Multi-class speed-density relationship for pedestrian traffic

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Outline

1. Introduction

2. Methodology

3. Case study
   - Model specification
   - Model estimation and performance analysis

4. Conclusion and future work
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4. Conclusion and future work
Fundamental relationships

- Play an important role in the field: design and planning; model input or calibration criterion
- Modeling assumption: the traffic system is at equilibrium - homogenous and stationary
Speed-density relationships for pedestrian traffic

**Deterministic approach**

- **Empirically derived models** [Older, 1968; Tregenza, 1976; Weidmann, 1993; Rastogi et al., 2013]
- **Simulation-based models** [Blue and Adler, 1998]
- **Theory-based models** [Flötteröd and Lämmel, 2015]

**Empirical observations**

- Scatter: violation of the equilibrium assumptions

**Probabilistic approach**

- **Data-driven PedProb-vk** [Nikolić et al., 2016]
- Superior compared to deterministic approaches from the literature
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Behavioral approach

Assumptions

- Pedestrian population is heterogeneous (e.g. trip purpose, age, gender, etc.)
- Heterogeneity leads to the existence of multiple pedestrian classes
- Classes are characterized by different types of behavior
- Latent class modeling approach to capture unobserved heterogeneity
Multi-class speed-density relationship (MC-vk)

Model structure

\[
P(v_i|k_i) = \sum_{c=1}^{C} P(v_i|k_i, c)P(c|X_i)
\]

- \(P(v_i|k_i, c)\): class-specific model
- \(P(c|X_i)\): class membership model

- \(i\): pedestrian identifier, \(i = 1, ..., N\)
- \(v_i\): speed of pedestrian \(i\)
- \(k_i\): density for pedestrian \(i\)
- \(c\): class identifier, \(C\) - number of classes
- \(X_i\): characteristics associated to pedestrian \(i\)
Class-specific speed-density relationship

Social Force Model

\[ \ddot{a}_i = \frac{\vec{v}_{i}^f - \vec{v}_i}{\tau_i} - C_i \sum_j \exp\left(-\frac{R_{ij}}{B_i}\right) \vec{n}_{ij}(\lambda_i + (1 - \lambda_i) \frac{1 + \cos(\phi_{ij})}{2}) \]

[Helbing and Molnar, 1995]
Class-specific speed-density relationship

Isotropy ($\lambda_i = 1$)

$$a_i = \frac{v_i^f - v_i}{\tau_i} - C_i \sum_j \exp\left(- \frac{R_{ij}}{B_i}\right) = \frac{v_i^f - v_i}{\tau_i} - C_i k_i$$

Stationarity ($a_i = 0$)

$$v_i = v_i^f - \gamma_i k_i$$

Homogeneity (all pedestrians have the same movement parameters)

$$v_i = v = v_f - \gamma k_i$$
Class membership model

- It cannot be deterministically identified to which class a pedestrian belongs.
- Probability that a pedestrian $i$, associated with characteristics $X_i$ (e.g. trip purpose, age, gender, etc.), belong to a latent class $c$: for each pedestrian there is a utility associated to each class $c$.

**Specification of utilities**

$$U_{i}^{c} = \underbrace{ASC_{c}^{c} + \beta_{c}^{c} X_{i} + \xi_{i}^{c}}_{V_{i}^{c}}$$

$V_{i}^{c}$: deterministic part of utilities

$\xi_{i}^{c}$: error term
Multi-class speed-density relationship (MC-vk)

Class-specific model: \( P(v_i|k_i, c) \)

\[ v_i^c = v_f^c - \gamma^c k_i + \epsilon_i^c \]

\( P(v_i|k_i, c) \) is determined by \( \epsilon_i^c \)

Class membership model: \( P(c|X_i) \)

\[ U_i^c = ASC^c + \beta^c X_i + \xi_i^c \]

\( P(c|X_i) \) is determined by \( \xi_i^c \)

