Waste collection inventory routing with non-stationary stochastic demands

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University of Applied Sciences Western Switzerland (HES-SO)

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Outline

1. Introduction
2. Related Literature
3. Formulation
4. Numerical Experiments
5. Conclusion
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Sensorized containers for recyclables periodically send waste level data to a central database.
Introduction

Setup

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- Level data is used for container selection and route planning.
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- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.
Sensorized containers for recyclables periodically send waste level data to a central database.

Level data is used for container selection and route planning.

Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm.

Efficient waste collection thus depends on the ability to:
- forecast container levels,
- select the containers to collect each day,
- and route the vehicles in an (near-)optimal way.
The setup falls within the framework of the Stochastic Inventory Routing Problem (SIRP) with:

- stochastic demands,
- Order-Up-to level (OU) policy,
- no allowed expected overflows,
- single-day backorder limit (i.e. if a container overflows on a given day, it must be collected on that day).
The setup falls within the framework of the Stochastic Inventory Routing Problem (SIRP) with:

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The routing component includes:

- intermediate facility visits (recycling plants),
- heterogeneous capacitated vehicles,
- site dependencies,
- vehicle-to-period availabilities,
- time windows,
- maximum tour duration.
Routing Component

Figure 1: Example of a Collection Tour

c = container

depot

dump

dump
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SIRP Research Directions

- Early research on optimal replenishment policies in a stochastic setting:

- Robust optimization:
  - Solyalı et al. (2012).

- Chance constraints:

- Scenario based:
Motivation and Contribution

- We use an approach with dynamic probabilistic information on container overflows and route failures:
  - scenario-based approaches are computationally expensive,
  - we can frequently revisit the states of random variables unlike in robust optimization,
  - we have a monetary value associated with the realization of undesirable events.
Motivation and Contribution

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- Rich routing features rarely considered in the IRP literature.
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- Rich routing features rarely considered in the IRP literature.

- Probabilistic approach superior wrt alternative practical policies.
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Selected Notation

Sets

- **$o$**: origin
- **$d$**: destination
- **$\mathcal{D}$**: set of dumps
- **$\mathcal{P}$**: set of containers
- **$\mathcal{N}$**: $\{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P}$
- **$\mathcal{T}$**: $\{0, \ldots, u\}$
- **$\mathcal{T}^+$**: $\{1, \ldots, u+1\}$

Parameters

- **$\pi_{ij}$**: travel distance of arc $(i, j)$
- **$\rho_{it}$**: demand of container $i$ on day $t$ (random variable)
- **$\sigma_{it}$**: $= 1$ if container $i$ is in a state of full and overflowing on day $t$, 0 otherwise
- **$\varsigma$**: forecasting model error (st. dev. of the fit's residuals)
- **$\omega_i$**: capacity of container $i$
- **$\chi$**: container overflow cost (monetary)
- **$\zeta$**: emergency collection cost (monetary)
Selected Notation

Sets

- $o$: origin
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- $\mathcal{D}$: set of dumps
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- $\mathcal{N} = \{o\} \cup \{d\} \cup \mathcal{D} \cup \mathcal{P}$
- $\mathcal{T} = \{0, \ldots, u\}$
- $\mathcal{T}^+ = \{1, \ldots, u + 1\}$

Parameters

- $\varphi_k$: daily deployment cost of vehicle $k$ (monetary)
- $\beta_k$: unit-distance running cost of vehicle $k$ (monetary)
- $\theta_k$: unit-time running cost of vehicle $k$ (monetary)
- $\Omega_k$: capacity of vehicle $k$
Selected Notation

Decision variables: binary

\[ x_{ijkt} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses arc } (i,j) \text{ on day } t \\ 0 & \text{otherwise} \end{cases} \]

\[ y_{ikt} = \begin{cases} 1 & \text{if vehicle } k \text{ visits point } i \text{ on day } t \\ 0 & \text{otherwise} \end{cases} \]

\[ z_{kt} = \begin{cases} 1 & \text{if vehicle } k \text{ is used on day } t \\ 0 & \text{otherwise} \end{cases} \]

Decision variables: continuous

\[ Q_{ikt} \quad \text{expected cumulative quantity on vehicle } k \text{ at point } i \text{ on day } t \]

\[ I_{it} \quad \text{expected inventory in container } i \text{ at the start of day } t \]

\[ S_{ikt} \quad \text{start-of-service time of vehicle } k \text{ at point } i \text{ on day } t \]
Demand is the amount deposited in a container on each day, and is random and non-stationary.
Forecasting Model

- Demand is the amount deposited in a container on each day, and is random and non-stationary.

- We can use any forecasting model that gives us:
  - the expected container demands $\mathbb{E}(\rho_{it})$ on each day,
  - a consistent estimate of the forecasting error $\varsigma$. 

