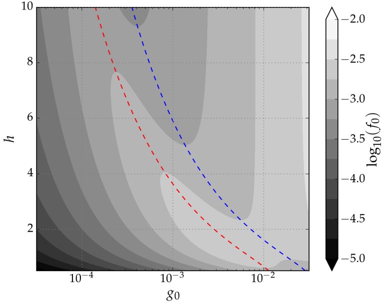
…after non-dimensionalisation, and some algebra…

$$rh^2eg - \left(e + h^2g\right)\left(\frac{h^2}{1+h^2} - e\right) = 0,$$

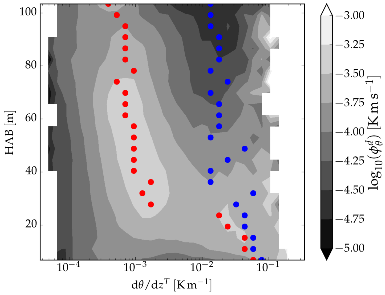
with r a non-dimensional constant.

A framework for interpretation

Model



Observations



$$\text{equilibrium flux:} \quad f_0 = l e_0^{1/2} g_0 = \frac{h e_0 g_0}{\left(e_0 + h^2 g_0\right)^{1/2}}.$$

Cimatoribus and van Haren (2016)

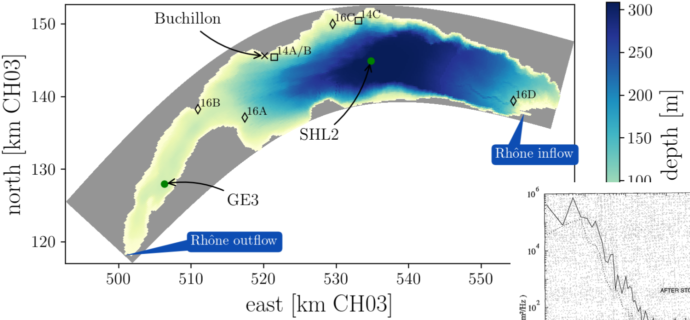
Conclusions (partial)

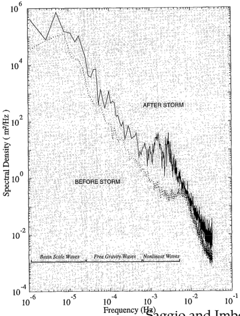
- A detailed “statistical” view on turbulence in the deep ocean.
- Generalised structure functions have some points of contact with laboratory results…
- …but show some specific behaviour too (“outer intermittency”).
- Scaling break suggests that the forcing, from internal waves, takes place at a specific length scale.
- Flux estimates show smooth, simple *average* behaviour,
  - supports idea of “spontaneous” layer formation by stratified turbulence.
- The model suggests:
  - validity of mixing length hypothesis,
  - seafloor limits both the mixing length and the forcing,
  - irrelevance of friction at the seafloor.
- Convection: in the statistics, but not in the model!


And Now For Something
Completely Different

A simpler (?) case: internal variability in a lake

Lake Geneva





Saggio and Imberger (1998)

"Standard" interpretation of observations:

- combination of long internal waves (seiches)
- linear or weakly nonlinear

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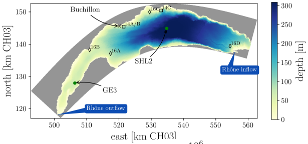
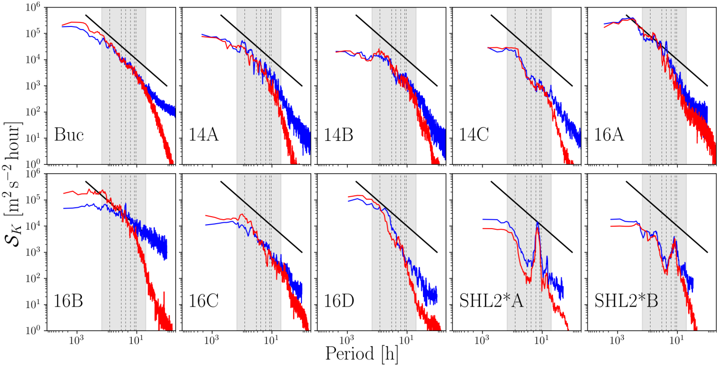


