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Punching shear strength and behaviour of compact reinforced concrete footings

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Abstract

Punching shear has been observed to be in many cases the governing failure mode of reinforced concrete footings with and without transverse reinforcement. Even though that punching shear failures have already been deeply investigated, there is still scanty experimental information on full-scale reinforced concrete footings failing in punching shear. The results of an experimental programme addressed to this issue (punching of reinforced concrete footings with and without shear reinforced) are presented in this paper. A total number of 5 square specimens were tested with a nominal thickness of 550 mm, where the influence of the side length of the footings, the column size (and thus shear slenderness) and the presence of top and transverse reinforcement were investigated. The experimental results show that punching strength increases with the increase of the column size and with the decrease of the size of the footing. It also results as experimental evidence that a significant flexural-shear interaction might occur for high shear loads, leading to much lower flexural strengths than those calculated according to yield-line theory.

Limit analysis might be a consistent tool to investigate the failure of concrete struts provided that cracking is properly accounted for, particularly with reference to the effective concrete compressive strength as a function of the state of deformations. In this sense, the kinematical theorem of limit analysis is used to investigate on the governing failure mechanisms and to calculate failure loads. The theoretical results show that an important flexural-shear interaction occurs, leading to a smooth transition between the flexural and punching shear failures. The experimental results are analysed based on theoretical considerations resulting from the application of limit analysis. It is shown that a consistent agreement can be found between such theoretical approach and the experimental evidences.

Keywords: footings, shear reinforcement, punching strength, flexural-shear interaction, limit analysis

1 Introduction

Reinforced concrete footings are one of the most popular foundation types. Its structural design is usually performed based on its flexural and punching shear strength, neglecting however their interaction. In addition, a general consensus with respect to the punching behaviour of reinforced concrete (RC) footings still remains to be achieved. Despite several experimental and theoretical works have already been presented with respect to this topic (e.g. Tabolt, 1913; Richart, 1948; Kordina and Nölting, 1981; Dieterle and Rosásty, 1987; Dieterle, 1987; Hallgren et al., 1998; Hallgren and Bjerke, 2002; Timm, 2003; Hegger et al., 2006, 2007, 2009; Ricker, 2009; Netopilik, 2012; Urban et al., 2013a, 2013b; Siburg and Hegger, 2014; Siburg 2014, Krakowski et al., 2015), further investigations are still needed in order to achieve an agreement.

An experimental campaign with 5 full-scale footings (nominal thickness of 550 mm) is presented in this paper, where the in-plane dimensions of the footings, the column size, the presence of top horizontal and shear reinforcement are the geometrical and mechanical properties investigated. The experimental results show that crushing of the concrete struts near the column could be identified as the phenomenon triggering the punching failure of compact reinforced concrete footings.

With respect to the analysis of these members, limit analysis is a suitable approach to investigate failures governed by the yielding of the reinforcement or by the crushing of concrete struts (e.g. Nielsen and Hoang, 2011). In this paper, the kinematical theorem of this theory is applied to analyse the strength of compact footings without shear reinforcement. It is shown that a smooth transition between pure flexural and punching regimes occurs, which represents a flexural-shear regime. A good agreement between theoretical and experimental results (Dieterle and Rostásy, 1987) is observed. The experimental results presented in this paper are analysed based on the theoretical considerations resulting from the approach presented and consistent agreement is found.

