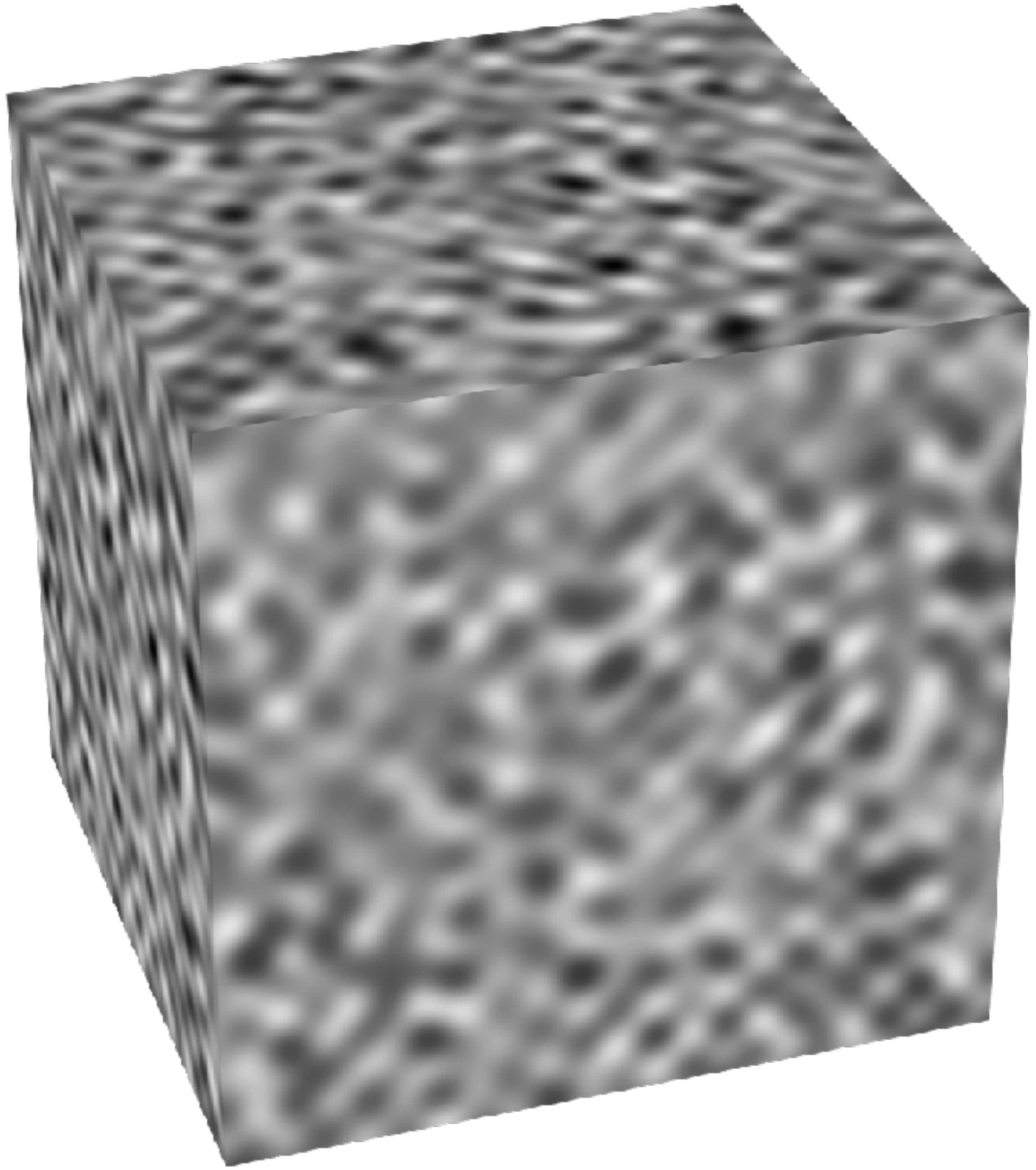
Shape from bandwidth

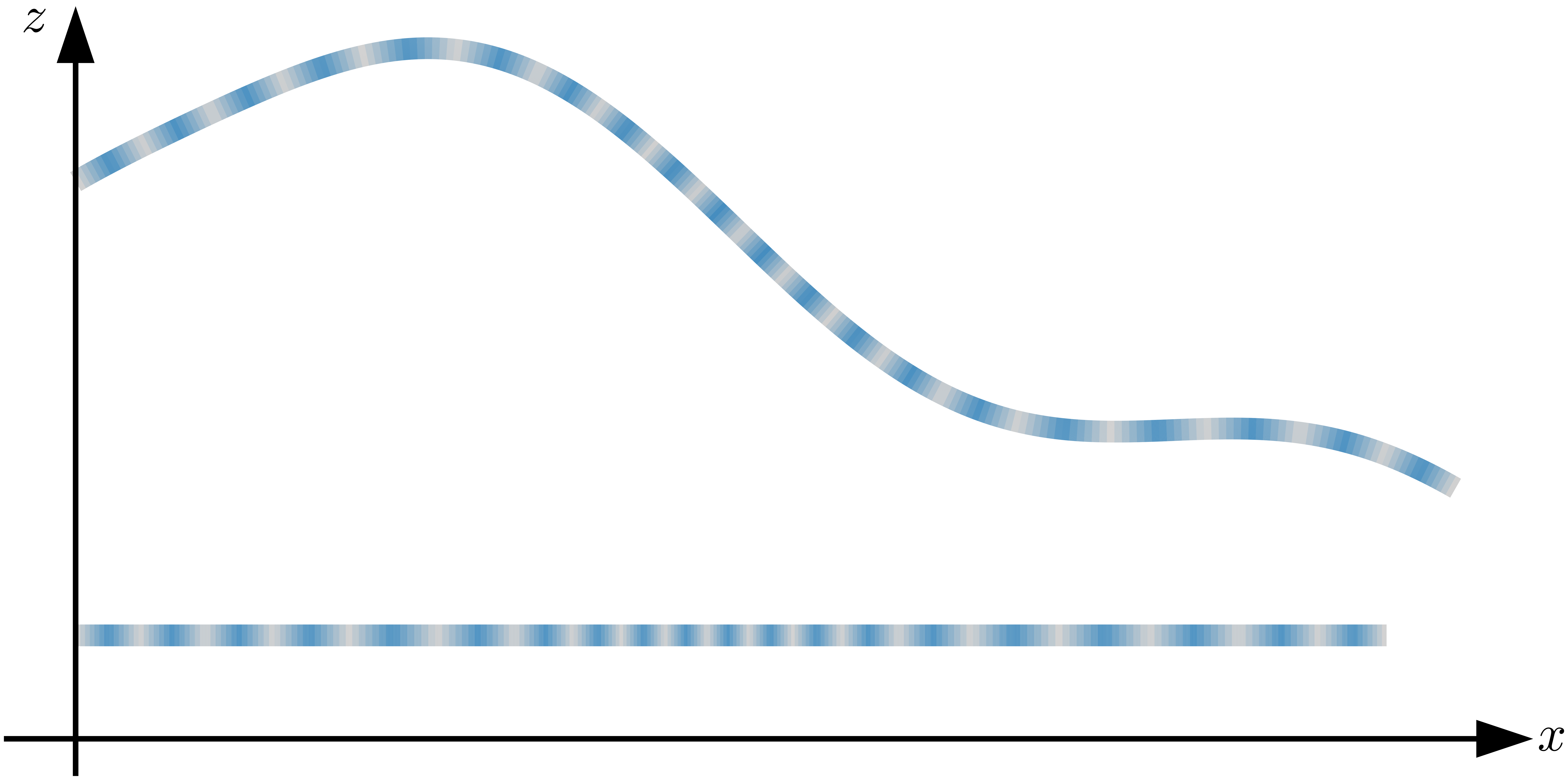
The 2-D orthogonal projection case

Adam Scholefield, Benjamín Béjar Haro and Martin Vetterli

Bandwidth



2-D Orthogonal projection case

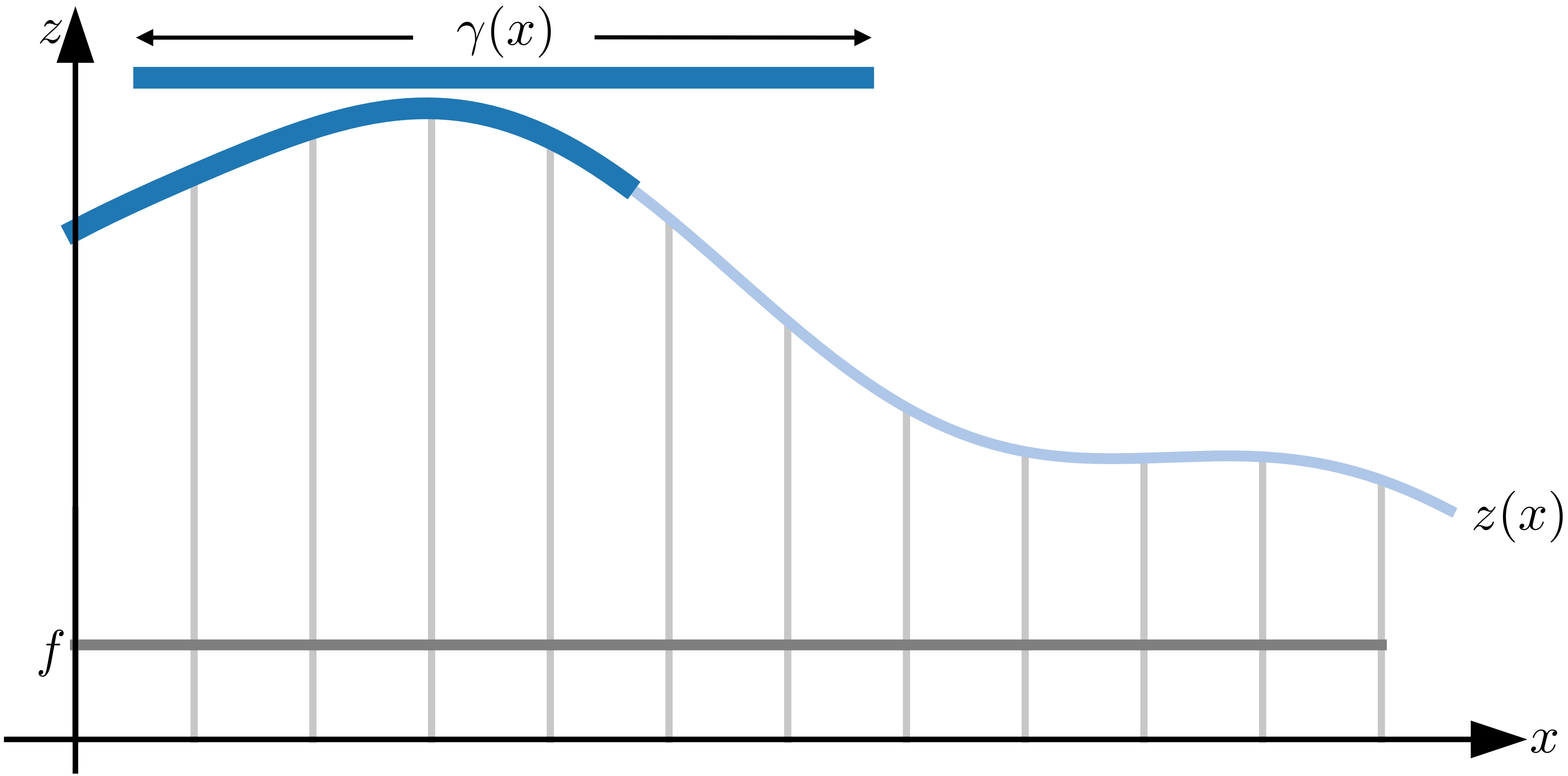


Outline

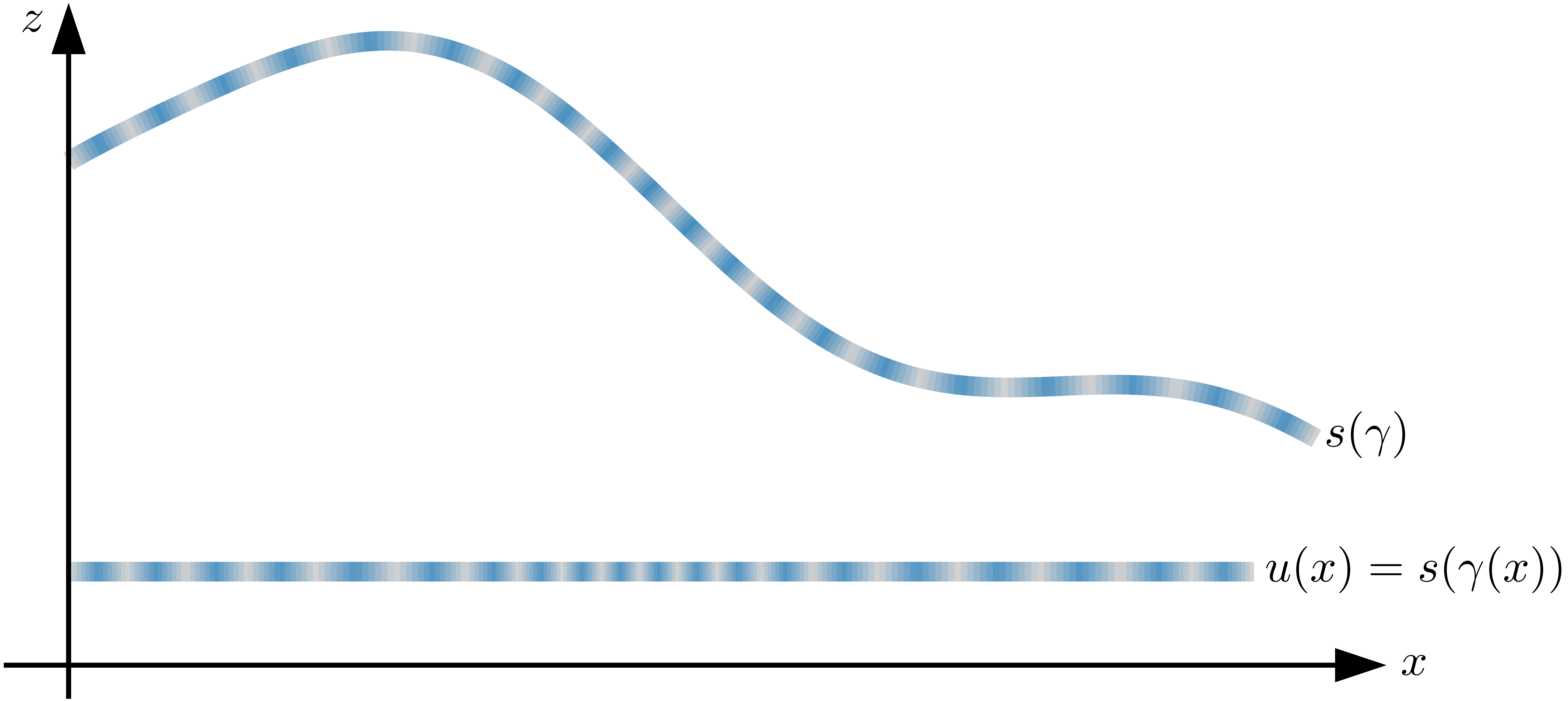
- Problem setup
- Algorithm overview:
  - Observation to arc-length
  - Arc-length to surface (ambiguities)
- Uniqueness result
- “Practical” experiment
- Conclusions and future work

Setup

Setup



Setup



z – surface, γ – arc-length, s – texture, u – observed

Algorithm overview

Observation to arc-length

$$u(x) = s(\gamma(x))$$

$$u(x; x_0) := s\left(\gamma(x_0) + \gamma'(x_0)(x - x_0)\right)$$

