Hybrid Cyclicity: Combining The Benefits Of Cyclic And Non-Cyclic Timetables
The main product of a Train Operating Company is a train timetable.
What is a timetable?
A railway timetable is defined as a set of arrival and departure times of every train from each of its stopping stations.
In our case, the travel times and dwell times are fixed. Therefore, we denote a timetable as a set of departure times of every train from its origin station ($d_{v}^L$).
Two types of timetables exist: Non-Cyclic and Cyclic.
The cyclic timetable originates from the Periodic Event Scheduling Problem (PESP), which was first defined by Serafini and Ukovich (1989).
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A set of events is scheduled in an equally spaced intervals, e.g. TRISTAN - every 3 years.
A special subset of cyclicity is the **clockfaced timetables**:

Event every xx:15.
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Event every xx:15. Especially popular within:
Issue: The demand is not uniformly distributed.
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Passengers find the regularity of a timetable easier to be memorized (Wardman et al. (2004), Johnson et al. (2006)).
Therefore one is not superior to the other.
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Why not both?
What we want to combine and how:

Regularity: Taken care of by the design

Figure: Ursus Wehrli
What we want to combine and how:

**Flexibility:** Passenger satisfaction, maximized by solving the Passenger Centric Train Timetabling Problem

**Figure:** Ursus Wehrli

**Regularity:** Taken care of by the design
\[ S_{i}^{tp} = -VOT \cdot \left( \sum_{\ell \in L^p} r_{i}^{\ell} + \beta_{W} \cdot w_{i}^{tp} + \beta_{T} \cdot (|L^p| - 1) + \beta_{E} \cdot \delta_{ip}^{t} + \beta_{L} \cdot \gamma_{ip}^{t} \right), \quad \forall i \in I, \forall t \in T_{i}, \forall p \in P_{i}, \]

- \( r_{i}^{\ell} \) – running time/in-vehicle time
- \( w_{i}^{tp} \) – waiting time
- \(|L^p| - 1\) – number of transfers
- \( \delta_{ip}^{t} \) – early schedule passenger delay
- \( \gamma_{ip}^{t} \) – late schedule passenger delay
- \( -VOT \) – value of time
- \( \beta_{W}, \beta_{T}, \beta_{E}, \beta_{L} \) – estimates from literature
What are the combinations?
\[ \theta \text{ Shifted Cyclic Timetable} \]

For a cycle of 60 minutes:

- \( \theta = 0 \) is equivalent to the cyclic timetable
- \( \theta = 30 \) is the maximum deviation without overlapping trains
- We test values between 0 and 30 in 3 minute intervals
\( \theta \) Shifted Cyclic Timetable

\[ \Delta^\ell_v \in (-\theta, \theta) \]

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\[ \theta \text{ Shifted Cyclic Timetable} \]

For a cycle of 60 minutes:
- \( \theta = 0 \) is equivalent to the cyclic timetable
- \( \theta = 30 \) is the maximum deviation without overlapping trains
- We test values between 0 and 30 in 3 minute intervals
\( \xi \) Partially Cyclic Timetable

\( \eta = \max(|\mathcal{V}|) \cdot \frac{\xi}{100} \)

\( \eta \) trains per line have a cyclic departure time, the rest is free

For a cycle of 60 minutes:

- \( \xi = 100 \) is equivalent to the cyclic timetable
- \( \xi = 0 \) is equivalent to the non-cyclic timetable
- We test values between 0 and 100 in 10% intervals
A cycle can have:

- no train
- a cyclic train
- a cyclic train and one or more non-cyclic ones
Model

\[
\text{max } \text{satisfaction} \quad (1)
\]

\[
\text{satisfaction function} \quad (2)
\]

\[
\text{at most one path per passenger} \quad (3)
\]

\[
\text{link trains with paths} \quad (4)
\]

\[
\text{cyclicity} \quad (5)
\]

\[
\text{train scheduling} \quad (6)
\]

\[
\text{train capacity} \quad (7)
\]

\[
\text{schedule delay} \quad (8)
\]

\[
\text{waiting time} \quad (9)
\]
Methodology: Simulated Annealing
Israel 2008

- OD Matrix for an average working day (Sunday to Thursday) in Israel during 2008
- 47 Stations
- 2162 ODs
- 34 (unidirectional) lines
- 380 trains
- Min. transfer – 4 mins
- VOT – 21.12 NIS per hour
- 126 036/193 886 Passengers
### Table: Computational results of the existing timetables for the 2008 demand

<table>
<thead>
<tr>
<th></th>
<th>IR 13/14 as Strictly Cyclic</th>
<th>IR 13/14</th>
<th>cyclic</th>
<th>non-cyclic</th>
<th>perfect service</th>
</tr>
</thead>
<tbody>
<tr>
<td>satisfaction [NIS]</td>
<td>-704 904</td>
<td>-537 503</td>
<td>-476 774</td>
<td>-424 529</td>
<td>-2 089 049</td>
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<tr>
<td>drivers [-]</td>
<td>470</td>
<td>388</td>
<td>388</td>
<td>388</td>
<td>48 960</td>
</tr>
<tr>
<td>rolling stock [-]</td>
<td>940</td>
<td>776</td>
<td>776</td>
<td>776</td>
<td>48 960</td>
</tr>
<tr>
<td>covered [%]</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>time [sec]</td>
<td>12</td>
<td>6</td>
<td>24 997</td>
<td>25 613</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table: Computational results of the existing timetables for the 2014 demand

<table>
<thead>
<tr>
<th></th>
<th>IR 13/14 as Strictly Cyclic</th>
<th>IR 13/14</th>
<th>cyclic</th>
<th>non-cyclic</th>
<th>perfect service</th>
</tr>
</thead>
<tbody>
<tr>
<td>satisfaction [NIS]</td>
<td>-3 792 733</td>
<td>-3 379 596</td>
<td>-2 392 909</td>
<td>-1 365 779</td>
<td>-3 171 721</td>
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<td>388</td>
<td>388</td>
<td>388</td>
<td>48 960</td>
</tr>
<tr>
<td>rolling stock [-]</td>
<td>940</td>
<td>776</td>
<td>776</td>
<td>776</td>
<td>48 960</td>
</tr>
<tr>
<td>covered [%]</td>
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<td>99.32</td>
<td>99.32</td>
<td>99.23</td>
<td>100</td>
</tr>
<tr>
<td>time [sec]</td>
<td>11</td>
<td>8</td>
<td>86 627</td>
<td>88 342</td>
<td>2</td>
</tr>
</tbody>
</table>
Figure: Breakdown of the passenger satisfaction for various timetables under the 2014 demand
Passenger Satisfaction [mNIS]

-4.4
-4.6
-4.8
-5.0
-5.2
-5.4
-5.6

0 5 10 15 20 25 30

\theta [\text{min}]

--- non-cyclic

--- \theta shifted

--- cyclic
Passenger Satisfaction [mNIS]

-4.4

-4.6

-4.8

-5.0

-5.2

-5.4

-5.6

0 20 40 60 80 100

\[\xi\] [%]

- - - non-cyclic

- - - cyclic

\[\xi\] partial
Conclusion
Case Study

- Difference in Pax. Sat. between cyclic and non-cyclic timetable: 18.5%
- $\theta$ Shifted Timetable can reduce the difference to a half
- $\xi$ Partially Cyclic can diminish the difference already at $\xi = 60$ with a train ratio $3:1$
- Hybrid Cyclic finds the same ratio, provides good level of service

General

- As the demand is time dependent, purely cyclic timetable is not a good option
- Hybrid cyclic timetable can diminish the impact of the cyclicity constraints
Future Work
• Elastic Demand
• Need of an opt-out
• Maximize Profit
• Adapt Pricing Scheme
The regularity of a habit is generally in proportion to its absurdity.

(Marcel Proust)
References