2 Experimental programme

2.1 Specimens and materials

An experimental campaign was performed by the authors to investigate on the punching behaviour of reinforced concrete footings. Five square specimens were tested and the investigated parameters were the size of the footings (2.12 m for PS11 and PS12; 1.59 m to PS13 to PS15), the size of the square column (0.30 m for PS11 and PS13 to PS15; 0.45 m to PS12), the influence of top reinforcement (PS11 to PS13 with and PS14 to PS15 without) and shear reinforcement (PS15). A nominal thickness of 550 mm was used in all the specimens. Nominal values of bottom flexural reinforcement ratio (0.75%) and top reinforcement ratio (0.40%; with exception of specimen PS14 and PS15 – without top reinforcement) were kept constant (refer to Fig. 1). Specimen PS15 was shear reinforced with 2 rows of 16 double-headed studs of 25 mm diameter (refer to Fig. 1 (c) and (d)). Steel reinforcement bars with nominal yield strength of 500 MPa were used. The nominal value of the cylinder concrete compressive strength was kept constant and equal to 30 MPa (maximum aggregate size of 16 mm). Detailed description of the experimental campaign can be found elsewhere (Simões et al., 2016b).

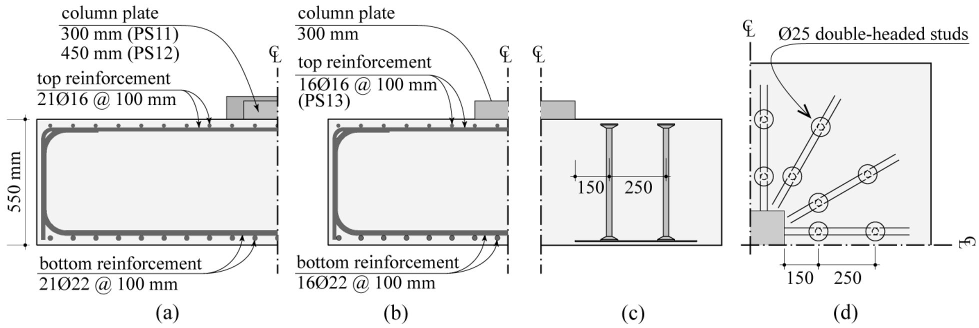

Fig. 1 Reinforcement layout of (a) PS11 to PS12 and (b) PS13 to PS15; (c) shear reinforcement layout of PS15; (d) plan view of shear reinforcement layout of PS15 (adapted from Simões et al. (2016b)).

2.2 Test setup

As shown in Fig. 2, the test setup used allowed the application of a uniform soil pressure in the bottom surface of the specimens. The lower part of the test setup consisted on the loading system, which was composed by a group of flat jacks (9 for the smaller and 16 for the larger specimens) connected in series and placed in the bottom of a rigid box (composed by 4 wood plates against 4 horizontal steel U-profiles, confined by 16 vertical steel U-profiles). The box was filled with a layer of sand and a Teflon sheet together with aluminium plates were placed between the sand and the specimen (to reduce the soil-structure friction). The upper part of the test setup was a reaction frame, consisting on two steel beams supported on a round high-strength steel column (placed on the top of a square steel plate with the investigated column size). Four hydraulic jacks were placed on top of the two steel beams, enabling the application of part of the initial load. The steel beams were fixed to the laboratory strong floor with 4 high-strength steel bars. The loading rate applied was of 50 kN/min.

3 Experimental results

The specimens were cut in half (at least in the weak direction) after the experimental tests, to confirm that all the specimens failed in punching. Fig. 3 shows the saw-cuts of specimens (left) PS14 and (right) PS15. Signs of crushing could be observed in the failure surface near the column for the specimens without shear reinforcement (see Fig. 3 (left) for specimen PS14). Fig. 3 (right) confirms that the specimen with shear reinforcement (PS15) failed by crushing of concrete between the column edge and the first row of shear reinforcement.

