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Karim

First

Shalash

Last

karim_shalash@hotmail.com

E-mail

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Authors: Karim Shalash, Jürg Schiffmann

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ON THE MANUFACTURING OF COMPLIANT FOIL BEARINGS

K. Shalash, J. Schiffmann

Laboratory for Applied Mechanical Design (LAMD)
École Polytechnique Fédérale de Lausanne (EPFL)
Rue de la Maladière 71b, Neuchâtel 2002, Switzerland

ABSTRACT

The fabrication of foil bearings is challenging due to the dependence on sheet metal forming to produce the compliant structure. This paper is an attempt to shed light into the foil bearing manufacturing know-how. Design of experiments techniques are used to quantify the effects of the different manufacturing parameters, as well as defining an optimum manufacturing procedure. The effect of manufacturing noise on the static performance of foil bearings is quantified using a Monte Carlo simulation of a bump foil stiffness model. A non-intrusive optical measurement technique which has been developed to measure the formed bump foils is also presented. An uncertainty quantification was performed for the produced foils, showing large uncertainty in the bump dimensions, which significantly affect both nominal bearing clearance and compliance. Finite element simulations are used to model the bump foil forming process that would present potential problems during fabrication, suggesting sharp bends in the bump foil as the main driver for manufacturing deviations. Based on this outcome an improved design for the compliant bump foil with reduced curvature is proposed, manufactured and measured. The novel design allows to reduce springback error by 69% compared to classical bump foils and thus offers an equivalent yet more robust foil bearing design.

1. INTRODUCTION

Foil bearings are compliant self-acting gas lubricated bearings, which are known for their extended lifetime, high load capacity, extreme temperature operation, and their ability to cope with a large variety of process fluids [1, 2]. Foil bearings are mainly used to support high speed turbomachines. AiResearch (now Honeywell) pioneered the commercial use of foil bearings in air cycle machines in several aircrafts [3]. There is also a number of proof of concepts for other systems running on foil bearings. Examples are microturbines and auxiliary power units [4-7], turbocompressors [8-11], and turbo expanders [12, 13].

The classical foil bearing is constructed of three main components, which are the bearing sleeve, the bump foil, and the top foil - Figure 1. The first is considered the main holder of the foils, the second is a corrugated foil which acts like a compliant structure to support the rotor, and the latter is a cylindrical foil that creates the fluid film wedge along with the rotating shaft. Even though foil bearings are a relatively old technology [14], they still suffer from some practical issues, among them is the problem of fabrication. The literature is very limited on the topic of foil bearing manufacturing, the authors could mention four references [9, 15-17]. However, the presented know-how in these manuscripts is considered alike and relies mainly on experience and trial and error. It was recently shown that such techniques produce inaccurate foils in terms of dimensions [18]. The lack of available manufacturing know-how and analysis is hindering the scientific development, as well as affecting the performance of foil bearings.

Figure 1 – Foil bearing construction elements

1.1 Nature of the Issue.

The key role of the bump foil is to provide a compliant underlying structure. The foil bearing compliance is governed by the bump foil material and geometry. Iordanoff [19] deduced an analytical formula for a single bump compliance, where he accounted for the effect of the welded and the free bumps. Iordanoff used the bump length, height, pitch, and thickness which are considered the standard descriptive variables of the bump foil geometry as shown in Figure 2:

Figure 2 - Geometrical design variables of a bump which influence its compliance

Fixing the bump pitch and the foil thickness, and using basic Euclidean geometry, the bump could be also described using the bump radius, and angle. The reason behind adopting different descriptive variables is that the latter are more easily measurable.

$$h_b = t + R - R \cos(\alpha/2) \quad (1)$$

$$l_0 = R \sin(\alpha/2) \quad (2)$$

A sensitivity analysis on the bump radius and angle has been performed in view of assessing the propagation of the bump geometry uncertainty on compliance. The mean bump radius and angle are used as input variables to model the bump compliance both varying on a hypothetical range of $\pm 5\%$ around a mean design value [16]. Figure 3 shows a normalized bump foil performance map that highlights the effect of bump radii and angle deviations on bump stiffness.

