A Note on the Expressiveness of BIP

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We extend our previous algebraic formalisation of the notion of component-based framework in order to formally define two forms—strong and weak—of the notion of full expressiveness. Our earlier result shows that the BIP (Behaviour-Interaction-Priority) framework does not possess the strong full expressiveness. In this paper, we show that BIP has the weak form of this notion and provide results detailing weak and strong full expressiveness for classical BIP and several modifications, obtained by relaxing the constraints imposed on priority models.

1 Introduction

In our previous work [1], we have formalised some of the properties that are desirable for component-based design frameworks, namely: incrementality, flattening, compositionality and modularity [17, 25]. We have also discussed the full expressiveness property, although without providing a formal definition for it. The formalisation is based on a very simple, abstract algebraic definition of the notion of component-based framework, which we extend below in order to also provide such formal definition of full expressiveness.

Intuitively, flattening requires that, for any component obtained by hierarchically applying two composition operators to a finite set of sub-components, there must exist an equivalent component obtained by applying a single composition operator to the same sub-components. Full expressiveness w.r.t. a given set of operators—e.g. those defined by a particular Structural Operational Semantics (SOS) rule format—requires that all operators in that set be expressible as composition operators in the component-based framework.

In [1], we have studied the satisfaction of the above properties by BIP (Behaviour-Interaction-Priority), which is a component-based framework for the design of correct-by-construction concurrent software and systems based on the separation of concerns between coordination and computation [2, 3]. BIP systems consist of components modelled as Labelled Transition Systems (LTS). Transitions are labelled by ports, which are used for synchronisation with other components. Composition operators defining such synchronisations are obtained by combining interaction and priority models. Operational semantics of the BIP composition operators is defined by SOS rules in a format, which is a restriction of GSOS [7]. Below we refer to this format as BIP-like SOS. We focus on the flattening and full expressiveness w.r.t. BIP-like SOS of BIP with the classical semantics defined in [4] and used in the language and code-generation tool-set developed by VERIMAG.¹

In [1], we have provided a counter-example showing that the classical semantics of BIP does not possess flattening, which implies that it does not possess full expressiveness w.r.t. BIP-like SOS either. This shows that the often encountered informal statement: “BIP possesses the expressiveness of the universal glue” (or its equivalent in slightly different formulations) is based on an erroneous proposition in previous work [5, Proposition 4]. The fundamental reasons for this absence of full expressiveness lie in the definition of the priority models. A priority model is a strict partial order on the underlying

¹http://www-verimag.imag.fr/New-BIP-tools.html
interaction model (set of allowed interactions). This definition guarantees that applying a priority model does not introduce deadlocks in the otherwise deadlock-free system. However, such deadlocks can be introduced by certain operators respecting BIP-like SOS.

In [1], we have shown that relaxing the restrictions on priority models to allow arbitrary relations on interactions—rather than strict partial orders on interactions in the interaction model—provides full expressiveness w.r.t. the full class of BIP-like SOS operators.

In this paper, we refine this discussion as follows

- We formally define two notions—weak and strong—of full expressiveness. Weak full expressiveness means that any operator that can be defined by a set of BIP-like SOS rules can be expressed as a hierarchical composition of BIP composition operators, as opposed to only one composition operator for strong full expressiveness.

- We provide a syntactic characterisation of a subset of operators defined by BIP-like SOS rules.

- We show that BIP has weak full expressiveness with respect to this subset of operators.

- We show that relaxing the partial order restriction in the definition of priority models allows us to recover weak full expressiveness w.r.t. the full class of BIP-like SOS operators.

The rest of the paper is structured as follows: Section 2 presents the algebraic formalisation of the notion “component-based framework” and defines its basic properties, namely flattening, strong and weak full expressiveness. Section 3 introduces the BIP component-based framework and its formal semantics. Section 4 presents the main results of the paper as stated above. Section 5 briefly discusses some related work. Finally, Section 6 concludes the paper.

2 Algebraic formalisation of component-based frameworks

Each component-based design framework can be viewed as an algebra \( \mathcal{A} \) of components equipped with a semantic mapping \( \sigma \) and an equivalence relation \( \simeq \), satisfying a set of basic properties, which we list below. More precisely, \( \mathcal{A} \) is an algebraic structure generated by a behaviour type \( \mathcal{B} \) and a set \( \mathcal{G} \) of composition (glue) operators:

\[
\mathcal{A} := B \mid f(C_1, \ldots, C_n),
\]

with \( B \in \mathcal{B}, C_1, \ldots, C_n \in \mathcal{A} \) and \( f \in \mathcal{G} \). We call the elements of \( \mathcal{A} \) components and the elements of \( \mathcal{B} \) behaviours. The algebraic structure \( \mathcal{A} \) represents the set of all systems constructible within the framework. Behaviour type \( \mathcal{B} \) defines the semantic nature of the components manipulated by the framework.

