Incorporating advanced behavioral models in integer optimization

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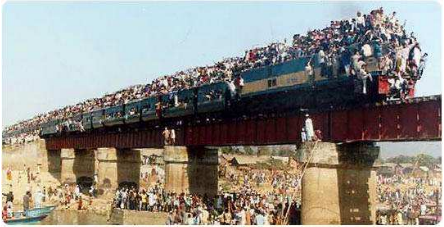
Outline

Operations research



- Usually in OR:
- optimization of the supply
- for a given (fixed) demand

Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand functions: $P = f(Q)$
- Inverse demand: $Q = f^{-1}(P)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

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Demand-supply interactions

Operations Research

- Given the demand...
- configure the system

Johnson City Enterprise.

Published Every Saturday,

$1. per year—Advance Payment.

SATURDAY, APRIL 7, 1883.

TIME TABLE

E. T., V. & G. R. R.

PASSENGER,	ARRIVES,
No. 1, West,	6:37, a. m.
No. 2, East,	9:45, p. m.
No. 3, West,	11:51, p.m.
No. 4, East,	3:56, a. m.
LOCAL FREIGHT,	ARRIVES,
No. 5,	7:20, a. m.
No. 8,	6:20, p. m.

JNO. W. EAKIN, Agent.

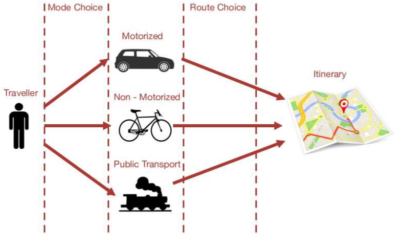
E. T. & W. N. C. R. R.

Passenger, leaves,	7, a. m.
" arrives,	6, p. m.

J. C. HARDIN, Agent.

Behavioral models

- Given the configuration of the system...
- predict the demand



Demand-supply interactions

Multi-objective optimization

Minimize costs

Maximize satisfaction

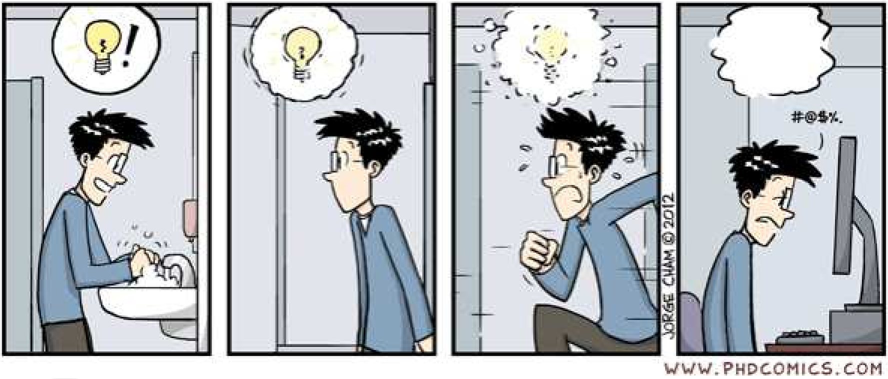
Disaggregate demand in optimization

Issues

- Highly non linear
- Highly non convex
- Literature contains some successful instances
- Relatively easy when decision variables = availability
- Difficult when decision variables in utility (e.g. pricing)

Outline

The main idea



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The main idea

Linearization

Hopeless to linearize the logit formula (we tried...)

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- We define lower and upper bounds

$$\ell_{inr} \leq U_{inr} \leq m_{inr}, \ \forall i, n, r.$$

and

$$M_{inr} = m_{inr} - \ell_{inr}.$$

Availability

Decision variable (supply)

$$y_{in} = 1 \text{ if } i \in \mathcal{C}_n, 0 \text{ otherwise}$$

Capacity reached (demand)

$$y_{inr} = 1 \text{ if } i \text{ is full for scenario } r, 0 \text{ otherwise}$$

Relation

$$y_{inr} \leq y_{in}, \ \forall i, n, r.$$

Utility and availability

New variable

$$\bar{U}_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{inr} & \text{if } y_{inr} = 0 \end{cases}$$

