A compressed-sensing approach for ultrasound imaging

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***Abstract*—Ultrasonography uses multiple piezo-electric element probes to image tissues. Current time-domain beamforming techniques require the signal at each transducer-element to be sampled at a rate higher than the Nyquist criterion, resulting in an extensive amount of data to be received, stored and processed. In this work, we propose to exploit sparsity of the signal received at each transducer-element. The proposed approach uses multiple compressive multiplexers for signal encoding and solves an ℓ_1-minimization in the decoding step, resulting in the reduction of 75 % of the amount of data, the number of cables and the number of analog-to-digital converters required to perform high quality reconstruction.**

Medical ultrasonography is a widely used modality nowadays due to its non-invasiveness and real-time capability. In many ultrasound (US) systems, an array of transducer-elements is used to transmit acoustic pulses, which, when reflected back by the medium inhomogeneities, are sensed by the same array. According to the Shannon-Nyquist theorem, the sampling rate at each element must be at least twice the bandwidth of the received signal. In practice, time-domain beamforming techniques require sampling rates between 3 and 10 times the center frequency to minimize the delay-quantization errors [1]. The large number of transducer-elements and the high central frequency required in medical ultrasonography motivate the research towards sampling rate reduction.

Let us consider an US probe made of N_{el} transducer-elements and call $r_i(t)$ with $i \in \{1, ..., N_{el}\}$ the corresponding echo signals received by each transducer-element at time t. If we consider a medium made of K inhomogeneities, then $r_i(t) = \sum_{k=1}^{K} a_{ik} \psi(t - t_k)$, with (a_{ik}, t_k) amplitudes and times-of-arrival of the K echo-pulses to the i^{th} transducer-element and $\psi(t)$ the elementary waveform. Assuming a linear propagation, we can state that $\psi(t) = (e * h_{Tx} * h_{Rx})(t)$ where $e(t)$ denotes the excitation and $h_{Tx}(t)$ and $h_{Rx}(t)$ are the transmit and receive impulse responses of the transducer-elements, respectively. Given the model described above, it can be stated that the vector $\boldsymbol{r_i} = [r_i(t_1), ..., r_i(t_{N_t})]^T \in \mathbb{R}^{N_t}$, where N_t denotes the number of time samples, obeys a K-sparse synthesis model in an overcomplete dictionary $\Psi \in \mathbb{R}^{N_t \times N_t}$ made of all the shifted replicas of the pulse [2]. Thus, $\boldsymbol{r_i} = \Psi \boldsymbol{a_i}$ with $||\boldsymbol{a_i}||_0 = K$, which can be exploited under the compressed sensing (CS) framework. Many efforts have been made in order to provide sub-Nyquist acquisition systems which result in different CS architectures such as the Random Demodulator [3] and the Random-Modulator Pre-Integrator [4] for single-channel signals, and the compressive multiplexer for multi-channel signals [5], [6]. In US imaging, Chernyakova *et al.* have recently proposed a hardware architecture based on finite rate of innovation and Xampling ideas [7].

In this work, we propose a proof of concept for a compressive multiplexer (CMUX) applied to US imaging. The architecture, denoted as US-CMUX is derived from the work of Kim *et al.* [6]. The idea is to split the N_{el} channels of the US probe into L groups of M channels and to mix each group according to the CMUX framework. In the decoding step, a convex problem is solved.

The CMUX, described on Figure 1, is based on modulating each channel $r_i(t)$ of a given group by a chipping sequence $p_i(t)$ sampled from a Rademacher distribution. The modulated channels are then summed to $y(t) = \sum_{i=1}^{M} p_i(t) r_i(t)$ and sampled at f_s.

The US-CMUX, described on Figure 2, uses L CMUX sharing the same chipping sequences to perform the signal encoding, giving rise to the matrix $\mathsf{Y} = [\boldsymbol{y_1}, ..., \boldsymbol{y_L}] \in \mathbb{R}^{N_t \times L}$. In the decoding step, the following convex problem is solved:

$$\min_{\bar{\mathsf{A}} \in \mathbb{R}^{MN_t \times L}} ||\bar{\mathsf{A}}||_{11} \text{ subject to } \|\mathsf{Y} - \Psi_{\mathsf{P}} \bar{\mathsf{A}}\|_F \leq \epsilon, \tag{1}$$