Kinetic energy spectra



Observations, model, slope = -1, dashed lines: linear modes frequencies

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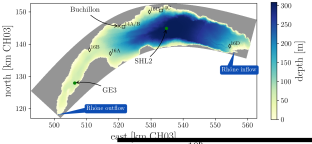
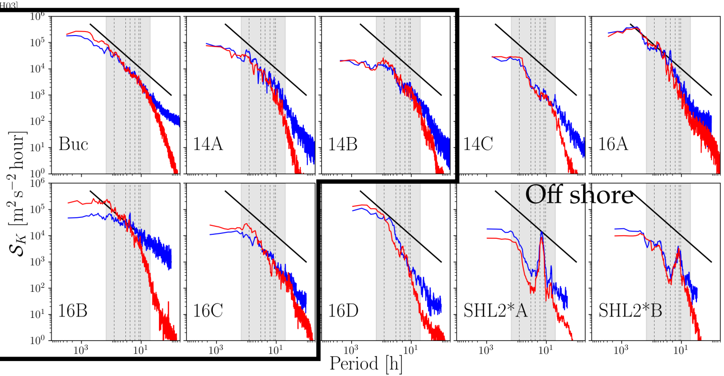


Kinetic energy spectra

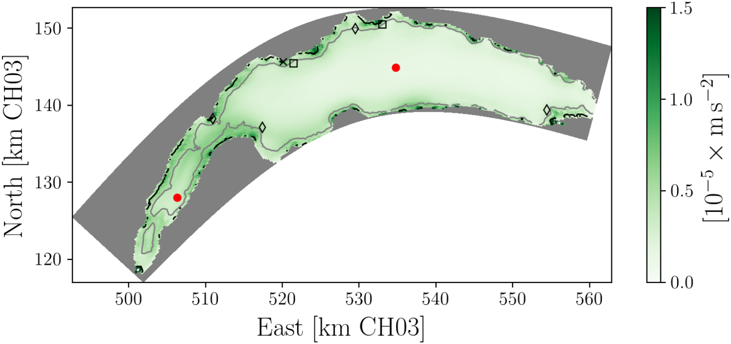
Near shore



Observations, model, slope = -1, dashed lines: linear modes frequencies

A simpler (?) case: internal variability in a lake

$$\frac{\partial \mathbf{v}_h}{\partial t} = \mathcal{N} + \mathcal{C} + \mathcal{P} + \mathcal{F} - \mathcal{D} = -\mathbf{v} \cdot \nabla \mathbf{v}_h - 2\mathbf{\Omega} \times \mathbf{v}_h - \nabla_h p + \mathit{forcing} - \mathit{dissipation}$$



Greens: $\|\mathcal{N}\|$, gray contour: $\|\mathcal{F} - \mathcal{D}\| = 10^{-6}\,\mathrm{ms}^{-2}$, black contour: $\|\mathcal{F} - \mathcal{D}\| = 10^{-5}\,\mathrm{ms}^{-2}$

Conclusions

- Field observations begin to enable the characterisation of probability density functions of different quantities
- A statistical description enables to test theories in a natural (uncontrolled) environment
- Sometimes, statistical quantities can surprise:
  - Simple behaviour out of highly turbulent environments
  - Nonlinear behaviour (instabilities? vortices?) in a low energy environment
    - from very common power spectra!

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A. Saggio and J. Imberger, Limnology and Oceanography **43**, 1780 (1998).

Thanks for listening.