$$\mathcal{F}\left[u(x; x_0)\right] = \frac{1}{\gamma'(x_0)} S\left(\frac{\omega}{\gamma'(x_0)}\right) e^{j\omega\left(\gamma(x_0)/\gamma'(x_0) - x_0\right)}$$

s has bandwidth $B \;\Rightarrow\; u(x; x_0)$ has bandwidth $\gamma'(x_0)B$

z – surface, γ – arc-length, s – texture, u – observed

Observation to arc-length

$$u(x) = s(\gamma(x))$$

$$u(x; x_0) := s\left(\gamma(x_0) + \gamma'(x_0)(x - x_0)\right)$$

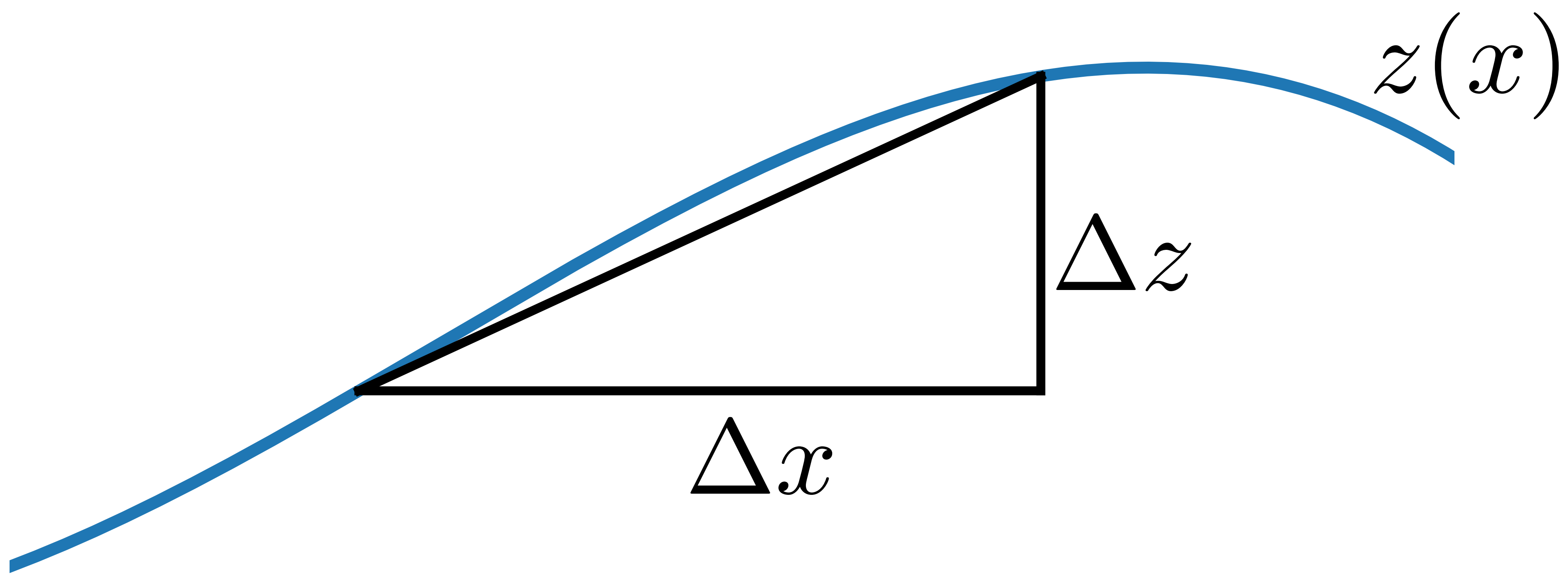
$$\mathcal{F}\left[u(x; x_0)\right] = \frac{1}{\gamma'(x_0)} S\left(\frac{\omega}{\gamma'(x_0)}\right) e^{j\omega\left(\gamma(x_0)/\gamma'(x_0) - x_0\right)}$$

s has bandwidth B $\Rightarrow$ u has local bandwidth $\gamma'(x_0)B$ at x_0

z – surface, γ – arc-length, s – texture, u – observed

Arc-length to surface gradient

s has bandwidth $B \quad \Rightarrow \quad u$ has local bandwidth $\gamma'(x_0)B$ at x_0