where $||.||_{11}$ accounts for the ℓ_{11}-norm, $||.||_F$ is the Frobenius norm, $\Psi_{\mathsf{P}} = [\Psi_{\mathsf{p1}}, ..., \Psi_{\mathsf{pM}}] \in \mathbb{R}^{N_t \times MN_t}$ in which $\Psi_{\mathsf{pi}} = [\boldsymbol{p_i} \otimes \Psi_1, ..., \boldsymbol{p_i} \otimes \Psi_{N_t}] \in \mathbb{R}^{N_t \times N_t}$, where $\otimes$ denotes the Hadamard product, and

$$\mathsf{A} = \begin{bmatrix} \boldsymbol{a_1} & \boldsymbol{a_{M+1}} & \cdots & \boldsymbol{a_{N_{el}-M+1}} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{a_M} & \boldsymbol{a_{2M}} & \cdots & \boldsymbol{a_{N_{el}}} \end{bmatrix} \in \mathbb{R}^{MN_t \times L},$$

with $\boldsymbol{a_i} \in \mathbb{R}^{N_t}$ the representation coefficients of $\boldsymbol{r_i}$.

Problem (1) is solved using the primal-dual forward backward algorithm [8] and each channel $\boldsymbol{r_i}$ is recovered from A as: $\boldsymbol{r_i} = \Psi \boldsymbol{a_i}$.

In order to validate the proposed method, we present numerical results on an *in-vitro* hyperechoic inclusion phantom (Model 54GS, Computerized Imaging Reference Systems Inc., Norfolk, USA) and an *in-vivo* carotid acquired with a Verasonics research scanner (V1-128, Verasonics Inc., Redmond, WA). The US probe used for the different experiments is a L12-5 $50\,\mathrm{mm}$ probe, with 128 active transducer-elements, working at $5\,\mathrm{MHz}$ with 100% bandwidth. The sampling frequency is $31.2\,\mathrm{MHz}$, corresponding to 4 times the bandwidth. The architecture is simulated on MATLAB thus the experiments are carried out on a digital setting. The hardware implementation of the proposed scheme will be investigated in future work. The architecture is tested for $L = 2, 4$ which means a reduction of 50 % and 75 % of the sampling rate, respectively. In the decoding process, ϵ is set to $10^{-6}\ ||\mathsf{Y}||_F$ and 1500 iterations of the algorithm are run. Radio-frequency images are computed with a classical delay-and-sum (DAS) algorithm, with a linear interpolation for the delay calculation and without apodization, and the B-mode image is obtained by Hilbert demodulation, normalization and log-compression with a dynamic range of $40\,\mathrm{dB}$.

The performance of the proposed method is quantified by the signal-to-noise ratio (SNR) and the structural similarity index (SSIM) against the computed image using 100 % of the data, calculated on the B-mode image without log-compression.

The results, displayed on Table I, as well as a visual evaluation on Figure 3 and Figure 4 show that the proposed architecture leads to high quality reconstruction with 25 % of data only since anechoic, hyperechoic and speckle regions are preserved.

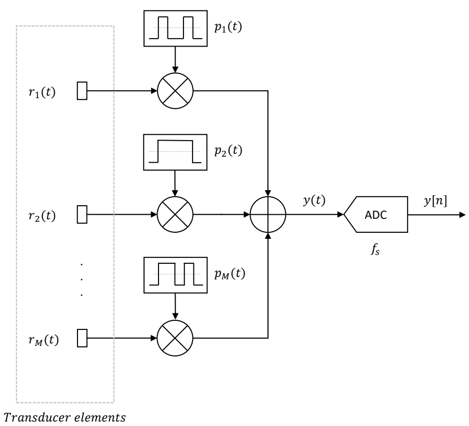


Figure 1 Compressive multiplexer (CMUX) architecture for M transducer-elements.

