Incorporating advanced behavioral models in integer optimization

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September 13, 2016
Demand in optimization

1. Linear representation of demand
   - Availability
   - Choice
   - Aggregate indicators
   - Capacities

2. Regret minimization
3. A simple example
   - Example: one theater
   - Example: two theaters
   - Example: two theaters with capacities
4. Conclusion
Operations research

- Usually in OR:
- optimization of the supply
- for a given (fixed) demand
Aggregate demand

- Homogeneous population
- Identical behavior
- Price \((P)\) and quantity \((Q)\)
- Demand functions: \(P = f(Q)\)
- Inverse demand: \(Q = f^{-1}(P)\)
Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.
Demand-supply interactions

Operations Research
- Given the demand...
- configure the system

Behavioral models
- Given the configuration of the system...
- predict the demand

Johnson City Enterprise.
Published Every Saturday,
$1 per year—Advance Payment.
Saturday, April 7, 1883.

TIME TABLE
E. T., V. & G. R.

E. T. & W. N. C. R.
Passenger, leaves, 7 a.m.
arrives, 8 p.m.
Jno. W. Eakin, Agent.

E. T. & W. N. C. R.
No. 1, West, 6:37 a.m.
No. 2, East, 9:45 p.m.
No. 3, West, 11:51 p.m.
No. 4, East, 3:36 a.m.
Local Freight, arrives, 7:20 a.m.
No. 5, 6:20 p.m.
No. 8, Jno. W. Eakin, Agent.

Mode Choice
- Motorized
- Non-Motorized
- Public Transport

Route Choice
- Itinerary

Traveler

Bierlaire (EPFL)
Demand-supply interactions

Multi-objective optimization

Minimize costs

Maximize satisfaction

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Disaggregate demand in optimization

Issues

- Highly non linear
- Highly non convex
- Literature contains some successful instances
- Relatively easy when decision variables = availability
- Difficult when decision variables in utility (e.g. pricing)
Outline

1 Demand in optimization
   • Availability
   • Choice
   • Aggregate indicators
   • Capacities

2 Linear representation of demand
   • Availability
   • Choice
   • Aggregate indicators
   • Capacities

3 Regret minimization
   • A simple example
     • Example: one theater
     • Example: two theaters
     • Example: two theaters with capacities

4 Conclusion
The main idea
The main idea

Linearization

Hopeless to linearize the logit formula (we tried...)

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability
A linear formulation

Utility function

\[ U_{in} = V_{in} + \varepsilon_{in} = \sum_{k} \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}. \]

Simulation

- Assume a distribution for \( \varepsilon_{in} \)
- E.g. logit: i.i.d. extreme value
- Draw \( R \) realizations \( \xi_{inr} \), \( r = 1, \ldots, R \)
- The choice problem becomes deterministic
### Scenarios

#### Draws

- Draw $R$ realizations $\xi_{inr}$, $r = 1, \ldots, R$.
- We obtain $R$ scenarios

\[
U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.
\]

- We define lower and upper bounds

\[
\ell_{inr} \leq U_{inr} \leq m_{inr}, \forall i, n, r.
\]

and

\[
M_{inr} = m_{inr} - \ell_{inr}.
\]
Availability

Decision variable (supply)

\[ y_{in} = 1 \text{ if } i \in C_n, 0 \text{ otherwise} \]

Capacity reached (demand)

\[ y_{inr} = 1 \text{ if } i \text{ is full for scenario } r, 0 \text{ otherwise} \]

Relation

\[ y_{inr} \leq y_{in}, \forall i, n, r. \]
Utility and availability

New variable

\[ \bar{U}_{inr} = \begin{cases} U_{inr} & \text{if } y_{inr} = 1 \\ \ell_{inr} & \text{if } y_{inr} = 0 \end{cases} \]