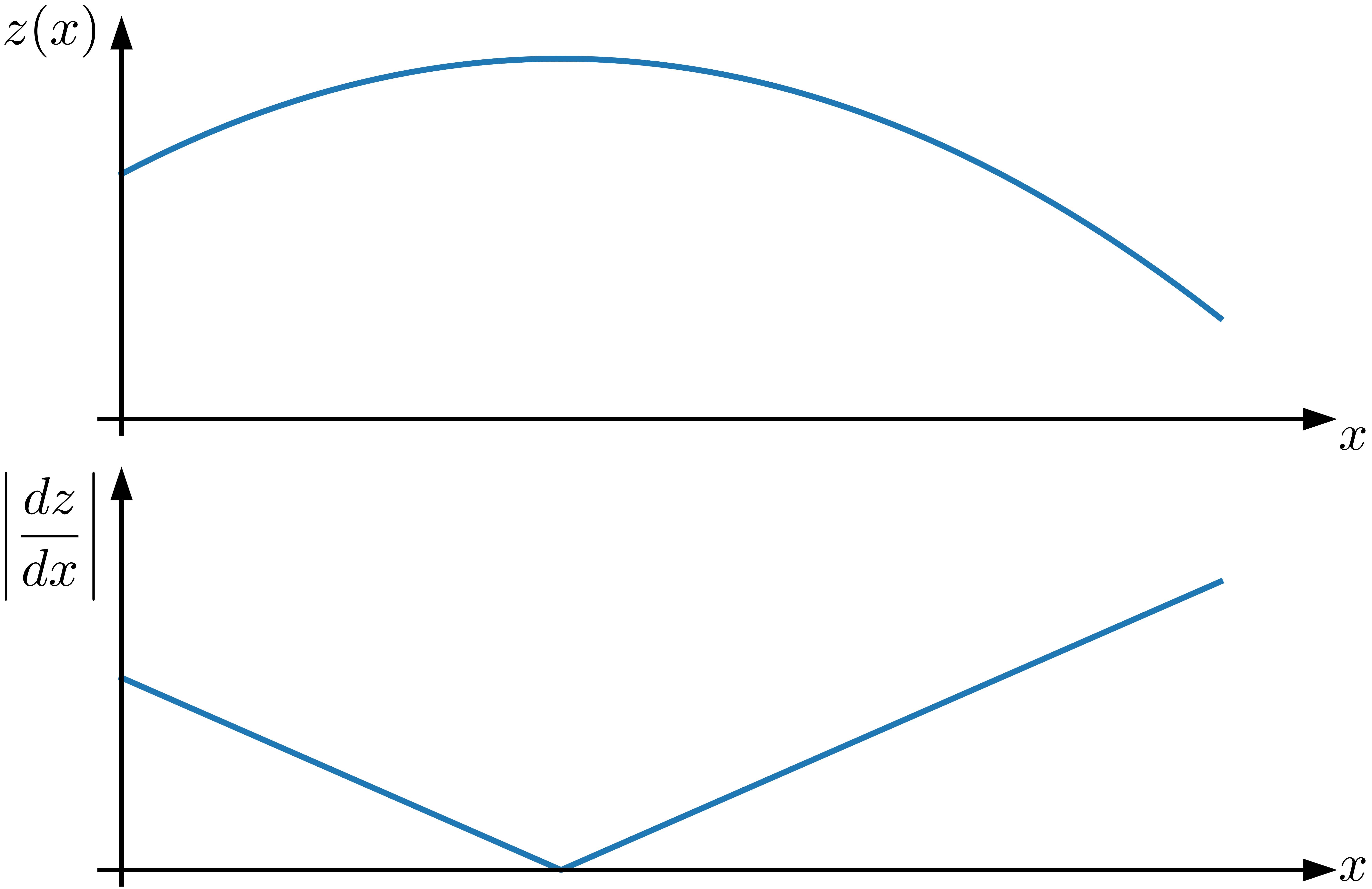


$$(\Delta\gamma)^2 = (\Delta x)^2 + (\Delta z)^2$$

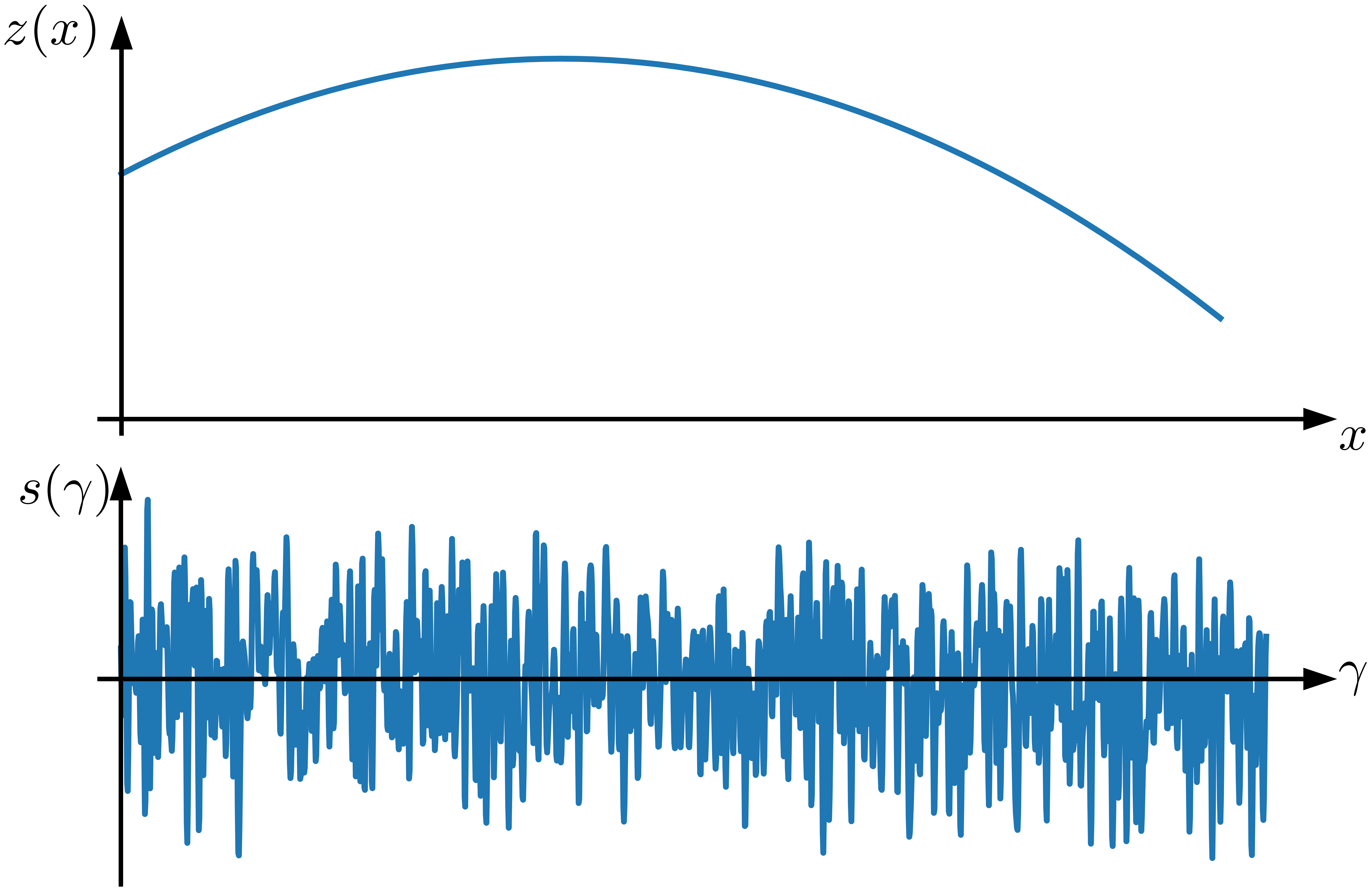
$$\left(\frac{d\gamma}{dx}\right)^2 = 1 + \left(\frac{dz}{dx}\right)^2 \quad \Rightarrow \quad \left|\frac{dz}{dx}\right| = \sqrt{\left(\frac{d\gamma}{dx}\right)^2 - 1}$$

