Interaction of neutral atoms and plasma turbulence in the tokamak edge region

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In brief

- First fully turbulent SOL simulations self-consistently coupled to a kinetic neutral model implemented in GBS.
- Neutral kinetic equation with Krook operators for ionization, recombination and charge-exchange.
- Two fluid drift-reduced Braginskii equations are solved for the plasma.
- Development of a more refined two-point model in very good agreement with GBS
- Initial study of gas puff imaging with fluctuating neutrals.
- The details of the model in [C. Wersal and P. Ricci, 2015 *Nucl. Fusion* **55** 123014].

A model for neutral atoms in the SOL

Kinetic equation with Krook operators

$$\frac{\partial f_n}{\partial t} + \mathbf{v}\cdot\frac{\partial f_n}{\partial \mathbf{x}} = -\nu_{iz} f_n - \nu_{cx} n_n\left(\frac{f_n}{n_n} - \frac{f_i}{n_i}\right) + \nu_{rec} f_i \qquad (1)$$

$$\nu_{iz} = n_e r_{iz} = n_e\langle v_e \sigma_{iz}(v_e)\rangle, \quad \nu_{cx} = n_i r_{cx} = n_i \langle v_{rel}\sigma_{cx}(v_{rel})\rangle, \quad \nu_{rec} = n_e r_{rec} = n_e\langle v_e \sigma_{rec}(v_e)\rangle$$

Boundary conditions: particle conservation, i.e.

$$f_n(\mathbf{x}_b, \mathbf{v}) = (1-\alpha_{refl})\Gamma_{out}(\mathbf{x}_b)\chi_{in}(\mathbf{x}_b,\mathbf{v}) + \alpha_{refl}[f_n(\mathbf{x}_b, \mathbf{v} - 2\mathbf{v}_p) + f_i(\mathbf{x}_b, \mathbf{v} - 2\mathbf{v}_p)] \qquad (2)$$

with Γ_{out} the ion and neutral particle outflow, α_{refl} the reflection coefficient, $\mathbf{v}_p$ the velocity perpendicular to the wall. The distribution function of absorbed and re-emitted particles is

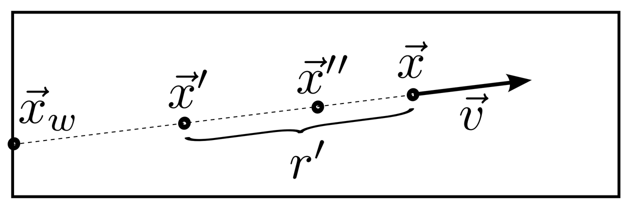
$$\chi_{in}(\mathbf{x}_b, \mathbf{v}) = \frac{3}{4\pi}\frac{m^2}{T_b^2} cos(\theta) \exp\left(-\frac{mv^2}{2T_b}\right) \qquad (3)$$

with θ the angle between $\mathbf{v}$ and vector normal to the surface, and wall temperature T_b.

Two assumptions: $\tau_{\text{neutral losses}} < \tau_{\text{turbulence}}$ and $\lambda_{\text{mfp, neutrals}} \ll L_{\parallel,\text{plasma}}$.

The method of characteristics
The formal solution of Eq. (1) is

$$f_n(\mathbf{x}_\perp, \mathbf{v}) = \int_0^{r_{\perp b}}\left[\frac{S(\mathbf{x}'_\perp, \mathbf{v})}{v_\perp} + \delta(r'_\perp - r_{\perp b}) f_n(\mathbf{x}'_{\perp b}, \mathbf{v})\right]\exp\left[-\frac{1}{v_\perp}\int_0^{r'_\perp}\nu_{eff}(\mathbf{x}''_\perp)dr''_\perp\right]dr'_\perp \qquad (4)$$



$$S(\mathbf{x}, \mathbf{v}) = \nu_{cx}(\mathbf{x}) n_n(\mathbf{x})\Phi_i(\mathbf{x}, \mathbf{v}) + \nu_{rec}(\mathbf{x}) f_i(\mathbf{x}, \mathbf{v})$$
$$\nu_{eff}(\mathbf{x}) = \nu_{iz}(\mathbf{x}) + \nu_{cx}(\mathbf{x})$$
$$r' = |\mathbf{x} - \mathbf{x}'|$$

