Effect of static and kinematic boundary conditions on the out-of-plane response of brick masonry walls

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ABSTRACT: The Unreinforced Wall (URM) elements that are most susceptible to out-of-plane failure are wall elements of the upper storeys where the accelerations are largest. In modern URM buildings with reinforced concrete slabs, the out-of-plane mechanism involves typically a storey-high wall element, which is subjected at its base and top to the accelerations of the corresponding floor slabs. The acceleration time histories of the two slabs differ as a result of the deformability of the in-plane loaded walls and their rigid body rotations. Existing studies on the out-of-plane response of URM walls analysed walls that were subjected to the same input motion at the top and bottom of the wall. In few studies, the motion was modified by a spring modelling the in-plane flexibility of the slabs. This article investigates how the response of walls subjected to different accelerations at the top and bottom differ from the response of walls that are subjected to the same, mean acceleration at the top and bottom. The topic is investigated by means of a parametric study using discrete element modelling and by means of simple mechanical models using rigid body mechanisms. The article concludes that in particular the relative displacement between the top and base of the wall contributes to the reduced out-of-plane resistance. The non-constant force distribution over the wall height has only a lesser influence. The paper concludes with recommendations for future research.

1 PROBLEM STATEMENT

Under earthquake loading, Unreinforced Masonry (URM) buildings are among the most vulnerable buildings (Grünthal 1998). Damage to URM walls can be classified as damage due to in-plane or out-of-plane loading. In-plane failure typically occurs in the bottom stories where shear forces are largest. Walls of the upper stories are particular vulnerable to out-of-plane loading since these walls are subjected to the largest accelerations and have—due to the small vertical forces that act upon them—also the smallest resistance to out-of-plane loads.

This article examines the out-of-plane behaviour of URM walls in modern brick masonry buildings with Reinforced Concrete (RC) slabs. It investigates in particular the effect of the static and kinematic boundary conditions on the out-of-plane stability of the walls. In engineering practice, the out-of-plane stability is typically assessed by means of maximum wall slenderness ratios for which masonry walls are considered as stable (e.g. SIA 2014; CEn 2004).

Griffith and co-workers proposed a nonlinear static analysis method that defines the out-of-plane capacity by means of a pushover curve (Doherty et al. 2002; Griffith et al. 2003; Derakhshan et al. 2013).

The paper starts with a brief presentation of the method by Griffith and a discussion of the assumptions (Section 2). One assumption relates to the accelerations at the wall base and wall head, which in the original method are assumed to be the same. This assumption is in the following reviewed by means of a parametric study using nonlinear time-history analysis (Section 3). The effect of a non-constant acceleration over the height of the wall and the effect of a relative displacement between top and base of the wall is investigated (Section 5). The article concludes with a discussion and an outlook on future studies.

2 ASSESSMENT BY GRIFFITH ET AL.

2.1 Approach

Griffith represents the capacity of a wall that is loaded out-of-plane by means of a pushover curve (Figure 1, Doherty et al. 2002). The actual pushover curve is approximated by a trilinear curve.
The third branch of this curve, which has a negative slope, is determined by means of equilibrium considerations of a rigid body mechanism \((F_0, \Delta_0)\). The elasticity of brick and mortar influences in particular the first and second branch of the pushover curve. These two branches can therefore not be obtained directly from the rigid body mechanism but Griffith et al. (2003) propose empirical relationships that relate \(\Delta_1\) and \(\Delta_2\) to \(\Delta_u\).

The displacement demand is determined by means of an equivalent elastic single degree of freedom system. Griffith et al. (2003) investigate different assumptions with regard to the choice of the stiffness of the equivalent single degree of freedom system. These are not reviewed here since the focus of this paper is on the capacity of out-of-plane loaded walls.

2.2 Assumptions

The method by Griffith belongs to the nonlinear static analysis methods. An earthquake is of course a dynamic excitation but nonlinear static methods are widely used in earthquake engineering, e.g. the N2-method in Eurocode 8, Part 3 (CEN 2005a). Further simplifications concern the static system of the out-of-plane loaded wall and the forces that act on the wall. Doherty et al. (2002) analyse four different static systems (Figure 2):

- System 1: A free standing wall that is not supported at the top.
- System 2: A wall that is laterally supported at the top but that is not subjected to an axial load (non-load bearing wall).
- System 3: A wall that carries a slab, which can introduce the vertical load to the wall at any position over the width of the wall.
- System 4: A wall that carries a slab that introduces the vertical load to the wall at a fixed position (typically at the centre of the wall).