Likelihood of the sample

\[ \mathcal{L} = \prod_{i=1}^{N} P(v_i|k_i) = \prod_{i=1}^{N} \sum_{c=1}^{C} P(v_i|k_i, c)P(c|X_i) \]
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Lausanne railway station
Pedestrian underpass West

1: South entrance, 3: Coop Pronto Supermarket
2 - 4: Stairs (resp. ramp) to platform 9
5 - 6: Stairs (resp. ramp) to platform 7 and 8
7 - 8: Stairs (resp. ramp) to platform 5 and 6
9 - 10: Stairs (resp. ramp) to platform 3 and 4
11: Stairs to platform 1 and out of the station
12: Access ramp
13: Stairs to or out of the train station and to buses
14: Pathway leading to buses and metro (M2)
Data set

Pedestrian underpass

- A large-scale network of smart sensors: a sparsity driven tracking framework [Alahi et al., 2014]
- Dataset: 25,603 trajectories, collected between 07:00 and 08:00 on February 12, 13, 14, 15 and 18, 2013
- The average length of the trajectories: 78 meters
- The duration of a pedestrians’ stay: from 15 seconds to 2.2 minutes
Speed-density relationship
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Specification issues

Panel data
- Data collected over multiple time periods for the same sample of individuals

Serial correlation
- The observations across time for a single pedestrian are likely to be correlated, due to the unobserved factors related to a pedestrian that exist over time
- $\epsilon_{i(t-1)}^C$ cannot be assumed independent from $\epsilon_{it}^C$
- If ignored - consistent but not efficient estimators
Multi-class speed-density relationship (MC-vk)

Class-specific model: \( P(v_i|k_i, c) \)

\[
v^c_{it} = v^c_f - \gamma^c k_{it} + \alpha^c_i + \epsilon'^c_{it}
\]

\( P(v_i|k_i, c) \) is determined by \( \epsilon'^c_{it} \), \( \alpha^c_i \) is an agent effect

Class membership model: \( P(c|X_i) \)

\[
U^c_i = ASC^c + \beta^c X_i + \xi^c_i
\]

\( P(c|X_i) \) is determined by \( \xi^c_i \)

Likelihood of the sample

\[
\mathcal{L} = \prod_{i=1}^{N} \sum_{c=1}^{C} \left\{ \frac{1}{R} \sum_{r} \exp \left( \sum_{t=1}^{T} \log P(v_i|k_i, c, \alpha^c_r) \right) \right\} P(c|X_i)
\]
Assumptions

Number of classes

1. Pedestrians sensitive to congestion
2. Pedestrians non-sensitive to congestion

Class specific model

- The same functional form of v-k for each class
- $\epsilon_{it}^{c} \sim \text{Rayleigh distribution}$
- $\alpha_{i}^{c} \sim \text{Rayleigh distribution}$

Class membership model

- Logit model
- Explanatory variables: type of pedestrian, time to departure, OD distance, peak periods
Pedestrian types

Classification based on origins and destinations

1: Arriving passenger - pedestrians originating from a platform and exiting the station

2: Departing passenger - pedestrians walking to a platform to embark on their trains

3: Transferring passenger - pedestrians whose origin and destination are different platforms

4: Non-passenger - pedestrians whose origin and destination are different from a platform (e.g. pedestrians that go shopping in the station)
Pedestrian types

Number of pedestrians per pedestrian type

![Bar chart showing the number of pedestrians per type in different months: arriving, departing, transferring, and non-passenger. The chart compares data for Feb 12, Feb 13, Feb 14, Feb 15, and Feb 18.]
Pedestrian types

Speed distribution per pedestrian type
Train timetable

Time to departure

![Train illustration](image)

![Bar chart](chart)
OD distance
Peak periods
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Estimation results