The semantic mapping \( \sigma : \mathcal{A} \to \mathcal{B} \) assigns to each component its meaning in terms of the behaviour type \( \mathcal{B} \). The semantic mapping must be consistent: for all \( B \in \mathcal{B} \), must hold the equality \( \sigma(B) = B \). The semantic mapping is called structural, if it is defined by associating to each \( n \)-ary glue operator \( f : \mathcal{A}^n \to \mathcal{A} \) a corresponding operator \( \tilde{f} : \mathcal{B}^n \to \mathcal{B} \) and putting

\[
\sigma(f(C_1, \ldots, C_n)) = \tilde{f}(\sigma(C_1), \ldots, \sigma(C_n)), \quad \text{for all } C_1, \ldots, C_n \in \mathcal{A} \text{ and } f \in \mathcal{G}.
\]

Finally, the equivalence relation \( \simeq \subseteq \mathcal{A} \times \mathcal{A} \)—that allows comparing components in terms, for example, of their functionality, observable behaviour or capability of interaction with the environment—must respect the semantics: for all \( C_1, C_2 \in \mathcal{A} \), must hold the implication \( \sigma(C_1) = \sigma(C_2) \Rightarrow C_1 \simeq C_2 \).

In this context, the flattening property mentioned in the introduction is formalised by requiring that

\[
\forall i, j \in [1, n] (i \leq j), \forall C_1, C_2, \ldots, C_n \in \mathcal{A}, \forall f, g \in \mathcal{G}, \exists h \in \mathcal{G} : f(C_1, \ldots, C_{i-1}, g(C_i, \ldots, C_j), C_{j+1}, \ldots, C_n) \simeq h(C_1, \ldots, C_n).
\]
In other words, $\mathcal{G}$ must be closed under composition. Similarly, given a set of operators $\mathcal{O} \subseteq \bigcup_{m=0}^{\infty} (\mathcal{B}^m \rightarrow \mathcal{B})$, we say that the component-based framework $(\mathcal{A}, \sigma, \simeq)$ has strong full expressiveness w.r.t. $\mathcal{O}$ iff

$$\forall o \in \mathcal{O}^n, \exists \tilde{\sigma} \in \mathcal{G} : \forall B_1, \ldots, B_n \in \mathcal{B}, \sigma(\tilde{\sigma}(B_1, \ldots, B_n)) = o(B_1, \ldots, B_n),$$

where $\mathcal{O}^n = \mathcal{O} \cap (\mathcal{B}^n \rightarrow \mathcal{B})$. We say that $(\mathcal{A}, \sigma, \simeq)$ has (weak) full expressiveness w.r.t. $\mathcal{O}$ iff,

$$\forall o \in \mathcal{O}^n, \exists \tilde{\sigma} \in \mathcal{G}[Z_1, \ldots, Z_n] : \forall B_1, \ldots, B_n \in \mathcal{B}, \sigma(\tilde{\sigma}[B_1/Z_1, \ldots, B_n/Z_n]) = o(B_1, \ldots, B_n),$$

where $\mathcal{G}[Z_1, \ldots, Z_n]$ is the set of expressions on variables $Z_1, \ldots, Z_n$ obtained by hierarchically applying the glue operators from $\mathcal{G}$; whereas $\tilde{\sigma}[B_1/Z_1, \ldots, B_n/Z_n] \in \mathcal{A}$ is the component obtained by substituting in $\tilde{\sigma}$ the variables $Z_i$ by components $B_i$, for all $i \in [1, n]$.

3 The BIP component-based framework

In this section, we briefly recall BIP and its classical operational semantics, as initially published in [4]. The behaviour type in BIP is the set of Labelled Transition Systems (LTS).

**Definition 3.1.** A labelled transition system (LTS) is a triple $(Q, P, \rightarrow)$, where $Q$ is a set of states, $P$ is a set of ports, and $\rightarrow \subseteq Q \times (2^P \setminus \{\emptyset\}) \times Q$ is a set of transitions labelled by interactions, i.e. non-empty sets of ports. For $q, q' \in Q$ and $a \in 2^P$, we write $q \xrightarrow{a} q'$ iff $(q, a, q') \in \rightarrow$. A label $a \in 2^P$ is active in a state $q \in Q$ (denoted $q \xrightarrow{a}$), iff there exists $q' \in Q$ such that $q \xrightarrow{a} q'$. We abbreviate $q \xrightarrow{a}\rightarrow = \neg(q \xrightarrow{a})$.

**Note 3.2.** In the rest of the paper, whenever we speak of a set of LTS $B_i = (Q_i, P_i, \rightarrow_i)$, for $i \in [1, n]$, we assume that all $P_i$ are pairwise disjoint, i.e. $i \neq j$ implies $P_i \cap P_j = \emptyset$. We denote $P = \bigcup_{i=1}^{n} P_i$. When the indices are clear from the context, we drop them on transition relations and denote $\rightarrow$.

Glue operators are separated in two categories: interaction models define the sets of allowed interactions, that is synchronisations between the transitions of their operand components; priority models define the scheduling—or conflict resolution—policies, reducing non-determinism when several synchronisations allowed by the interaction model are enabled simultaneously.