These static systems are based on the following assumptions:

1. The wall can be represented by a 2D model.
2. The third hinge of systems 2–4 forms at midheight of the storey.
3. The axial force that acts at the wall head \((W\) in Figure 2, later in this article \(P\)) is constant throughout the response of the wall.
4. Systems 2–4 are analysed for lateral load distributions with equal forces at the top and the base, i.e., it is assumed that the excitation at the base and head of the wall are equal.

Modelling the walls by means of 2D systems might be too simplistic in particular for existing buildings (D'Ayala & Speranza 2003) and approaches for computing the resistance of walls that are spanning in two directions have been developed (Willis et al. 2004; Griffith et al. 2005). In modern URM buildings with RC slabs there are some walls that span only in one direction (e.g. walls between storey-high window and door openings); for all other walls 3D-mechanisms need to be considered.

The second assumption relates to the height of the third hinge, which originally was assumed to be at midheight. Derakhshan et al. (2013) generalized the model and derived the height that leads to the smallest out-of-plane resistance (see also Section 5).

The third assumption relates to the axial force, which is assumed to be constant throughout the pushover analysis while this is typically not the case in real buildings where the framing action provided by spandrels or stiff slabs can lead to a change in axial force. Furthermore, the rocking of the out-of-plane loaded wall leads to an elongation of the wall, which in modern buildings is at least partially
restraint by the RC slabs and therefore leads to an increase of the axial force. This was confirmed when analysing the out-of-plane wall response of a shake table test on a 4-storey building with URM and RC walls (Figure 3 and 4, Tondelli et al. 2015, Beyer et al. 2015).

The fourth assumption relates to the distribution of the horizontal accelerations over the height of the wall. Griffith investigates force distributions with equal values at the top and bottom of the wall (Doherty et al. 2002). This reflects walls whose top and bottom are subjected to the same accelerations. In reality, this is, however, typically not applicable since the storey accelerations vary from storey to storey. In general, the storey accelerations increase with increasing height in the building. This leads to the following boundary conditions:

- For small wall displacements, i.e., before the wall starts to rock, the distribution of accelerations over the wall height is trapezoidal and not constant.
- The top and bottom of the wall are subjected to a certain relative displacement.

Both the acceleration distribution and the relative displacement between top and bottom of the wall vary over the duration of the earthquake. In addition, the frequency content may change and excitations at the top and base of the wall might not be exactly in phase.

3 PARAMETRIC STUDY—INPUT DEFINITION

To investigate the influence of the boundary conditions on the out-of-plane response a parametric study on single spanning walls was conducted. The models represent a wall of the top storey of four-storey buildings. The acceleration demand on these walls was defined as the storey accelerations of reference buildings. These time histories were determined by means of Tremuri-analyses (Lagomarsino et al. 2013) of different four storey reference buildings.

These buildings are represented by 2D models, which consist of two URM walls, each of a length of 3 m, which are coupled by 0.2 m thick RC slabs. The storey heights are 3 m and the clear distance between the walls 2 m. The structure is regular over the height and the storey masses M are tuned in such a way that the first effective period of the structure is 0.30 s, 0.45 s, and 0.60 s, respectively. The axial load at each storey due to gravity loads is computed as \( N = gM/2 \) assuming that the mass is largely concentrated in the slabs and half the weight of the slab is carried by walls orthogonal to the direction of excitation.

