From complex travel behavior to optimization: the methodological challenges

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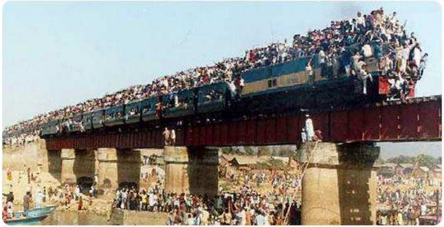
Outline

Demand models

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch

Demand models



- Usually in OR:
- optimization of the supply
- for a given (fixed) demand

Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand functions: $P = f(Q)$
- Inverse demand: $Q = f^{-1}(P)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

Demand-supply interactions

Operations Research

- Given the demand...
- configure the system


Johnson City Enterprise.
Published Every Saturday,
$1. per year—Advance Payment.
SATURDAY, APRIL 7, 1883.

TIME TABLE
E. T., V. & G. R. R.

PASSENGER,	ARRIVES,
No. 1, West,	6:37, a. m.
No. 2, East,	9:45, p. m.
No. 3, West,	11:51, p.m.
No. 4, East,	3:56, a. m.
LOCAL FREIGHT,	ARRIVES,
No. 5,	7:20, a. m.
No. 8,	6:20, p. m.

JNO. W. EAKIN, Agent.

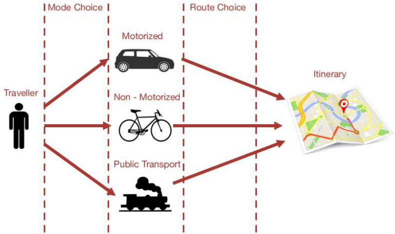
E. T. & W. N. C. R. R.

Passenger, leaves,	7, a. m.
" arrives,	6, p. m.

J. C. HARDIN, Agent.

Behavioral models

- Given the configuration of the system...
- predict the demand



Demand-supply interactions

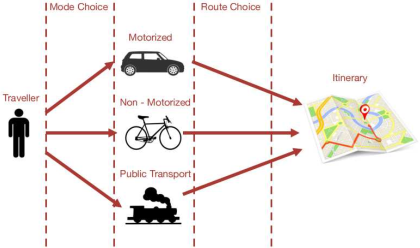
Multi-objective optimization

Minimize costs

Maximize satisfaction

Outline

Choice models



Behavioral models

- Demand = sequence of choices
- Choosing means trade-offs
- In practice: derive trade-offs from choice models

Choice models

Theoretical foundations

- Random utility theory
- Choice set: $\mathcal{C}_n$
- $y_{in} = 1$ if $i \in \mathcal{C}_n$, 0 if not
- Logit model:

$$P(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}$$

2000

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Logit model

Utility

$$U_{in} = V_{in} + \varepsilon_{in}$$

Choice probability

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}.$$

- Decision-maker n
- Alternative $i \in \mathcal{C}_n$

Variables: $x_{in} = (z_{in}, s_n)$

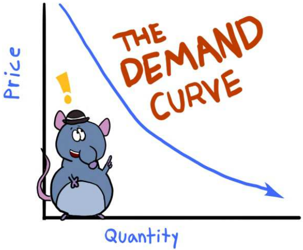
Attributes of alternative i: z_{in}

- Cost / price
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker n: s_n

- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.

Demand curve



Disaggregate model

$$P_n(i|c_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|c_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in c_{in} and z_{in}

Outline

Models

Decision variables

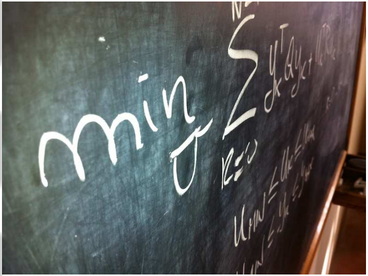
$x \in \mathbb{R}^n$

Objective function

$f(x) \in \mathbb{R}$

Constraints

$g(x) \leq 0, \; g : \mathbb{R}^n \to \mathbb{R}^m, \; x_i \in \mathbb{N}, \; x_j \in \{0, 1\}$



Models

Models in transportation

Decision variables

$x \in \mathbb{R}^n$: n is large

Objective function

$f(x) = \sum_{i=1}^{n} c_i x_i$: linear

Constraints

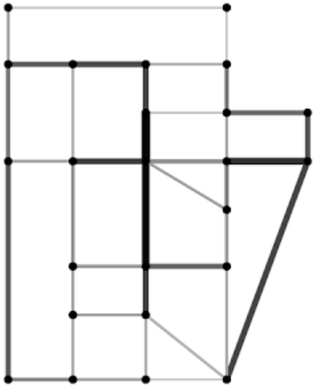
g linear, and $x_j \in \{0, 1\}$

Tractability

Importance of linear specification

Outline

Stochastic traffic assignment



Features

- Nash equilibrium
- Flow problem
- Demand: path choice
- Supply: capacity

Selected literature

- [Dial, 1971]: logit
- [Daganzo and Sheffi, 1977]: probit
- [Fisk, 1980]: logit
- [Bekhor and Prashker, 2001]: cross-nested logit
- and many others...