Modeling

\[ \begin{align*}
\ell_{inr} & \leq \bar{U}_{inr} \\
\bar{U}_{inr} & \leq \ell_{inr} + M_{inr}y_{inr} \\
U_{inr} - M_{inr}(1 - y_{inr}) & \leq \bar{U}_{inr} \\
\bar{U}_{inr} & \leq U_{inr}.
\end{align*} \]
Utility and availability

Modeling

\[
\ell_{inr} \leq \bar{U}_{inr}
\]
\[
\bar{U}_{inr} \leq \ell_{inr} + M_{inr}y_{inr}
\]
\[
U_{inr} - M_{inr}(1 - y_{inr}) \leq \bar{U}_{inr}
\]
\[
\bar{U}_{inr} \leq U_{inr}.
\]

\[y_{inr} = 1\]
\[\bar{U}_{inr} \leq \ell_{inr} + M_{inr} = m_{inr}\]
\[U_{inr} \leq \bar{U}_{inr}\]
\[U_{inr} = \bar{U}_{inr}\]

\[y_{inr} = 0\]
\[\bar{U}_{inr} \leq \ell_{inr}\]
\[U_{inr} - M_{inr} \leq \bar{U}_{inr}\]
\[\bar{U}_{inr} = \ell_{inr}\]
Maximum utility

\[ U^*_n = \max_{i \in C_n} \bar{U}_{inr}. \]

Choice

\[ w_{inr} = \begin{cases} 
1 & \text{if } U^*_n = \bar{U}_{inr} \\
0 & \text{otherwise}
\end{cases} \]
Choice

Modeling

\[ \begin{align*}
M^*_{nr} &= \max_i M_{inr} \\
\bar{U}_{inr} &\leq U^*_nr \\
U^*_nr &\leq \bar{U}_{inr} + M^*_{nr} (1 - w^*_{inr}) \\
\sum_i w^*_{inr} &= 1
\end{align*} \]

\[ w^*_{inr} = 1 \quad \text{or} \quad w^*_{inr} = 0 \]

\[ U^*_nr \leq \bar{U}_{inr} \quad \text{or} \quad U^*_nr = \bar{U}_{inr} \]

\[ U^*_nr \leq \bar{U}_{inr} + M^*_{nr} \]
Demand and revenues

Demand

\[ D_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} p_{in} \sum_{r=1}^{R} w_{inr}. \]

Revenues

\[ R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} w_{inr}. \]
Revenues

Non linear specification

\[ R_i = \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}. \]

Predetermined price levels

Price levels: \( p_{in}^\ell, \ell = 1, \ldots, L_{in} \)

\[ p_{in} = \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} p_{in}^\ell. \]

New decision variables

\( \lambda_{in\ell} \in \{0, 1\} \)

\[ \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} = 1. \]
Revenues

Non linear function

\[ R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{\ell=1}^{L_{in}} \lambda_{in\ell} p_{in}^{\ell} \sum_{r=1}^{R} w_{inr}. \]

Linearization

\[ \alpha_{inr\ell} = \lambda_{in\ell} w_{inr}. \]
Linear specification of revenues

\[ R_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} \sum_{\ell=1}^{L_{in}} \alpha_{inr\ell} p_{in}^{\ell}, \]

with

\[ \lambda_{in\ell} + w_{inr} \leq 1 + \alpha_{inr\ell}, \forall i, n, r, \ell, \]
\[ \alpha_{inr\ell} \leq \lambda_{in\ell}, \forall i, n, r, \ell, \]
\[ \alpha_{inr\ell} \leq w_{inr}, \forall i, n, r, \ell. \]
Dealing with capacities

- Demand may exceed supply
- Not every choice can be accommodated
- Difficulty: who has access?
- Assumption: priority list is exogenous
Priority list

Application dependent
- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework
The list of customers must be sorted
Dealing with capacities

