Multi-class speed-density relationship for pedestrian traffic

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Outline

1. Introduction
2. Methodology
3. Case study
   - Empirical analysis
   - Model specification and estimation
4. Conclusion and future work
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1 Introduction

2 Methodology

3 Case study
   - Empirical analysis
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4 Conclusion and future work
Fundamental relationships

- Play an important role in the field: design and planning; model input or calibration criterion
- Modeling assumption: the traffic system is at equilibrium - homogenous and stationary
Speed-density relationships for pedestrian traffic

Deterministic approach

- **Empirically derived models** [Older, 1968; Tregenza, 1976; Weidmann, 1993; Rastogi et al., 2013]
- **Simulation-based models** [Blue and Adler, 1998]
- **Theory-based models** [Flötteröd and Lämmel, 2015]

Empirical observations

- Scatter: violation of the equilibrium assumptions

Probabilistic approach

- **Data-driven PedProb-vk** [Nikolić et al., 2016]
- Superior compared to deterministic approaches from the literature
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Behavioral approach

Assumptions

• Pedestrian population is heterogeneous (e.g. trip purpose, age, gender, etc.)
• Heterogeneity leads to the existence of multiple pedestrian classes
• Classes are characterized by different types of behavior
• Latent class modeling approach to capture unobserved heterogeneity
Multi-class speed-density relationship (MC-vk)

Model structure

\[ P(v_i|k_i) = \sum_{c=1}^{C} P(v_i|k_i, c)P(c|X_i) \]

- \( P(v_i|k_i, c) \): class-specific model
- \( P(c|X_i) \): class membership model

- \( i \): pedestrian identifier, \( i = 1, ..., N \)
- \( v_i \): speed of pedestrian \( i \)
- \( k_i \): density for pedestrian \( i \)
- \( c \): class identifier, \( C \) - number of classes
- \( X_i \): characteristics associated to pedestrian \( i \)
Class-specific speed-density relationship

Social Force Model

\[
\ddot{a}_i = \frac{\vec{v}_i^f - \vec{v}_i}{\tau_i} - C_i \sum_j \exp\left(-\frac{R_{ij}}{B_i}\right)\vec{n}_{ij}(\lambda_i + (1 - \lambda_i)\frac{1 + \cos(\phi_{ij})}{2})
\]

[Helbing and Molnár, 1995]
Class-specific speed-density relationship

**Isotropy** \((\lambda_i = 1)\)

\[
a_i = \frac{v_i^f - v_i}{\tau_i} - C_i \sum_j \exp\left(-\frac{R_{ij}}{B_i}\right) = \frac{v_i^f - v_i}{\tau_i} - C_i k_i
\]

**Stationarity** \((a_i = 0)\)

\[
v_i = v_i^f - \gamma_i k_i
\]

**Homogeneity** (all pedestrians have the same movement parameters)

\[
v_i = v = v_f - \gamma k_i
\]
Class membership model

- It cannot be deterministically identified to which class a pedestrian belongs
- Probability that a pedestrian $i$, associated with characteristics $X_i$ (e.g. trip purpose, age, gender, etc.), belong to a latent class $c$: for each pedestrian there is a utility associated to each class $c$

**Specification of utilities**

$$U_i^c = ASC^c + \beta^c X_i + \xi_i^c$$

$V_i^c$: deterministic part of utilities
$\xi_i^c$: error term
Multi-class speed-density relationship (MC-vk)

Class-specific model: \( P(v_i|k_i, c) \)

\[
v_i^c = v_f^c - \gamma^c k_i + \epsilon_i^c
\]

\( P(v_i|k_i, c) \) is determined by \( \epsilon_i^c \)

Class membership model: \( P(c|X_i) \)

\[
U_i^c = ASC^c + \beta^c X_i + \xi_i^c
\]

\( P(c|X_i) \) is determined by \( \xi_i^c \)

Likelihood of the sample

\[
\mathcal{L} = \prod_{i=1}^{N} P(v_i|k_i) = \prod_{i=1}^{N} \sum_{c=1}^{C} P(v_i|k_i, c)P(c|X_i)
\]
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Lausanne railway station
Data set

Pedestrian underpass

- A large-scale network of smart sensors: a sparsity driven tracking framework [Alahi et al., 2014]
- Dataset: 25,603 trajectories, collected between 07:00 and 08:00 on February 12, 13, 14, 15 and 18, 2013
- The average length of the trajectories: 78 meters
- The duration of a pedestrians’ stay: from 15 seconds to 2.2 minutes
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Speed-density relationship
Pedestrian types