An **integral equation** for neutral density is obtained by integrating Eq. (4) over $\mathbf{v}$.

$$n_n(\mathbf{x}_\perp) = \int dv\, f_n(\mathbf{x}_\perp, \mathbf{v}) = \int_D n_n(\mathbf{x}'_\perp)\nu_{cx}(\mathbf{x}'_\perp) K_{p\to p}(\mathbf{x}_\perp, \mathbf{x}'_\perp) dA' + n_{n,rec}(\mathbf{x}_\perp) + n_{n,walls}(\mathbf{x}_\perp) \qquad (5)$$

$$K_{p\to p}(\mathbf{x}_\perp, \mathbf{x}'_\perp) = \int_0^\infty \frac{1}{r'_\perp}\Phi_{\perp i}(\mathbf{x}'_\perp, \mathbf{v}_\perp)\exp\left[-\frac{1}{v_\perp}\int_0^{r'_\perp}\nu_{eff}(\mathbf{x}''_\perp)dr''_\perp\right]dv_\perp \qquad (6)$$

$K_{p\to p}$ only depends on plasma quantities. Equation (5) and boundary conditions are spatially discretized, leading to a linear system of equations

$$\begin{bmatrix} n_n \\ \Gamma_{out}\end{bmatrix} = \begin{bmatrix} K_{p\to p} & K_{b\to p} \\ K_{p\to b} & K_{b\to b}\end{bmatrix}\cdot\begin{bmatrix} n_n \\ \Gamma_{out}\end{bmatrix} + \begin{bmatrix} n_{n,rec} \\ \Gamma_{out,rec} + \Gamma_{out,i}\end{bmatrix} \qquad (7)$$

which is solved with standard methods. n_n is used to compute f_n and its moments using Eq. (4).

The GBS code

GBS is a **3D**, **flux-driven**, **global** turbulence code in limited geometry.
GBS solves the **two fluid drift-reduced Braginskii equations** [Ricci *et al.*, PPCF 2012], $k_\perp^2 \gg k_\parallel^2$, $d/dt \ll \omega_{ci}$

$$\frac{\partial n}{\partial t} = -\frac{1}{B}[\phi, n] + \frac{2}{eB}[C(p_e) - enC(\phi)] - \nabla_\parallel(nv_{\parallel e}) + \mathcal{D}_n(n) + S_n + n_n\nu_{iz} - n\nu_{rec} \qquad (8)$$

$$\frac{\partial\tilde{\omega}}{\partial t} = -\frac{1}{B}[\phi, \tilde{\omega}] - v_{\parallel i}\nabla_\parallel\tilde{\omega} + \frac{B^2}{m_i n}\nabla_\parallel j_\parallel + \frac{2B}{m_i n}C(p) + \mathcal{D}_{\tilde{\omega}}(\tilde{\omega}) - \frac{n_n}{n}\nu_{cx}\tilde{\omega} \qquad (9)$$

$$\frac{\partial v_{\parallel e}}{\partial t} = -\frac{1}{B}[\phi, v_{\parallel e}] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{e}{\sigma_\parallel m_e}j_\parallel + \frac{e}{m_e}\nabla_\parallel\phi - \frac{T_e}{m_e n}\nabla_\parallel n - \frac{1.71}{m_e}\nabla_\parallel T_e + \mathcal{D}_{v_{\parallel e}}(v_{\parallel e}) + \frac{n_n}{n}(\nu_{en} + 2\nu_{iz})(v_{\parallel n} - v_{\parallel e}) \qquad (10)$$

$$\frac{\partial v_{\parallel i}}{\partial t} = -\frac{1}{B}[\phi, v_{\parallel i}] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{1}{m_i n}\nabla_\parallel p + \mathcal{D}_{v_{\parallel i}}(v_{\parallel i}) + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})(v_{\parallel n} - v_{\parallel i}) \qquad (11)$$