z – surface, γ – arc-length, s – texture, u – observed

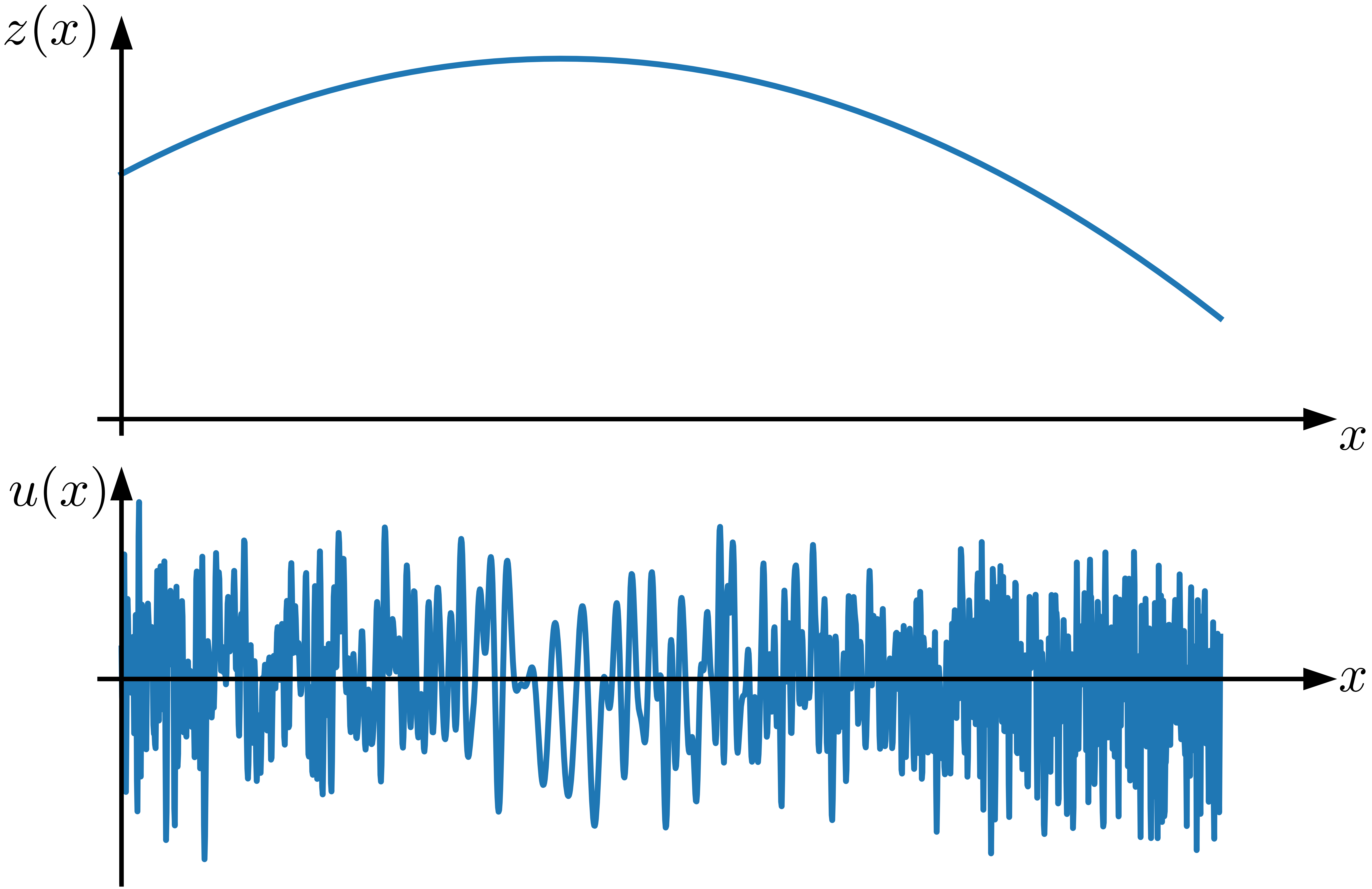
Example



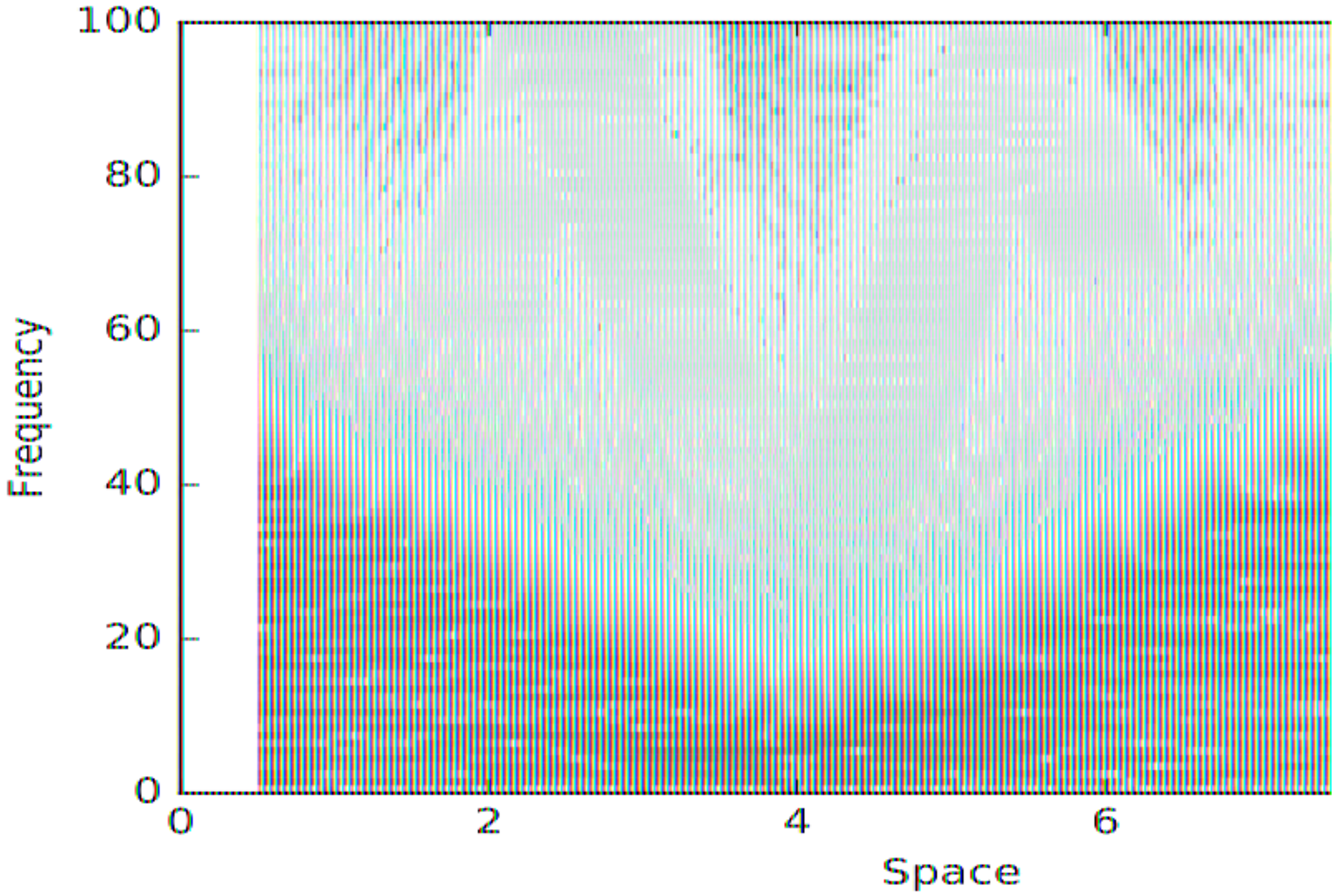
Example



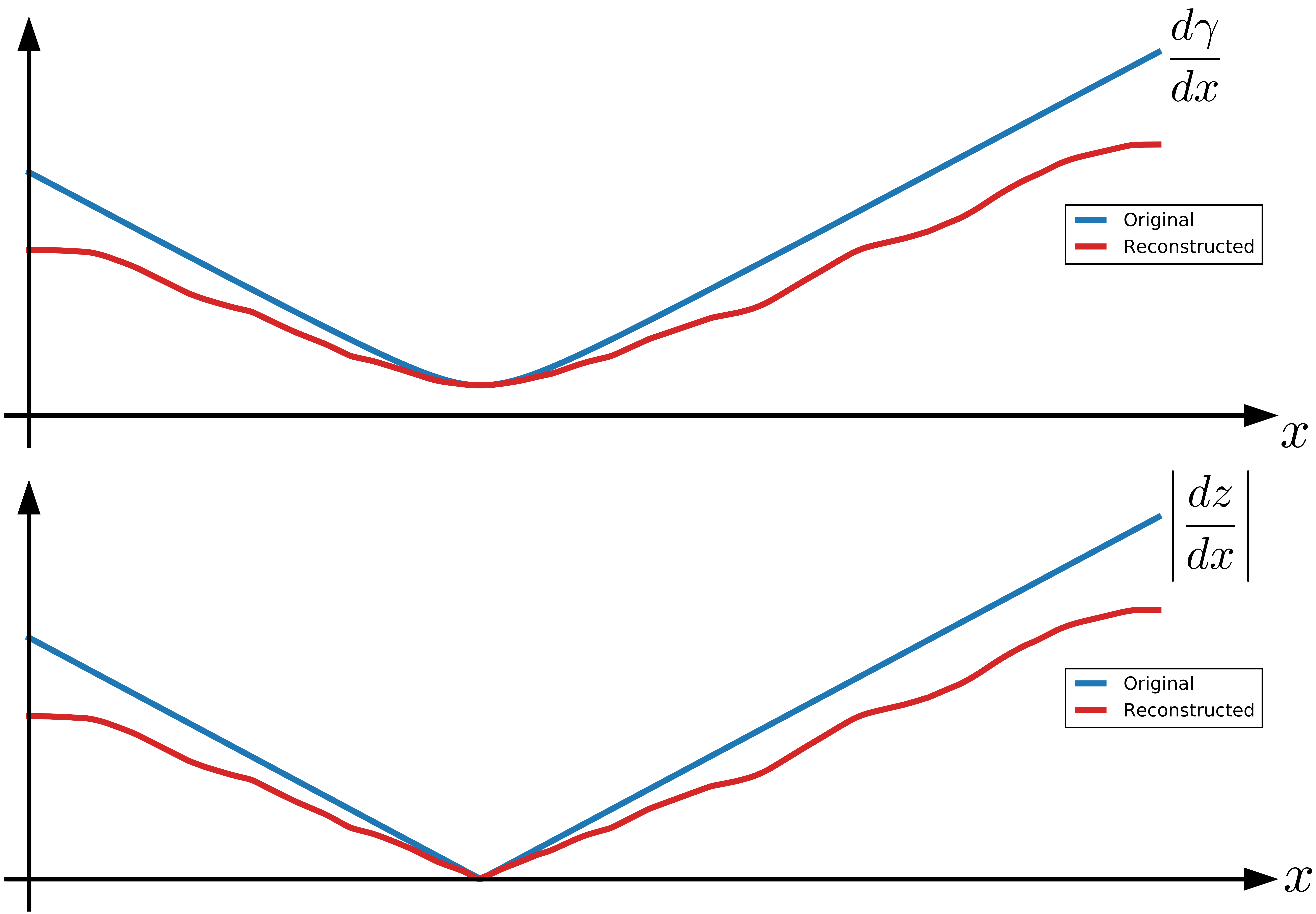
Example



Example

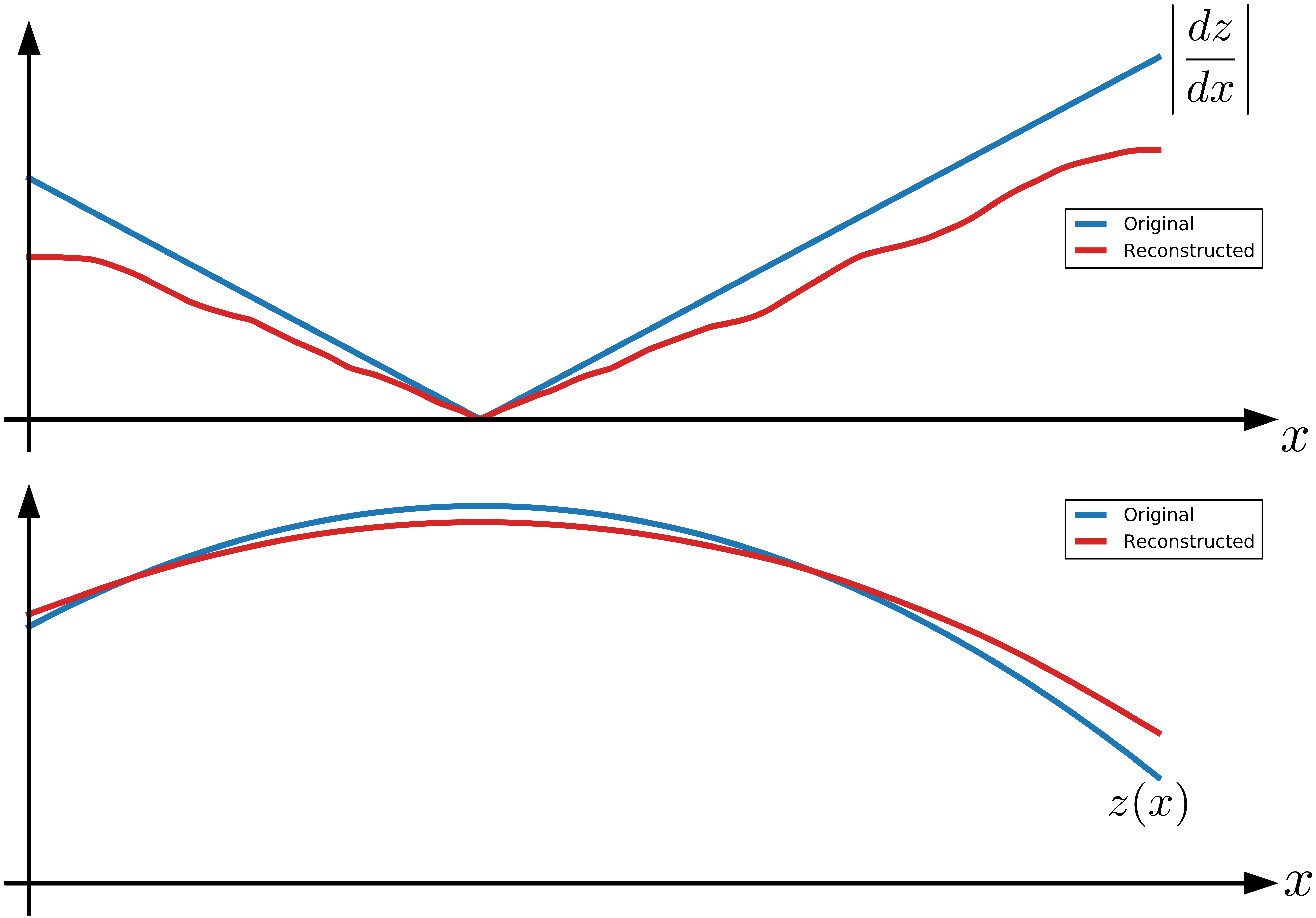


Example



$\frac{d\gamma}{dx}$
Original
Reconstructed
x
$\left|\frac{dz}{dx}\right|$
Original
Reconstructed
x

Example



$\left|\frac{dz}{dx}\right|$
Original
Reconstructed
x
Original
Reconstructed
$z(x)$
x

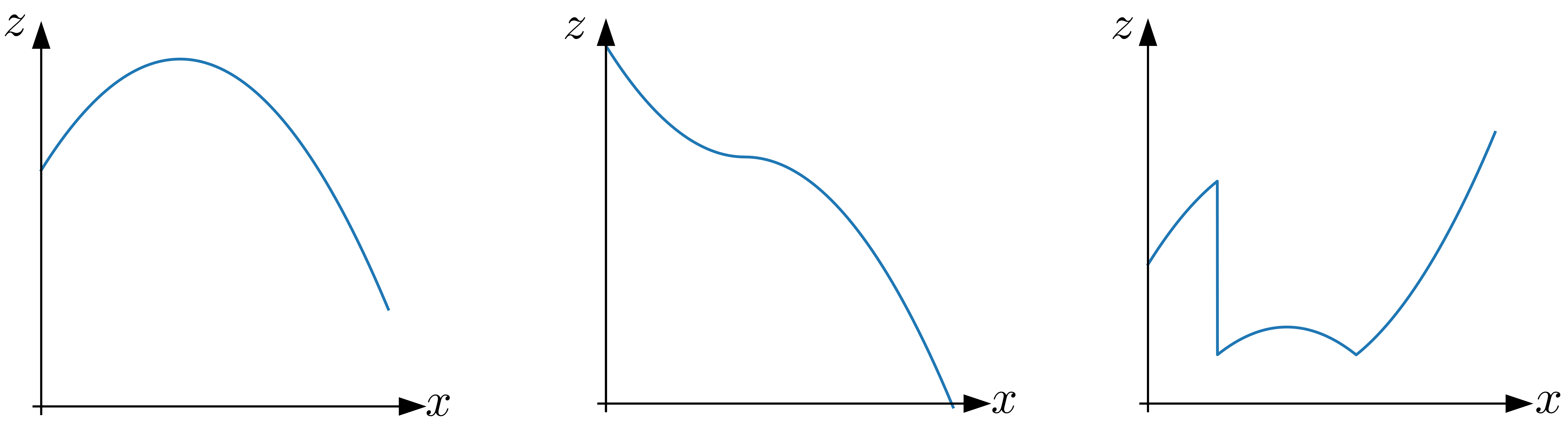
Arc-length to surface
(ambiguities)

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

LCAV

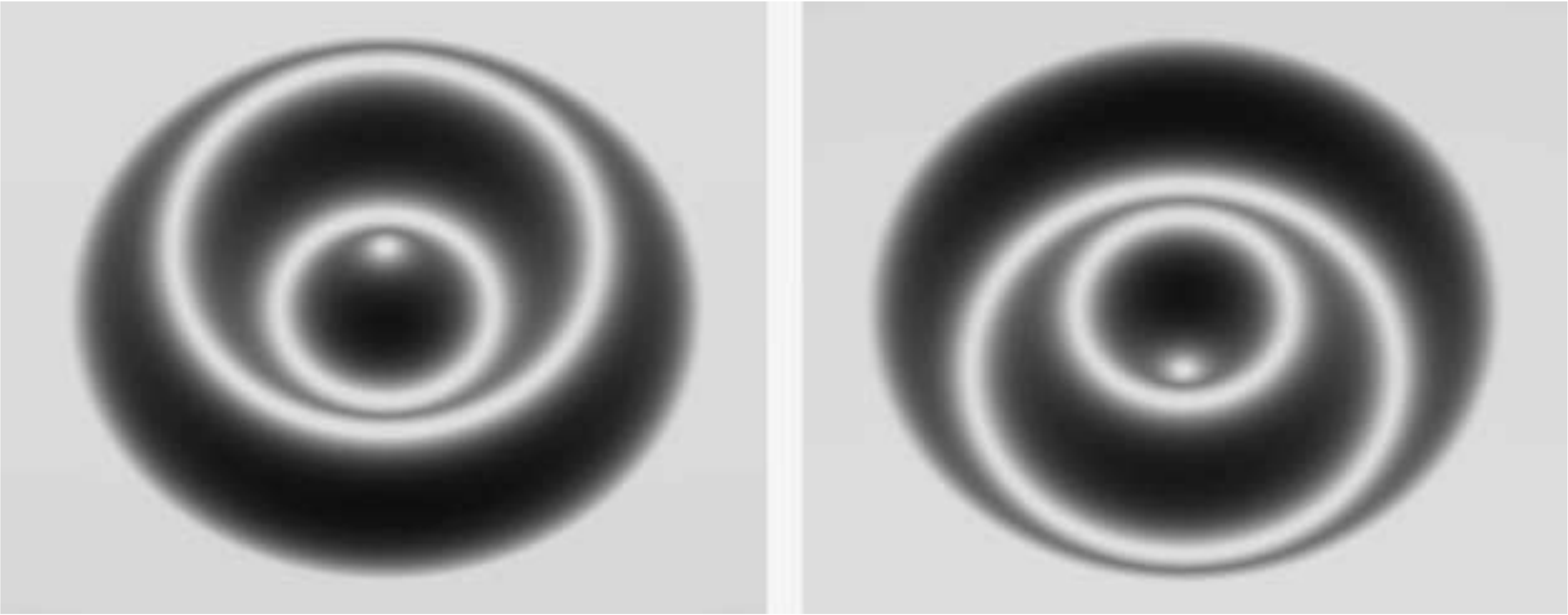
Arc-length to surface

$$\left(\frac{d\gamma}{dx}\right)^2 = 1 + \left(\frac{dz}{dx}\right)^2 \quad \Rightarrow \quad \hat{z}(x) = \int_{x_{min}}^{x} \left|\frac{dz}{dx'}\right| dx'$$