### Class membership model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ASC^{NS} )</td>
<td>-0.258</td>
<td>5.18e(^{-6})</td>
</tr>
<tr>
<td>( \beta^{NS} ) Arriving pass.</td>
<td>-0.641</td>
<td>1.03e(^{-5})</td>
</tr>
<tr>
<td>( \beta^{NS} ) Departing pass.</td>
<td>58.5</td>
<td>2.11e(^{-5})</td>
</tr>
<tr>
<td>( \beta^{NS} ) Transferring pass.</td>
<td>63.5</td>
<td>1.73e(^{-5})</td>
</tr>
<tr>
<td>( \beta^{S} ) Time to departure</td>
<td>0.236</td>
<td>1.57e(^{-5})</td>
</tr>
<tr>
<td>( \beta^{S} ) Peak period</td>
<td>0.125</td>
<td>1.54e(^{-5})</td>
</tr>
<tr>
<td>( \beta^{S} ) OD distance</td>
<td>0.0328</td>
<td>1.93e(^{-5})</td>
</tr>
</tbody>
</table>

### Class specific model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_f^{NS} )</td>
<td>1.13</td>
<td>1.32e(^{-5})</td>
</tr>
<tr>
<td>( \gamma^{NS} )</td>
<td>0.0812</td>
<td>1.73e(^{-5})</td>
</tr>
<tr>
<td>( v_f^{S} )</td>
<td>0.949</td>
<td>9.37e(^{-5})</td>
</tr>
<tr>
<td>( \gamma^{S} )</td>
<td>0.178</td>
<td>1.28e(^{-5})</td>
</tr>
<tr>
<td>( \eta^{NS} )</td>
<td>0.0104</td>
<td>2.67e(^{-5})</td>
</tr>
<tr>
<td>( \eta^{S} )</td>
<td>0.102</td>
<td>1.66e(^{-5})</td>
</tr>
</tbody>
</table>

S - Pedestrians sensitive to congestion
NS - Pedestrians non-sensitive to congestion
How many classes?

Bayesian information criterion - \textit{BIC}

<table>
<thead>
<tr>
<th>Model</th>
<th>1 class</th>
<th>2 classes</th>
<th>3 classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>log $L$</td>
<td>-527491.289</td>
<td>-524094.577</td>
<td>-523726.125</td>
</tr>
<tr>
<td>#observations</td>
<td>747385</td>
<td>747385</td>
<td>747385</td>
</tr>
<tr>
<td>#parameters</td>
<td>3</td>
<td>13</td>
<td>23</td>
</tr>
<tr>
<td>\textit{BIC}</td>
<td>1055023.152</td>
<td>1048364.971</td>
<td>1047763.309</td>
</tr>
</tbody>
</table>
Average time to departure

- Average time to departure [s]
  - Non-sensitive to congestion
  - Sensitive to congestion
- Data points for:
  - All
  - Departing
  - Transferring
Average OD distance
Scenario: time table change (decrease of the time to departure)
Model comparison

Average behavior

\[ \bar{v}_{MC-vk} = \sum_{c=1}^{C} \left\{ \frac{1}{N} \sum_{i=1}^{N} P(c \mid X_i; \beta^c) v^c(k; \theta^c) \right\} \]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Model} & \text{Weidmann} & \text{Tregenza} & \text{Rastogi} & \text{Linear} & \text{PedProb-vk} & \text{MC-vk} \\
\hline
\text{MSE} & 4.81e^{-03} & 3.63e^{-03} & 3.95e^{-03} & 4.99e^{-03} & 3.17e^{-03} & 2.12e^{-03} \\
\text{R}^2 & 2.64e^{-01} & 4.45e^{-01} & 3.96e^{-01} & 2.37e^{-01} & 5.16e^{-01} & 6.76e^{-01} \\
\hline
\end{array}
\]
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Conclusion and future work

Conclusion

• MC-vk: latent class modeling approach to capture heterogeneity in pedestrian population
• Satisfying behavioral interpretation
• Good performance at the aggregate level

Future work

• Additional factors: walking in groups, attractiveness of origins/destinations
• Additional scenarios: train reallocation
• Accounting for dynamics
Thank you

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