Variational phase field model for dynamic brittle fracture

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Mechanisms of dynamic fracture

Variational phase-field model of brittle fracture

Crack branching in homogeneous medium

Crack propagation in heterogeneous medium
Limiting crack velocity: in theory, $v_{\text{lim}} = c_R$ for mode I
never attained in experiments, rarely exceed $0.4 - 0.7 c_R$
seems to depend on experimental setup (geometry, loading conditions)
**Limiting crack velocity**: in theory, $v_{lim} = c_R$ for mode I never attained in experiments, rarely exceed $0.4 - 0.7 c_R$ seems to depend on experimental setup (geometry, loading conditions) explained by **crack tip instabilities** [Sharon and Fineberg, 1996]:

- microbranching ($\sim 0.4 c_R$): small (1-100 $\mu$m in PMMA) short-lived micro-cracks, highly localized
- mirror, mist, hackle patterns
Crack branching

Macroscopic branching at even higher velocities

[Ramulu and Kobayashi, 1984]  [Kobayashi and Mall, 1977]
Crack branching

**Macroscopic branching** at even higher velocities

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**Criterion** for branching? question is still open...

- experiments and numerical simulations seem to exclude a criterion based (only) on crack tip velocity
- existence of a critical SIF or ERR?
Velocity-toughening mechanism

Experiments on PMMA report a strong increase of apparent fracture energy with velocity: velocity-toughening mechanism

- a large part is attributed to an increase of created fracture surface due to microbranching

- recent experiments show an increase from 400 J/m² to 1 200 J/m² between 0.11\(c_R\) and 0.18\(c_R\) [Scheibert et al., 2010]
Outline

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Crack branching in homogeneous medium

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Phase-field approach

- alternative to cohesive elements or XFEM for simulating crack propagation
- non-local approach: continuous scalar field $d(x)$ representing the crack + a regularization length $l_0$ [Bourdin et al., 2000]
- can be formulated as a damage gradient model

![Diagram showing the phase-field approach with a continuous scalar field $d(x)$ and a regularization length $l_0$.](image)
Phase-field approach

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- can be formulated as a damage gradient model

- convergence to Griffith theory when $l_0/L \rightarrow 0$, at least for quasi-static propagation
Many constitutive modeling choices are possible, we follow [Li et al., 2016]

- elastic strain energy density:

\[
\psi(\varepsilon, d) = (1 - d)^2 \left( \frac{\kappa}{2} \langle \text{tr} \varepsilon \rangle_+ + \mu \varepsilon^d : \varepsilon^d \right) + \frac{\kappa}{2} \langle \text{tr} \varepsilon \rangle_-
\]

Remark: existence of an elastic phase for this model

Numerical resolution using a staggered approach:

- minimization of total energy with respect to \( u \): explicit dynamics
- minimization with respect to \( d \): quadratic function with bound constraints (\( d_n \leq d_{n+1} \leq 1 \)) to enforce damage irreversibility
Phase-field approach

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- non-local fracture energy:

\[ w_{\text{frac}}(d, \nabla d) = \frac{3G_c}{8l_0} \left( d + l_0^2 \| \nabla d \|^2 \right) \]

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Phase-field approach

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Prestrained plate geometry

Prestrained PMMA plate, fixed boundaries [Zhou, 1996]

\[ E = 3.09 \text{ GPa}, \, \nu = 0.35, \, \rho = 1180 \text{ kg/m}^3, \, G_c = 300 \text{ J/m}^2, \, c_R = 906 \text{ m/s} \]
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\[ \Gamma = \frac{2E(\Delta U)^2}{h} \Rightarrow \text{crack should accelerate to } c_R \]

- transition from straight propagation to branched patterns
- apparent toughness increases with loading/crack velocity
Prestrained plate geometry

(a) $\Delta U = 0.035 \text{ mm at } t = 40 \mu s$

(b) $\Delta U = 0.038 \text{ mm at } t = 40 \mu s$

(c) $\Delta U = 0.040 \text{ mm at } t = 40 \mu s$

(d) $\Delta U = 0.045 \text{ mm at } t = 20 \mu s$

However, branching occurs at smaller load levels than in experiments, the crack is too fast $\Rightarrow$ same problem with CZM, non-local integral approach.
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Crack velocities

no evident decrease of crack speed after branching limiting velocity around $0.68c_R$
Damage zone thickening