Revenue management

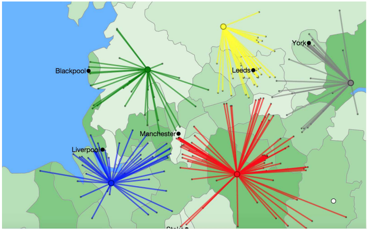
Features

- Stackelberg game
- Bi-level optimization
- Demand: purchase
- Supply: price and capacity

Selected literature

- [Labbé et al., 1998]: bi-level programming
- [Andersson, 1998]: choice-based RM
- [Talluri and Van Ryzin, 2004]: choice-based RM
- [Gilbert et al., 2014a]: logit
- [Gilbert et al., 2014b]: mixed logit
- [Azadeh et al., 2015]: global optimization
- and many others...

Facility location problem



Features

- Competitive market
- Opening a facility impact the costs
- Opening a facility impact the demand
- Decision variables: availability of the alternatives

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}.$$

Selected literature

- [Hakimi, 1990]: competitive location (heuristics)
- [Benati, 1999]: competitive location (B & B, Lagrangian relaxation, submodularity)
- [Serra and Colomé, 2001]: competitive location (heuristics)
- [Marianov et al., 2008]: competitive location (heuristic)
- [Haase and Müller, 2013]: school location (simulation-based)

Outline

The main idea

Linearization

Hopeless to linearize the logit formula (we tried...)

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Variables

Availability

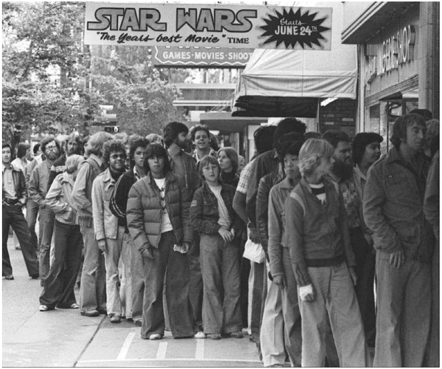
$$y_{in} = \begin{cases} 1 & \text{if alt. } i \text{ available for } n, \\ 0 & \text{otherwise.} \end{cases}$$

Choice

$$w_{inr} = \begin{cases} 1 & \text{if } y_{in} = 1 \text{ and } U_{inr} = \max_{j | y_{jn}=1} U_{jnr}, \\ 0 & \text{if } y_{in} = 0 \text{ or } U_{inr} < \max_{j | y_{jn}=1} U_{jnr}. \end{cases}$$

Capacities

- Demand may exceed supply
- Each alternative i can be chosen by maximum c_i individuals.
- An exogenous priority list is available.
- The numbering of individuals is consistent with their priority.



Priority list

Application dependent

- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework

The list of customers must be sorted

Optimization

Decision variables

Supply: assortment, capacity, price, level of service, etc.

Objective function

A combination of revenues, costs, users satisfaction, etc.

Constraints (for each customer)

- Capacity: availability of alternatives
- Choice: preferred alternative is chosen

References

- Technical report: [Bierlaire and Azadeh, 2016]
- Conference proceeding: [Pacheco et al., 2016]

Outline

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_i
- What price? p_{in}

Back to the example: pricing

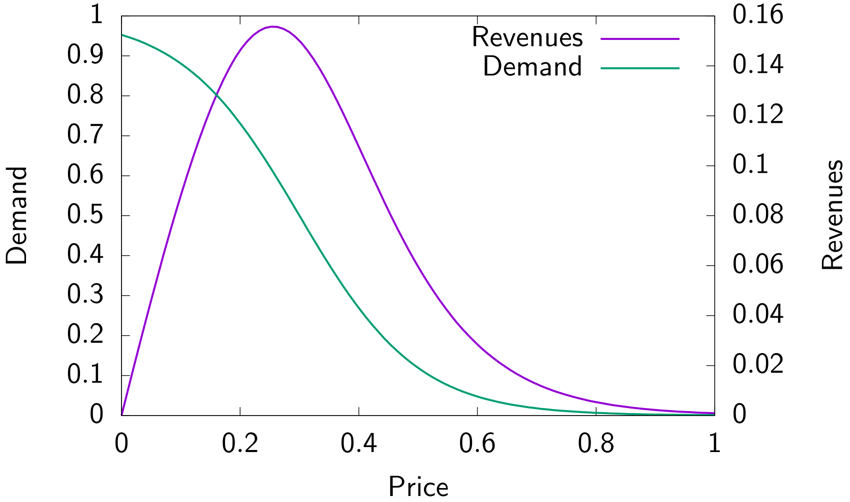
Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an homogeneous population of N individuals

$$U_c = 0 + \varepsilon_c$$
$$U_m = \beta_c p_m + \varepsilon_m$$

- $\beta_c < 0$
- Logit model: ε_m i.i.d. EV

Demand and revenues



Optimization (with GLPK)