Figure 3 - Effect of bump radius and angle deviation on the normalized bump stiffness – bump foil performance map

The sensitivity is around 5% change in stiffness for each 1% change in the bump angle. Similar conclusions could be drawn on the effect of the bump radius deviation. It was found that a 1% change in bump radius yields a 5% change in stiffness. A theoretical robust design, however, should exhibit a sensitivity slope equal or close to zero (changes in geometry shall not affect the performance variable). It is worth mentioning that the manufacturing tolerance in bump radii of foil forming dies can be within 1% without taking into account for other process related deviations. It is therefore concluded that the bump foil compliance is far from robust for a classical bump foil design, as it is highly sensitive to minimal changes in the geometry, thus highlighting the importance of the accurate manufacturing of bump foils.

In foil bearing systems, the main manufacturing complication is the fabrication of the bump foil, as the sleeve and the rotor can be manufactured with good accuracy. The foil forming process includes stamping and heat treatment of metal sheets of a certain thickness. Metal forming processes, however, suffer from an elastic-driven change in the shape after the load release, which is due to the finite modulus of elasticity and the yield strength of the material, a phenomenon called “springback”. Therefore, geometrical uncertainty of a foil bearing due to manufacturing noise will generate uncertainty in its compliance, and hence will affect the dynamic performance of the rotor bearing system. These uncertainties might have a positive effect on the system, such as breaking symmetry, however they should be quantified.

Springback is usually geometrically compensated for in tooling design, and/or by over bending of metal sheets. Increasing creep and decreasing yield strength by increasing temperature during loading is expected to decrease the springback [20-22]. There has been an extensive amount of research in the area of springback compensation [23-26], however none of it was directly concerned with foil bearings. Hence, manufacturing guidelines for accurate foil bearings are missing. For the simple case of a flat foil formed to an arc shape, springback is geometrically quantified by the change in radius and angle of the arc after the load is removed. Figure 4 shows the final formed top foils for different heat treatments at a constant loading pressure and highlights both the springback and the sensitivity of the foil shape to the manufacturing process. The foil at the bottom of the figure has the largest springback and the lowest heat treatment temperature, while the foil at the top has the lowest springback and the highest heat treatment temperature.

Figure 4 - Effect of heat treatment temperature and duration on springback showing reduced residual error with increasing heat treatment temperature

1.2 Goal and Objectives.

The goal of this paper is to shed light into the foil bearing manufacturing know-how and to propose a more robust bump foil design than the classical one. The objectives are to (1) assess known manufacturing procedures, (2) investigate the effect of process variables on springback, (3) identify the springback distribution on classical bump foils and (4) propose a new, more robust corrugated foil design.

1.3 Scope of the Paper.

The technological roadmap of this work is based on a Design of Experiment (DOE) approach to investigate the significance of the different process variables such as pressure, temperature and exposure time on springback. The optimum process is then used in a second step to produce a number of bump foils, which are individually measured using a non-intrusive optical measurement technique developed for this purpose. The statistical evaluation of the measured bump radii and angles further allows to identify error distributions. The effect of this manufacturing noise on the performance of foil bearings is presented using a Monte Carlo simulation with the foil bearing stiffness model developed by Iordanoff [19]. A finite element simulation is then used to simulate the bump foil forming process to isolate the locations yielding the most pronounced springback effects. Finally, an improved bump foil design is proposed to reduce manufacturing errors based on these results. The benchmark bearing design parameters are summarized in Table 1.

Table 1 - Nominal dimensions of bump foil bearing

Bearing Parameter	Unit	Value
Bearing Inner Diameter, D	mm	40
Bearing Axial Length, L	mm	40
Nominal Assembly Clearance, C	mm	0.075
Number of Bumps, N	-	27
Bump Pitch, S_0	mm	4.41
Bump length, $2l_0$	mm	1.76
Foil Thickness, t	mm	0.1
Bump Height, h_b	mm	0.607
Bump Radius, R	mm	3.32
Bump Foil Overall Radius, R_0	mm	
Bump Angle, α	°	63.15
Bump Intersection Angle, θ	°	
Stainless Steel 1.4310	-	-

Young's Modulus, E	MPa	193
Poisson Ratio, ν	-	0.29
Bump Calculated Stiffness	MN/m	1.7854

