Laplace Beamshapes for Phased-Array Imaging

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Abstract — The imaging capabilities of phased-array systems are governed by the properties of their array beamshape. We propose the use of a new beamshape, called the **Laplace beamshape**, built with the **Flexibeam** framework. This beamshape trades spatial resolution for smoother sidelobes, resulting in an artifact-free image that is much easier to process.

1. Beamforming

Beamforming combines networks of antennas *coherently* to achieved desirable radiation patterns:

$$y(t) = \boldsymbol{w}^H \boldsymbol{x}(t) = \sum_{i=1}^{L} w_i^* x_i(t).$$

Steering the antenna array towards specific directions is done with **matched beamforming**:

$$w_i(\theta) = \left(e^{-j\frac{2\pi}{\lambda}\|\boldsymbol{p}_i\|\cos(\theta)}\right)/\sqrt{L}.$$

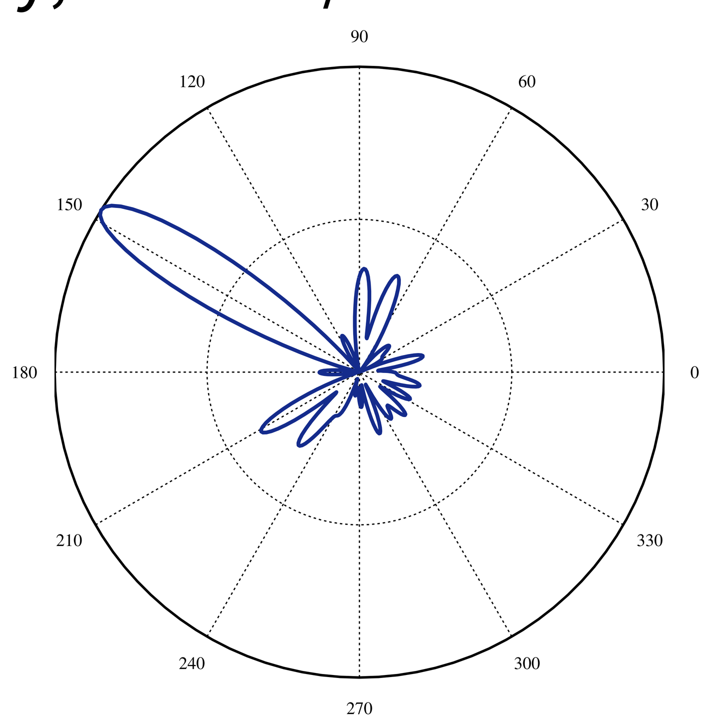
Variance of the signal for various DOA can be estimated as:

$$I(\theta) = \mathbb{E}[y(t)y(t)^*] = \boldsymbol{w}^H(\theta)\Sigma\boldsymbol{w}(\theta).$$

This procedure is called *imaging by beamforming*. It is used in radio-astronomy, sonar/radar and ultrasound.

Quality of the image can be assessed through the **beamshape** of the array:

$$b(\theta) = \sum_{i=1}^{L} w_i e^{j2\pi p_i \cos(\theta)}.$$



Typical beamshape.

Sidelobes can induce severe artifacts into the image. Two competing metrics: *resolution* and *contrast*.