**Interaction models** An interaction model is a set of interactions $\gamma \subseteq 2^P \setminus \{\emptyset\}$. The semantics of the application of an interaction model $\gamma$ is defined by putting $\sigma(\gamma(B_1, \ldots, B_n)) \overset{\text{def}}{=} (Q, P, \rightarrow_{\gamma})$, with $Q = \prod_{i=1}^{n} Q_i$ and the minimal transition relation $\rightarrow_{\gamma}$ satisfying the rule

$$a \in \gamma \quad \left\{ q_i \xrightarrow{a P_i} q_i' \mid i \in I \right\} \quad \left\{ q_i = q_i' \mid i \notin I \right\},$$

where $I = \{i \in [1, n] \mid a \cap P_i \neq \emptyset\}$. Intuitively, this means that an interaction $a$ allowed by the interaction model $\gamma$ can be fired when all the components involved in $a$ are ready to fire the corresponding transitions. All the components that are not involved in $a$ remain in their current state.

**Priority models** For a behaviour $B = (Q, P, \rightarrow)$, a priority model is a strict\(^2\) partial order $\pi \subseteq 2^P \times 2^P$ (we write $a \prec b$ as a shorthand for $(a, b) \in \pi$). The semantics of the application of a priority model $\pi$ is

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\(^2\)As opposed to a (non-strict) partial order, which is a reflexive, antisymmetric and transitive relation, a strict partial order is an irreflexive and transitive (hence also asymmetric) one.
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Figure 1: Component behaviours for Example 3.6

defined by putting $\sigma(\pi(B)) \overset{\text{def}}{=} (Q, P, \rightarrow_\pi)$, with the minimal transition relation $\rightarrow_\pi$ satisfying the rule

$$
\frac{q \rightarrow q'}{q' \overset{a \rightarrow_\pi}{} q'}.
$$

(2)

Intuitively, this means that an interaction can be fired only if no higher-priority interaction is enabled.

Notice that the semantic mapping $\sigma$ defined by (1) and (2) is structural, since it is defined by associating to both interaction and priority models operators on behaviours.

**Note 3.3.** The rules (1) and (2) defining the semantics of BIP operators require that a partition $\bigcup_{i=1}^{n} P_i = P$ be defined, but they do not depend on the specific behaviours $B_1, \ldots, B_n$.

**Definition 3.4.** An $n$-ary BIP glue operator is a triple $((P_i)_{i=1}^{n}, \gamma, \pi)$, where $(P_i)_{i=1}^{n}$ are disjoint sets of ports and, denoting $P = \bigcup_{i=1}^{n} P_i$, the remaining two elements $\gamma \subseteq 2^P$ and $\pi \subseteq \gamma \times \gamma$ are, respectively, interaction and priority models on $P$. (In the remainder of the paper, we omit the sets of ports $(P_i)_{i=1}^{n}$ when they are clear from the context.)

To simplify the notation, we denote the component obtained by applying the glue operator $((P_i)_{i=1}^{n}, \gamma, \pi)$ to sub-components $B_1, \ldots, B_n$ by $\pi\gamma(B_1, \ldots, B_n)$ instead of $((P_i)_{i=1}^{n}, \gamma, \pi)(B_1, \ldots, B_n)$. Furthermore, when $\pi = \emptyset$, we write directly $\gamma(B_1, \ldots, B_n)$, omitting $\pi$.

**Definition 3.5.** Two behaviours $B_i = (Q_i, P_i, \rightarrow_i)$, for $i = 1, 2$ are equivalent if $P_1 = P_2$, and the two LTS are bisimilar, i.e. there exists a bisimulation [23] relation $R \subseteq Q_1 \times Q_2$ total on both $Q_1$ and $Q_2$.

**Example 3.6.** Consider the two components $B_1$ and $B_2$ shown in Figures 1(a) and 1(b), with $P_1 = \{p, q\}$ and $P_2 = \{r\}$, and put $\gamma = \{p, q, r, qr\}$ and $\pi = \{q \prec r\}$. The glue operator defined by the combination of the interaction model $\gamma$ and the priority model $\pi$ is given by the following four rules, obtained by composing rules of forms (1) and (2) and removing premises, wherever satisfaction does not depend on the state of the operand components (e.g. the premise $a \in \gamma$ is satisfied in all states):

$$
\frac{q_1 \rightarrow q_1'}{q_1 q_2 \rightarrow q_1' q_2}, \quad \frac{q_2 \rightarrow q_2'}{q_1 q_2 \rightarrow q_1' q_2'}, \quad \frac{q_1 \rightarrow q_1'}{q_1 q_2 \rightarrow q_1' q_2'}, \quad \frac{q_1 q_2 \rightarrow q_1 q_2'}{q_1 q_2 \rightarrow q_1' q_2'}. \quad (3)
$$

The composed component $\pi\gamma(B_1, B_2)$ is shown in Figure 1(c). The dashed arrow $21 \overset{q}{\rightarrow} 31$ shows the transition present only in $\gamma(B_1, B_2)$, but not in $\pi\gamma(B_1, B_2)$. Solid arrows show the transitions of $\pi\gamma(B_1, B_2)$.