Variables
- $y_{in}$: decision of the operator
- $y_{inr}$: availability

Constraints
\[
\sum_{n=1}^{N} w_{inr} \leq c_i \\
y_{inr} \leq y_{in} \\
y_{i(n+1)r} \leq y_{inr}
\]
Dealing with capacities

Modeling

\[ c_i (1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + c_i (1 - y_{in}) \]

\[ \sum_{m=1}^{n-1} w_{imr} \leq (c_i - 1) y_{inr} + (n - 1) (1 - y_{inr}) \]

\[ y_{in} = 0 \text{ so that } y_{inr} = 0 \]

\[ c_i \leq \sum_{m=1}^{n-1} w_{imr} + c_i \]

\[ \sum_{m=1}^{n-1} w_{imr} \leq (n - 1) \]
Dealing with capacities

Modeling

\[
\begin{align*}
  c_i(1 - y_{inr}) & \leq \sum_{m=1}^{n-1} w_{imr} + c_i(1 - y_{in}) \\
  \sum_{m=1}^{n-1} w_{imr} & \leq (c_i - 1)y_{inr} + (n - 1)(1 - y_{inr})
\end{align*}
\]

- \( y_{in} = 1, y_{inr} = 1 \)

- \( y_{in} = 1, y_{inr} = 0 \)

\[
\begin{align*}
  0 & \leq \sum_{m=1}^{n-1} w_{imr} \\
  \sum_{m=1}^{n-1} w_{imr} & \leq c_i - 1 \\
  c_i & \leq \sum_{m=1}^{n-1} w_{imr} \\
  \sum_{m=1}^{n-1} w_{imr} & \leq n - 1
\end{align*}
\]
Choice models

- logit
- MEV
- mixtures
- hybrid
- panel effects
- random regret
- etc.
Regret minimization

Model specification

\[
R_{ij}^m = \max \left\{ 0 + \nu_{0m}, \beta_m(x_{jm} - x_{im}) + \nu_{xm} \right\}
\]

\[
R_{ij} = \sum_m R_{ij}^m
\]

\[
R_i = \sum_{j \neq i} R_{ij}
\]

\[
i^* = \arg\min_i R_i - \varepsilon_i = \arg\max_i -R_i + \varepsilon_i
\]
Regret minimization

\[ R_{ij}^m = \max \{0 + \nu_0m, \beta_m(x_{jm} - x_{im}) + \nu_{xm}\} \]

Modeling

\[ R_{ij}^m \geq \nu_0m \]
\[ R_{ij}^m \geq \beta_m(x_{jm} - x_{im}) + \nu_{xm} \]
\[ R_{ij}^m \leq \nu_0m + \delta_{ij}^m M_{ij}^m \]
\[ R_{ij}^m \leq \beta_m(x_{jm} - x_{im}) + \nu_{xm} + (1 - \delta_{ij}^m) M_{ij}^m \]
Regret minimization

Generalization of the framework

- The linear formulation of maximum utility is easily extended
- The generalization is still linear
- Increase in complexity (more variables and constraints).
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Data
- $C$: set of movies
- Population of $N$ individuals
- Utility function:
  \[ U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in} \]

Decision variables
- What movies to propose? $y_i$
- What price? $p_{in}$
Data

- Two alternatives: my theater \( (m) \) and the competition \( (c) \)
- We assume an homogeneous population of \( N \) individuals

\[
U_c = 0 + \varepsilon_c \\
U_m = \beta_c p_m + \varepsilon_m
\]

- \( \beta_c < 0 \)

- Logit model: \( \varepsilon_m \) i.i.d. EV
Demand and revenues
Optimization (with GLPK)

Data

- \( N = 1 \)
- \( R = 100 \)
- \( U_m = -10p_m + 3 \)
- Prices: 0.10, 0.20, 0.30, 0.40, 0.50

Results

- Optimum price: 0.3
- Demand: 56%
- Revenues: 0.168
Heterogeneous population

Two groups in the population

\[ U_{in} = -\beta_n p_i + c_n \]