2. MANUFACTURING PROCESS ASSESSMENT

2.1 Material selection.

Choosing the proper foil material is of a great importance to the foil bearing design, as the material limitations will impact the overall bearing performance [27, 28]. The foils should be compliant, withstand heat, and offer the required damping to the rotordynamic system. It was shown in the literature that Inconel is an adequate solution, as it exhibits an acceptable modulus of elasticity, and can withstand high temperature which is important if the foil bearing is to be implemented in high temperature applications [29-31]. Stainless Steel is a cheaper alternative, with some drawbacks like lower operational temperature when compared to Inconel, and sensitivity to certain coatings [32-38]. San Andrés et al. [39] used copper in their metal mesh foil bearing for its enhanced Coulomb damping characteristics when compared to an equivalent Stainless Steel metal mesh. San Andrés et al. [40] and Kim et al. [41] reported using chromium molybdenum steel for the top and bump foils. Xu et al. [42] used Beryllium-Copper alloy in an oil lubricated leaf type foil bearing. Also, Kulkarni et al. [43] used the same material for bump foils, signifying its use for its self-lubricating properties. Although the Beryllium-Copper alloy exhibits excellent metal working capabilities, the existence of such a toxic material in a foil bearing is debatable for some applications. In this paper Stainless Steel 1.4310 was selected as the foil material.

2.2 Foil shaping.

In order to shape the foils, the cut foils are pressed inside a tooling (die) with the required foil geometry and then exposed to high temperature. The die is usually manufactured of a heat resistant alloy (Inconel – Stainless steel) to allow the toolings to withstand thermal fatigue. DellaCorte et al. [15] and Ruscitto [16] proposed a rule of thumb using flat dies for the bump foils, where the foils are pressed, and produced in a flat form, then rolled around a mandrel to get the required curvature. A mandrel diameter $2/3$ of the required final diameter of the bump foils is suggested to compensate for the springback effect. For the top foils a roller is proposed to produce the required curvature.

DellaCorte et al. [15] also executed a qualitative sensitivity study on the required forming load to achieve adequate bump foil deformation, and concluded that a unit load of 28 MPa is satisfactory to achieve the required foil shapes. Dykas et al. [17] published a similar fabrication guideline where 20 MPa were used to form annealed Inconel foils X-750. Better foil quality was claimed using Polytetrafluoroethylene (PTFE). However, no quantitative measurements were done on the springback of the foils, assuming that Inconel foils would maintain their formed shape. Yet, a major problem with the cold forming of the foils is the springback effect, which is an unceasing threat to the foil accuracy.

In this paper the forming technique uses dies which are designed to produce the required curvature and features of both the top and bump foils as shown in Figure 5. The die should be manufactured to perfection and exhibit a good surface finish which would be reflected on the produced foils, for EDM cut dies, a surface roughness R_a of 0.2 is achievable. The foil is then carefully placed and fixed inside the tooling, then pressed before undergoing the heat treatment. It is worth mentioning that a drawback for this technique is the necessity to manufacture a die for each bearing diameter and each foil thickness.

Figure 5 - Forming dies used to form the top (right) and the bump foils (left). The cut foils are placed into the dies, pressed and then submitted to the heat treatment process.

2.3 Heat Treatment.

The heat treatment is a critical step in the foil fabrication. Depending on the material of the foil, as well as the state of the formed material, the heat treatment would significantly change. For Inconel foils, precipitation hardening transforms the shaped foils from the annealed state and increases the material strength. There are several heat treatment recipes possible for any material [44], and for foil bearings it is desirable to select the recipes yielding high spring properties and fatigue resistance to remain constrained by the functionality of the foils as compliant structures. DellaCorte et al. [15] summarized some of the possible heat treatment recipes for Inconel X-750 foils. For Stainless Steel, the formed foil requires stress relief annealing, that is the heat treatment to reduce residual stresses after cold working. One major difference between heat treatment of Stainless Steel and Inconel is the fact that Stainless Steel requires being heat treated inside the pressed die, as the available Stainless Steel in the market is usually partially hardened, and the annealed stainless steels are usually not heat treatable for hardening purposes.

2.4 Coating.

A drawback of foil bearings is running at low speeds where mechanical contact occurs between the rotor and the top foil. For this reason the top foils are usually coated with a dry lubricant to accommodate for the friction between the two elements during startup and shutdown. Coating is the last process of manufacturing the foils. Rubio and San Andrés [45] used Teflon® as a dry lubricant, also, Song and Kim [33] used it to coat one side of the top foil with an extra thin layer of Chromium Nitride on the top of it. Shafts are also coated with dry lubricants, DellaCorte et al. [46] coated a 35mm diameter shaft with a 0.35mm thick layer of PS304 running on foil bearings. The large arena of solid lubrication technology is beyond the scope of this paper, however, advancements in this technology goes inseparably with the foil bearing development [47-49].