z – surface, γ – arc-length, s – texture, u – observed

Perceptual biases



Liu, Todd. “Perceptual biases in the interpretation of 3D shape from shading”, 2004.

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

LCAV

Algorithm

1. Compute spectrogram of observed signal
2. For each spatial value, estimate local BW
3. Divide local BW by minimum observed BW
4. Calculate $\left|\frac{dz}{dx}\right| = \sqrt{\left(\frac{d\gamma}{dx}\right)^2 - 1}$
5. Reconstruct all surfaces in the equivalence class

Uniqueness from Clark's conjecture

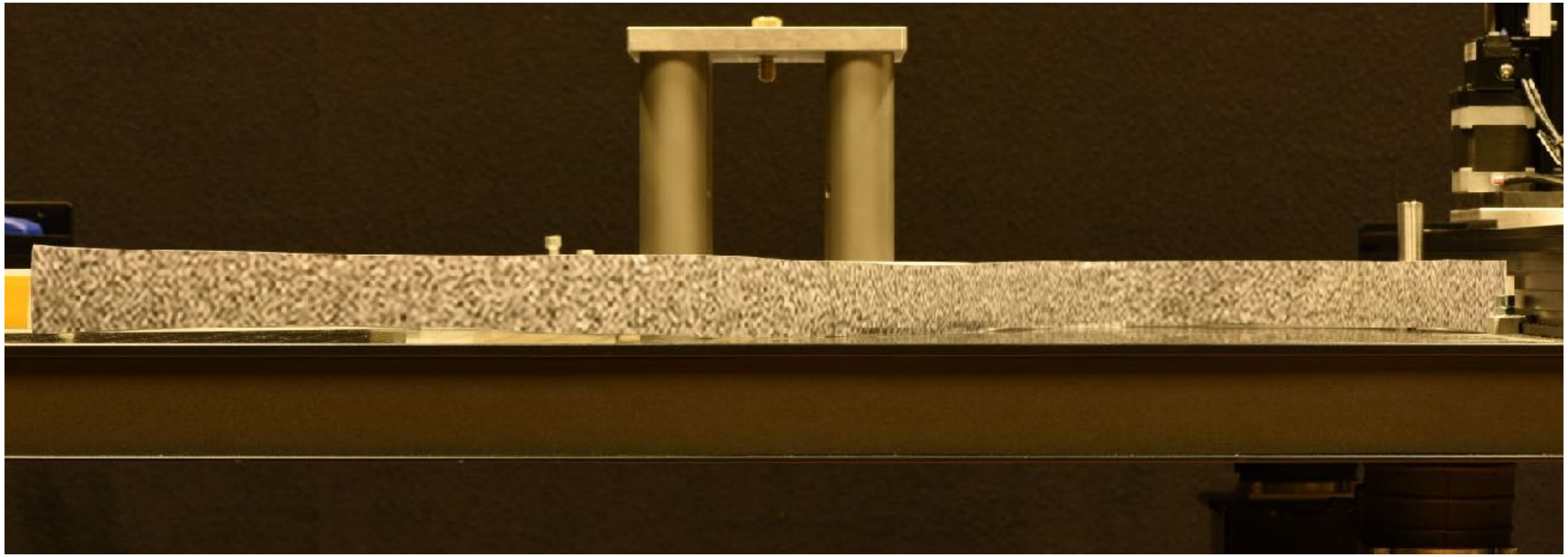
Conjecture (Clark 1989). *A warped bandlimited signal will be bandlimited if and only if the warping is affine.*

