The region of a hydraulic fracture driven by a power law fluid

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1. Introduction

The problem of a fluid-driven fracture propagating in an elastic isotropic medium is a subject of great importance in the hydraulic fracturing process and also in the oil and gas industry. We present here numerical tip region solutions for fracture width and pressure when a non-newtonian fluid drives a plane strain fracture in an impermeable linear solid. We account for the presence of lag of a priori unknown length between the fluid front and the crack tip. The fluid studied is a power law shear thinning fluid formulated using the lubrication flow model. For such a rheology, in simple shear, the fluid shear stress \( \tau \) is linked to the shear rate \( \dot{\gamma} \) as follows:

\[ \tau = -\mu \dot{\gamma} \]

where \( \tau \) is the fluid index \((n < 1 \text{ for shear-thinning fluid})\) and \( \mu \) is the consistency index.

2. Problem statement

Consider propagation of a semi-infinite two-dimensional fracture with a constant velocity \( V \) in an impermeable elastic isotropic medium characterised by plane strain elastic modulus \( E \) and fracture toughness \( K \). The fracture is loaded by the internal power law fluid pressure \( p(x) \) and by the far-field confining stress \( \sigma_o \). The fracture is assumed to be in mobile equilibrium \((K_1 = K_2)\) and propagating along the \( x \)-axis [2].

Our numerical results include the fracture opening \( w(x) \) and the net pressure \( p(x) = p_f(x) - \sigma_o \) profiles over the whole fracture as well as the corresponding value of the fluid lag size. The solid deformation is given by the equations of linear elastic fracture mechanics which links the net pressure to the fracture opening using the integral equation below:

\[ p(x) = \frac{E}{2\pi K_1} \int_0^L \frac{w(z)}{|x-z|} dz \]

The fluid flow inside the fracture is described by the lubrication law, for \( x \in [0, \infty] \):

\[ \dot{\gamma} = \frac{2 \mu}{n} \left( \frac{\partial p}{\partial x} \right) \]

where \( \dot{\gamma} = \frac{d \theta}{d t} \).

In the moving system, the flow rate per unit width is simply reduced to \( q = V w \).

And the propagation condition \( K_1 = K_2 \) can be prescribed as:

\[ w = \frac{V}{E} \xi^{1/2}, \quad x \to 0 \]

3. Results

We introduce the dimensionless variables of the problem using two length scales \( L_o \) and \( L_w \):

\[ \Omega = \frac{x}{L_o}, \quad \Pi = \frac{p - \sigma_o}{\sigma_o}, \quad \xi = \frac{\dot{\gamma}}{\dot{\gamma}_o}, \quad \xi = \frac{\dot{\gamma}}{\dot{\gamma}_o} \]

where

\[ L_o = V \left( \frac{E}{\sigma_o \dot{\gamma}_o} \right)^{1/5} \quad \text{and} \quad L_w = \left( \frac{K_1}{\sigma_o} \right)^{1/2} \]

The dimensionless form of the governing equations will only depend on the value of the power law index \( n \) and a dimensionless toughness \( \kappa \) defined as:

\[ \kappa = \left( \frac{1}{\sigma_o} \right)^{1/2} \left( \frac{\dot{\gamma}_o^2}{E} \right)^{1/2} \]

The nonlinear system of equations is discretised using the Gauss-Chebyshev polynomials [3]. This technique uses trigonometric values for the abscissas and the collocation points as made when using the Gauss-Chebyshev for solving singular integral equations corresponding to finite cracks. Therefore, we will transform the coordinate from the semi-infinite interval \([0, \infty]\) to the finite interval \([-1, 1]\). The resulting non-linear system of equations is solved via a quasi-Newton root-finding scheme using the dimensionless net pressure at the collocation points as the primary unknown variables.

4. References

