A new mathematical formulation to integrate supply and demand within a choice-based optimization framework

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Outline

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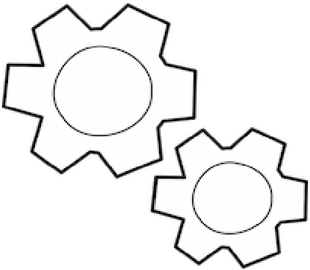
Motivation

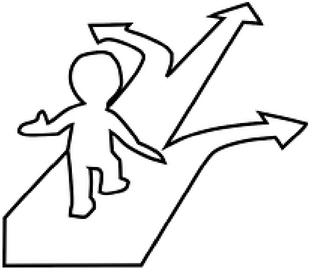
Supply and demand

Supply



- Decision variables to design and configurate the supply
- Maximize revenues
- **Here:** MILP

Demand



- Formalization of preferences for demand forecasting
- Maximize satisfaction
- **Here:** discrete choice models

State of the art: Integration paradigms

Linear choice-based optimization models

- Decision variables are not in the utility function
- Exogeneous utility

Nonlinear choice-based optimization models

- Endogeneous utility
- Nonlinearity and nonconvexity to the optimization model

General observations

- The assumption of exogeneously given demand is in most of the cases unrealistic
- **Motivation:** consider utility as endogeneous to the optimization model (better representation of the demand)
- Complexity increases
  - Mathematical model
  - Resolution approach

Integration of supply and demand

- Integration of discrete choice models in MILP
  - Probabilistic
  - Nonlinearity and nonconvexity
- Linear approach addressing
  - Nonconvex representation of probabilities
  - Wide class of discrete choice models

Outline

Utility

Supply and demand

- Population of N individuals
- Set of alternatives $\mathcal{C}$
  - artificial *opt-out* alternative
- $\mathcal{C}_n \subseteq \mathcal{C}$ subset of available alternatives to individual n

Utility

$U_{in} = V_{in} + \varepsilon_{in}$: associated score with alternative i by individual n

- V_{in}: deterministic part
- ε_{in}: error term

Behavioral assumption: n chooses i if U_{in} is the highest in $\mathcal{C}_n$

Probabilistic model

Availability

$$y_{in} = \begin{cases} 1 & \text{if } i \in \mathcal{C}_n \\ 0 & \text{otherwise} \end{cases}$$

Choice

$$w_{in} = \begin{cases} 1 & \text{if } n \text{ chooses } i \\ 0 & \text{otherwise} \end{cases}$$

Probabilistic model

- $\Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$ and i available ($y_{in} = 1$)
- $D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1)$, in general non linear

Simulation

Behavioral scenarios

- Assume a distribution for ε_{in}
- Generate R draws $\xi_{in1} \dots \xi_{inR}$
- The choice problem becomes **deterministic**

Demand model

$$U_{inr} = V_{in} + \xi_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr} \tag{1}$$

Endogeneous part of V_{in}

- Decision variables x_{ink}
- Assumption: linear

Exogeneous part of V_{in}

- Other variables z_{in}
- f not necessarily linear

Availability of alternatives

Operator level

y_{in} decision of the operator

$$y_{in} = 0 \ \forall i \notin \mathcal{C}_n \tag{2}$$

Scenario level

y_{inr} availability at scenario level (*e.g.* demand exceeding capacity)

$$y_{inr} \leq y_{in} \tag{3}$$

Choice of alternatives

Choice at scenario level

$$w_{inr} = \begin{cases} 1 & \text{if } i = \underset{j|y_{jnr}=1}{\arg\max}\{U_{jnr}\} \\ 0 & \text{otherwise} \end{cases}$$

Choice and availability

$$w_{inr} \leq y_{inr} \tag{4}$$

Linearization of the choice (I)

Auxiliary variables

$$\nu_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ l_{nr} & \text{if } y_{inr} = 0 \end{cases}$$

Linearizing constraints

$$l_{nr} \leq \nu_{inr} \tag{5}$$
$$\nu_{inr} \leq l_{nr} + M_{inr} y_{inr} \tag{6}$$
$$U_{inr} - M_{inr}(1 - y_{inr}) \leq \nu_{inr} \tag{7}$$
$$\nu_{inr} \leq U_{inr} \tag{8}$$

where $l_{nr} = \min_{i \in \mathcal{C}}\{U_{inr}\}, m_{inr} \geq U_{inr}, M_{inr} = m_{inr} - l_{nr}$