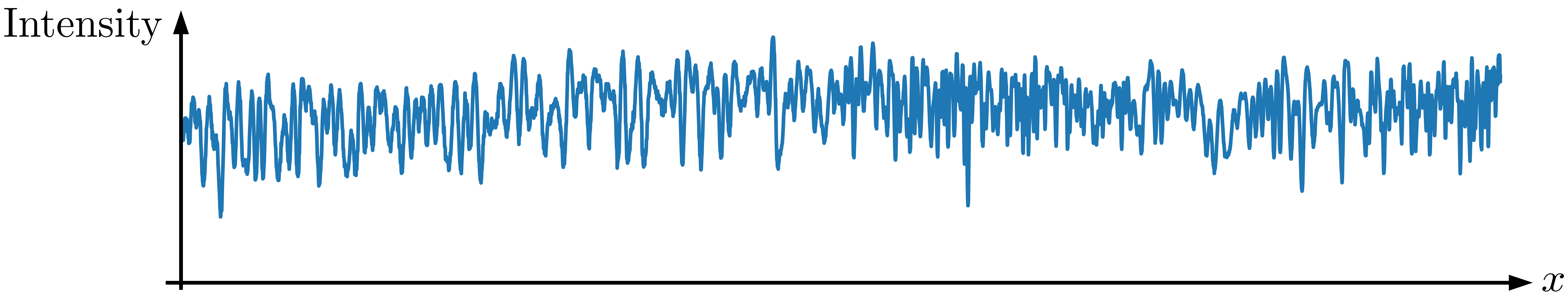
Theorem (Xia and Zhang 1992). The c*onjecture is true if we restrict the warping to be entire on an interval.*

Counter example (Azizi et. al.1999). *Yves Meyer came up with a peculiar counter-example.*