The forecasting error is the standard deviation of the residuals based on a historical fit. Its distribution can be approximated as a normal, and is used to calculate probabilities of container overflows and route failures.
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Objective Function

Routing cost + Expected overflow and emergency collection cost + Expected route failure cost

Lower routing cost is counterbalanced by more overflows and route failures, and vice versa.
Our goal is to minimize the expected monetary value of all components.
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Objective Function: Main Concepts

- Two container states:
  - $\sigma_{it} = 0$: not full,
  - $\sigma_{it} = 1$: full and overflowing.
Objective Function: Main Concepts

- **Two container states:**
  - $\sigma_{it} = 0$: not full,
  - $\sigma_{it} = 1$: full and overflowing.

- **Two types of container collection:**
  - regular collection of container $i$ on day $t$: $\exists k \in K : y_{ikt} = 1$,
  - emergency collection of container $i$ on day $t$: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in K$. 

- Overflow cost $\chi$: paid in state $\sigma_{it} = 1$,
- Emergency collection cost $\zeta$: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0, \forall k \in K$. 

Objective Function: Main Concepts

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- Two types of container collection:
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  - emergency collection of container $i$ on day $t$: $\sigma_{it} = 1$ and $y_{ikt} = 0, \forall k \in K$.

- Related costs:
  - overflow cost $\chi$: paid in state $\sigma_{it} = 1$,
  - emergency collection cost $\zeta$: paid in state $\sigma_{it} = 1$ when $y_{ikt} = 0, \forall k \in K$. 
Figure 2: Container State Probability Tree

\[
\begin{align*}
\sigma_{10} = 0 & \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 + \rho i_2 < \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 < \omega_i \quad \sigma_{i1} = 0 \\
& \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 + \rho i_2 \geq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 < \omega_i \quad \sigma_{i1} = 1 \\
\sigma_{i1} = 0 & \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 \leq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i2} = 0 \\
& \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 + \rho i_2 < \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i2} = 1 \\
\sigma_{i1} = 1 & \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 \leq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i2} = 0 \\
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\sigma_{i2} = 0 & \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 \leq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i3} = 0 \\
& \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 + \rho i_2 \geq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i3} = 1 \\
\sigma_{i2} = 1 & \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 \leq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i3} = 0 \\
& \quad \Rightarrow \quad P(l_0 + \rho i_0 + \rho i_1 + \rho i_2 \geq \omega_i) \quad | l_0 + \rho i_0 + \rho i_1 \leq \omega_i \quad \sigma_{i3} = 1 \\
\sigma_{i3} = 0 & \quad \Rightarrow \quad P(0 + \rho i_2 < \omega_i) \quad \sigma_{i4} = 1 \\
\sigma_{i3} = 1 & \quad \Rightarrow \quad P(0 + \rho i_2 > \omega_i) \quad \sigma_{i4} = 1 \\
\end{align*}
\]
Objective Function: Formulation

- **Routing Cost (RC):**

\[
\sum_{t \in T} \sum_{k \in K} \left( \varphi_k z_{kt} + \beta_k \sum_{i \in N} \sum_{j \in N} \pi_{ij} x_{ijkt} + \theta_k (S_{dkt} - S_{okt}) \right)
\] (1)
Objective Function: Formulation

- **Routing Cost (RC):**

  \[
  \sum_{t \in T} \sum_{k \in K} \left( \varphi_k z_{kt} + \beta_k \sum_{i \in N} \sum_{j \in N} \pi_{ij} x_{ijkt} + \theta_k \left( S_{dkt} - S_{okt} \right) \right) \quad (1)
  \]

- **Expected Overflow and Emergency Collection Cost (EOECC):**

  \[
  \sum_{t \in T \cup T^+} \sum_{i \in P} \left( \mathbb{P} (\sigma_{it} = 1 \mid \max (0, g < t : \exists k \in \mathcal{K}: y_{ikg} = 1)) \left( \chi + \zeta - \zeta \sum_{k \in \mathcal{K}} y_{ikt} \right) \right) \quad (2)
  \]
Objective Function: Formulation

- Expected Route Failure Cost (ERFC):

\[
\sum_{t \in T \setminus 0} \sum_{k \in K} \sum_{S \in \mathcal{S}_k} \left( \psi C_S \Pr \left( \sum_{s \in S} (I_{st} > \Omega_k \mid \max(0, g < t: y_{skg} = 1)) \right) \right),
\]

where
- \( \mathcal{S}_k \) is the set of depot-to-dump or dump-to-dump trips for vehicle \( k \) on day \( t \),
- \( S \) is the set of containers in a particular trip,
- \( C_S \) is the average routing cost of going from \( S \) to the nearest dump and back to \( S \),
- \( \psi \) is the Route Failure Cost Multiplier (RFCM), controlling the degree of conservatism wrt this component.
Objective Function: Formulation