Modeling

$$\begin{aligned} \ell_{inr} &\leq \bar{U}_{inr} \\ \bar{U}_{inr} &\leq \ell_{inr} + M_{inr} y_{inr} \\ U_{inr} - M_{inr}(1 - y_{inr}) &\leq \bar{U}_{inr} \\ \bar{U}_{inr} &\leq U_{inr}. \end{aligned}$$

Utility and availability

Modeling

$$
\begin{aligned}
\ell_{inr} &\leq \bar{U}_{inr} \\
\bar{U}_{inr} &\leq \ell_{inr} + M_{inr} y_{inr} \\
U_{inr} - M_{inr}(1 - y_{inr}) &\leq \bar{U}_{inr} \\
\bar{U}_{inr} &\leq U_{inr}.
\end{aligned}
$$

$y_{inr} = 1$

$$
\begin{aligned}
\bar{U}_{inr} &\leq \ell_{inr} + M_{inr} = m_{inr} \\
U_{inr} &\leq \bar{U}_{inr} \\
U_{inr} &= \bar{U}_{inr}
\end{aligned}
$$

$y_{inr} = 0$

$$
\begin{aligned}
\bar{U}_{inr} &\leq \ell_{inr} \\
U_{inr} - M_{inr} &\leq \bar{U}_{inr} \\
\bar{U}_{inr} &= \ell_{inr}
\end{aligned}
$$

Choice

Maximum utility

$$U^*_{nr} = \max_{i \in \mathcal{C}_n} \bar{U}_{inr}.$$

Choice

$$w_{inr} = \begin{cases} 1 & \text{if } U^*_{nr} = \bar{U}_{inr} \\ 0 & \text{otherwise} \end{cases}$$

Choice

Modeling

$$
\begin{aligned}
M^*_{nr} &= \max_i M_{inr} \\
\bar{U}_{inr} &\leq U^*_{nr} \\
U^*_{nr} &\leq \bar{U}_{inr} + M^*_{nr}(1 - w_{inr}) \\
\sum_i w_{inr} &= 1
\end{aligned}
$$

$w_{inr} = 1$

$$U^*_{nr} \leq \bar{U}_{inr}$$
$$U^*_{nr} = \bar{U}_{inr}$$

$w_{inr} = 0$

$$U^*_{nr} \leq \bar{U}_{inr} + M^*_{nr}$$

Demand and revenues

Demand

$$D_i = \frac{1}{R} \sum_{n=1}^{n} \sum_{r=1}^{R} w_{inr}.$$

Revenues

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}.$$

Revenues

Non linear specification

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}.$$

Predetermined price levels

Price levels: p_{in}^{ℓ}, $\ell = 1, \ldots, L_{in}$

$$p_{in} = \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} p_{in}^{\ell}.$$

New decision variables

$\lambda_{in\ell} \in \{0, 1\}$

$$\sum_{\ell=1}^{L_{in}} \lambda_{in\ell} = 1.$$

Revenues

Non linear function

$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} p_{in}^{\ell} \sum_{r=1}^{R} w_{inr}.$$

Linearization

$$\alpha_{inr\ell} = \lambda_{in\ell} w_{inr}$$

Linear specification of revenues

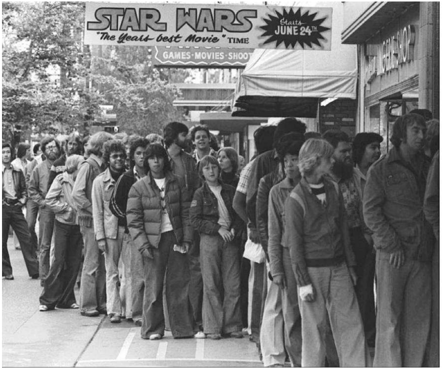
$$R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} \sum_{\ell=1}^{L_{in}} \alpha_{inr\ell} p_{in}^{\ell},$$

with

$$\lambda_{in\ell} + w_{inr} \leq 1 + \alpha_{inr\ell}, \forall i, n, r, \ell,$$
$$\alpha_{inr\ell} \leq \lambda_{in\ell}, \forall i, n, r, \ell,$$
$$\alpha_{inr\ell} \leq w_{inr}, \forall i, n, r, \ell.$$