Linearization of the choice (II)

Highest utility

$$U_{nr} = \max_{i \in \mathcal{C}_n} \nu_{inr}$$

Linearizing constraints

$$\nu_{inr} \leq U_{nr} \tag{9}$$

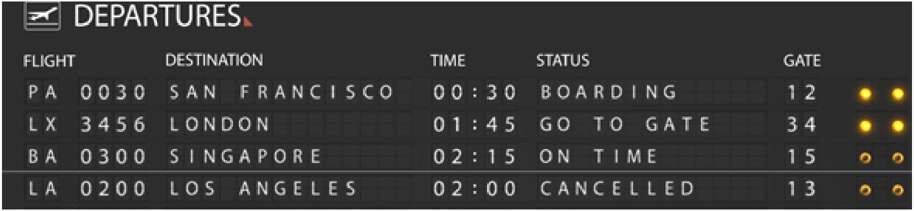
$$U_{nr} \leq \nu_{inr} + M_{nr}(1 - w_{inr}) \tag{10}$$

$$\sum_{i \in \mathcal{C}} w_{inr} = 1 \tag{11}$$

where $M_{nr} = \max_{i \in \mathcal{C}}\{U_{inr}\} - l_{nr}$

Outline

Maximization of revenues



Application

- Operator selling services to a market, each service:
    - Price
    - Capacity (number of individuals)
- $i = 0$ denotes the opt-out option
- Demand is price elastic and heterogenous
- **Goal:** best strategy in terms of capacity allocation and pricing

Pricing (I)

Revenues per alternative

- p_{in} price that individual n has to pay to access to alternative i
- Endogeneous variable in the utility function (1)

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}$$

Linearization (I)

- Discretization of the price: $p_{in}^1, \ldots, p_{in}^{L_{in}}$
- Binary variables λ_{inl} such that $p_{in} = \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^l$ and

$$\sum_{\ell=1}^{L_{in}} \lambda_{in\ell} = 1, \forall i > 0 \tag{12}$$

Pricing (II)

Linearization (II)

- Revenues from alternative i:

$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{l=1}^{L_{in}} \lambda_{inl} p_{in}^{l} \sum_{r=1}^{R} w_{inr}$$

- Still non linear $\Rightarrow \alpha_{inrl} = \lambda_{inl} w_{inr}$ to linearize it

$$\lambda_{in\ell} + w_{inr} \leq 1 + \alpha_{inr\ell} \; \forall i > 0 \tag{13}$$

$$\alpha_{inr\ell} \leq \lambda_{in\ell} \; \forall i > 0 \tag{14}$$

$$\alpha_{inr\ell} \leq w_{inr} \; \forall i > 0 \tag{15}$$

Capacity (I)

Overview

- c_i capacity of service i
- Who has access if the capacity is reached?
- The model favors customers bringing higher revenues
- ... but generally customers arrive in a random order

Priority list

- An individual is served only if all individuals before her in the list have been served
- Can account for fidelity programs, VIP customers, etc.
- We assume it given

$$y_{inr} \geq y_{i(n+1)r} \; \forall i > 0 \tag{16}$$

Capacity (II)

Capacity must not be exceeded

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr}) \; \forall i > 0, n > c_i \tag{17}$$

- $y_{inr} = 1 \Rightarrow 1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i$
- $y_{inr} = 0 \Rightarrow \sum_{m=1}^{n-1} w_{imr} \leq n - 1$

Capacity has been reached

$$c_i(y_{in} - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} \; \forall i > 0 \tag{18}$$

- $y_{in} = 1, y_{inr} = 0 \Rightarrow \sum_{m=1}^{n-1} w_{imr} \geq c_i$

Full model

Objective function

$$\max \sum_{i>0} \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} p_{in}^{\ell} \sum_{r=1}^{R} \alpha_{inr\ell} \tag{19}$$

Constraints

- Utility: (1)
- Availability of alternatives: (2) and (3)
- Choice: (4), (5), (6), (7),(8), (9), (10) and (11)
- Pricing: (12), (13), (14) and (15)
- Capacity allocation: (16), (17) and (18)

Outline

Parking choices

Original experiment

- [Ibe, 2014] *Modelling parking choices considering user heterogeneity*
- Stated preferences survey
- Analyze viability of an underground car park
- Mixed logit model (random taste parameters)

Free on-Street Parking (FSP)

Free (opt-out)

Paid on-Street Parking (PSP)