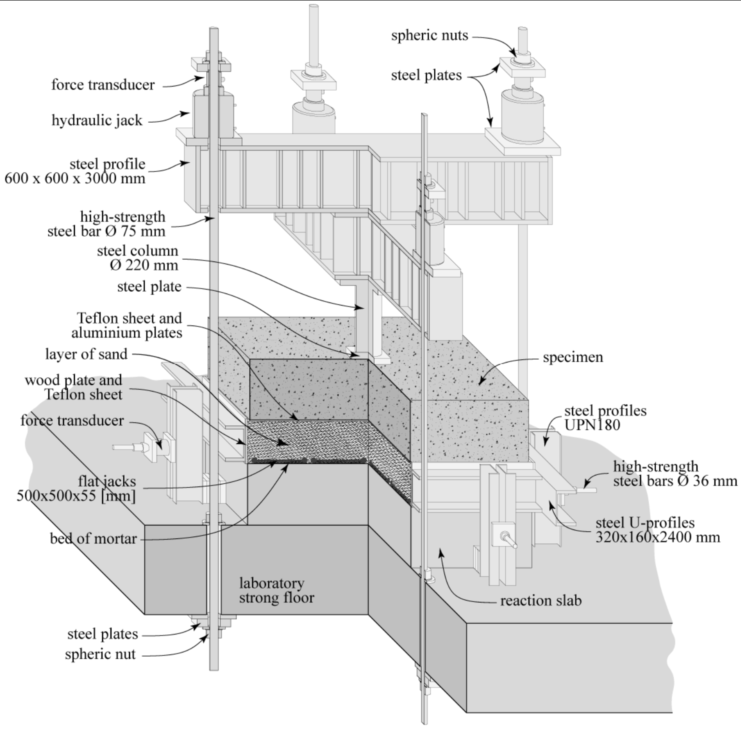

Fig. 2 Schematic representation of test setup (adapted from Simões et al. (2016b)).

The load-rotation curves of the specimens without shear reinforcement and with top reinforcement are shown in Fig. 4 (left). It can be seen that an increase of the column size (PS11 to PS12) and a decrease of the side length of the footing (PS11 to PS13) lead to an increase of the punching strength and of the rotation capacity. Fig. 4 (right) presents the load-rotation curves of two equivalent specimens without (PS14) and with shear reinforcement (PS15). It is experimentally shown that the use of double-headed shear studs as shear reinforcement may be efficient (increase of approximately 39% of the strength and even more of rotation capacity), also in the case of compact footings. It is also important to note that the load-rotation curve of the specimen with shear reinforcement seems to have reached a plateau for a load level much lower than the one that can be calculated using the yield line theory (Johansen, 1962). Experimental investigations have shown that this can also occur in flat slender slabs with large amounts of shear reinforcement (e.g. Lips et al., 2012).

It should be noted that the use of top horizontal reinforcement (PS13 compared to PS14) leads to an increase of approximately 8% of the normalized strength. Vertical displacements measured at different locations of the footing and column allowed to define separately the flexural deformations from shear deformations and column penetration. Based on the recorded measurements, it may be shown that in addition to the rotation, also the shear deformation and the column penetration present an important contribution to the total deformation of the footing (see Simões et al. (2016b) for detailed explanations).

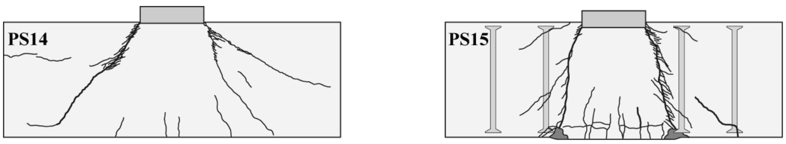

Fig. 3 Schematic representation of saw-cuts of specimens PS14 (left) and PS15 (right) (Simões et al. (2016b)).

Different regimes have been observed in the punching behaviour of reinforced concrete footings without shear reinforcement presented in this paper (refer to Simões et al. (2016b) for details). In the last regime (approximately from 85% to 100% of total load), a significant change in the thickness of the specimens and an important increase in the column penetration were measured, accompanied by a stabilization of the tangential strains and a decrease of the radial strains (both measured on concrete top surface in the vicinity of the column). According to Simões et al. (2016b), this behaviour might be related to the crushing of the concrete struts close to the column (justifying the significant increase in the column penetration and the rather constant value of the tangential compression).