- **Expected Route Failure Cost (ERFC):**

\[
\sum_{t \in T \setminus 0} \sum_{k \in K} \sum_{S \in \mathcal{S}_{kt}} \left( \psi C_S \mathbb{P} \left( \sum_{s \in S} (l_{st} > \Omega_k \mid \max(0, g < t: y_{skg} = 1) \right) \right), \tag{3}
\]

where
- \( \mathcal{S}_{kt} \) is the set of depot-to-dump or dump-to-dump trips for vehicle \( k \) on day \( t \),
- \( S \) is the set of containers in a particular trip,
- \( C_S \) is the average routing cost of going from \( S \) to the nearest dump and back to \( S \),
- \( \psi \) is the Route Failure Cost Multiplier (RFCM), controlling the degree of conservatism wrt this component.

- The objective function becomes:

\[
\min z = RC + EOECC + ERFC \tag{4}
\]

and is non-linear, thus resulting in an MINLP.
Constraints

- Basic routing constraints.
- Inventory constraints:
  - container inventory tracking,
  - no expected overflows for $T^+$,
  - single-day backorder limit,
  - OU policy.
- Capacity tracking and renewal constraints.
- Time tracking and time window constraints.
- Domain definition.
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Setup

- We solve the problem using an Adaptive Large Neighborhood Search (ALNS).
Numerical Experiments

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- We solve the problem using an Adaptive Large Neighborhood Search (ALNS).
- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.
Numerical Experiments

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- 63 instances, each covering a week of white glass collections in Geneva, Switzerland in 2014, 2015, or 2016.
- Planning horizon of 7 days.
- Up to 2 heterogeneous vehicles.
- Up to 53 containers (41 on average).
- 2 dumps located far apart from each other.
- 10 runs for each instance.
- Simulation-based validation to assess probability information captured by the objective function.
Probabilistic Policies

- **We consider two types of objective function:**
  - **complete**: minimizes the full probabilistic objective defined by expression (4).
  - **routing-only**: minimizes the routing cost only, as defined by expression (1), disregarding all probability information.
Numerical Experiments

Probabilistic Policies

- We consider two types of objective function:
  - **complete**: minimizes the full probabilistic objective defined by expression (4).
  - **routing-only**: minimizes the routing cost only, as defined by expression (1), disregarding all probability information.

- Probability-related costs:
  - overflow cost $\chi$: 100 CHF (fixed by municipality),
  - emergency collection cost $\zeta$: 100 CHF, 50 CHF, 25 CHF (does not apply to routing-only = 0 CHF),
  - Route Failure Cost Multiplier (RFCM) $\psi$: 1.00, 0.50, 0.25 (does not apply to routing-only = 0 CHF).
## Probabilistic Policies

### Table 1: Cost and KPI

<table>
<thead>
<tr>
<th>Objective</th>
<th>ECC</th>
<th>RFCM</th>
<th>Avg Routing Cost (CHF)</th>
<th>Avg Overflow Cost (CHF)</th>
<th>Avg Rte Failure Cost (CHF)</th>
<th>Avg Collected Volume (L)</th>
<th>Liters Per Unit Cost</th>
<th>Liters Per Unit Routing Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete</td>
<td>100.00</td>
<td>1.00</td>
<td>579.78</td>
<td>99.73</td>
<td>0.03</td>
<td>47,234.59</td>
<td>69.51</td>
<td>81.47</td>
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<td>Complete</td>
<td>100.00</td>
<td>0.50</td>
<td>579.46</td>
<td>99.33</td>
<td>0.05</td>
<td>47,225.62</td>
<td>69.57</td>
<td>81.50</td>
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<td>577.84</td>
<td>99.93</td>
<td>0.04</td>
<td>47,455.19</td>
<td>70.01</td>
<td>82.13</td>
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<td>578.83</td>
<td>98.28</td>
<td>0.00</td>
<td>47,662.90</td>
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<tr>
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<td>559.44</td>
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<td>0.02</td>
<td>45,646.48</td>
<td>68.93</td>
<td>81.59</td>
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<td>82.18</td>
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<td>547.74</td>
<td>103.46</td>
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<td>44,682.00</td>
<td>68.61</td>
<td>81.57</td>
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<td>548.10</td>
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<td>546.34</td>
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<td>44,773.34</td>
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<td>81.95</td>
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<td>Routing-only</td>
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<td>0.00</td>
<td>425.08</td>
<td>0.00</td>
<td>0.00</td>
<td>25,286.94</td>
<td>59.49</td>
<td>59.49</td>
</tr>
</tbody>
</table>
# Probabilistic Policies