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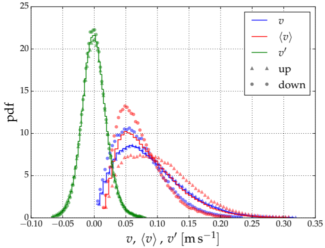
[La Palma, Islas Canarias]

Taylor's hypothesis of frozen turbulence

Time series (time rate 1 s) $\rightarrow$ Spatial series (spatial resolution 0.2 m)

Pdf of velocity

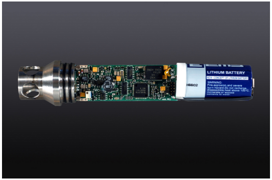


$$v = \langle v \rangle + v'$$

$\langle v \rangle$: lowpass filter
$1/\sigma = 3000\,\mathrm{s}$

- Lowpass filter defines the mean flow component
- Average velocity within each segment is used
- Velocity is not constant, thus the spatial time series obtained has variable step
- Interpolation to have a constant spatial step
- Dataset size is reduced to 1/2
- Increments are computed close in time (approximately one hour maximum)

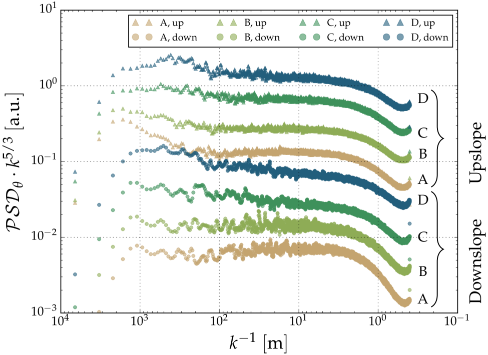
NIOZ-HST thermistors data processing



1. Read raw data from the thermistor memory (integer numbers)
   - Often subsampling is necessary due to the large amount of data recorded
2. Calibrate raw data using data from a calibration bath, or CTD data
3. Remove sensor drift by requiring a stable (or at least “smooth”) stratification on “long” time scales

Wavenumber spectra

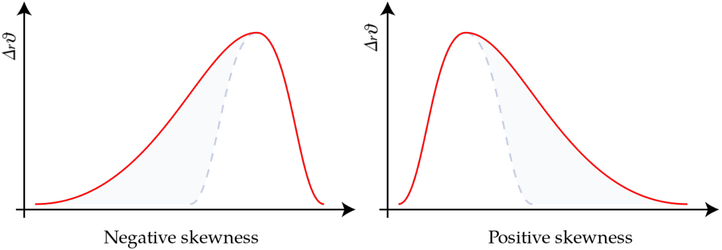
Spectra averaged by tidal phase and mooring segment



A, B, C, D, from bottom to top of the mooring

Skewness of temperature increments

$$\mu_3\left(\Delta_r\theta\right) = \frac{\left\langle \left(\Delta_r\theta - \langle\Delta_r\theta\rangle\right)^3 \right\rangle}{\left\langle \left(\Delta_r\theta - \langle\Delta_r\theta\rangle\right)^2 \right\rangle^{\frac{3}{2}}}, \text{ for increments } \Delta_r\theta = \theta(r+r_0) - \theta(r_0)$$



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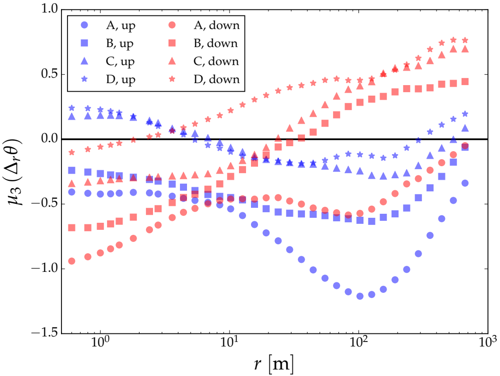


Convective structures and plus/minus increments

Convective structures have been studied using “plus” and “minus” increments:

$$\text{horizontal: } \Delta_r\theta^{\pm} = \left(|\Delta_r\theta| \pm \Delta_r\theta\right)/2$$
$$\text{vertical: } \Delta_z\theta^{\pm} = \left(|\Delta_z\theta| \pm \Delta_z\theta\right)/2$$

- Convective plume = sharp front, gentle tail
- The skewness of plus and minus increments is sensitive to this difference