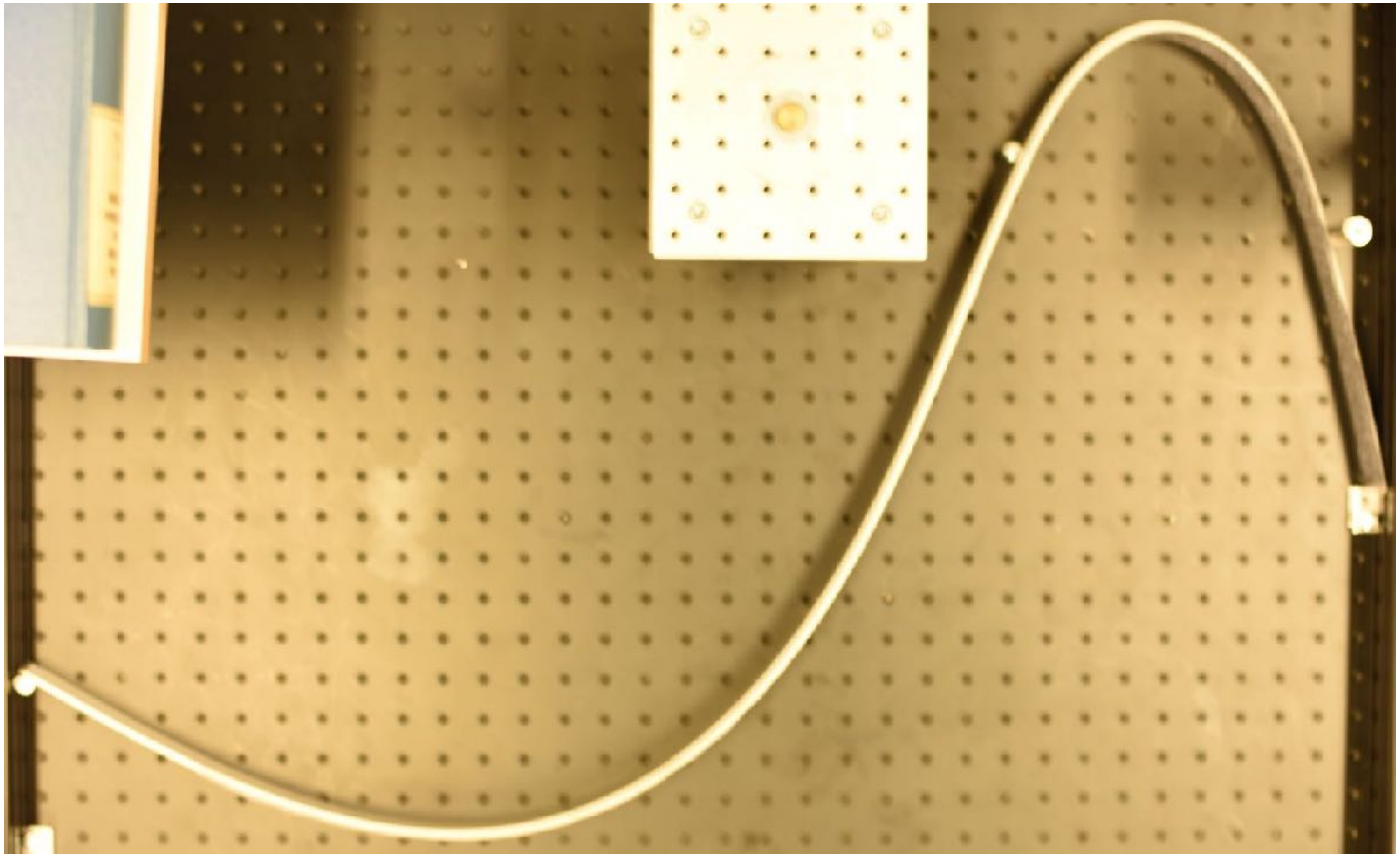
Experiment

Experiment

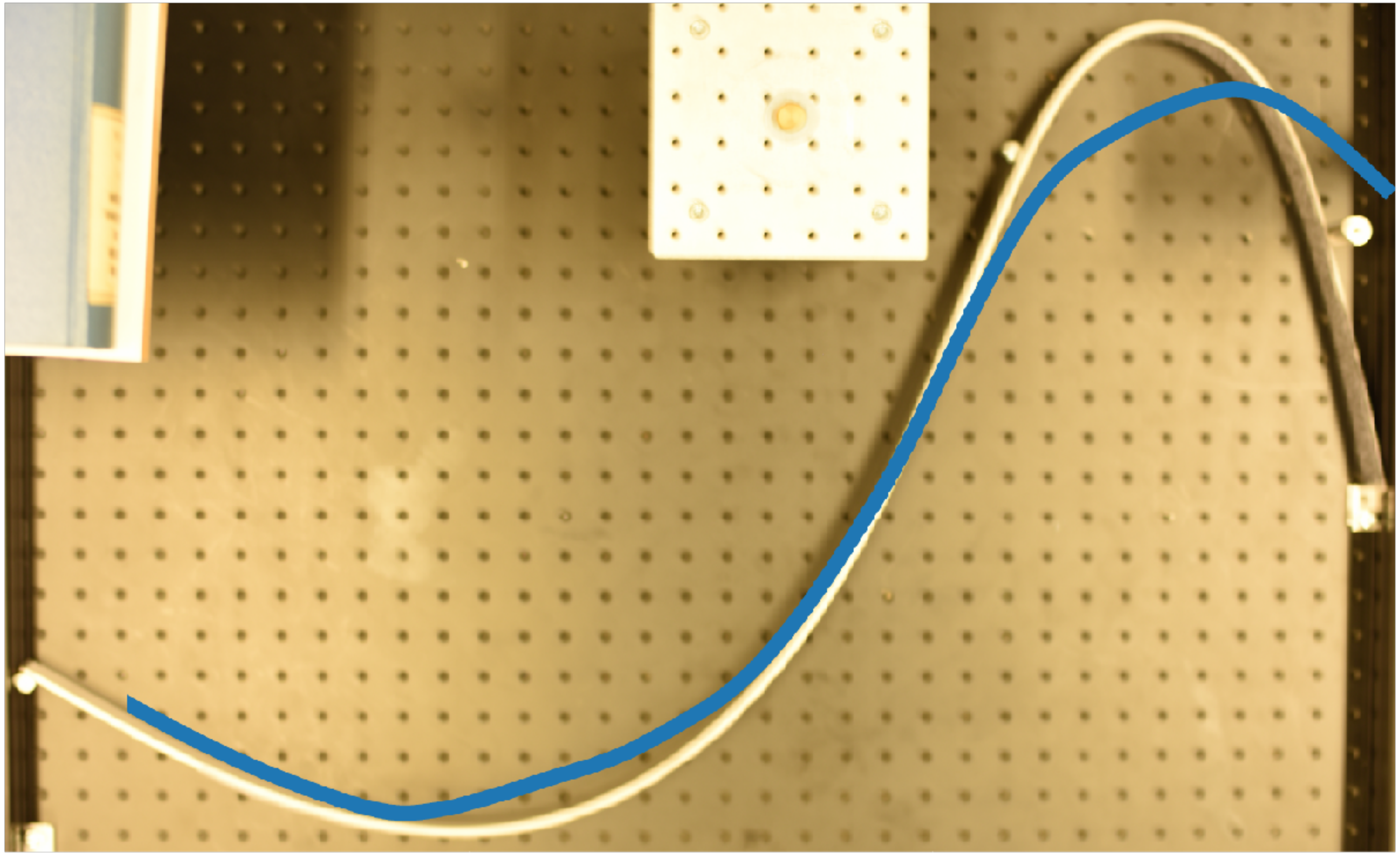




Experiment



Experiment



Conclusions and future work

- Basic algorithm to retrieve surface from bandwidth

Future work

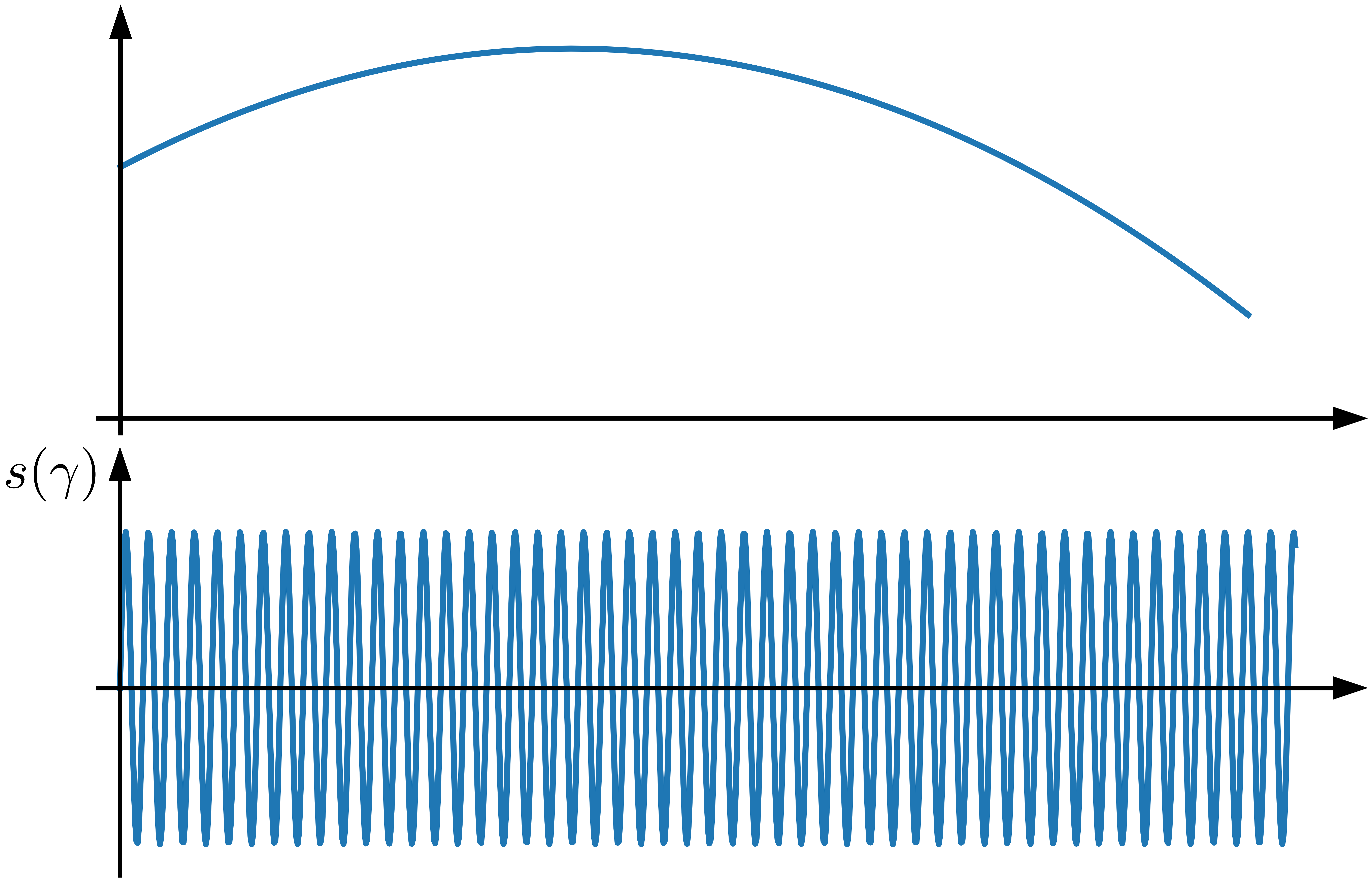
- Extensions - Central projection and 3D
- Algorithmic improvements - window size …
- Sampling
- Uniqueness and recovery guarantees
- Related - calibration, structured light depth sensing …

Thank you

An IPython notebook is available reproducing all the results of the paper: https://infoscience.epfl.ch/record/224065?ln=en