Classification based on origins and destinations

1: Arriving passenger - pedestrians originating from a platform and exiting the station
2: Departing passenger - pedestrians walking to a platform to embark on their trains
3: Transferring passenger - pedestrians whose origin and destination are different platforms
4: Non-passenger - pedestrians whose origin and destination are different from a platform (e.g. pedestrians that go shopping in the station)
Pedestrian types

Number of pedestrians per pedestrian type

![Bar chart showing the number of pedestrians per type (arriving, departing, transferring, non-passenger) for different months (Feb 12 to Feb 18).]
Pedestrian types

Speed distribution per pedestrian type
Train timetable

Time to departure

![Graph showing distribution of time to departure]
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Specification issues

Panel data
- Data collected over multiple time periods for the same sample of individuals

Serial correlation
- The observations across time for a single pedestrian are likely to be correlated, due to the unobserved factors related to a pedestrian that exist over time
- \( \epsilon_{i(t-1)}^c \) cannot be assumed independent from \( \epsilon_{it}^c \)
- If ignored - consistent but not efficient estimators
Multi-class speed-density relationship (MC-vk)

Class-specific model: \( P(v_i|k_i, c) \)

\[
v^c_{it} = v^c_f - \gamma^c k_{it} + \alpha_i^c + \epsilon^c_{it}
\]

\( P(v_i|k_i, c) \) is determined by \( \epsilon^c_{it} \), \( \alpha_i^c \) is an agent effect

Class membership model: \( P(c|X_i) \)

\[
U^c_i = ASC^c + \beta^c X_i + \xi^c_i
\]

\( P(c|X_i) \) is determined by \( \xi^c_i \)

Likelihood of the sample

\[
\mathcal{L} = \prod_{i=1}^{N} \sum_{c=1}^{C} \left\{ \frac{1}{R} \sum_{r} \exp \left( \sum_{t=1}^{T} \log P(v_i|k_i, c, \alpha^c_r) \right) \right\} P(c|X_i)
\]
Assumptions

Number of classes

1. Pedestrians sensitive to congestion
2. Rushing pedestrians
3. Pedestrians non-sensitive to congestion

Class membership model

• Explanatory variables: time to diparture, type of pedestrian
• Logit model

Class specific model

• The same functional form of v-k for each class
• $\epsilon'_{it} \sim N(\mu, \sigma^c)$
• $\alpha'_{i} \sim N(\mu, \eta^c)$
Estimation results

Class membership model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Std.err.</th>
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</table>

S - Pedestrians sensitive to congestion
R - Rushing pedestrians
NS - Pedestrians non-sensitive to congestion

Class specific model

<table>
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</tbody>
</table>
How many classes?

Bayesian information criterion - $BIC$

<table>
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<th>Model</th>
<th>1 class</th>
<th>2 classes</th>
<th>3 classes</th>
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</thead>
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<td>1069424.602</td>
</tr>
</tbody>
</table>
Class profiling

Shares

- Sensitive to congestion: 86%
- Rushing: 9%
- Non-sensitive to congestion: 5%

Bar charts for Non-passerger, Transferring, Departing, Arriving categories.
Class profiling

Average time to departure

![Diagram showing average time to departure for different classes: sensitive to congestion, Rushing, and non-sensitive to congestion. The chart illustrates the average time in seconds for each category, with varying data points for each class.]

- **Sensitive to congestion**
- **Rushing**
- **Non-sensitive to congestion**
Model comparison

Average behavior

\[ \bar{v}_{MC-vk} = \sum_{c=1}^{C} \left\{ \frac{1}{N} \sum_{i=1}^{N} P(c|X_i; \beta^c) v^c(k; \theta^c) \right\} \]

<table>
<thead>
<tr>
<th>Model</th>
<th>Weidmann</th>
<th>Tregenza</th>
<th>Rastogi</th>
<th>Linear</th>
<th>PedProb-vk</th>
<th>MC-vk</th>
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</thead>
<tbody>
<tr>
<td>( M SE )</td>
<td>( 5.34 \times 10^{-3} )</td>
<td>( 4.82 \times 10^{-3} )</td>
<td>( 4.42 \times 10^{-3} )</td>
<td>( 5.59 \times 10^{-3} )</td>
<td>( 4.02 \times 10^{-3} )</td>
<td>( 1.72 \times 10^{-3} )</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>( 2.38 \times 10^{-1} )</td>
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<td>( 4.29 \times 10^{-1} )</td>
<td>( 7.54 \times 10^{-1} )</td>
</tr>
</tbody>
</table>
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Conclusion and future work