Dealing with capacities

- Demand may exceed supply
- Not every choice can be accommodated
- Difficulty: who has access?
- Assumption: priority list is exogenous



Priority list

Application dependent

- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework

The list of customers must be sorted

Dealing with capacities

Variables

- y_{in}: decision of the operator
- y_{inr}: availability

Constraints

$$\sum_{n=1}^{N} w_{inr} \leq c_i$$

$$y_{inr} \leq y_{in}$$

$$y_{i(n+1)r} \leq y_{inr}$$

Dealing with capacities

Modeling

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + c_i(1 - y_{in})$$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr})$$

$y_{in} = 0$ so that $y_{inr} = 0$

$$c_i \leq \sum_{m=1}^{n-1} w_{imr} + c_i$$

$$\sum_{m=1}^{n-1} w_{imr} \leq (n-1)$$

Dealing with capacities

Modeling

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + c_i(1 - y_{in})$$

$$\sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1)y_{inr} + (n-1)(1 - y_{inr})$$

$y_{in} = 1, \; y_{inr} = 1$

$$0 \leq \sum_{m=1}^{n-1} w_{imr}$$

$$\sum_{m=1}^{n-1} w_{imr} \leq c_i - 1$$

$y_{in} = 1, \; y_{inr} = 0$

$$c_i \leq \sum_{m=1}^{n-1} w_{imr}$$

$$\sum_{m=1}^{n-1} w_{imr} \leq n - 1$$

General framework

Choice models

- logit
- MEV
- mixtures
- hybrid
- panel effects
- random regret
- etc.

Regret minimization

Model specification

$$R_{ij}^m = \max\left\{0 + \nu_{0m}, \beta_m(x_{jm} - x_{im}) + \nu_{xm}\right\}$$

$$R_{ij} = \sum_m R_{ij}^m$$

$$R_i = \sum_{j \neq i} R_{ij}$$

$$i^* = \operatorname{argmin}_i R_i - \varepsilon_i = \operatorname{argmax}_i -R_i + \varepsilon_i$$

Regret minimization

$$R_{ij}^m = \max\left\{0 + \nu_{0m}, \beta_m(x_{jm} - x_{im}) + \nu_{xm}\right\}$$

Modeling

$$
\begin{aligned}
R_{ij}^m &\geq \nu_{0m} \\
R_{ij}^m &\geq \beta_m(x_{jm} - x_{im}) + \nu_{xm} \\
R_{ij}^m &\leq \nu_{0m} + \delta_{ij}^m M_{ij}^m \\
R_{ij}^m &\leq \beta_m(x_{jm} - x_{im}) + \nu_{xm} + (1 - \delta_{ij}^m) M_{ij}^m
\end{aligned}
$$

Regret minimization

Generalization of the framework

- The linear formulation of maximum utility is easily extended
- The generalization is still linear
- Increase in complexity (more variables and constraints).

Outline

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_i
- What price? p_{in}

Back to the example: pricing

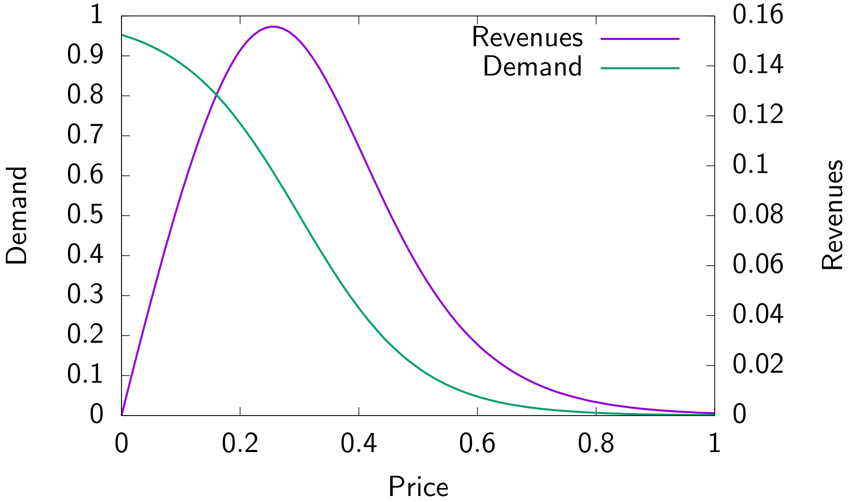
Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an homogeneous population of N individuals

$$U_c = 0 + \varepsilon_c$$
$$U_m = \beta_c p_m + \varepsilon_m$$

- $\beta_c < 0$
- Logit model: ε_m i.i.d. EV

Demand and revenues



Optimization (with GLPK)