Data

- $N = 1$
- $R = 100$
- $U_m = -10p_m + 3$
- Prices: 0.10, 0.20, 0.30, 0.40, 0.50

Results

- Optimum price: 0.3
- Demand: 56%
- Revenues: 0.168

Heterogeneous population

Two groups in the population

$$U_{in} = -\beta_n p_i + c_n$$

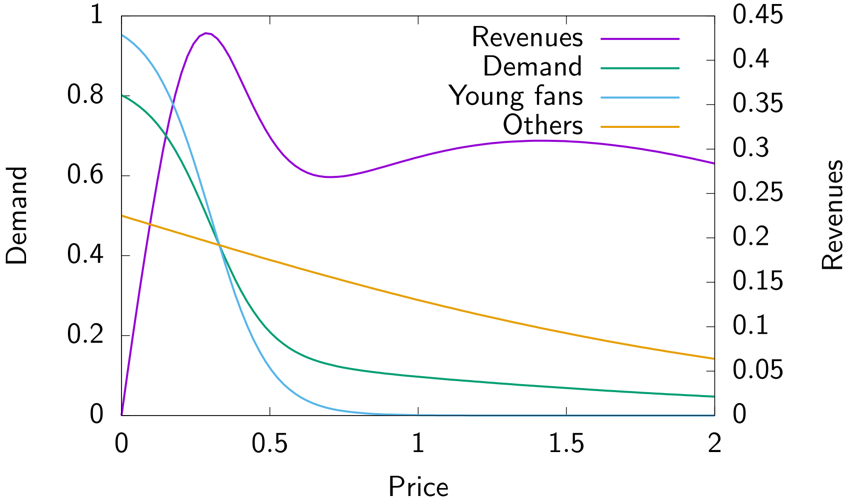
Young fans: 2/3

$\beta_1 = -10, \; c_1 = 3$

Others: 1/3

$\beta_1 = -0.9, \; c_1 = 0$

Demand and revenues



Optimization

Data

- $N = 3$
- $R = 100$
- $U_{m1} = -10p_m + 3$
- $U_{m2} = -0.9p_m$
- Prices: 0.3, 0.7, 1.1, 1.5, 1.9

Results

- Optimum price: 0.3
- Customer 1 (fan): 60% [theory: 50 %]
- Customer 2 (fan) : 49% [theory: 50 %]
- Customer 3 (other) : 45% [theory: 43 %]
- Demand: 1.54 (51%)
- Revenues: 0.48

Two theaters, different types of films

Two theaters, different types of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Tinker Tailor Soldier Spy

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, different types of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + 4$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + 0$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.6
- 4 young customers: 0
- 2 old customers: 0.5
- Demand: 0.5 (8.3%)
- Revenues: 0.8

Theater k

- Optimum price m: 0.5
- Young customers: 0.8
- Old customers: 1.5
- Demand: 2.3 (38%)
- Revenues: 1.15

Two theaters, same type of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Star Wars Episode VIII

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, same type of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + 4$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + 4$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Young customers: 0
- Old customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater k

Closed

Two theaters with capacity, different types of films

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4,\ n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m,\ n = 3, 6$
- $U_{kn} = -10p_k + 0,\ n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k,\ n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Demand: 0.2 (3.3%)
- Revenues: 0.36

Theater k

- Optimum price m: 0.5
- Demand: 2 (33.3%)
- Revenues: 1.15

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	k	2	1
4	0	2	1
5	0	2	1
6	k	2	0

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	k	2	1
3	0	2	1
4	k	2	0
5	0	2	0
6	0	2	0

Outline

Parking management

Alternatives

- paid on-street parking (PSP) [20]
- paid parking in an underground car park (PUP) [20]
- free on-street parking (FSP) [unlimited]