Young fans: 2/3
\[ \beta_1 = -10, \ c_1 = 3 \]

Others: 1/3
\[ \beta_1 = -0.9, \ c_1 = 0 \]
Demand and revenues

![Graph showing demand and revenues as functions of price. The graph includes lines representing different groups (Young fans, Others) with corresponding revenues and demand.](image)
A simple example
Example: one theater

Optimization

Data
- $N = 3$
- $R = 100$
- $U_{m1} = -10p_m + 3$
- $U_{m2} = -0.9p_m$
- Prices: 0.3, 0.7, 1.1, 1.5, 1.9

Results
- Optimum price: 0.3
- Customer 1 (fan): 60% [theory: 50 %]
- Customer 2 (fan): 49% [theory: 50 %]
- Customer 3 (other): 45% [theory: 43 %]
- Demand: 1.54 (51%)
- Revenues: 0.48
Two theaters, different types of films
Two theaters, different types of films

**Theater $m$**
- Expensive
- Star Wars Episode VII

**Theater $k$**
- Cheap
- Tinker Tailor Soldier Spy

**Heterogeneous demand**
- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)
Two theaters, different types of films

Data

- Theaters $m$ and $k$
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + 4$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + 0$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices $m$: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices $k$: half price

Theater $m$

- Optimum price $m$: 1.6
- 4 young customers: 0
- 2 old customers: 0.5
- Demand: 0.5 (8.3%)
- Revenues: 0.8

Theater $k$

- Optimum price $m$: 0.5
- Young customers: 0.8
- Old customers: 1.5
- Demand: 2.3 (38%)
- Revenues: 1.15
Two theaters, same type of films

**Theater** \( m \)
- Expensive
- Star Wars Episode VII

**Theater** \( k \)
- Cheap
- Star Wars Episode VIII

Heterogeneous demand
- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)
Two theaters, same type of films

Data

- Theaters \( m \) and \( k \)
- \( N = 6 \)
- \( R = 10 \)
- \( U_{mn} = -10p_m + 4, \quad n = 1, 2, 4, 5 \)
- \( U_{mn} = -0.9p_m, \quad n = 3, 6 \)
- \( U_{kn} = -10p_k + 4, \quad n = 1, 2, 4, 5 \)
- \( U_{kn} = -0.9p_k, \quad n = 3, 6 \)
- Prices \( m \): 1.0, 1.2, 1.4, 1.6, 1.8
- Prices \( k \): half price

Theater \( m \)

- Optimum price \( m \): 1.8
- Young customers: 0
- Old customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater \( k \)

Closed
Two theaters with capacity, different types of films

Data

- Theaters \( m \) and \( k \)
- Capacity: 2
- \( N = 6 \)
- \( R = 5 \)
- \( U_{mn} = -10p_m + 4, \ n = 1, 2, 4, 5 \)
- \( U_{mn} = -0.9p_m, \ n = 3, 6 \)
- \( U_{kn} = -10p_k + 0, \ n = 1, 2, 4, 5 \)
- \( U_{kn} = -0.9p_k, \ n = 3, 6 \)
- Prices \( m \): 1.0, 1.2, 1.4, 1.6, 1.8
- Prices \( k \): half price

Theater \( m \)

- Optimum price \( m \): 1.8
- Demand: 0.2 (3.3%)
- Revenues: 0.36

Theater \( k \)

- Optimum price \( m \): 0.5
- Demand: 2 (33.3%)
- Revenues: 1.15
## Example of two scenarios

<table>
<thead>
<tr>
<th>Customer</th>
<th>Choice</th>
<th>Capacity $m$</th>
<th>Capacity $k$</th>
</tr>
</thead>
<tbody>
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Bierlaire (EPFL)

Choice and optimization

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Choice models in integer optimization

Discrete choice models
- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation
- Linear in the decision variables
- Large scale
- Fairly general
- Easily extended for regret minimization