## Table 2: Container Overflows and Route Failures

<table>
<thead>
<tr>
<th>Objective</th>
<th>ECC</th>
<th>RFCM</th>
<th>Avg Num Overflows</th>
<th>Avg Num Route Failures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>75th Perc.</td>
<td>90th Perc.</td>
<td>95th Perc.</td>
<td>99th Perc.</td>
</tr>
<tr>
<td>Complete</td>
<td>100.00</td>
<td>1.00</td>
<td>0.98</td>
<td>1.78</td>
</tr>
<tr>
<td>Complete</td>
<td>100.00</td>
<td>0.50</td>
<td>0.99</td>
<td>1.78</td>
</tr>
<tr>
<td>Complete</td>
<td>100.00</td>
<td>0.25</td>
<td>0.97</td>
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<td>0.00</td>
<td>0.94</td>
<td>1.77</td>
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<td>Complete</td>
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<td>1.00</td>
<td>1.26</td>
<td>2.19</td>
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<tr>
<td>Complete</td>
<td>50.00</td>
<td>0.50</td>
<td>1.28</td>
<td>2.19</td>
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<td>0.25</td>
<td>1.28</td>
<td>2.18</td>
</tr>
<tr>
<td>Complete</td>
<td>50.00</td>
<td>0.00</td>
<td>1.31</td>
<td>2.23</td>
</tr>
<tr>
<td>Complete</td>
<td>25.00</td>
<td>1.00</td>
<td>1.48</td>
<td>2.46</td>
</tr>
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<td>1.48</td>
<td>2.46</td>
</tr>
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<td>25.00</td>
<td>0.25</td>
<td>1.51</td>
<td>2.50</td>
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<tr>
<td>Routing-only</td>
<td>0.00</td>
<td>0.00</td>
<td>16.97</td>
<td>20.45</td>
</tr>
</tbody>
</table>
Probabilistic Policies

Figure 3: Average Number of Overflows for All Instances

(a) Complete Objective with ECC=100, RFCM=1

(b) Routing-only Objective

Percentiles: 75th, 90th, 95th, 99th
Alternative Policies

- An alternative practical policy is the use of artificially low capacities in the solution process:
  - Container Effective Capacity (CEC): the fraction of the usable container capacity,
  - Truck Effective Capacity (TEC): the fraction of the usable truck capacity,
  - tests for values of 1.00, 0.90 and 0.75.
Alternative Policies

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  - Container Effective Capacity (CEC): the fraction of the usable container capacity,
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  - tests for values of 1.00, 0.90 and 0.75.

- The simulation experiments are wrt the original capacities.

- The objective is always routing-only.
### Alternative Policies

#### Table 3: Cost and KPI

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<td>59.49</td>
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</tr>
<tr>
<td>Routing-only</td>
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### Alternative Policies

**Table 4: Container Overflows and Route Failures**

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Figure 4: Comparison of Routing Cost
Policy Comparison

Figure 5: Comparison of Container Overflows and Route Failures

(a) Overflows

(b) Route Failures
Outline

1. Introduction
2. Related Literature
3. Formulation
4. Numerical Experiments
5. Conclusion
Conclusions

- A rich stochastic IRP with the relevant dynamic uncertainty components in the objective.

- Computational experiments on real-data instances demonstrate:
  - the relevance of the probabilistic information captured in the objective,
  - the superiority of the probabilistic approach in comparison to alternative policies.

Future research directions:
- generalizations for solving other problems,
- chance constraints/robust optimization,
- value of stochastic information.
Conclusions

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- Future research directions:
  - generalizations for solving other problems,
  - chance constraints/robust optimization,
  - value of stochastic information.
Thank you.

Questions?
### Table 5: Average Number of Collections by Day

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A2: Rolling Horizon Approach

- In practice, our SIRP will be solved on a rolling horizon basis:
  - container information is dynamically revealed each day,
  - the problem is solved for a planning horizon $T$,
  - the tours planned for day $t = 0$ are executed,
  - the horizon is rolled over by a day and the procedure is repeated.

- The problem described above is referred to as a Dynamic and Stochastic Inventory Routing Problem (DSIRP).
A2: Rolling Horizon Approach

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- We hypothesize that the solution cost of a DSIRP is bounded:
  - below by the solution of a static IRP with true demands,
  - above by the solution of a static SIRP with forecast demands.
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  - above by the solution of a static SIRP with forecast demands.

- Tests on 41 instances, each covering two weeks of white glass collections in the canton of Geneva, Switzerland in 2014, 2015, or 2016.
### A2: Rolling Horizon Approach

#### Table 6: Analysis of Rolling Horizon DSIRP Bounds

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Note: The four instances for which the hypothesized bounds do not hold are shown in bold.


