Laplace Beamshapes for Phased-Array Imaging

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***Abstract*—The imaging capabilities of phased-array systems are governed by the properties of their array beamshape, directly linked to the instrument impulse response. To ensure good spatial resolution, beamshapes are designed with a very narrow main lobe, at the cost of a complex sidelobe structure, potentially leading to severe image artifacts. We propose the use of a new beamshape, called the Laplace beamshape, built with the Flexibeam framework. This beamshape trades spatial resolution for smoother sidelobes, resulting in an artifact-free image that is much easier to process. This tradeoff can be optimally assessed through a single parameter of the beamshape, allowing the analyst to perform a multi-scale analysis.**

Extended Abstract

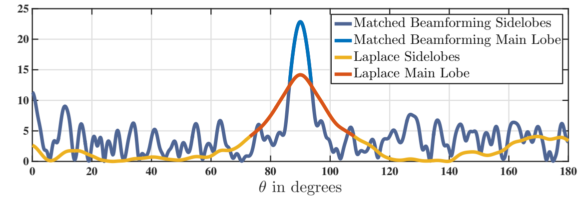
Beamforming combines networks of antennas coherently so as to achieve specific radiation patterns with desirable properties. For simplicity, we restrict our attention to 2D-beamforming in all that follows. Assuming hence an array of L antennas with positions $\boldsymbol{p}_1, \dots, \boldsymbol{p}_L \in \mathbb{R}^2$, the *beamformed signal* $y(t)$ is obtained by combining linearly the antenna signals $x_i(t)$:

$$y(t) = \boldsymbol{w}^H \boldsymbol{x}(t) = \sum_{i=1}^{L} w_i^* x_i(t), \quad \forall t \in \mathbb{R},$$

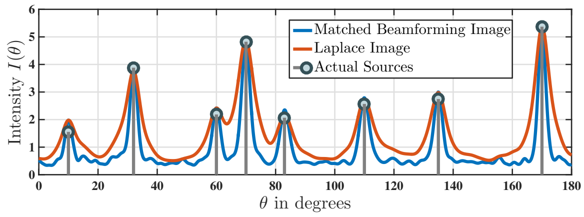
where $\boldsymbol{x}(t) := [x_1(t), \cdots, x_L(t)]$ and $\boldsymbol{w} := [w_1, \cdots, w_L] \in \mathbb{C}^L$. By properly choosing the beamforming weights w_i, it is possible to steer the antenna array towards specific directions in the sky. This is typically done via the popular *matched beamforming* method, which sets the weights to $w_i(\theta) = \left(e^{-j\frac{2\pi}{\lambda}\|\boldsymbol{p}_i\|\cos(\theta)}\right)/\sqrt{L}$, for a signal of wavelength $\lambda \in \mathbb{R}$ and a direction $\theta \in [0, 2\pi]$. By computing the variance of the beamformed signal $y(t)$, one can then obtain an estimate of the sky intensity at this location

$$I(\theta) = \mathbb{E}[y(t)y(t)^*] = \boldsymbol{w}^H(\theta)\Sigma\boldsymbol{w}(\theta), \tag{1}$$

where $\Sigma := \mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}^H(t)]$. Ranging across directions $\theta \in [0, 2\pi]$ produces an estimate $I(\theta)$ of the sky intensity field. This procedure is known as *imaging by beamforming*, or *B-scan imaging*, and is commonly used in radio-astronomy, sonar/radar and ultrasound imaging. The imaging capabilities of the instrument can then be assessed through its *point spread function*, response of the tool to an idealised point-source. We can show that this function is directly proportional to the squared magnitude of the array far-field radiation pattern, also called *beamshape*. Properties of this beamshape hence completely determine the quality of the image in Eq. (1). For optimal performance, two competing features must be optimised: the main lobe width, which controls the achievable angular resolution, and the sidelobes structure, which can translate into severe artifacts within the image. As described in [1], matched beamforming is attempting to achieve a beamshape as close as possible to a Dirac $\delta(\theta - \theta_0)$. As such, it typically performs quite well in terms of angular resolution, with a very narrow main lobe around the direction of focus θ_0, but often demonstrates strong sidelobes (see Fig. 1a). These prominent sidelobes are a consequence of the ill-defined nature of the Dirac function, which makes it a very difficult object to approximate. To avoid such complications, we hence propose to target a much better



(a) Matched beamforming versus Laplace beamshape (squared magnitude in logarithmic scale).



(b) B-scan Images.

Fig. 1: Imaging with Laplace and matched beamforming.

behaved function, namely a circular Laplace function:

$$\mathcal{L}(\theta) = \exp\left[-\frac{\sqrt{(\cos\theta - \cos\theta_0)^2 + (\sin\theta - \sin\theta_0)^2}}{\Theta}\right], \tag{2}$$

where $\Theta > 0$ controls the width of the main lobe. Being continuous and quickly decaying from the direction of focus θ_0, this function has a well-behaved Fourier spectrum, and can hence be easily approximated with a finite number of complex plane-waves. Moreover, its sharp central peak permits the very accurate estimation of source locations. To construct the *Laplace beamshape*, we used the very general Flexibeam framework introduced in [1]. Beamforming weights were obtained by sampling the so-called *beamforming function*, which, for the circular Laplace function is given by:

$$w_i = \omega(\boldsymbol{p}_i) = \frac{2\pi\Theta^2}{\left(1 + 4\pi^2\Theta^2\|\boldsymbol{p}_i\|^2\lambda^{-2}\right)^{3/2}} e^{-j\frac{2\pi}{\lambda}\|\boldsymbol{p}_i\|\cos(\theta_0)}.$$

As expected, the resulting beamshape exhibits much smoother sidelobes (see Fig. 1a), with most of its energy contained in the main lobe ($\sim$86% against 74% for matched beamforming). As a result, the image obtained with the Laplace beamshape appears much smoother, facilitating enormously the recovery of the actual sources within the field. Observe that this smoother behaviour was obtained at the cost of lower angular resolution. This fundamental tradeoff can be formally assessed by varying the parameter Θ, leading to a multi-scale analysis of the image. The Laplace beamforming strategy above described readily extends to 3D beamforming, and this will be the subject of a future publication.

References

[1] P. Hurley and M. Simeoni, “Flexibeam: analytic spatial filtering by beamforming,” in *International Conference on Acoustics, Speech and Signal Processing (ICASSP), IEEE*, March 2016.