4. NON-INTRUSIVE GEOMETRY MEASUREMENT

A crucial step in the assessment of manufacturing deviations is an accurate measurement technique. The foil bearing fabrication literature relies either on the accuracy of the forming dies, or adopts ad-hoc measurements techniques, both techniques will not quantify springback. The structure of the formed bump foil requires a profilometer to measure its full profile. Most of high accuracy profilometers are mechanical devices, which use a stylus to probe and measure dimensions. Such technique, however, is not feasible due to the foil compliance. Hence, an optical non-intrusive geometrical measurement technique has been established to quantify the springback. A numerical code has been developed to assess the manufactured foils in an inexpensive, fast, and fully automated manner [50]. The code receives high resolution two dimensional scans of the formed foils – 4800 dpi, and treats the images to measure the overall radius of the foil, and the radii of each of the bumps constructing the foil. The algorithm relies on treating the grey scaled pixels representing the scanned foil cross-sections as data points, subsequently recognizes the bumps and fits circles through them, as well as the overall foil radius using a Nelder–Mead method algorithm - Figure 6. The fitting of the circles identifies the radii and their corresponding center coordinates. Consequently, the bump angle (α), the intersection angle (θ), the bump height, and clearance are calculated. Similar but less accurate techniques could be found in [51-53].

Figure 6 - Optical measurement of manufactured foils

5. OPTIMIZATION OF THE MANUFACTURING PROCESS

Since there is no reliable published manufacturing procedure for foil bearings, it was decided to discover the optimum procedure to minimize springback. When experimenting for different factors affecting a certain response, the one factor at a time approach is proven to be expensive, time consuming, and does not necessarily fully explain the observed phenomena [54]. Founded on these facts, Design of Experiment (DoE) procedures have produced different algorithms to plan and analyze experiments. Factorial design is a DoE algorithm developed by Fisher in the 1930s, where a number of levels is selected for each factor (variable), and then experiments are executed in all possible combinations of these factors [55]. Such algorithm is useful in system characterization, which would permit the optimization of the studied manufacturing process.

A DoE approach is used to decide and plan the necessary experiments to produce sufficient data to understand the different manufacturing process variables (factors) affecting the springback measured as the bump foil overall radius (response). After executing the data collection procedure, a model is developed to quantify the effects of each factor. Afterwards, an analysis of variance inference is performed to identify the significance of each factor. The three factors under investigation are: (i) the forming pressure, (ii) the heat treatment temperature, and (iii) the heat treatment duration.

5.1 Full Factorial and Modified Composite Designs.

A three factor two level factorial design has been chosen for the initial exploration of the experimental domain. The levels of forming pressures are 300 and 400 bars, heat treatment temperatures of 500°C and 600°C, and heat treatment durations of 3 and 5 hours. The chosen measured response is the overall radius of the bump foil error compared to the design value of 20.73mm. It can be shown that the overall radius of the bump foil is a comprehensive measure of the springback effect.

Table 2 - Full factorial experimental design

Experiment	Forming pressure	Heat treatment temperature	Heat treatment duration	Springback Error
[-]	[bar]	[°C]	[h]	[%]
1	300	500	3	-13.65
2	400	500	3	-9.69
3	300	600	3	-2.67
4	400	600	3	-4.23
5	300	500	5	-9.26
6	400	500	5	-9.41
7	300	600	5	-4.63
8	400	600	5	-3.33

The results in Table 2 shows that a minimum springback error of -2.67% is obtained from experiment 3. The best contenders are results from experiments 3, 4, 7, and 8. The common factor between those points is a heat treatment temperature of 600°C.

As the factors are three different physical quantities, a normalization is required for the different factors. The upper and lower values of each factor are normalized to 1 and -1 consecutively. A linear model with interaction is developed taking the following form:

$$Y = \beta_{mean} + X_P\beta_P + X_T\beta_T + X_D\beta_D + X_PX_T\beta_{PT} + X_PX_D\beta_{PD} + X_TX_D\beta_{TD} + \varepsilon \quad (3)$$

Table 3 summarizes the estimation results, presenting the model coefficients, the relative effects, the squared errors, t-statistics, and p-values. It can be seen that the temperature has the highest relative effect with a magnitude of -47.74%. The negative sign of the effect indicates that increasing the temperature factor would decrease the springback error. Pressure and time factors also have negative effects on the response, but they are much lower in magnitude as well as the effects of interactions. The results suggests a high insignificance of the pressure factor (P value above 70%). Hence, it was omitted and the regression was repeated only for the temperature and duration factors as well as their interaction and the results summarized in Table 4. The model and the temperature were both significant above 99%. However, the time and interaction factors were shown insignificant.