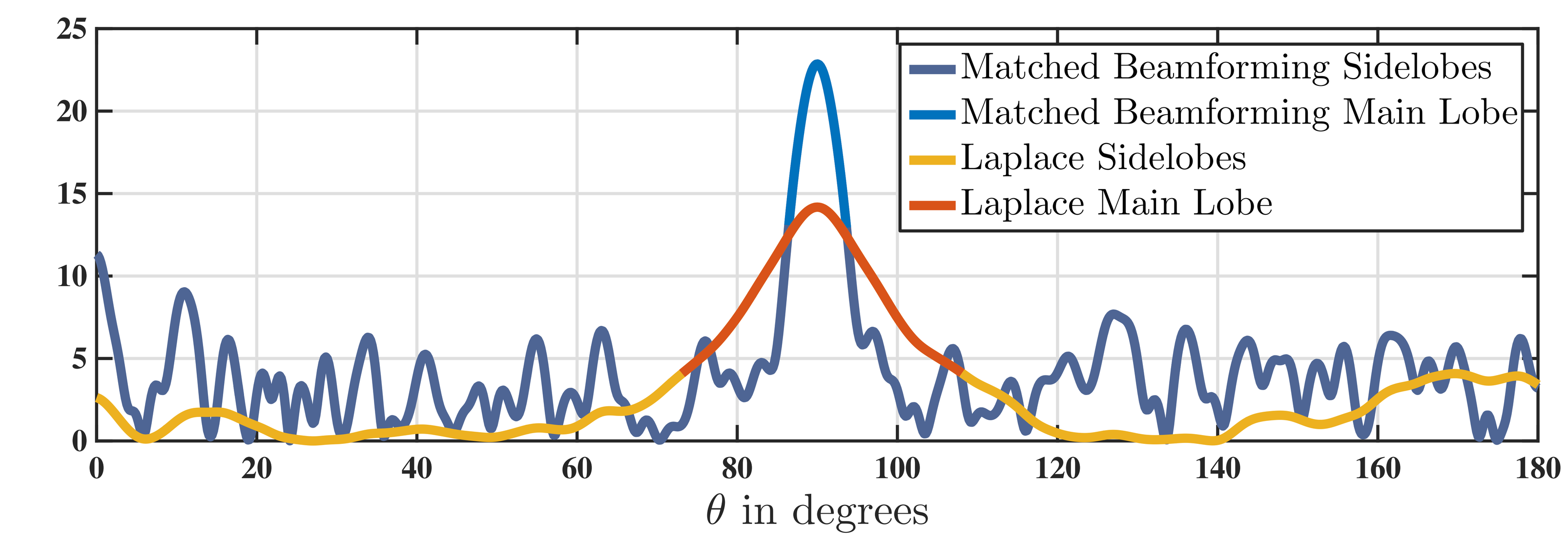
2. Laplace Beamshape

Matched beamforming is attempting to achieve a *Dirac* beamshape. **Too hard!**
We propose to target the **circular Laplace function**, much *better behaved*:

$$\mathcal{L}(\theta) = \exp\left[-\frac{\sqrt{(\cos\theta - \cos\theta_0)^2 + (\sin\theta - \sin\theta_0)^2}}{\Theta}\right].$$

Beamforming weights can be obtained using the **Flexibeam** framework [1]:

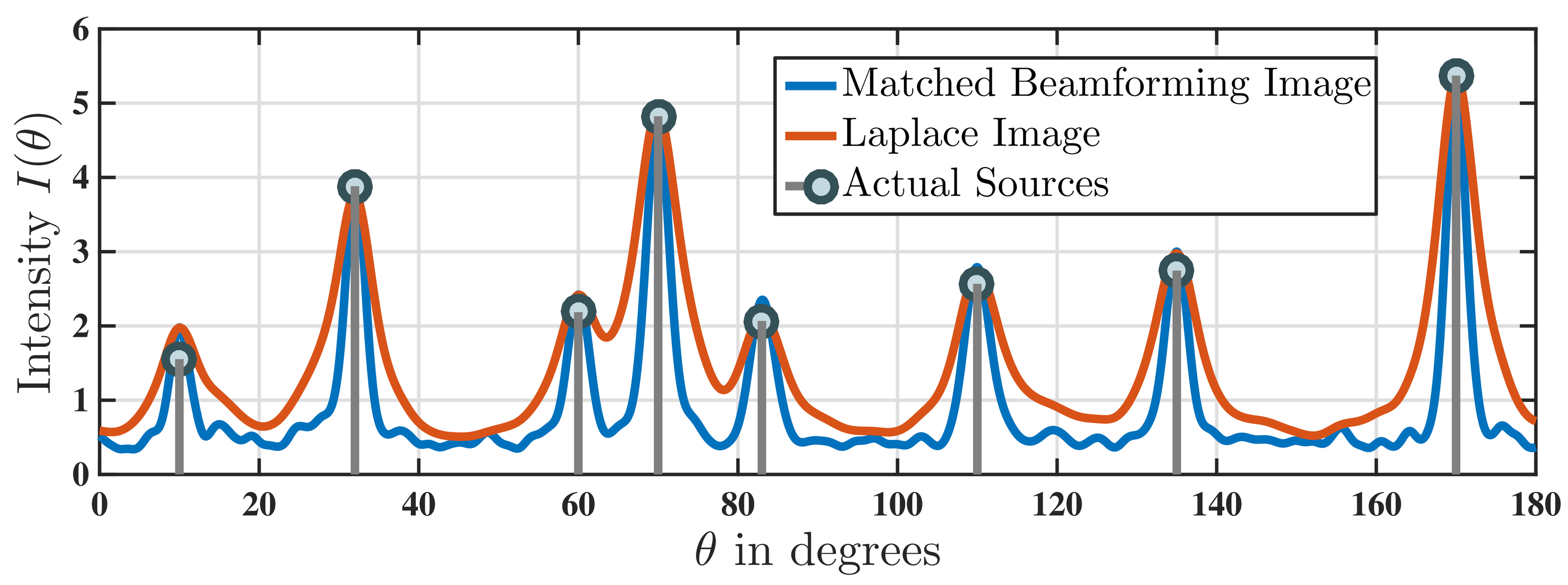
$$w_i = \omega(\boldsymbol{p}_i) = \frac{2\pi\Theta^2}{\left(1 + 4\pi^2\Theta^2\|\boldsymbol{p}_i\|^2\lambda^{-2}\right)^{3/2}} e^{-j\frac{2\pi}{\lambda}\|\boldsymbol{p}_i\|\cos(\theta_0)}.$$



Matched beamforming vs. Laplace Beamshape

3. Imaging with Laplace Beamshape

The Laplace beamshape being much **smoother**, the resulting image appears *artifact-free*. There is a **tradeoff** between resolution and smoothness.



Imaging with Laplace and matched beamforming

4. References

1. P. Hurley and M. Simeoni, "Flexibeam: analytic spatial filtering by beamforming," in *International Conference on Acoustics, Speech and Signal Processing (ICASSP), IEEE*, March 2016.