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3To simplify the notation we use the juxtaposition $\gamma = \{p, q, r, qr\}$ instead of the set notation $\gamma = \{\{p\}, \{q\}, \{r\}, \{q, r\}\}$ for interactions. Similarly, we directly write $\pi = \{q \prec r\}$ instead of $\pi = \{(q, r)\}$.
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Among the transitions labelled by \( q \), only the transition \( 22 \overset{a}{\Rightarrow} 32 \) is enabled and not \( 21 \overset{a}{\Rightarrow} 31 \) (Figure 1(c)). Indeed, the negative premise in the fourth rule of (3), generated by the priority \( q \prec r \), suppresses the interaction \( q \) when a transition labelled \( r \) is possible in the second component.

After merging rules of forms (1) and (2) and the simplification by removing the constant premises, all rules used to define the semantics of BIP glue operators follow the format

\[
\begin{align*}
\{ q_i & \overset{a\cdot b}{\Rightarrow} q_i' \mid i \in I \} & \quad \{ q_i = q_i' \mid i \notin I \} & \quad \{ q_j \overset{b_i}{\Rightarrow} j \in J, k \in K_j \} \\
q_1 \ldots q_n & \rightarrow q_1' \ldots q_n'
\end{align*}
\]

where \( I = \{ i \in [1, n] \mid a \cap P_i \neq \emptyset \} \), whereas \( J, K_j \subseteq [1, n] \) and, for each \( j \in J \) and \( k \in K_j \), holds \( b_i^j \in 2^P \).

Let us now recall an important property of the BIP glue operators with the above semantics, which was originally shown in [16]: application of a priority model does not introduce deadlocks.

**Definition 3.7.** Let \( B = (Q, P, \rightarrow) \) be a behaviour. A state \( q \in Q \) is a deadlock iff holds \( \forall a \subseteq P \), \( q \not\rightarrow a \).

**Lemma 3.8** ([16]). Let \( B_i = (Q_i, P_i, \rightarrow) \), for \( i \in [1, n] \), be a set of behaviours, \( \gamma \) and \( \pi \) be respectively interaction and priority models on \( P = \bigcup_{i=1}^n P_i \). A state \( q \in \prod_{i=1}^n Q_i \) is a deadlock in \( \pi \gamma (B_1, \ldots, B_n) \) if and only if it is a deadlock in \( \gamma (B_1, \ldots, B_n) \).

**Proof.** The “if” implication is trivial. To prove the “only if” implication, assume that, for some \( a \in \gamma \), we have \( q \overset{a}{\rightarrow} \gamma \). Let \( b \subseteq P \) be an interaction, maximal w.r.t. \( \pi \), such that \( b \in \gamma \), \( a \prec b \) and \( q \overset{b}{\rightarrow} \gamma \). If such \( b \) exists, holds \( q \overset{b}{\rightarrow} \pi \). Otherwise holds \( q \overset{a}{\rightarrow} \pi \). In both cases, \( q \) is not a deadlock in \( \pi \gamma (B_1, \ldots, B_n) \).

Notice that this proof does not rely on \( \pi \) being a strict partial order. The lemma can be generalised to any acyclic relation \( \pi \subseteq \gamma \times \gamma \).

### 4 Expressiveness

We now consider full expressiveness of BIP w.r.t. the set \( O \) of operators defined as pairs \( (\mathcal{P}_i)_{i=1}^n, \mathcal{R} \), where \( n \) is the arity of the operator, \( \mathcal{P}_i \) are pair-wise disjoint sets of ports and \( \mathcal{R} \) is a set of SOS rules in the format (4). In [1], we have shown that the operator defined by the following four rules, which respect the format (4), cannot be expressed as a BIP glue operator in the classical semantics:

\[
\begin{align*}
q_1 \overset{p_1}{\rightarrow} q_1' & \quad q_2 \overset{f_3}{\rightarrow}, & q_1q_2q_3 \overset{p_3}{\rightarrow} q_1'q_2q_3 \\
q_1q_2q_3 \overset{q_2q_2q_3}{\rightarrow} q_1'q_2q_3 & \quad q_2 \overset{s_2}{\rightarrow} q_2' & q_1q_2q_3 \overset{s_2}{\rightarrow} q_1q_2q_3 \\
q_2q_3 \overset{q_3}{\rightarrow} q_3' & \quad q_1q_2q_3 \overset{q_3}{\rightarrow} q_1q_2q_3.
\end{align*}
\]

We conclude that the classical semantics of BIP does not have neither flattening, nor strong full expressiveness w.r.t. \( O \).

Furthermore, the example below shows that the classical semantics of BIP does not have even weak full expressiveness.

**Example 4.1.** Consider a composition operator defined by the following two rules:

\[
\begin{align*}
q_1 \overset{p_1}{\rightarrow} q_1' & \quad q_1 \overset{f_3}{\rightarrow} \\
q_1 \overset{p_1}{\rightarrow} q_1' & \quad q_1 \overset{f_3}{\rightarrow}
\end{align*}
\]

applied to the component in Figure 2. Assume that there exists a hierarchy of BIP glue, such that applying them to the component in Figure 2 results in an equivalent composed component. States 1

\footnote{For the details of this example and the associated discussion, we refer the reader to [1].}
Proposition 4.2. If by be inhibited by the negative premises, one such premise must be involved for each rule in where, as above, the information used by the priority model refers only to interactions authorised by the underlying interaction model—all the information about transitions enabled in sub-components is lost [1];

• the priority model \( \pi \) must be a strict partial order.