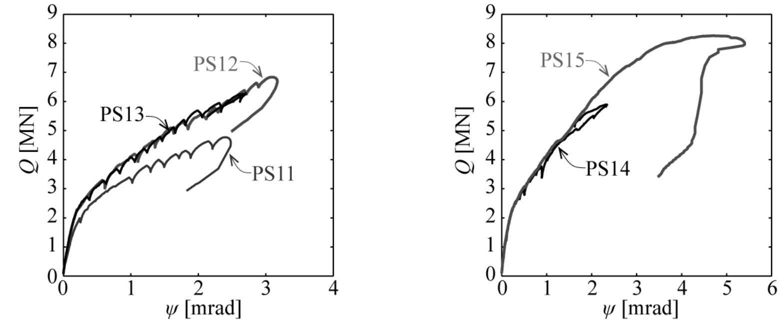

Fig. 4 Load-rotation curves of: specimens PS11 to PS13 (left); specimens PS14 and PS15 (right) (adapted from Simões et al., 2016b).

4 Strength of footings based on the kinematical theorem of Limit Analysis

4.1 Introduction

Based on experimental results, it was previously stated that the failure of reinforced concrete footings may be associated to crushing of concrete struts (Simões et al., 2016b). Limit analysis may be used to investigate on this type of failures (Nielsen and Hoang, 2011). In this section, the kinematical theorem of limit analysis is used to study the governing failure mechanisms and to calculate failure loads of RC footings without shear reinforcement. This section is ground on the work of Simões et al. (2016a), where thoroughly description of the approach presented below can be found.

The kinematical theorem of limit analysis has already been applied by other researchers to investigate the punching strength of slabs (originally by Braestrup et al. (1976) and later, among others, by Jiang and Shen(1986)). However, the referred works have considered a failure mechanism characterized by a vertical displacement of the outer portion of the slab, investigating only the punching regime (not accounting for the influence of rotations on the failure mechanism and, consequently, neglecting the influence of the flexural reinforcement). A more general approach is presented in this paper, allowing to investigate both the flexural and punching regimes, as well as their interaction.

As shown in Fig. 5, concrete is considered to have a rigid-plastic behaviour, respecting a Mohr-Coulomb yield criterion with a tension cut-off (tensile strength neglected due to very brittle behaviour in tension). Normality condition is also adopted. A plastic concrete compressive strength is used (Fig. 5(a)), accounting for the brittleness of high-strength concrete (η_{fc} calculated according to Muttoni (1990)) and for the potential presence of transverse cracking (η_ε with constant values in the present work). Uniaxial rigid-plastic behaviour is considered for the reinforcement (Fig. 5(c)).

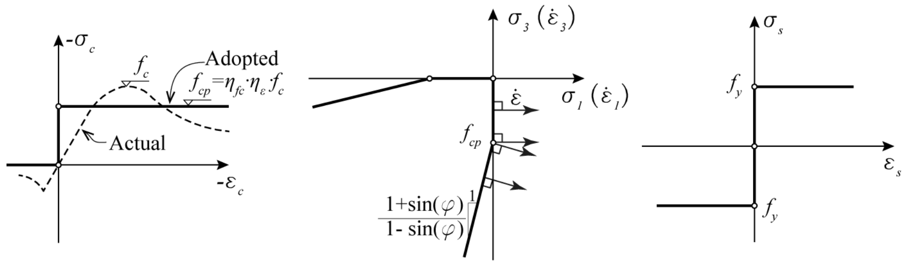

Fig. 5 (a) Rigid-plastic behaviour adopted for concrete; (b) Mohr-Coulomb yield criterion with tension cut-off; (c) uniaxial rigid-plastic behaviour considered for the reinforcement (adapted from Simões et al. (2016a)).

