The tip region of a hydraulic fracture driven by a power law fluid

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1. Introduction

The problem of a fluid-driven fracture propagating in an elastic isotropic medium is a subject of great importance in the hydraulic fracturing process and also in the oil and gas industry. We present here numerical tip region solutions for fracture width and pressure when a Non-newtonian fluid drives a plane strain fracture in an impermeable linear solid. We account for the presence of lag of a priori unknown length between the fluid front and the crack tip. The fluid studied is a power law shear thinning fluid formulated using the lubrication flow model. For such a rheology, in simple shear, the fluid shear stress τ is linked to the shear rate $\dot{\gamma}$ as follows:

$$\tau = M\dot{\gamma}^n$$

where n is the fluid index ($n < 1$ for shear-thinning fluid) and M is the consistency index.

2. Problem statement

Consider propagation of a semi-infinite two-dimensional fracture with a constant velocity V in an impermeable elastic isotropic medium characterised by plane strain elastic modulus E' and fracture toughness $K' = \sqrt{\frac{32}{\pi}}K_{Ic}$. The fracture is loaded by the internal power law fluid pressure $p_f(x)$ and by the far-field confining stress σ_o. The fracture is assumed to be in mobile equilibrium ($K_I = K_{Ic}$) and propagating along x-axis [2].

Our numerical results include the fracture opening $w(x)$ and the net pressure $p(x) = p_f(x) - \sigma_o$ profiles over the whole fracture as well as the corresponding value of the fluid lag size.

The solid deformation is given by the equations of linear elastic fracture mechanics which links the net pressure to the fracture opening using the integral equation below

$$p(x) = \frac{E'}{4\pi}\int_0^\infty \frac{\partial w}{\partial x}\frac{\partial s}{x - s}$$

The fluid flow inside the fracture is described by the lubrication law, for $x \in]\lambda, \infty[$

$$q|q|^{n-1} = \frac{w^{2n+1}}{M'}\frac{\partial p}{\partial x}$$

where $M' = \frac{2^{n+1}(2n+1)^n}{n^n}M$

In the moving system, the flow rate per unit width is simply reduced to $q = Vw$.

In the lag region, the net pressure is neglected: $p = 0$ for $x \in [0, \lambda]$

And the propagation condition $K_I = K_{Ic}$ can be prescribed as: $w = \frac{K'}{E'}x^{1/2}$, $x \to 0$

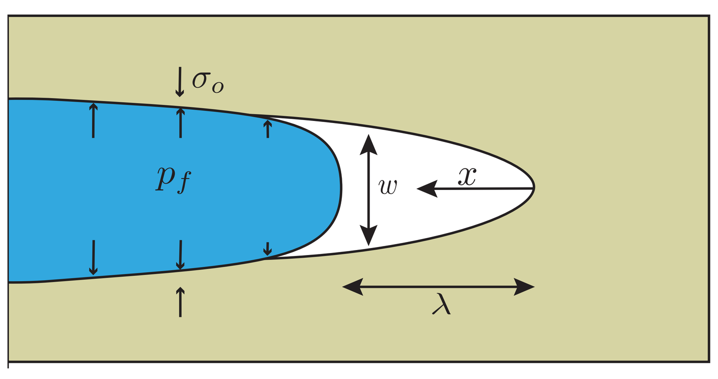

Figure 1: *Sketch of semi-infinite fluid-driven fracture propagating at constant velocity V in an impermeable medium*

3. Results

We introduce the dimensionless variables of the problem using two lengthscales L_μ and L_κ

$$\Omega = \frac{E'w}{\sigma_o L_\mu}, \qquad \Pi = \frac{p - \sigma_o}{\sigma_o}, \qquad \xi = \frac{x}{L_\mu}$$

where

$$L_\mu = V\left(\frac{M'}{\sigma_o}\right)^{1/n}\left(\frac{E'}{\sigma_o}\right)^{\frac{n+1}{n}}, \qquad L_k = \left(\frac{K'}{\sigma_o}\right)^2$$

The dimensionless form of the governing equations will only depend on the value of the power law index n and a dimensionless toughness κ defined as:

$$\kappa = \left(\frac{1}{V}\right)^{1/2}\left(\frac{\sigma_0^{2-n}}{M'E'^{n+1}}\right)^{1/2n} K'$$

The nonlinear system of equations is discretised using the Gauss-Chebyshev polynomials [3]. This technique uses trigonometric values for the abscissas and the colocation points as made when using the Gauss-Chebyshev for solving singular integral equations corresponding to finite cracks. Therefore, we will transform the coordinate from the semi-infinite interval $[0, \infty[$ to the finite interval $[-1, 1[$. The resulting non-linear system of equations is solved via a quasi-Newton root-finding scheme using the dimensionless net pressure at the collocation points as the primary unknown variables.