Conclusion

• MC-vk: latent class modeling approach to capture heterogeneity in pedestrian population
• Satisfying behavioral interpretation
• Good performance at the aggregate level

Future work

• Additional factors
  – Walking in groups
  – Peak intervals
  – Attractiveness of origins/destinations
Thank you

9th TRIENNIAL SYMPOSIUM ON TRANSPORTATION ANALYSIS (TRISTAN IX), Aruba:
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Pedestrian underpass West

1: South entrance
2 - 4: Stairs (resp. ramp) to platform 9
3: Coop Pronto Supermarket
5 - 6: Stairs (resp. ramp) to platform 7 and 8
7 - 8: Stairs (resp. ramp) to platform 5 and 6
9 - 10: Stairs (resp. ramp) to platform 3 and 4
11: Stairs to platform 1 and out of the station
12: Access ramp
13: Stairs to or out of the train station and to buses
14: Pathway leading to buses and metro (M2)
Group behavior

A group of pedestrians walking together

Given spatial threshold $\varepsilon$, speed threshold $\theta$, directional threshold $\varphi$ and temporal threshold $k$ a group of at least 2 pedestrians that are density-connected w.r.t. $\varepsilon$, $\theta$, $\varphi$ during at least $k$ time periods (not necessarily consecutive time periods) represent a group of pedestrians walking together.

Spatial clustering

Density-based clustering - grouping of data into categories based on $\varepsilon$ (2.1336m), $\theta$ (0.1524m/s), $\varphi$ ($3^\circ$)

Temporal clustering

Frequent pattern analysis - finds sets of density-based clusters that are frequently observed together (w.r.t $k$ - temporal threshold, relative to the total time a pedestrian travels in the corridor)
Peak periods during morning rush hour

Number of pedestrians over time
# Peak periods per day

## February 12
07:10 - 07:15, 07:25 - 07:30, 07:50 - 07:55

## February 13
07:15 - 07:20, 07:40 - 07:45

## February 14
07:10 - 07:15, 07:40 - 07:45

## February 15
07:10 - 07:15, 07:25 - 07:30, 07:40 - 07:45

## February 18
07:10 - 07:15, 07:40 - 07:45
Peak/off-peak analysis
## Weather

<table>
<thead>
<tr>
<th>Day</th>
<th>Temperature</th>
<th>Rain/Sun</th>
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</thead>
<tbody>
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<td>Sun</td>
</tr>
<tr>
<td>13 February</td>
<td>-1.6°C</td>
<td>Rain</td>
</tr>
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<td>14 February</td>
<td>-3.2°C</td>
<td>Rain</td>
</tr>
<tr>
<td>15 February</td>
<td>0.5°C</td>
<td>Sun</td>
</tr>
<tr>
<td>18 February</td>
<td>-0.3°C</td>
<td>Sun</td>
</tr>
</tbody>
</table>
OD pattern
Number of pedestrians per origin

![Graph showing time series of pedestrian counts per origin from February 2012 to 2018.](image-url)
Number of pedestrians per destination
OD distances
OD distances analysis
Indicators

Trajectory - a finite collection of triples

\[ p_{is} = (x_{is}, y_{is}, t_s), t_s = (t_0, t_1, \ldots, t_f) \]

Density

\[ k_{is} = \frac{n_{is}^{\text{real}} + n_{is}^{\text{imputed}}}{|V_{is}|} \]
Indicators

Trajectory - a finite collection of triples

\[ p_{is} = (x_{is}, y_{is}, t_s), t_s = (t_0, t_1, \ldots, t_f) \]

Speed

\[ v_{is} = \sqrt{\left( \frac{\Delta x_{is}}{\Delta t} \right)^2 + \left( \frac{\Delta y_{is}}{\Delta t} \right)^2} \]

\[ \Delta x_{is} = x_{i,s+1} - x_{i,s-1}, \quad \Delta y_{is} = y_{i,s+1} - y_{i,s-1} \]

\[ \Delta t = t_{s+1} - t_{s-1} \]