As we explain below, among these two reasons, the first one is easily addressed to achieve weak, rather than strong, full expressiveness, whereas the second one presents the main difficulty.

What can be done without changing the BIP glue? Consider an \( n \)-ary operator \( o : \text{LTS}^n \to \text{LTS} \) defined by \( (P_i)_{i=1}^n \) and the set of rules

\[
\begin{align*}
q_i & \xrightarrow{a \cap p_i} q_i' \quad & i \in I', \\
q_i & = q_i' \quad & i \notin I', \\
q_j & \xrightarrow{b_j^l} j \in J', k \in K_j^{l_j} \quad & j \in J', k \in K_j^{l_j}, \text{ for } l \in [1, m],
\end{align*}
\]

where, as above, \( I' = \{ i \in [1, n] \mid a' \cap p_i \neq \emptyset \} \). For an interaction \( a \in \{ a' \mid l \in [1, m] \} \), denote \( R_a \overset{\text{def}}{=} \{ l \in [1, m] \mid a = a' \} \) the set of rules with the conclusion labelled by \( a \). Clearly, for the interaction \( a \) to be inhibited by the negative premises, one such premise must be involved for each rule in \( R_a \). We denote by \( j : R_a \overset{\text{def}}{=} J \) the choice mappings \( j : R_a \to \bigcup_{l=1}^m J' \), such that \( j(l) \in J' \), for all \( l \in R_a \).

We define the inhibiting relation \( \pi \subseteq 2^P \times 2^P \) (where \( P = \bigcup_{l=1}^n P_i \)) by putting

\[
\pi = \bigcup_{l=1}^m \{ (a', b) \mid b = \bigcup_{s \in J} b_{J(s), k(s)}^{s}, \text{ for some } j : R_a \overset{\text{def}}{=} J, k(s) \in K_{J(s)}^{s_j} \}.
\]

Proposition 4.2. If \( \pi \) has cycles, then the operator \( o \) cannot be realised by any hierarchical composition of BIP glue operators.

Proof. Consider a cycle in the inhibiting relation \( \pi : a_1 \prec a_2 \prec \cdots \prec a_l \prec a_1 \).

Let \( P = \bigcup_{j=1}^n P_j \), where \( P_j = \{ p_1, \ldots, p_m \} \). Let \( c_i^l = a_i \cap P_j \) for \( i \in [1, l], j \in [1, n] \) and \( C_j = \{ c_i^l \mid c_i^l \neq \emptyset \} \). For each \( j \) consider a behaviour as shown in Figure 3. There are no transitions from state 0; from each

\[\text{Figure 2: Component behaviour for Example 4.1}\]
state $i$ such that $c_i^j \neq \emptyset$, there is a single transition to state $m_j$ with labels $c_i^j \in C_j$, respectively, and loop transitions in state $m_j$ with labels $c_i^j \in C_j$.

The composition of such behaviours with the operator $o$ allows a single transition $a_i$ from the state $q_1 \ldots q_n$, where $q_j = i$ if $c_i^j \neq \emptyset$ or $q_j = 0$ otherwise. In order to allow these transitions, an interaction model of a BIP glue must contain all $a_i$. In the state $q_1 \ldots q_n$, with $q_j = m_j$, all interactions $a_1, \ldots, a_l$ are available. The operator $o$ forbids all of them from this state. Interaction models of BIP glues allow all these interactions and priority models cannot introduce deadlock in this state Lemma 3.8. Thus, this system is not expressible in BIP.

**Proposition 4.3.** If $\pi$ is acyclic, then the operator $o$ can be realised by a hierarchical composition of BIP glue operators.

**Proof.** Since $\pi$ is acyclic, we can associate a depth $d(a)$ to each interaction $a$ involved in $\pi$ as the length of the longest path leading to $a$ in the directed acyclic graph defined by $\pi$. Denote $d \overset{\text{def}}{=} \max_d d(a)$. Furthermore, for $i \in [1, d]$, denote $\pi_i \overset{\text{def}}{=} \{(a, b) \in \pi \mid d(a) = i - 1\}$.

Clearly all $\pi_i$ are strict partial orders. Furthermore $\pi_i \subseteq \pi \subseteq \psi_1 \times \psi_1$, for all $i \in [1, d]$ and

$$\psi_1 = \gamma_2 \cup \bigcup_{l=1}^{m} \left\{ \bigcup_{s \in R_i} B_{j(s), k(s)}^s \mid j : R_d \rightarrow J, k(s) \in K_j^s \right\},$$

$$\gamma_2 = \left\{ a' \mid l \in [1, m] \right\}.$$

Hence, for all $i \in [1, d]$, $(\psi_i, \pi_i)$ is a BIP glue operator.