Data

- $N = 1$
- $R = 100$
- $U_m = -10p_m + 3$
- Prices: 0.10, 0.20, 0.30, 0.40, 0.50

Results

- Optimum price: 0.3
- Demand: 56%
- Revenues: 0.168

Heterogeneous population

Two groups in the population

$$U_{in} = -\beta_n p_i + c_n$$

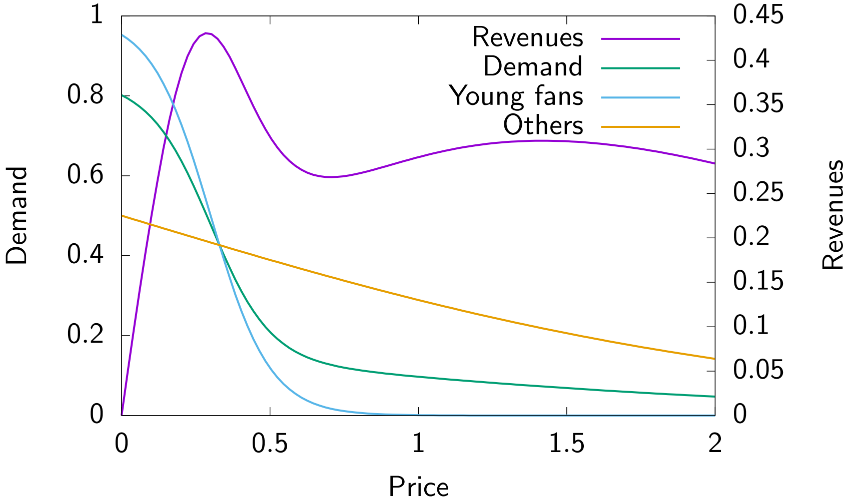
Young fans: 2/3

$\beta_1 = -10,\ c_1 = 3$

Others: 1/3

$\beta_1 = -0.9,\ c_1 = 0$

Demand and revenues



Optimization

Data

- $N = 3$
- $R = 100$
- $U_{m1} = -10p_m + 3$
- $U_{m2} = -0.9p_m$
- Prices: 0.3, 0.7, 1.1, 1.5, 1.9

Results

- Optimum price: 0.3
- Customer 1 (fan): 60% [theory: 50 %]
- Customer 2 (fan) : 49% [theory: 50 %]
- Customer 3 (other) : 45% [theory: 43 %]
- Demand: 1.54 (51%)
- Revenues: 0.48

Two theaters, different types of films

Two theaters, different types of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Tinker Tailor Soldier Spy

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, different types of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + 4$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + 0$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.6
- 4 young customers: 0
- 2 old customers: 0.5
- Demand: 0.5 (8.3%)
- Revenues: 0.8

Theater k

- Optimum price m: 0.5
- Young customers: 0.8
- Old customers: 1.5
- Demand: 2.3 (38%)
- Revenues: 1.15

Two theaters, same type of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Star Wars Episode VIII

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, same type of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + 4$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + 4$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Young customers: 0
- Old customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater k

Closed

Two theaters with capacity, different types of films

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4, \; n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m, \; n = 3, 6$
- $U_{kn} = -10p_k + 0, \; n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k, \; n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Demand: 0.2 (3.3%)
- Revenues: 0.36

Theater k

- Optimum price m: 0.5
- Demand: 2 (33.3%)
- Revenues: 1.15

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	k	2	1
4	0	2	1
5	0	2	1
6	k	2	0

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	k	2	1
3	0	2	1
4	k	2	0
5	0	2	0
6	0	2	0

Outline

Choice models in integer optimization

Discrete choice models

- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation

- Linear in the decision variables
- Large scale
- Fairly general
- Easily extended for regret minimization