$$\frac{\partial T_e}{\partial t} = -\frac{1}{B}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4T_e}{3eB}\left[\frac{T_e}{n}C(n) + \frac{7}{2}C(T_e) - eC(\phi)\right] + \frac{2T_e}{3n}\left[\frac{0.71}{e}\nabla_\parallel j_\parallel - n\nabla_\parallel v_{\parallel e}\right] + \mathcal{D}_{T_e}(T_e) + \mathcal{D}^\parallel_{T_e}(T_e) + S_{T_e} + \frac{n_n}{n}\nu_{iz}\left[-\frac{2}{3}E_{iz} - T_e + m_e v_{\parallel e}\left(v_{\parallel e} - \frac{4}{3}v_{\parallel n}\right)\right] - \frac{n_n}{n}m_e\frac{2}{3}\nu_{en}v_{\parallel e}(v_{\parallel n} - v_{\parallel e}) \qquad (12)$$

$$\frac{\partial T_i}{\partial t} = -\frac{1}{B}[\phi, T_i] - v_{\parallel i}\nabla_\parallel T_i + \frac{4T_i}{3eB}\left[C(T_e) + \frac{T_e}{n}C(n) - \frac{5}{3}C(T_i) - eC(\phi)\right] + \frac{2T_i}{3n}\left[\frac{1}{e}\nabla_\parallel j_\parallel - n\nabla_\parallel v_{\parallel i}\right] + \mathcal{D}_{T_i}(T_i) + \mathcal{D}^\parallel_{T_i}(T_i) + S_{T_i} + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})\left[T_n - T_i + \frac{1}{3}(v_{\parallel n} - v_{\parallel i})^2\right] \qquad (13)$$

$$\nabla_\perp^2\phi = \omega, \ \rho_* = \rho_s/R, \ \nabla_\parallel f = \mathbf{b}_0\cdot\nabla f, \ \tilde{\omega} = \omega + \tau\nabla_\perp^2 T_i, \ p = n(T_e + \tau T_i)$$

- A set of fluid boundary conditions applicable at the magnetic pre-sheath entrance where the magnetic field lines intersect the limiter is used [Loizu *et al.*, PoP 2012]

Some achievements of GBS (see also http://spc.epfl.ch/research_theory_plasma_edge):
- SOL width scaling as a function of dimensionless/engineering plasma parameters
- Origin and nature of intrinsic toroidal plasma rotation
- Non-linear turbulent regimes in the SOL
- Mechanism regulating the equilibrium electrostatic potential

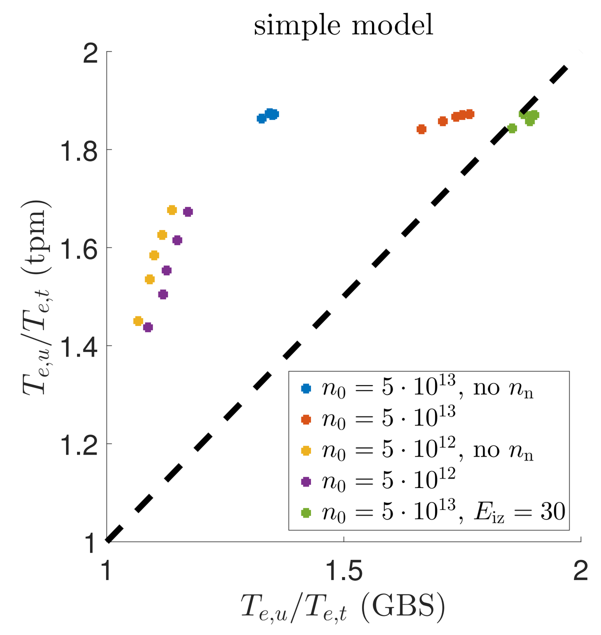
A refined two-point model

Two-point models describe the relation between target (HFS limiter) and upstream (LFS mid-plane) T_e. They are derived from 1-D models along B and widely used experimentally.