The operator $o$ is equivalent to the composition $(\gamma_2, \emptyset) \circ (\psi_1, \pi_1) \circ \cdots \circ (\psi_1, \pi_1)$. We show that for any set of behaviours $B_i = (Q_i, P_i, \rightarrow)$, with $i \in [1, n]$, holds $\sigma(\gamma_2(\pi_1 \gamma_1(B_1, \ldots, B_n))) = o(B_1, \ldots, B_n)$. We denote

$$B_o = o(B_1, \ldots, B_n), \quad B_{\pi'} = \sigma(\gamma_2(\pi_1 \gamma_1(B_1, \ldots, B_n))) .$$

The sets of states and ports of these behaviours are the same, thus we only need to check that their transitions coincide.

Let $q_1 \ldots q_n \overset{a_l}{\rightarrow} q_1' \ldots q_n'$ in $B_o$. This means that, among the rules defining $o$, i.e. for some $l \in [1, m]$, there is a rule

$$\left\{ q_i \overset{a \cap P_l}{\rightarrow} q_i' \mid i \in I_l \right\} \cup \left\{ q_i = q_i' \mid i \notin I_l \right\} \cup \left\{ q_j \overset{b^l}{\rightarrow} q_j' \mid j \in J_l, k \in K_j^l \right\} ,$$

(9)
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In [1], we have proposed the following notion of relaxed BIP operator. The operator \(\gamma\) is then equivalent to the composition \((\gamma_2, \emptyset) \circ (\gamma_1, \pi)\), where \(\pi\) is considered as a relaxed priority model.

Proposition 4.5. For any set of behaviours \(B_i = (Q_i, P_i, \rightarrow)\), with \(i \in [1, n]\), holds

\[
\sigma(\gamma_2(\pi \gamma_1(B_1, \ldots, B_n))) = o(B_1, \ldots, B_n).
\]
Proof. For a set of behaviours $B_i = (Q_i, P_i \rightarrow)$, with $i \in [1,n]$, denote

$$B_o = o(B_1, \ldots, B_n), \quad B_{\pi\gamma} = \sigma(\gamma_2(\pi_1(B_1, \ldots, B_n))).$$

The sets of states and ports of these behaviours are the same, thus we only need to check that their transitions coincide.

Let $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$ in $B_o$. This means that, among the rules defining $o$, i.e. for some $l \in [1,m]$, there is a rule

\begin{equation}
\begin{cases}
q_i \xrightarrow{a \in P_i} q'_i & \text{if } i \in I^l, \\
q_i = q'_i & \text{if } i \notin I^l, \\
q_j \xrightarrow{b_j^\pi_{\{j\}}} q'_i & \text{if } j \in J^l, k \in K_j^l
\end{cases},
\end{equation}

such that $q_i \xrightarrow{a \in P_i}$, for all $i \in I$, and $q_j \xrightarrow{b_j^\pi_{\{j\}}}$ for all $j \in J^l, k \in K_j^l$. By construction both $\pi_1$ and $\gamma_2$ contain $a$. Hence, $a$ is enabled in the state $q_1 \ldots q_n$ of $\gamma_1(B_1, \ldots, B_n)$ and in the same state of $\gamma_2(\pi_1(B_1, \ldots, B_n))$, provided that it is not disabled by the priority $\pi$. Thus, we have to show that no interaction available from this state has higher priority. Priority rules in $\pi$ that contain $a$ are of the form $a < b$, with $b = \bigcup_{s \in R_a} b_s^\pi_{\{s\},k(s)}$ for some $j : R_a \xrightarrow{\sim} J$ and $k(s) \in K_j^s$). Since all the premises of (11) are satisfied in $q_1 \ldots q_n$, interaction $b_j^\pi_{\{j\},k(l)}$ is disabled. Hence, $b$ is also disabled. Thus $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$ in $B_{\pi\gamma}$.

Let $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$ in $B_{\pi\gamma}$. This means that both $\pi_1$ and $\gamma_2$ contain the interaction $a$. Therefore, by the construction of $\gamma_2$, there is at least one rule

\begin{equation}
\begin{cases}
q_i \xrightarrow{a \in P_i} q'_i & \text{if } i \in I, \\
q_i = q'_i & \text{if } i \notin I, \\
q_j \xrightarrow{b_j^\pi_{\{j\}}} q'_i & \text{if } j \in J, k \in K_j
\end{cases},
\end{equation}

among the rules defining $o$. Furthermore, the priority model $\pi$ has to contain priorities of the form $a < b$, with $b = \bigcup_{s \in R_a} b_s^\pi_{\{s\},k(s)}$, for all $j : R_a \xrightarrow{\sim} J$ and $k(s) \in K_j^s$. Assuming now that none of rules defining $o$, with the conclusion labelled by $a$, applies in $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$. Since $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$ in $B_{\pi\gamma}$, this necessarily means that each of these rules has a negative premise that is not satisfied. Let $b = \bigcup_{s \in R_a} b_s^\pi_{\{s\},k(s)}$ with $b_j^\pi_{\{j\},k(l)}$ for all $s \in R_a$, being the labels of dissatisfied premises. Then $b$ is an enabled interaction such that $a < b$, which contradicts the assumption $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$ in $B_{\pi\gamma}$. Hence, there is at least one rule of the form (12) in the definition of $o$ with all premises satisfied in $q_1 \ldots q_n$ and, therefore, $q_1 \ldots q_n \xrightarrow{a} q'_1 \ldots q'_n$ in $B_o$.