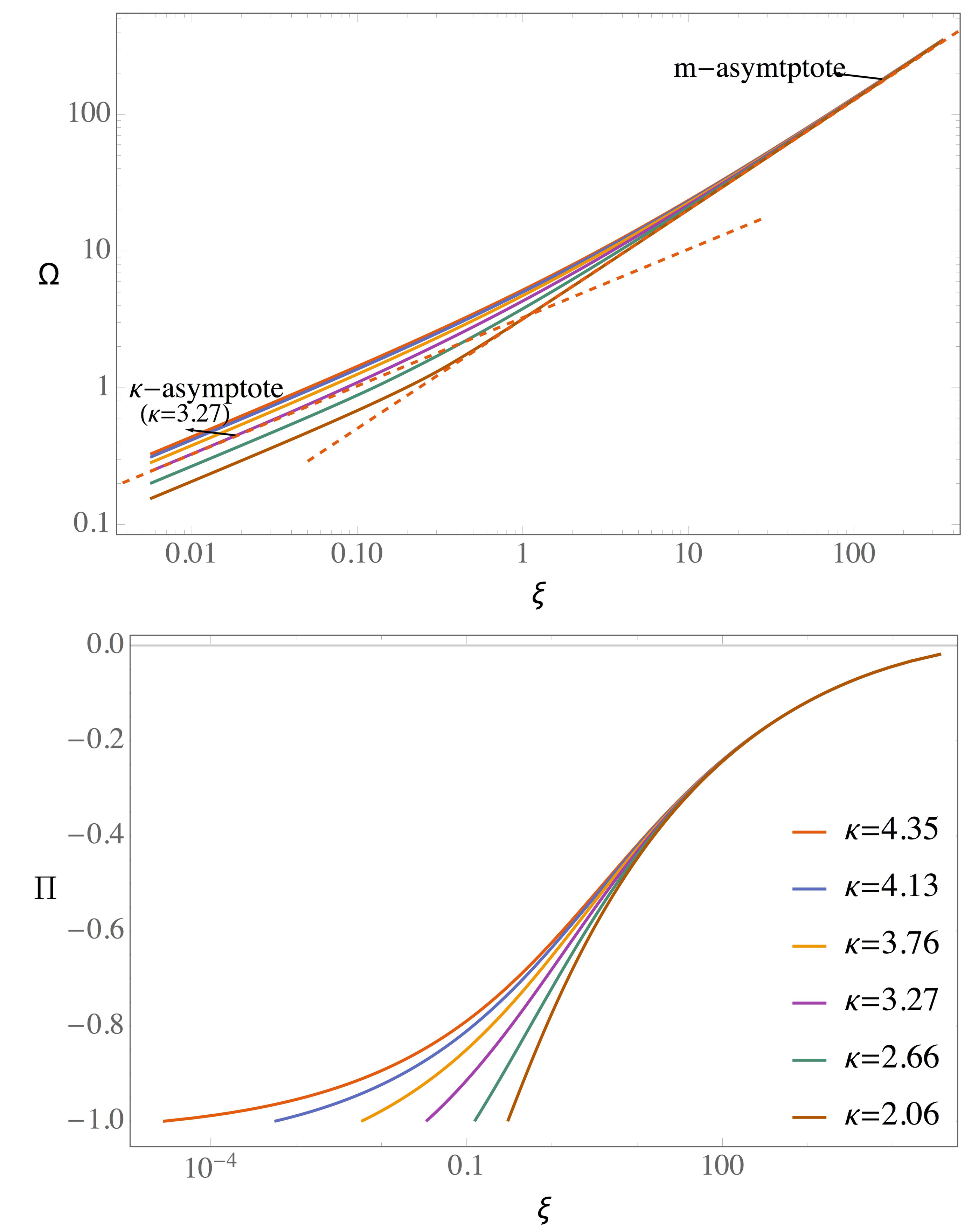

Figure 2: *Example of dimensionless opening Ω (in log-log scale) and net pressure Π (in semi-log scale) respectively along the fracture for different toughness ($n = 0.5$)*