Simplest two-point model for T_e in the limited SOL is

$$\nabla_\parallel\left(\frac{5}{2}\Gamma T_e\right) - \chi_{e0}\nabla_\parallel\left(T_e^{5/2}\nabla_\parallel T_e\right) = S_Q$$
$$\nabla_\parallel\Gamma = \nabla_\parallel(nv_\parallel) = S_n$$

with $\nabla_\parallel T_{e,u} = 0$, $Q_t = \gamma_e\Gamma_t T_{e,t}$, $\gamma_e \approx 5$, $\chi_{e0} = 3/2\bar{n}\kappa_{e\parallel}$, and constant S_Q and S_n.

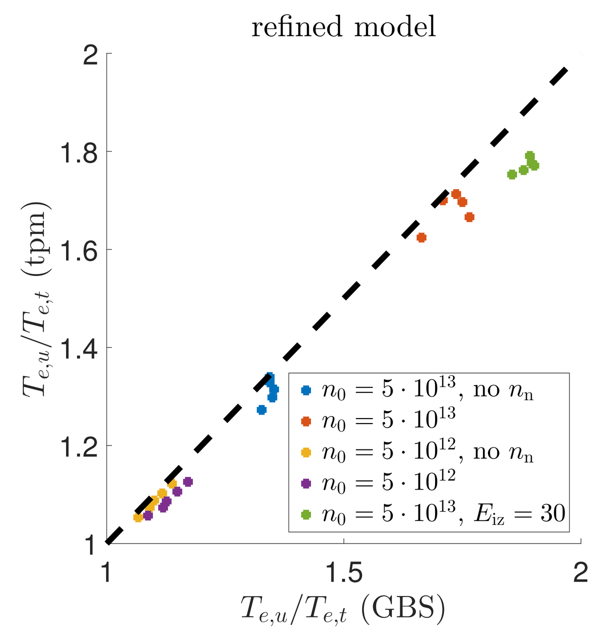


A **refined two-point model** is derived from the drift-reduced Braginskii equations

$$\nabla_\parallel\left(\frac{5}{2}\Gamma T_e\right) - \chi_{e0}\nabla_\parallel\left(T_e^{5/2}\nabla_\parallel T_e\right) - v_\parallel\nabla_\parallel(nT_e) = S_Q - n_n\nu_{iz}(T_e)E_{iz}$$
$$\nabla_\parallel\Gamma = \nabla_\parallel(nv_\parallel) = S_n + n_n\nu_{iz}(T_e)$$

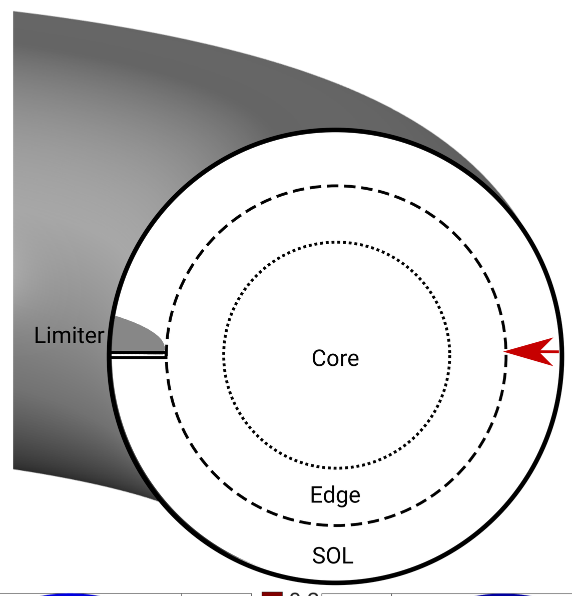
with $\nabla_\parallel T_{e,u} = 0$ and the assumptions
- $v_\parallel$ is linear from $-c_s$ to c_s
- $c_s = \sqrt{T_{e,t} + T_{i,t}} \approx \sqrt{2T_{e,t}}$
- Cosine-shaped S_Q and S_n
- n_n is decaying exponentially from limiter with λ_{mfp}
- Third input parameter, S_{iz}, the total ionization source