Table 3 – Statistical inference of full factorial design

	Coef. [β]	RE	SE	tStat	pValue
Mean	7.11		0.87	8.16	7.76%
P	-0.44	-6.24%	0.87	-0.51	70.01%
T	-3.39	-47.74%	0.87	-3.90	16.00%
D	-0.45	-6.35%	0.87	-0.52	69.58%
P:T	0.51	7.16%	0.87	0.58	66.35%
P:D	0.16	2.20%	0.87	0.18	88.70%
T:D	0.72	10.08%	0.87	0.82	56.20%

In conclusion, increasing the heat treatment temperature significantly reduces the springback. This is attributed to both an increase in creep and a decrease in the yield strength of the stainless steel. The forming pressure is an inert factor having an insignificant effect on the foil quality compared to the other factors, consequently it was removed from the studied factors in the following experiments. The heat treatment duration effects also exhibited low confidence levels, however it was decided to keep it under investigation for further experiments.

Table 4 - Statistical inference of modified full factorial design

	Coef. [β]	RE	SE	tStat	pValue
Mean	7.11		0.56	12.77	0.02%
T	-3.39	-47.74%	0.56	-6.10	0.37%
D	-0.45	-6.35%	0.56	-0.81	46.30%
T:D	0.72	10.08%	0.56	1.29	26.76%

A sequential augmentation was necessary to further explore the experimental domain, and to increase confidence intervals. For this purpose, a two-level modified composite design with single centered point is adopted, with an alpha value (distance of axial point to the center of the experimental domain) of 3 (instead of 1). The reason for changing the value of alpha is to confirm the positive effect of a high heat treatment temperature, while going beyond the uncertainty of the furnace. The model is modified to account for the removal of the pressure factor and the inclusion of second order effects as follows:

$$Y = \beta_{mean} + X_T\beta_T + X_D\beta_D + X_TX_D\beta_{TD} + X_T^2\beta_{TT} + X_D^2\beta_{DD} + \varepsilon \quad (4)$$

The results of experiments 1, 3, 5 and 7 are retained, where the forming pressure is 300bar. In order to be consistent, this same pressure is used in the additional experiments. This design produces variance inflation factors below 3, implying a low multicollinearity.

Table 5 - Composite experimental design

Experiment	Heat treatment temperature	Heat treatment duration	Springback Error
[-]	[°C]	[h]	
1	500	3	-13.65
3	600	3	-9.69
5	500	5	-2.67
7	600	5	-4.23
9	400	4	-20.77
10	700	4	-2.44
11	550	1	-9.15
12	550	7	-3.91
13	550	4	-8.25

The modified composite design domain was bounded by a heat treatment temperature between 400°C and 700°C, and a heat treatment duration between 1hr and 7hrs. Table 5 summarizes the results of the composite design. The minimum relative error of 2.44% comes from experiment 10. Table 6 shows the different relative effects and their corresponding p-values. Note the high relative effect of temperature and its corresponding low p-value, yielding a high confidence level. The highest relative effect is the interaction between temperature and duration, yet the p-value is very high, yielding a very low confidence level concerning the effect of this factor.

Table 6 - Statistical inference of composite design

	Coef. [β]	RE	SE	tStat	pValue
Mean	7.47		2.34	3.19	4.95%
T	-7.85	-105.06%	2.74	-2.87	6.42%
D	-4.38	-58.60%	2.74	-1.60	20.80%
T:D	12.67	169.58%	19.62	0.65	56.43%
T^2	4.08	54.57%	4.04	1.01	38.71%
D^2	-1.00	-13.34%	4.04	-0.25	82.10%

The model shows an optimum value of temperature equal to 675°C, which would yield a minimum foil radius springback error of 2.2%. However, since the optimum temperature is critically close to the recrystallization temperature of Stainless Steel at 700°C, the adopted optimum temperature was decided to be 650°C for a duration of 5 hours – Figure 7. Even though the duration's confidence level is relatively low, it is known that aging time inversely affect springback, also enough time is needed to heat the die equally, 5 hours resulted the minimum springback.