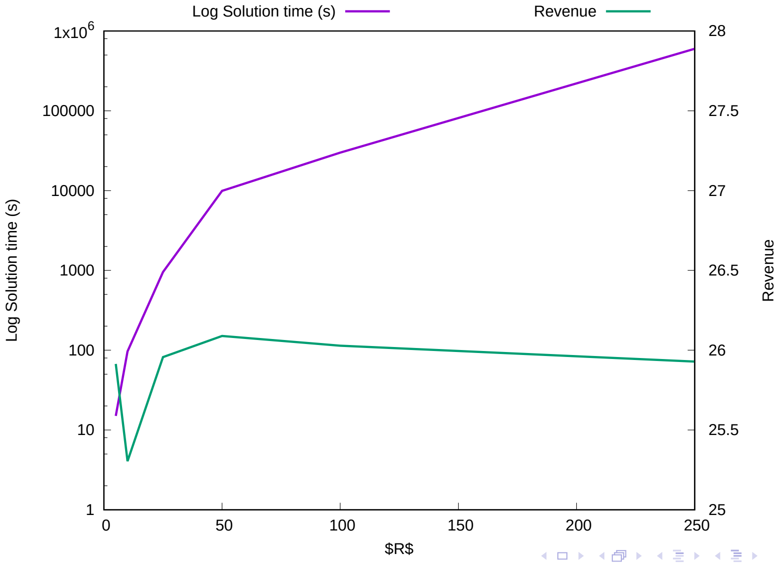
Demand model

[Ibeas et al., 2014]

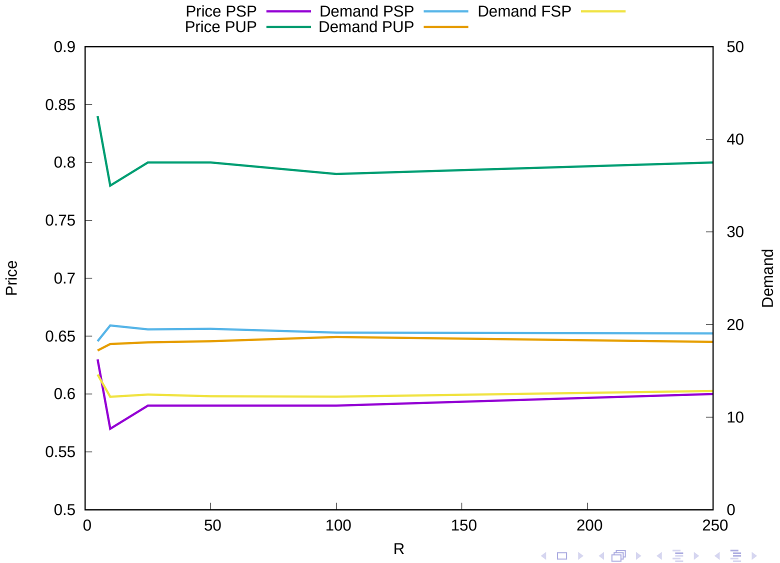
Scenario

- 50 customers
- Optimize revenues

Impact of the number of draws



Impact of the number of draws

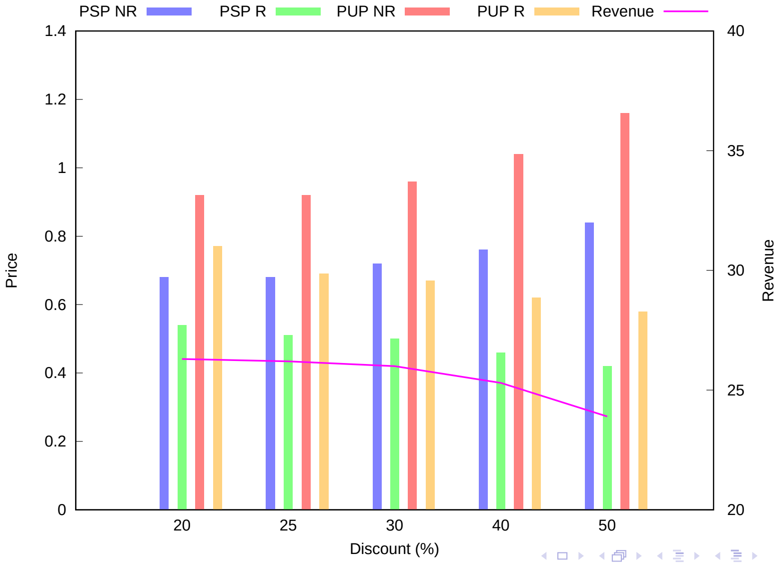


Heterogenous demand

Residents

- Residents pay less
- Operator is forces to apply reduced fees

Varying the amount of the reduction



Outline

Summary

Demand and supply

- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models

- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models

Optimization

Discrete choice models

- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation

- Linear in the decision variables
- Large scale
- Fairly general

Ongoing research

- Decomposition methods
- Scenarios are (almost) independent from each other (except objective function)
- Individuals are also loosely coupled (except for capacity constraints)

Thank you!

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