▶ progressive thickening of the damaged band before branching
▶ similar observation using peridynamics
▶ branching viewed as a progressive transition from a widening crack to two crack tips screening each other
▶ branching angle seems to depend on geometry
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Apparent fracture energy

Damage dissipation rate $\Gamma = \frac{dE_{\text{frac}}}{da}$ interpreted as the apparent fracture energy
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![Graph showing damage dissipation rate $\Gamma/G_c$ vs. crack tip horizontal position (mm)].

- Red dots: $\Delta U = 0.035$ mm (total)
- Blue squares: $\Delta U = 0.045$ mm (total)

Branching is indicated at approximately $\Gamma \approx 2G_c$. 
Damage dissipation rate $\Gamma = \frac{dE_{frac}}{da}$ interpreted as the apparent fracture energy
Apparent fracture energy

Damage dissipation rate $\Gamma = \frac{dE_{\text{frac}}}{da}$ interpreted as the apparent fracture energy

suggests a critical value of $\Gamma \approx 2G_c$ associated to branching
Velocity-toughening mechanism during propagation and before macroscopic branching
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during propagation and before macroscopic branching

existence of a well-defined $\Gamma(\nu)$ relationship associated to a velocity-toughening mechanism
the $\Gamma(v)$ relationship seems material-independent but geometry-dependent
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Propagation in constrained path

experiments report that crack can reach $c_R$ if constrained in a weak plane [Washabaugh and Knauss, 1994]

<table>
<thead>
<tr>
<th>Loading $\Delta U$ (mm)</th>
<th>Stored energy (N/m)</th>
<th>Crack velocity ($c_R$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>618</td>
<td>0.81</td>
</tr>
<tr>
<td>0.05</td>
<td>966</td>
<td>0.87</td>
</tr>
<tr>
<td>0.10</td>
<td>3,863</td>
<td>0.94</td>
</tr>
<tr>
<td>0.15</td>
<td>8,691</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Propagation in constrained path

idem for a series of holes on crack path

\[ D = 0.4 \text{ mm and } S = 0.9 \text{ mm} \]
Propagation in constrained path

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Propagation in constrained path

idem for a series of holes on crack path

- velocity of $0.9c_R$ for $\Delta U = 0.05$ mm
- shares qualitative similarities the nucleation and growth of microcracks interacting with defects
- the apparent fracture energy is much higher than the average toughness $G_{c,\text{weak}} = (1 - D/S)G_c \approx 0.56G_c$
Interaction with distant heterogeneities

crack passing near a hole

1mm from notch
Interaction with distant heterogeneities

crack passing near a hole

- Velocity of the crack tip is larger in the second case.
- Crack is more attracted: different near-tip stress fields?
- Faster crack looks for other ways of dissipating energy?
Interaction with distant heterogeneities

crack passing near a hole

- velocity of the crack tip is larger in the second case
- crack is more attracted: different near-tip stress fields? faster crack looks for other ways of dissipating energy?
Interaction with out-of-plane heterogeneities

Configuration with an array of holes located away from the middle plane

\[ B = 0.5 \text{ mm offset, } \Delta U = 0.04 \text{ mm} \]
\[ B = 0.5 \text{ mm offset, } \Delta U = 0.05 \text{ mm} \]
\[ B = 0.6 \text{ mm offset, } \Delta U = 0.04 \text{ mm} \]
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Conclusions and perspectives

**Conclusion**: some physical aspects of dynamic fracture can be reproduced with the phase-field approach

- propagation characterized by a damage band widening
- widening associated to an increase of the apparent fracture energy
- existence of a well-defined $\Gamma(\nu)$ relationship
- macroscopic branching observed when $\Gamma \geq 2G_c$
- existence of a limiting velocity around $0.7c_R$
- $c_R$ can be reached in constrained geometries
- strong influence of heterogeneities on branching process

Open questions

- rate-dependent model for PMMA?
- energy-based branching criterion?
- better understanding of 3D effects and role of defects
Conclusions and perspectives

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