Fig. 6 (a) presents the geometrical and mechanical properties of the investigated problem. Two different groups of failure mechanisms are studied. Inner and outer portions of the footing are divided by a failure surface developing through the entire effective depth (the concrete cover of the reinforcement is neglected). The velocity field along the failure surface is given by a relative rotation around the instantaneous centre of rotation (ICR). The two groups of mechanisms consider different locations of the ICR and different directions for the relative rotation. The group of mechanisms M1 (Fig. 6(b)) considers an ICR above the level of the bottom flexural reinforcement and radially behind the edge of the column, together with a counter clockwise relative rotation. The ICR in the group of mechanisms M2 (Fig. 6(c)) is considered to be radially located beyond the load introduction and below the level of the bottom flexural reinforcement, together with a clockwise relative rotation. The rate of total internal energy dissipated is given by the sum of different components, namely: shear transfer in the concrete; tangential bending in the outer portion of the footing; bottom and top reinforcement. The rate of external work is given by the applied load. Details on the calculation of each component of dissipation of energy can be found elsewhere (refer to Simões et al., 2016a). Using the work equation established in the kinematical theorem (rate of total internal energy dissipated balanced by rate of external work), the failure load can be obtained as a function of the geometry of the failure surface and location of the ICR. The solution of the problem can be numerically obtained through the minimization of the failure load (constrained non-linear optimization problem; see Simões et al., 2016a).

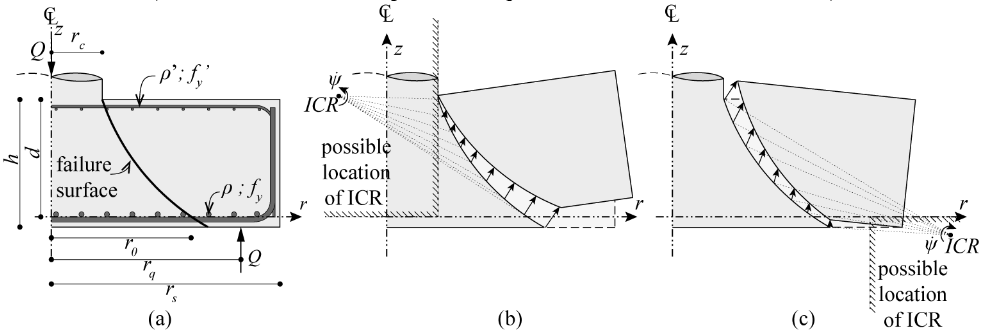

Fig. 6 (a) Definition of geometrical and material properties; general example of investigated mechanisms (b) M1 and (c) M2 (adapted from Simões et al., 2016a).

4.2 Application

Fig. 7(a) shows the normalized strength of a footing (characterized by r_s/d=1.77, r_c/d=0.38 and ω'=0) as a function of the bottom mechanical reinforcement ratio ω. The governing failure mechanisms for different values of ω are also plotted in Fig. 7 (b) to (d). With the help of these figures, it can be seen that:

- pure flexural regime occurs for very low amounts of ω (where the strength obtained is very close to the flexural capacity calculated according to yield line theory (Johansen, 1962));
- significant flexural-shear interaction occurs for intermediate values of ω, where the strength obtained is considerably lower than that obtained using yield line theory (pure flexural capacity overestimates the strength of the footing); this effect might be associated to the presence of diagonal concrete struts, which lead to an increase of the compression zone and, consequently, to a loss of lever arm (see also Fig. 7 (c));
- punching shear regime occurs for larger amounts of ω, where the governing failure mechanism may be other than a vertical translation of the outer portion of the footing (function of r_s/d, r_c/d and ω'; refer to Fig. 7 (d); see Simões et al. (2016a) for more details);
- the failure mechanism governing for low amounts of ω is characterized by an significant rotational component (ICR close to the footing), with an important contribution of the bottom reinforcement to the rate of total internal energy dissipated (Fig. 7 (b)); in the punching regime, the failure mechanism is either characterized by a vertical translation of the outer portion (ICR at the infinite, see Fig. 7 (e); large r_s/d, small r_c/d and large ω') or by a clockwise rotation of the outer portion of the footing (ICR close to footing, see Fig. 7 (d); small r_s/d, large r_c/d and reduced amounts of ω');
- according to the presented approach, the punching strength of small footings with large columns can be increased with the increase of top mechanical reinforcement ratio; for instance, the punching strength of the example presented in Fig. 7 would increase with the use of top mechanical reinforcement due to its activation as a consequence of the clockwise rotation of