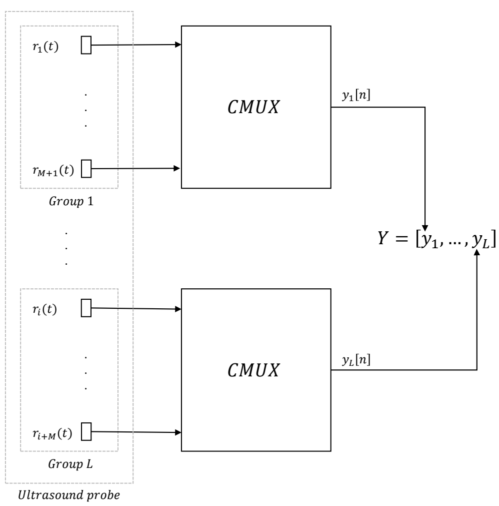
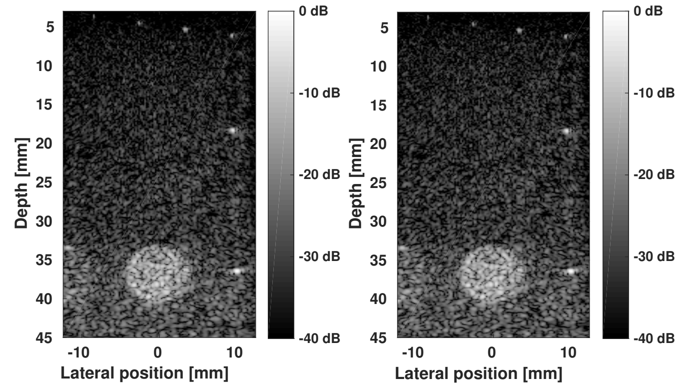


Figure 2 Ultrasound compressive multiplexer architecture using L CMUX.

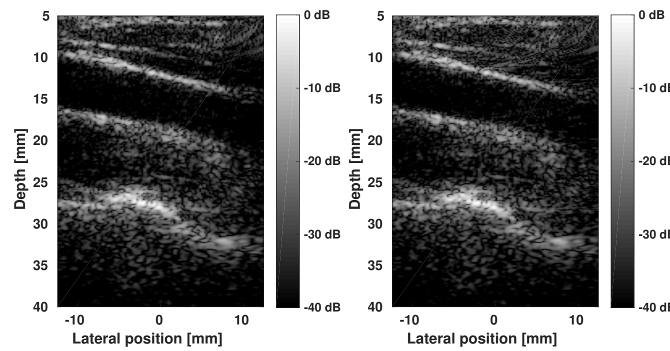
	Hyperechoic inclusion	*In-vivo* carotid
SNR - $L = 2$	39	36
SSIM - $L = 2$	0.94	0.87
SNR - $L = 4$	32	29
SSIM - $L = 4$	0.81	0.72

Table I Average values of the SNR (dB) and SSIM over 10 draws for the different images, for $L = 2$ and $L = 4$.



(a) Reference (b) CS reconstruction

Figure 3 B-mode image of the hyperechoic inclusion reconstructed with (a) 100 % of the data and (b) 25 % of the data acquired with the US-CMUX architecture.



(a) Reference (b) CS reconstruction

Figure 4 B-mode image of the carotid reconstructed with (a) 100 % of the data and (b) 25 % of the data acquired with the US-CMUX architecture.

References

[1] T. L. Szabo, *Diagnostic Ultrasound Imaging: Inside Out, second edition*. Elsevier, 2014.

[2] F. M. Naini, R. Gribonval, L. Jacques, and P. Vandergheynst, "Compressive sampling of pulse trains: Spread the spectrum!" in *2009 IEEE Int. Conf. Acoust. Speech Signal Process.*, 2009, pp. 2877–2880.

[3] J. A. Tropp, J. N. Laska, M. F. Duarte, J. K. Romberg, and R. G. Baraniuk, "Beyond Nyquist: Efficient sampling of sparse bandlimited signals," *IEEE Trans. Inf. Theory*, vol. 56, no. 1, pp. 520–544, 2010.

[4] S. R. Becker, "Practical compressed sensing: modern data acquisition and signal processing," Ph.D. dissertation, California Institute of Technology, 2011.

[5] J. P. Slavinsky, J. N. Laska, M. A. Davenport, and R. G. Baraniuk, "The compressive multiplexer for multi-channel compressive sensing," in *2011 IEEE Int. Conf. Acoust. Speech Signal Process.*, 2011, pp. 3980–3983.

[6] Y. Kim, W. Guo, B. V. Gowreesunker, N. Sun, and A. H. Tewfik, "Multichannel sparse data conversion with a single analog-to-digital converter," *IEEE J. Emerg. Sel. Top. Circuits Syst.*, vol. 2, no. 3, pp. 470–481, 2012.

[7] T. Chernyakova and Y. Eldar, "Fourier-domain beamforming: The path to compressed ultrasound imaging," *IEEE Trans. Ultrason. Ferroelectr. Freq. Control*, vol. 61, no. 8, pp. 1252–1267, 2014.

[8] P. L. Combettes, L. Condat, J.-C. Pesquet, and B. C. Vu, "A forward-backward view of some primal-dual optimization methods in image recovery," in *2014 IEEE Int. Conf. Image Process.*, 2014, pp. 4141–4145.