Figure 7 - Effect of heat treatment temperature on bump foil springback

6. MANUFACTURING UNCERTAINTY ASSESSMENT

After the identification of the optimum manufacturing process manufacturing uncertainty quantification was done statistically using 14 foil samples that would correspond to 126 bumps. Before the first use of the die, a profile measurement for the concave and convex parts was done using the mechanical probe measuring system Hommel T8000 of Jenoptik. The purpose of the measurement is to ensure the correct dimensions and tolerances of the foil shaping die after EDM machining. The standard deviation of the probe measurement is less than 1 micron for the bump radius. Three profiles were measured on each of the two parts of the die and the average values are used as reference. The measurement showed an average error of 0.09% in bump radius for the concave part, and 0.35% for the convex part compared to the design dimensions. These results were considered satisfactory to proceed the foil manufacturing procedure.

Figure 8 - Correlation between measured bump radius and angle springback errors suggesting a linear correlation

Measurements of the bump radii and their corresponding bump angles are represented in Figure 8. The results shows a linear correlation due to the finite length of the bump arc. To avoid the estimation procedure being sensitive to outliers in the measured data, the lowest and highest 5% of the bump radii were trimmed. The correlation shows a sensitivity between the bump radius and angle errors of approximately 1%/%. The geometrical interpretation of the correlation is presented in Figure 9.

Figure 9 - Effect of springback on the geometry of the bump

Figure 10 shows a scatter of the measured bump samples on the bump foil performance map. The resulting error in stiffness spans between -20% and 40% of the nominal design value and therefore suggests a significant deviation between the expected and the resulting bump stiffness. Hence, this result experimentally highlights the lack of robustness of the produced foils.

Figure 10 - Scatter of measured bumps on the bump foil performance map suggesting significant increase in bump stiffness as a result of manufacturing deviations

A non-parametric Kernel distribution was fitted to describe the statistical distribution of the 126 measured bump radii and their corresponding bump angles. Due to the linear inversely proportional relationship between the bump radius and angle, the normalized statistical distribution for both variables is identical in shape, yet one is positively skewed and the other is negatively skewed. Figure 11 shows the fitted distribution augmented to 100,000 samples and normalized by design value. The normalized mean and mode are 1.05 and 1.02 respectively. The larger mean compared to the mode signifies a clear positive skewness of the bump radii, which is a tendency to obtain oversize bump radii as a result from springback. The distribution of the normalized bump radii ranges between 0.95 to 1.3, which underlines the lack of robustness of the manufacturing process.

Figure 11 – Augmented statistical distribution of measured normalized bump radii

A Monte Carlo simulation using the augmented random sample distributions for the bump radii and corresponding angles as input for the Iordanoff model [19] has been performed. The analysis yields a normalized mean stiffness of 1.108 and a mode of 1.095. The distribution is close to normal, ranges between 0.7 and 1.6 with a relative standard deviation of 12.4%. This emphasizes the lack of robustness of the design and manufacturing process for classical bump foils and the significant impact of manufacturing deviation on foil bearing compliance.

An additional important measurement is the predicted error in the nominal bearing assembly clearance as a result of the measured deviation of bump radii and angles, i.e. bump height. Deviations in bearing clearance influence the fluid film the pressure field and therefore affect the bearing stiffness and damping properties. Figure 12 represents the obtained nominal clearance distribution, with a mean of approximately 50% increase in clearance and a relative standard deviation of 70% compared to the design value. This large variance in nominal bearing clearance might have a stabilizing effect on the bearing dynamic performance similar to selective shimming [56] or tailored compliance [57], yet and unlike these techniques, it is uncontrollable, and difficult to design a priori.

Figure 12 - Statistical distribution of nominal bearing clearance error as a result of manufacturing deviations

Based on the statistical evaluation of both bump radius and angle deviations it can be concluded that springback as a result from the manufacturing process yields a significant impact on both bearing compliance and clearance distribution and therefore significantly affect the rotordynamic bearing performance. With standard deviations up to 70% the bump foil design is far from robust.

7. FINITE ELEMENT SIMULATION OF BUMP FOIL FORMING PROCESS

In order to identify both the springback phenomenon and its distribution along the bump foil and to assess the effect of the bump foil geometry on springback and therefore on manufacturing deviation a Finite Element simulation for the forming process was implemented on ABAQUS® v6.14. The model simulates a 2D rigid tooling geometry and a 0.1mm thick stainless steel sheet as represented in Figure 13. The analysis of the forming process was divided to three phases where the first is the closing of the die, the second simulates the heat treatment, and the last is the die opening - Figure 14. The first and the third parts are dynamic simulations with high strains, plastic deformations and important friction effects, thus, explicit simulations were adopted. The heat treatment phase is a quasi-static simulation with a coupled temperature displacement analysis. To simulate the effect of the fixing binder, the foil area that should be in contact with the binder is fixed to the die. Two different surface-to-surface contacts have been created between the foil and the two part of the die.