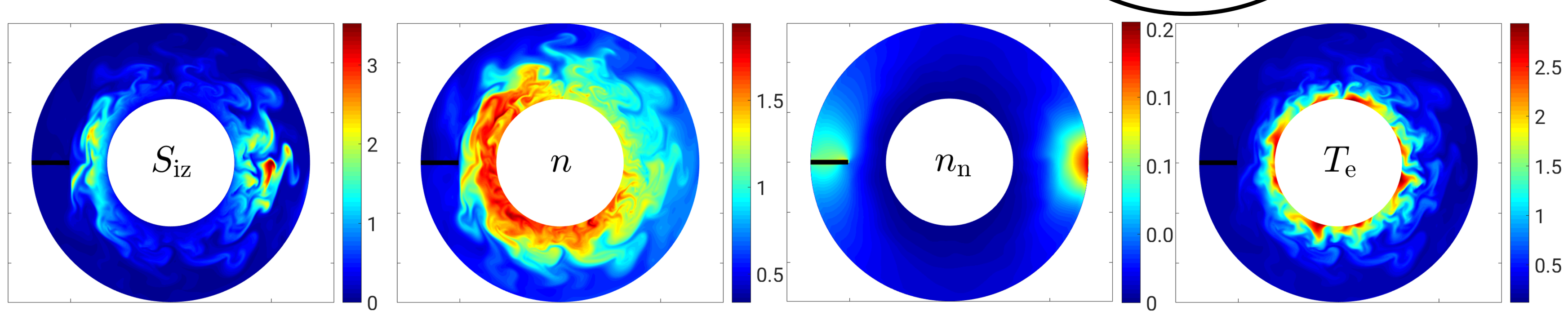
[C. Wersal, P. Ricci, and J. Loizu, 2016 submitted to *PPCF*]



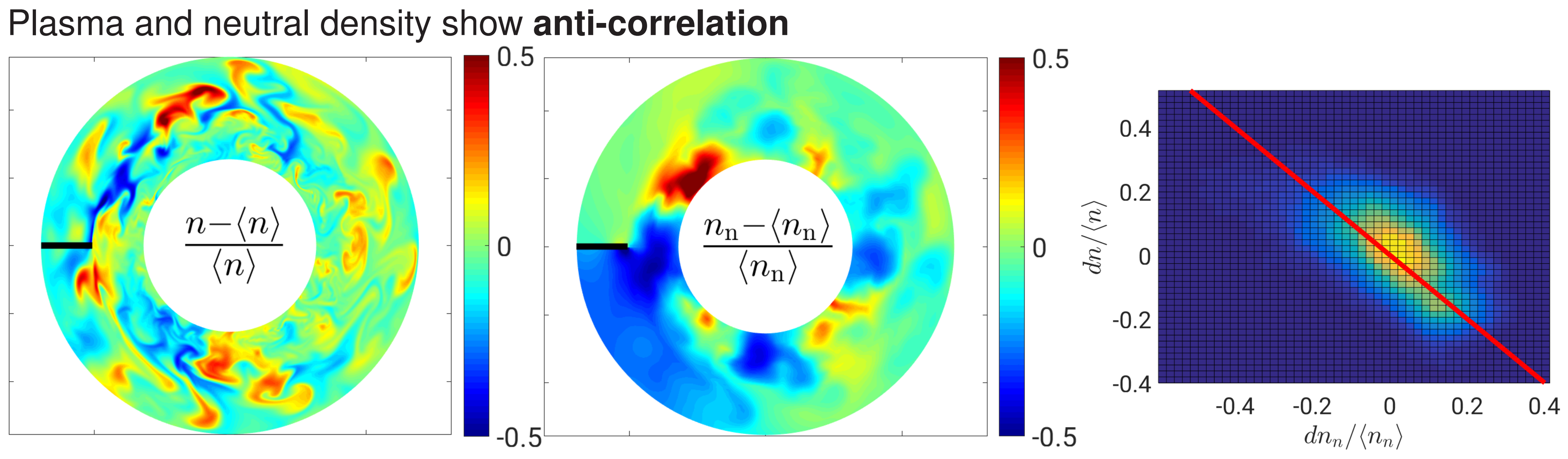
Neutral fluctuations and gas puff imaging