the governing failure mechanism; if the value of ω' is gradually increase, the governing failure mechanism (Fig. 7 (d) for ω'=0) progressively transforms to the one represented in Fig. 7 (e) and the punching strength increases (this is in agreement with the experimental results previously shown; increase of approximately 8% of punching strength from PS14 to PS13 with the introduction of the top reinforcement) ;

- the case investigated corresponds to the parameters of specimen PS14 (assuming equal column parameter and equal area of bottom surface); a good agreement can be found between theoretical results and experimental value assuming a value of η_ε=0.45 (parameter accounting for the reduction of strength due to transverse cracking of concrete).

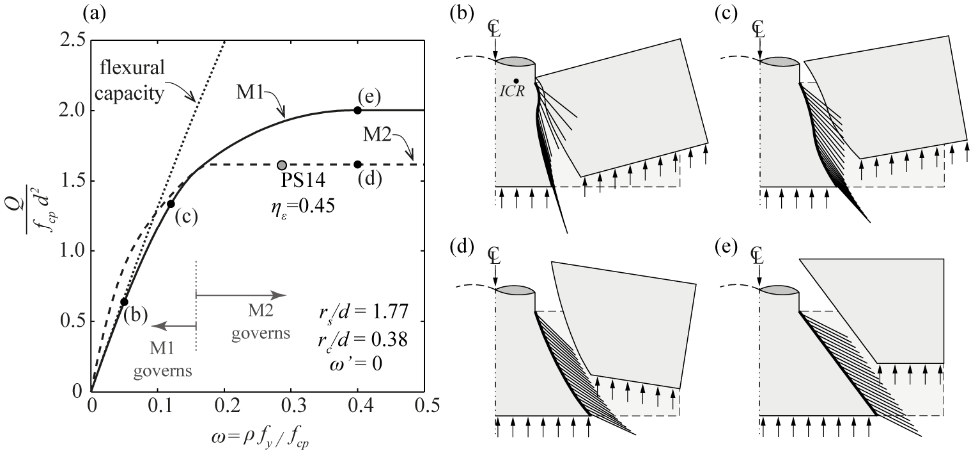

Fig. 7 (a) Strength as a function of the mechanical reinforcement for both mechanisms M1 and M2 for r_s/d=1.77, r_c/d=0.38 and ω'=0; (b) failure mechanism M1 for ω=0.05; (c) failure mechanism M1 for ω=0.12; (d) failure mechanism M2 for ω=0.40; (e) failure mechanism M1 for ω=0.40.

With the application of the approach here presented to different geometrical parameters, it can be shown that the secant inclination of the failure surface β in the punching regime decreases mainly as a function of the span-to-effective depth ratio *a/d* (which was already experimentally observed, e.g. Hegger et al. (2009)). A fairly good approximation of the theoretical results is given by β=90/(0.8+0.5 *a/d*) (Simões et al., 2016).

Dieterle and Rosásty (1987) have performed an experimental campaign containing a series of tests where the only variable changing was the bottom mechanical reinforcement ratio. A comparison between the theoretical and experimental results is shown in Fig. 8 for two different values of the reduction factor accounting for the presence of transverse strains.

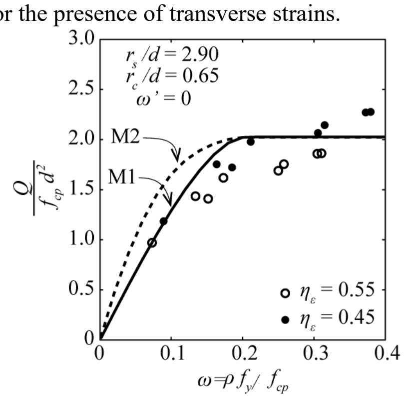

Fig. 8 Comparison between theoretical values and experimental results of Dieterle and Rostásy (1987) for two different values of the reduction factor accounting for transverse strains $\eta\varepsilon$ (adapted from Simões et al., 2016a).