Figure 13 – FE computational domain representing the two rigid dies and the undeformed foil before the die closing

The elasto-plastic relation of the 1.4310 stainless steel has been defined with a Ramberg-Osgood model at different temperatures [58]. A Norton power law is used to take into account for creep [59]. The parameters are calibrated based on experimental data from ambient temperature to 1000°C [60].

Figure 14 - Von Mises stress on one bump during the closing of the die

Coupled temperature-displacement plain strain with second order accuracy elements are adopted in this model. Linear elements are chosen for their better hourglass control for quadrilateral elements. As the simulation was mainly concerned with springback, the position of the free tip of the foil after the opening of the die was chosen as an indicator for grid independence. The displacement during the closure of the die converged with less than 2.5% relative error for 5 elements in the foil thickness, yielding an element thickness of 0.02 mm. It was shown that odd numbers of elements describes the physics with a better accuracy due to the existence of a neutral fiber. A sensitivity analysis was executed to determine the damping coefficient yielding a critically damped response. It was shown that a damping coefficient of 0.3 yields the minimum time to achieve a steady state position.

Figure 15 - Von Mises stress along the formed foil with local maxima indicating the transition between the bump and the land region and local minima occurring on the bumps themselves

The effect of friction on springback was investigated in prior work, which suggests that springback is sensitive to friction coefficients [61-66]. Hence, a sensitivity analysis was performed for friction level between the foil and the die. The standard Coulomb friction model was adopted and simulated for different levels of friction. The results confirm that springback is highly dependent on the friction coefficient. A friction coefficient of 0.05 yields the minimum springback - Figure 16. However, a friction coefficient of 0.1 is used in the model as it fits the experimental data, as well as the fact that friction coefficient below 0.1 between Stainless Steel and Inconel is very difficult to achieve experimentally.

Figure 16 - Effect of friction coefficient between the die and the formed foil on bump foil overall radius springback

Simulation results of the foils after load release and heat treatment were used to quantify springback with the same algorithms used for experimental investigation described above. Figure 17 represents the comparison of radius and intersection angle springback error for each bump for both the FE results and the experimental measurements of the manufactured bump foils. Bump number 1 is the first bump near the foil lip, and bump number 9 is last bump at the free end. The figure suggests that the two variables have similar trends along the foil. It is deduced that the bump radius springback is driven by the existence of the sharp intersection angle that exerts a pure bending on the formed foil, and hence playing a major role in springback. In addition, it can be seen that larger springback occurs near the foil lip (fixture), this is due to the large change in curvature at this point, which would influence the bumps in close proximity. The error bars on the figure represents the standard deviation (2-sigma) of the manufacturing process on each bump, and not of the measurement procedure, which is estimated to be negligible compared to that of manufacturing process. The FE results lie within the standard deviation of all the experimental measurements and follow the same trend. Note that the bumps towards the free end yield lower errors suggesting that the clamping of the foil at the lip itself is

responsible for part of springback. The resulting overall bump foil radius is plotted in Figure 7 thus suggesting good agreement between the numerical methodology and the experiments presented above.

Figure 17 - FE model validation with measured bump radius error (Top) and intersection angle θ (Bottom) for each bump

Figure 15 shows the von Mises stress distribution along the foil length after the closure of the die at a friction coefficient of 0.1. Since springback after the die opening is directly related to the stress level the latter can be used to identify the locations of major springback. The stress levels suggest that the main stress pike occurs at the main edge of the foil used for fixture. Local maxima appear in the sharp bends of the foil after each bump to follow the main circular profile, thus corroborating the θ -springback error represented in Figure 17, the sharper the intersection angle, the higher the stresses, hence, higher are the intersection angle springback errors - analogous to the bending of a metal sheet with large bend angles. Local minima are observed within the bump themselves. The results therefore suggest that the main driver for springback of the classical bump foils are the discontinuities of curvature at the bump-land transitions.