- Simulation with **SOL and edge**
- **Gas puff** from LFS
- Small constant main wall recycling
- $n_0 = 2\cdot10^{13}\mathrm{cm}^{-3}$, $T_0 = 20\mathrm{eV}$, $q_0 = 3.87$, $\rho_*^{-1} = 500$, $a_0 = 200\rho_s$, $\rho_s \approx 1\mathrm{mm}$, $R/c_s \approx 10\mu s$
- $S_{iz} = n_n n r_{iz}(T_e)$ is approximately proportional to light emission



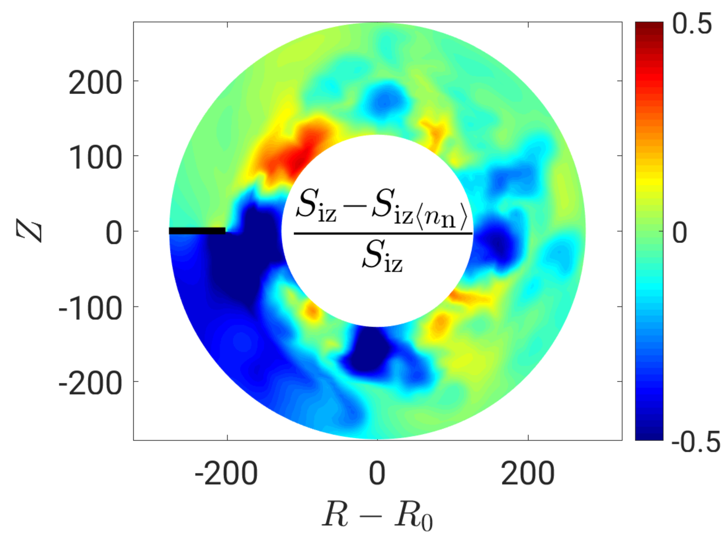


Plasma and neutral density show **anti-correlation**



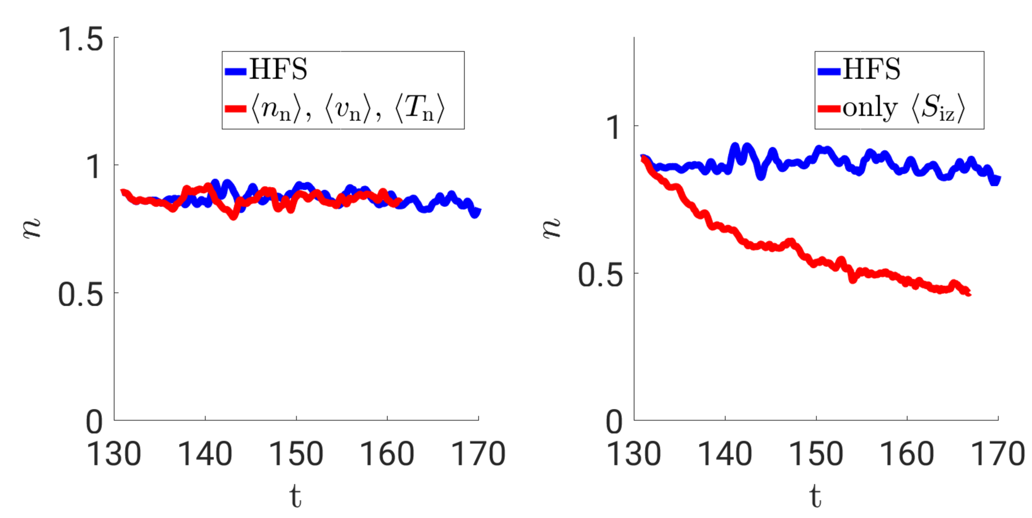
Evaluating S_{iz} without neutral density fluctuations leads to relative errors of up to 50% in the emission rate.

$$S_{iz} = n_n n r_{iz}(T_e)$$
$$S_{iz\langle n_n\rangle} = \langle n_n\rangle n r_{iz}(T_e)$$



Towards a simpler neutral model

Repeat a HFS gas puff simulation without neutral fluctuations
- (left) Average n_n, v_n, and T_n ($S_{iz} = \langle n_n\rangle n r_{iz}$) → no significant differences
- (right) Average $S_{iz} = \langle n_n n r_{iz}\rangle$ and neglect other neutral-plasma terms → large differences



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