A very good agreement can be observed between the theoretical and the experimental results for reasonable values of the η_ε (0.45 to 0.55, which are usual values for this parameter). It should be noted that a better agreement could be obtained if a variable value of the η_ε would be used (values close to 0.45 for low values of ω, associated to larger deformations; values close to 0.55 for larger values of ω, corresponding to small deformations). Therefore, it remains clear that a mechanical based formulation is needed to calculate the value of the reduction factor accounting for the presence of transverse strain for the case investigated in this paper (also the potential influence of size effect has to be studied). This is the topic of future work.

5 Interpretation of experimental results based on Limit Analysis

Fig. 9 presents a comparison between the cracking pattern observed in the saw-cuts of three specimens without shear reinforcement (PS11 to PS13) with the secant inclination of the failure surface and control section (located at a distance of $0.2 \cdot d \cdot \text{cotg}(\beta)$ from the column edge) defined according to Simões et al. (2016a). It can be seen in Fig. 9 that the secant inclination of the failure surface and the control section proposed by Simões et al. (2016a) agree very well with the observed cracking pattern and crushing zone (rather close to the column). It should also be noted that a consistent definition of the inclination of the failure surface is especially important in footings subjected to uniform soil pressure because it allows a correct definition of the amount of uniform soil pressure assumed to be directly equilibrated.

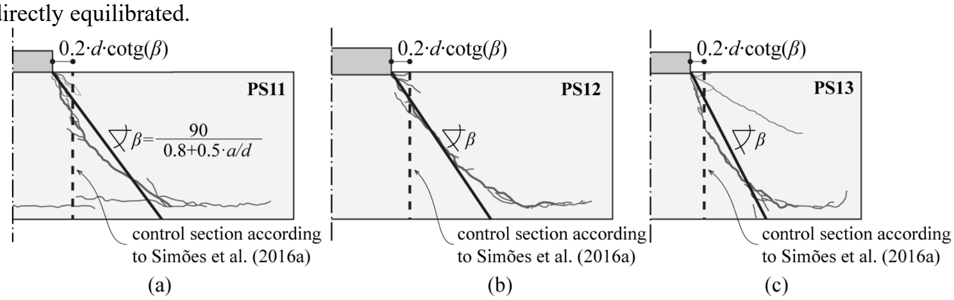

Fig. 9 Comparison of cracking pattern observed in the saw cuts and inclination of failure surface and position of control section calculated according to Simões et al. (2016b) for (a) PS11, (b) PS12 and (c) PS13.

6 Conclusions and outlook

An experimental testing programme investigating the punching behaviour and strength of reinforced concrete footings is presented in this paper. The following conclusions have to be highlighted:

- the punching strength increases with the decrease of the in-plane dimensions of the footings and with the increase of the column size;
- double headed shear-studs is an effective shear reinforcement system to increase the punching strength of RC footings;
- the load-rotation curve of the footing with shear reinforcement shows that a significant flexural-shear interaction occurs for high load levels;

The kinematical theorem of limit analysis is used to calculate the strength of RC footings without shear reinforcement. The main observations are:

- significant flexural-shear regime is observed for compact footings with large columns;
- good agreement is found between theoretical and experimental results for the experimental campaign of Dieterle and Rosásty (1987) adopting constant and reasonable values for the reduction factor accounting for the presence of transverse strains; a formulation with mechanical basis is absolutely needed to calculate this factor (this is the topic of future work).

It is still important to note that consistent agreement was found between the inclination of the failure surface obtained with theoretical approach here presented and the inclination of the failure surfaces experimentally observed.

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