0.6 and 0.8

Paid Underground Parking (PUP)

0.8 and 1.5

Choice model

Survey

- 197 respondents
- 8 scenarios based on
  - AT (access time to parking area)
  - TD (time to reach the destination)
  - FEE (price)

Mixed Logit model

- **Attributes:** TD
- **Random parameters:** AT, FEE
- **Socioeconomic characteristics:** residence, age of the vehicle
- **Interactions:** price and low income, price and residence

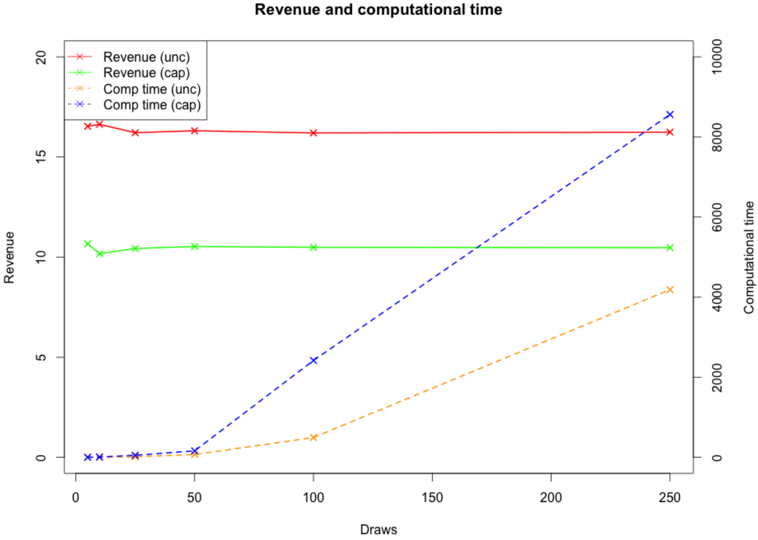
Computational results: overview

Assumptions

- Subset of 25 individuals
- Uncapacitated vs. capacitated case
- Capacity of 10 inviduals for both PSP and PUP
- 10 price levels from 0 to 3

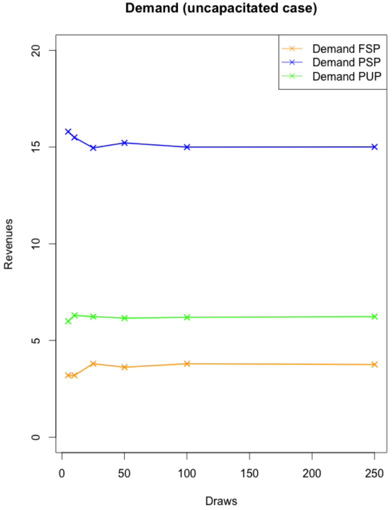
	FSP			PSP			PUP		
Scenario	AT	TD	FEE	AT	TD	FEE	AT	TD	FEE
5	15	15	0	10	10	0.6	5	10	1.5

Computational results: revenue and computational time

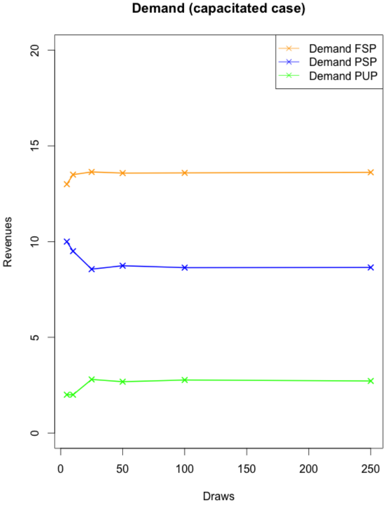


Computational results: demand

Uncapacitated case



Capacitated case



Outline

Conclusions

- General framework (*any* assumption can be made for ε_{in})
- Linear formulation integrating demand and supply
- High dimensionality of the problem (N and R)
- Need for speeding up the computational results

Future work

Decomposition techniques

- Decomposition sources:
  - Individual (own optimization problem)
  - Draw (independent scenarios)
- Decomposable structures in practice:
  - Complicating constraints (Lagrangian relaxation techniques)
  - Complicating variables (Benders decomposition)

Lagrangian decomposition

- Two interesting subproblems with common variables
  - Choice subproblem (user's side)
  - Pricing subproblem (operator's side)

Questions?

Modelling parking choices considering user heterogeneity. *Transportation Research Part A: Policy and Practice*, 70:41 – 49, 2014. ISSN 0965-8564.