Thus, we conclude that BIP with relaxed priority models has weak full expressiveness w.r.t. the set of all BIP-like SOS operators.

Notice that the relaxed priority model does not allow recovering strong full expressiveness. For instance, consider the operator defined by the single rule

\begin{equation}
\begin{array}{c}
q_1 \xrightarrow{p} q'_1 \\
q_1 \xrightarrow{p} q'_1
\end{array},
\end{equation}

applied to the behaviour in Figure 2. The composed component has a single transition $1 \xrightarrow{p} 3$. The interaction model of BIP cannot contain $r$, as it is not possible to exclude transition $2 \xrightarrow{r} 3$ with a priority model. The transition $3 \xrightarrow{p} 3$ has to be excluded by the priority model, however it cannot use $r$ in the priority relation.
Further relaxation of the definition of the BIP operator by removing the restriction \( \pi \subseteq \gamma \times \gamma \) requires a slight modification of the semantics. Clearly, the behaviour \( \gamma'(B_1, \ldots, B_n) \) does not have transitions that are not in \( \gamma \) and priority rules that can be applied to this behaviour are in \( \gamma \times \gamma \). Thus, we need to apply interaction and priority models simultaneously. The semantics of the simultaneous application of an interaction model \( \gamma \) and a priority model \( \pi \) is defined by putting \( \sigma(\pi \gamma(B_1, \ldots, B_n)) \equiv (Q, P, \rightarrow_{\pi\gamma}) \), with \( Q = \prod_{i=1}^n Q_i \) and the minimal transition relation \( \rightarrow_{\pi\gamma} \) inductively defined by the set of rules

\[
\left\{ \begin{array}{l}
q_1 \xrightarrow{a \in P_i} q'_1, \quad i \in I \\
q_i = q'_i, \quad i \notin I \\
q_j \xrightarrow{bc \in P_j} b \in K_a, \\
q_1 \cdots q_n \xrightarrow{\gamma} q'_1 \cdots q'_n
\end{array} \right\}, \quad a \in \gamma, \quad j : K_a \sim \{1, \ldots, n\},
\]

where \( I = \{i \in [1, n] \mid a \cap P_i \neq \emptyset\} \), \( K_a = \{b \mid a < b\} \) and \( j : K_a \sim [1, n] \) is a choice mapping \( j : K_a \rightarrow [1, n] \), such that, for all \( b \in K_a \), holds \( b \cap P_{j(b)} \neq \emptyset \).

With this relaxation we obtain strong full expressiveness, since the operator \( o \) is then clearly equivalent to \( (\gamma_2, \pi) \).

**What cannot be achieved?** Consider another relaxation of the definition of BIP glue operators, by considering operators \( (P_i)_{i=1}^n, \gamma, \pi \), with \( P = \bigcup_{i=1}^n P_i \) such that the priority model \( \pi \subseteq 2^P \times (2^P \setminus \{\emptyset\}) \) is a strict partial order, without requiring that it refer only to interactions (i.e. we do not impose \( \pi \subseteq \gamma \times \gamma \)). This relaxation does not recover even weak full expressiveness w.r.t. BIP-like SOS operators. Indeed, Example 4.1 is still no expressible.

### 5 Related Work

The results in this paper build mainly on our previous work. However, the following related work should also be mentioned.

Usually, comparison between formalisms and models is by flattening structure and reduction to a behaviorally equivalent model, e.g. automata and Turing machine. In this manner, all finite state formalisms turn out to be expressively equivalent independently of the features used for the composition of behaviors. Many models and languages are Turing-expressive, while their coordination capabilities are tremendously different. [5]

A first framework formally capturing meanings of expressiveness for sequential programming languages and taking into account not only the semantics but also the primitives of languages was provided in [13]. It allows formal reasoning about and distinguishing between *core elements* of a language and *syntactic sugar*. Although a number of studies have taken a similar approach in the context of concurrency, we will only point to [15] and the references therein. The key difference of our approach lies in the strong separation between the computation and coordination aspects of the behaviour of concurrent systems. Indeed, we consider that all sequential computation resides within the components of the system that are not subject to any kind of modification. Thus, we focus on the following question: *what system behaviour can be obtained by coordination of a given set of concurrent components?* In particular, this precludes the expression of parallel composition by choice operators, as in the expansion law [20].

An extensive overview of SOS formats is provided in [22], including some results comparing their expressiveness. More results comparing different formats of SOS can be found in [21]. The expressiveness property is closely related to the translation between languages. One of the definitions of encoding compared with other approaches can be found in [14]. It should be noted, however, that the above mentioned separation of concerns principle also leads to a very simple rule format. Indeed, the format that we
consider is a small subset of GSOS. Our focus in this paper, is more on the expressiveness of coordination mechanism provided by BIP than on that of the various SOS rule features.