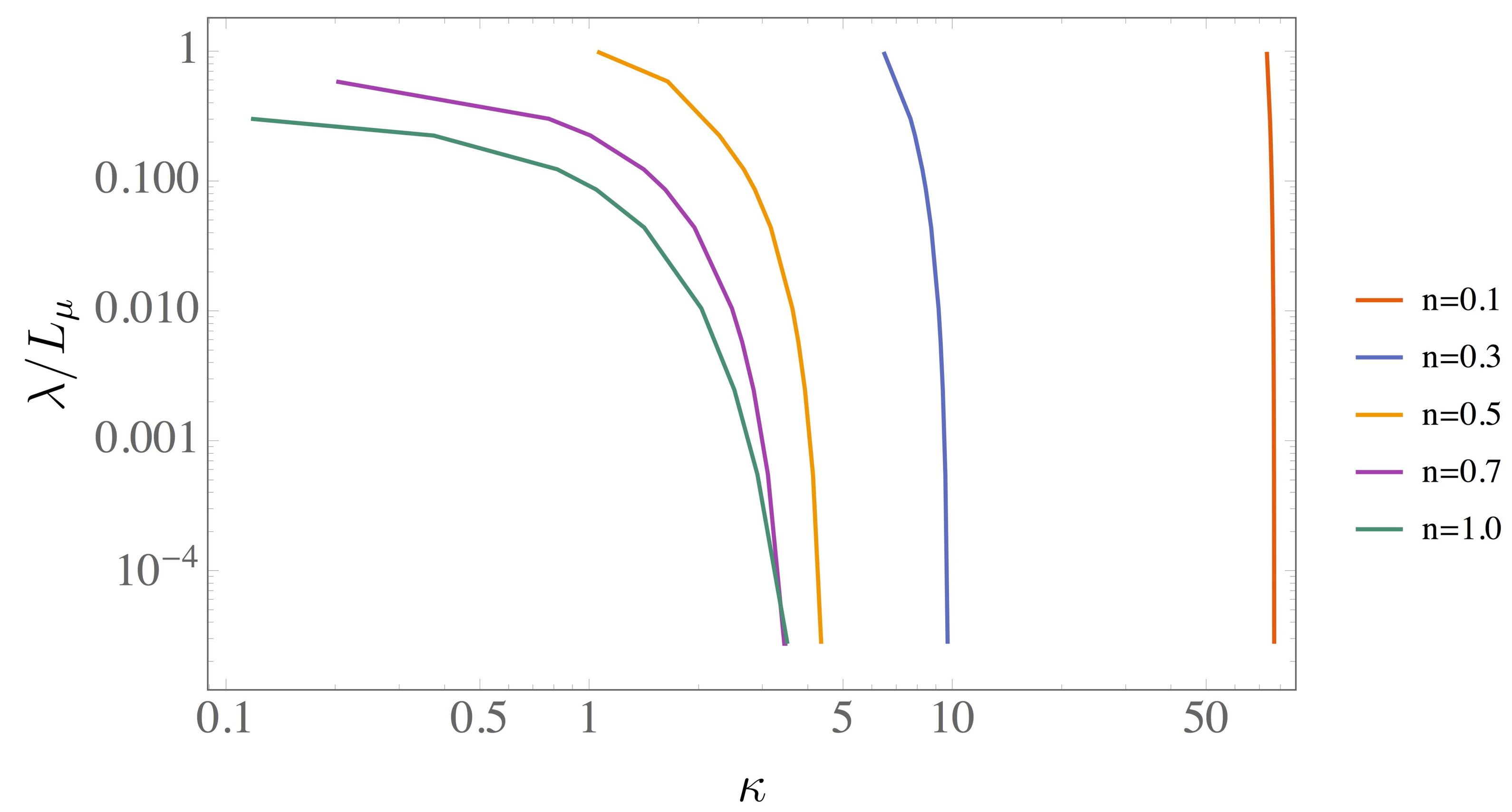

Figure 3: *Dimensionless fluid lag λ/L_μ (in log-log scale) with respect to the toughness κ for the different fluid index n*

κ-asymptote: $\Omega = \kappa\sqrt{\xi}$

m-asymptote: $\Omega = \left(\frac{2(n+2)^2}{n}\tan\left(\frac{-2\pi}{n+2}\right)\xi^2\right)^{\frac{1}{n+2}}$

We show that the solution is not only consistent with the square root singularity of linear elastic fracture mechanics near the tip ($x \ll L_\mu$) (κ-asymptote), but that its asymptotic behaviour in the far field ($x \gg L_\mu$) corresponds to the solution of a semi-infinite hydraulic fracture driven by a power-law fluid constructed on the assumptions of zero toughness and zero fluid lag (m-asymptote [1]). Our results also document how the power-law index modify the variation of the lag size as a function of the dimensionless toughness κ, and its disappearance for large value of κ.

References

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