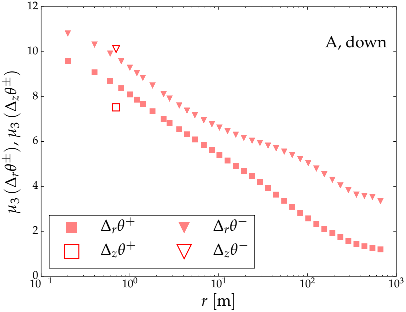
$$\text{warm plume} \begin{cases} \mu_3\left(\Delta_z\theta^+\right) < \mu_3\left(\Delta_z\theta^-\right) \\ \mu_3\left(\Delta_r\theta^+\right) > \mu_3\left(\Delta_r\theta^-\right) \end{cases}$$

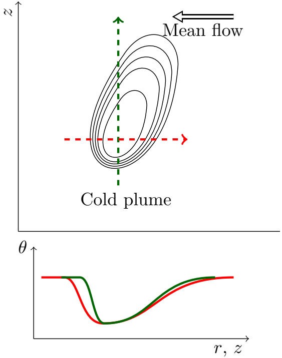
- More in general:
  characterise the spatial asymmetry of temperature anomalies

Zhou and Xia (2002)

Plus/minus increments – results

Bottom of the mooring





- Cold plumes during downslope phase
- No plumes during upslope phase

$$\mu_3(\Delta_r\theta) < 0 \ (?)$$

$$\mu_3(\Delta_z\theta^+) < \mu_3(\Delta_z\theta^-)$$

$$\mu_3(\Delta_r\theta^+) < \mu_3(\Delta_r\theta^-)$$

Plus/minus increments – results

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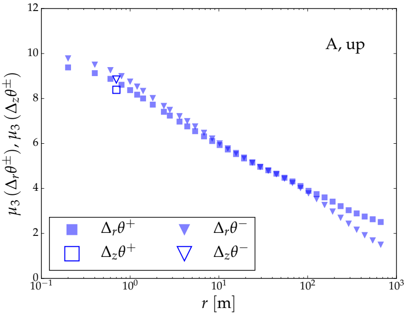

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$$\mu_3(\Delta_r\theta) \neq 0 \ (?)$$
$$\mu_3\left(\Delta_z\theta^+\right) \approx \mu_3\left(\Delta_z\theta^-\right)$$
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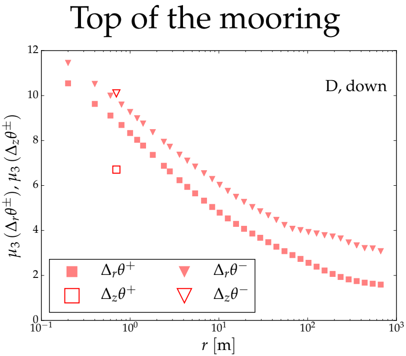
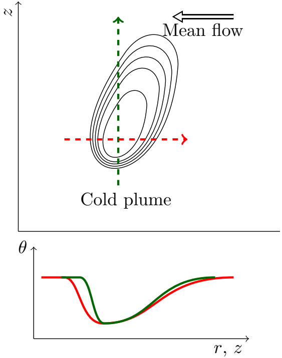
Plus/minus increments – results





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Plus/minus increments – results

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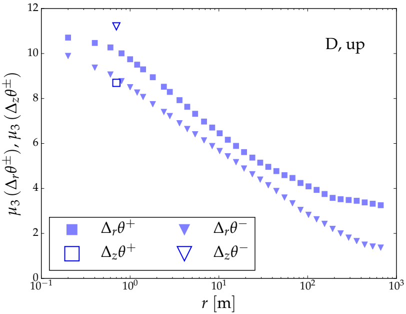

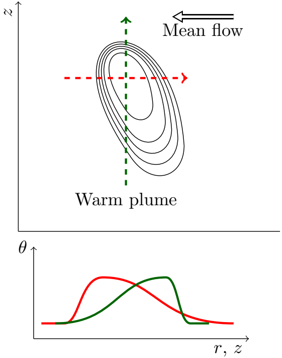

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- Warm plumes during upslope phase

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