Figure 18 - Original and modified die designs

8. IMPROVED DIE DESIGN

Based on this analysis, a second die was designed to reduce springback, while maintaining the same bump foil compliance and foil thickness. The rationale behind the new design is the reduction of the foil curvature by eliminating the 90° bend at the main edge for fixture, and replacing it with an edge that follows the curvature of the bearing. In addition, the sharp bends between the bumps are replaced by a smooth curve, resulting in a final shape similar to a sinusoidal wave along the overall bump radius - Figure 20. The geometrical modifications were a consequence of the reduction of the distributed stress along the foil, which is expected to reduce the springback as a consequence. A comparison between the two forming dies is represented in Figure 18 comparing the two fixtures (a and b) and the classical and sinusoidal bumps (c and d).

Figure 19 – Die curvature and Von Mises stress along the formed foil comparing original and modified die designs

Figure 19 shows the absolute curvature of both die geometries (a), and the comparison between the two geometries in terms of von Mises stress after the die closure (b). The mean stress along the foil was reduced by 17.4% compared to the original design. The springback was reduced by 69% in a cold forming simulation.

Figure 20 - Geometrical features of the modified bump foil eliminating the sharp theta angle

The modified foil die was prototyped in Inconel X750 using EDM. 6 foil samples were tested at the optimum conditions mentioned previously. The modified design produced better foils in terms of mean and variance. The mean normalized bump radius is 1.0096, and ranges between 0.95 and 1.068 compared to 0.95 to 1.3 for the classical bumps. The relative standard deviation drops from 12.4% to 1.7%. Figure 21 shows the probability density functions of the original and modified forming dies in terms of normalized bump radius. The reduction in variance suggests a significant improvement in accuracy and robustness.

Figure 21 – Kernel distribution of the normalized bump radius comparing the original die design to the modified die design and highlighting the improvement in robustness and accuracy

9. CONCLUSIONS

A review of the available knowledge in the fabrication of foil bearings was briefly presented. It was shown that the available know-how is not sufficient for the accurate manufacturing of foil bearings. The bump foil compliance was shown to be sensitive to the bump angle and radius, even for small errors of bump geometry (5% error in compliance for each 1% error in bump radius). A manufacturing procedure was selected using forming dies adopting the overall curvature of the bump foil. Due to the complexity of the bump foil shape, an optical measurement technique was developed for the purpose of the detailed geometrical measurement of the formed bump foils. Consequently, a DoE approach was used to identify the effect of heat treatment temperature, duration, and forming pressure on the foil springback. Finally, a FE model was developed to simulate the forming process of the foil inside the die. The following conclusions were made:

- It was shown that the heat treatment temperature is the significant factor affecting the measured springback. It was also shown that the optimum temperature for Stainless Steel (1.4310) foil forming is 650°C.
- The results of uncertainty quantification showed a 6% mean bump radius error, and the measured samples ranged between -5% to 30% error, which highlights the lack of robustness of both the manufacturing process and the bump foil design. The analysis showed major deviations in bearing clearance and compliance due to springback inducing significant effects on bearing performance.
- A linear correlation was found between the bump radius and bump angle errors, which is explained by the finite arc length of the bump.
- A Monte Carlo simulation was done with the quantified uncertainty distributions for the bump geometrical errors. The mean of obtained compared to design stiffness is an increase by 10% with a relative standard deviation of 12.4%.
- The FEA suggests that the stress levels within the foil are very sensitive to the shape of the die. It was shown that modifying the geometry of the forming die, by eliminating all the sharp bending locations, as well as aggressive changes in the bump foil curvature, the mean stress along the foil can be reduced by 17.4%, and springback by 69%.

- The modified die design was prototyped and tested and proving significant improvements. The accuracy improved in terms of the mean bump radius error, which reached a value 1% instead of 6% in the original design. The robustness also improved by having a significantly lower range of error for the different measured samples to range between -5% and 6% instead of ranging between -5% and 30% in the original die design, and a reduction in relative standard deviation from 12.4 % to 1.7%.

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Nomenclature

S	: Bump compliance	m/N
δ	: Bump deformation	m
F	: Loading force	N
S_0	: Bump pitch	m
L	: Bearing axial length	m
l_0	: Bump half length	m
ν	: Poisson ratio	-
E	: Young's modulus	Pa
t	: Foil thickness	m
α	: Bump angle	°
θ	: Bump intersection angle	°
μ_f	: Friction coefficient	-
h_b	: Bump height	m
R	: Bump radius	m
Y	: Model response	-
β	: Model coefficient	-
X	: Model variable	-
ε	: Model residual	-
P	: Forming Pressure	bar
T	: Heat Treatment Temperature	°C
D	: Heat Treatment Duration	Hr

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