There exist several works comparing BIP with various connector frameworks. A comparative study of three connector frameworks—tile model [8], wire calculus [26] and BIP [3]—was presented in [9]. Recently an attempt to relate BIP and Reo has been done [11]. From the operational semantics perspective, these comparisons only take in account operators with positive premises. In particular, priority in BIP is not considered.

Finally, in our formalisation of component-based frameworks, we rely on the notion of “behaviour type”. This can cover a very large spectrum, ranging from programs and labelled transition systems, through OSGi bundles and browser plug-ins, to systems of differential equations etc. Behaviour types can be organised in type systems and studied separately, as, for example, in the co-algebra theory [24]. However, this notion should be distinguished, for instance, from classes in object-oriented programming or session [10, 18] and behavioural [19] types for communication protocols. For instance, the notion of a class could be compared to that of a behaviour type in our sense as follows: a program would typically comprise a multitude of classes, whereas a component framework has only one underlying behaviour type. Although, in principle, component-based frameworks can be heterogeneous, e.g. Ptolemy II [12], that is rely on several distinct behaviour types for the design process, those aimed at the design of executable systems must have an underlying unifying behaviour type allowing the study and manipulation of a system as a whole.

6 Conclusion

Our previous investigations [1] of several properties that we consider fundamental for component-based design frameworks have revealed that the often encountered informal statement: “BIP possesses the expressiveness of the universal glue” (or its equivalent in slightly different formulations) is based on an erroneous proposition in previous work [5, Proposition 4]. We have, therefore, undertook an additional study of BIP expressiveness, whereof the results have been presented in this paper.

To achieve this goal, we rely on the algebraic formalisation of the notion of component-based design framework introduced in [1]. We have defined two new properties, weak and strong full expressiveness w.r.t. a given set of composition operators, which characterise whether these can be expressed by using the composition operators of the component-based framework under consideration. These two properties are very close to the weakly more expressive and strongly more expressive preorders introduced in [5]. In particular, for a component-based framework \((\mathcal{A}, \sigma, \simeq)\), with the underlying structure \(\mathcal{A}\) being generated by a set of glue operators \(\mathcal{G}\), the strong full expressiveness property w.r.t. a set of operators \(\mathcal{O}\) coincides with the statement that \(\mathcal{G}\) is strongly more expressive than \(\mathcal{O}\) in terms of [5]. However, the formal definition that we have provided in Section 2 is novel and has the advantage of fitting elegantly with that of the component-based design frameworks in [1]. Furthermore, the notion of weak full expressiveness is different from the weakly more expressive preorder: the former relaxes the strong form of the property by allowing hierarchical composition of glue operators, whereas the latter considers only flat operators, but allows a limited use of additional coordinating behaviour. Studying the combination of the two relaxations could be an interesting direction for future work.

We have studied the weak and full expressiveness of BIP w.r.t. operators defined by SOS rules in a particular format, which we call BIP-like SOS. The set of all the operators that can be defined in this format is the “universal glue”, w.r.t. which full expressiveness has been erroneously claimed in [5].

We observe that there are two obstacles to achieving strong full expressiveness: 1) a priority model is required to be a strict partial order on interactions and 2) by the definition of the BIP operational seman-
tics, priorities can only be applied to interactions that appear in the interaction model. The combination of these two requirements ensures that priorities cannot introduce new deadlocks. However, negative premises in BIP-like SOS rules—which correspond to priorities in BIP glue operators—can introduce deadlocks. To characterise this situation, we consider, for a set of BIP-like SOS rules, a corresponding inhibiting relation. In order to introduce deadlocks, this relation must have cycles. We show that BIP glue operators have weak full expressiveness w.r.t. BIP-like SOS operators that have acyclic inhibiting relations, with at most $d + 1$ layers of glue necessary to encode a BIP-like SOS operator, whereof the depth of the inhibiting relation is $d$.

A relaxation of both of the above requirements together recovers strong full expressiveness w.r.t. all BIP-like SOS. However, it calls for a definition of the operational semantics of BIP glue operators, which combines the interaction and the priority models, as opposed to the classical definition, where the interaction model is applied first, then the priority model is applied to the resulting component.

A relaxation of only the first requirement, which does not require any other modifications of the BIP semantics, leads to weak full expressiveness w.r.t. the set of all BIP-like SOS operators. Moreover, we have shown that at most two layers of glue are necessary to encode any operator.

As mentioned above, studying the combination of the two weak forms of full expressiveness—allowing both hierarchical glue and limited use of additional coordinating behaviour—could be an interesting direction for future work. Another direction for future work would consist in exploring the expressiveness of the full BIP framework, including the data manipulation and transfer, which has been recently formalised in [6]. Finally, a third extension could consist in studying larger SOS formats, including, for instance, witness premises, i.e. positive premises that allow testing the possibility of an action that does not, however, contribute to the conclusion of the rule.

References