Sinusoidal texture

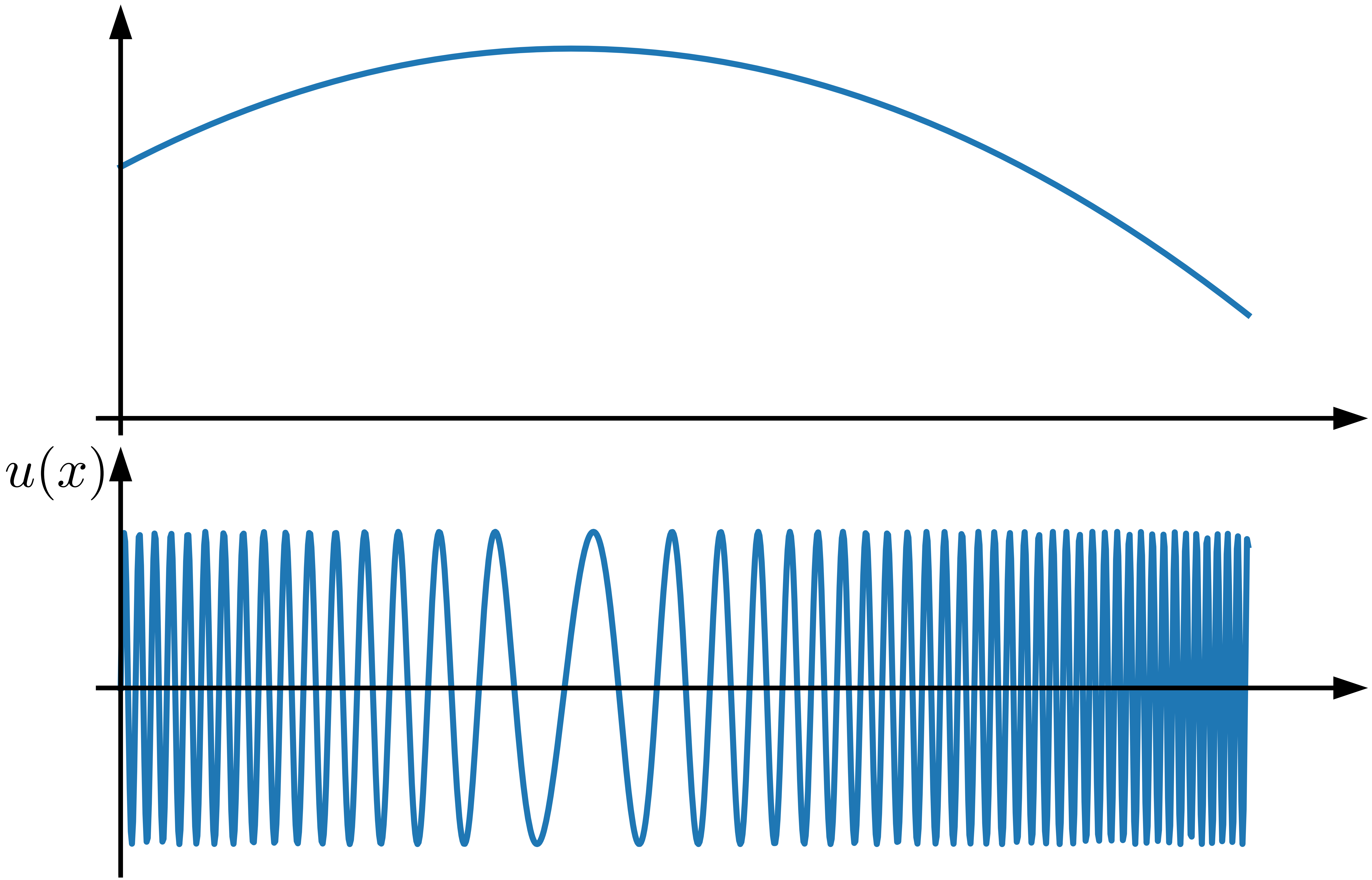
Sinusoidal texture



ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

LCAV

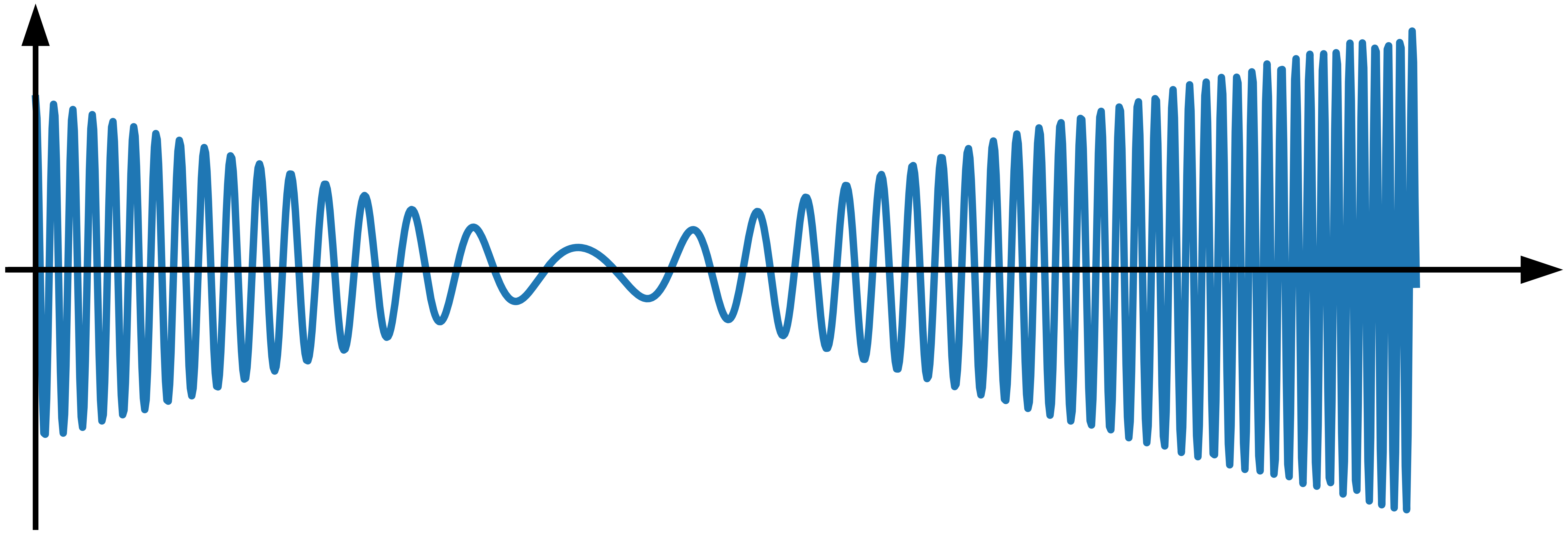
Sinusoidal texture



Sinusoidal texture

$$s(\gamma) = A\sin(\omega\gamma)$$

$$\frac{du}{dx} = A\omega\frac{d\gamma}{dx}\cos(\omega\gamma(x))$$

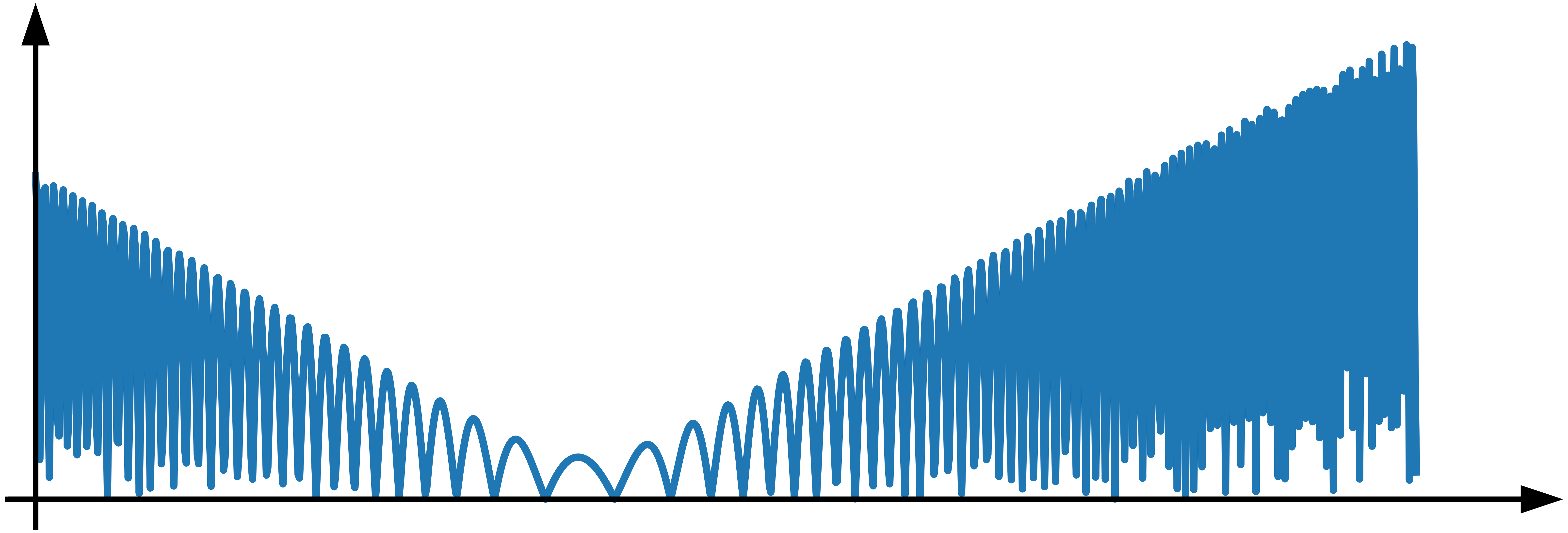


z – surface, γ – arc-length, s – texture, u – observed

Sinusoidal texture

$$s(\gamma) = A\sin(\omega\gamma)$$

$$\frac{du}{dx} = A\omega\frac{d\gamma}{dx}\cos(\omega\gamma(x))$$

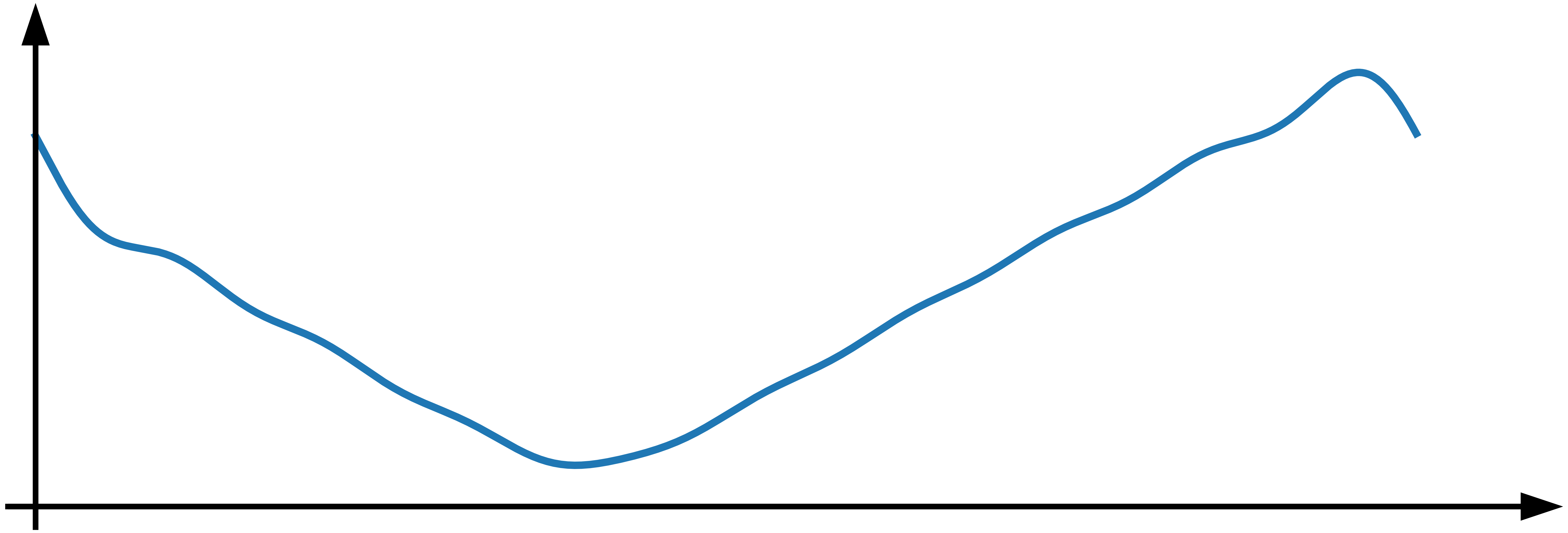


z – surface, γ – arc-length, s – texture, u – observed

Sinusoidal texture

$$s(\gamma) = A\sin(\omega\gamma)$$

$$\frac{du}{dx} = A\omega\frac{d\gamma}{dx}\cos(\omega\gamma(x))$$

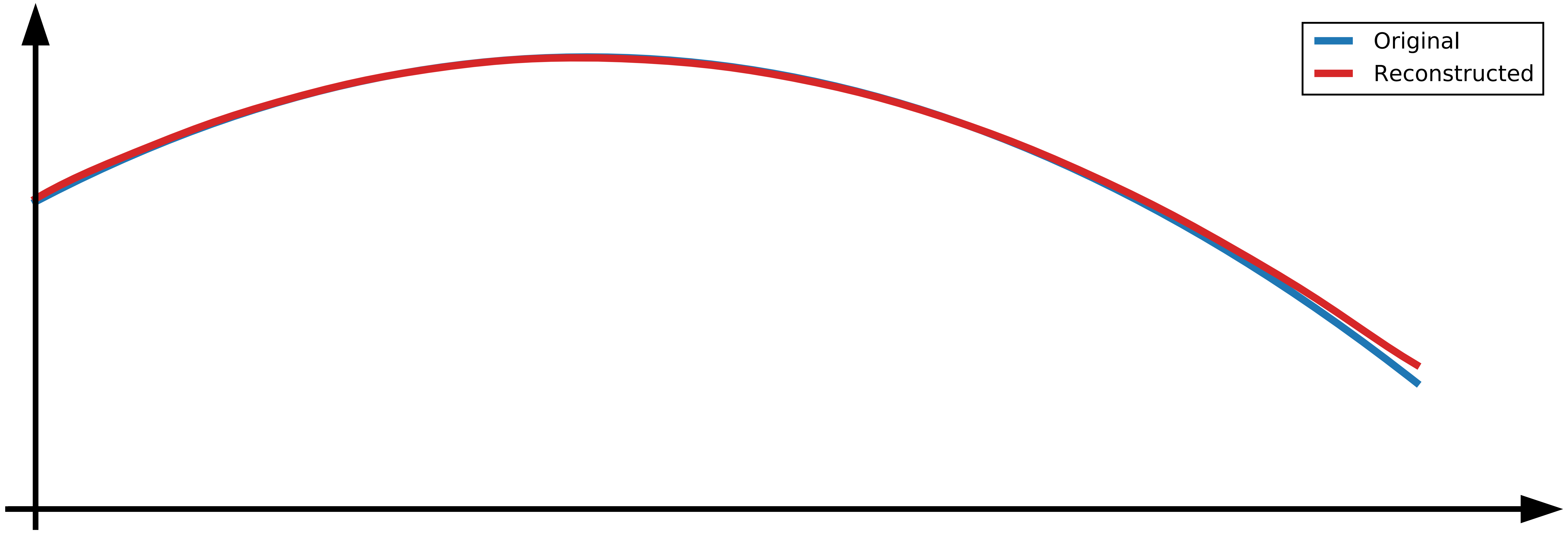


z – surface, γ – arc-length, s – texture, u – observed

Sinusoidal texture

$$s(\gamma) = A\sin(\omega\gamma)$$

$$\frac{du}{dx} = A\omega\frac{d\gamma}{dx}\cos(\omega\gamma(x))$$



z – surface, γ – arc-length, s – texture, u – observed