To derive the input motions at the 3rd and 4th storey level, inelastic and elastic analysis were carried out. To reach reasonable levels of inelasticity the following procedure was adopted: When designing standard buildings, a seismic hazard level corresponding to a return period of 475
years is considered and standard buildings are expected to reach for this return period the limit state “Significant Damage” (CEN 2005b). URM walls failing in shear are expected to reach at this limit state an interstorey drift of 0.4%. The analyses were carried out using the five real records of the study by Griffith et al. (2003). The accelerograms are scaled in such a way that the first storey drift corresponds to 0.4% (accepted variation: ±5%). For comparison, also the response of the elastic structure was computed. The ratio R of the maximum base shear of the elastic structure to the inelastic structure varies for the three structures with T = 0.30 s, 0.45 s and 0.60 s subjected to five records between 1.3 and 3.8 with one outlier of 5.2. The elastic analyses were carried out with the same scaling factor for the ground motion as the inelastic analyses.

In the parametric study, the out-of-plane response of a fourth storey wall will be analysed. Such a wall is supported at the base by the third floor slab and at the top by the fourth floor slab. To gauge the influence of using an average acceleration rather than defining the third and fourth floor accelerations at the bottom and top of the wall respectively, the average acceleration of the third and fourth storey is computed. This acceleration is in the following referred to as the 4th storey mid-height acceleration or short mid-height acceleration.

4 PARAMETRIC STUDY—OUT-OF-PLANE RESPONSE

In order to investigate the effect of different accelerations at the top and bottom of the wall, two different load cases were analysed for each wall and record configuration:

- “Average excitation“: For this load case, the same acceleration time history was applied at the top and bottom of the wall. The acceleration time history corresponded to the average acceleration of the third and fourth storey:

\[ a_b(t) = \frac{a_3(t) + a_4(t)}{2} \]  \hspace{1cm} (1)

- “Real excitation“: The excitation at the wall base corresponds to the third storey acceleration and the excitation at the wall head to the fourth storey excitation:

\[ a_b(t) = a_3(t), \quad a_t(t) = a_4(t) \]  \hspace{1cm} (2)

The wall response was analysed by means of the discrete element analysis program UDEC (Itasca 2014). The analyses are based on a 2D-model (Figure 5), which consists of 14 blocks for the bricks and two blocks for the slabs. The blocks are rigid and all deformations are concentrated in the mortar joints, which are modelled by means of nonlinear contact elements. This model is similar to the UDEC model in (Tonelli et al. 2015), which was validated against the experimental results of the shake table test. Next to the ground motion, also the wall width, the axial load ratio and fundamental period of the building was varied. In total, 960 analyses were conducted.

The results of the simulation show that the walls that were subjected to the real excitation failed more frequently than the walls that were subjected to the average excitation (Figure 6). A wall was assumed to have failed when the maximum out-of-
plane displacement $\Delta$ reached or exceeded the wall thickness $t$.

Previous studies analysed systems where the wall base and head were subjected to the same excitation. The results of Figure 6 show that this can be potentially non-conservative. The following sections of this paper investigate the consequences of the different excitation at the top and the bottom by means of a simple mechanical model subjected to static excitation, which is based on the model by Griffith.

5 MODEL TO ANALYSE A WALL THAT IS SUBJECTED TO DIFFERENT OUT-OF-PLANE EXCITATIONS AT THE TOP AND THE BOTTOM

In Section 2.2 it was outlined that different excitations at the top and bottom of the slab can be represented in static pushover analysis by a trapezoidal force distribution over the height of the wall and a relative displacement between top and bottom of the wall. To simplify the analyses, the two effects are investigated separately. Section 5.1 investigates the effect of different accelerations at the top and bottom of the wall and Section 5.2 shows the effect of a relative horizontal displacement between wall head and base.

5.1 Model to consider different accelerations at the wall base and wall head

In the pushover analysis the inertia forces resulting from the acceleration demand are represented by the horizontal loads that act over the height of the wall. Figure 7a shows a wall with a constant load and Figure 7b a wall with a trapezoidal load distribution. The static system corresponds to the third system in Figure 2. The horizontal loads in Figure 7 correspond to the inertia forces of an infinitely rigid wall before the third hinge is forming. Figure 7a represents the case of equal accelerations at wall head and base and Figure 7b the case when the acceleration at the wall head is larger than at the base. If the third hinge forms at midheight and if the tensile strength of the mortar joints is neglected, both load distributions lead to the same resultant force $F_0$ which triggers for the rigid system out-of-plane rocking ($t = \text{wall thickness}$, all other parameters are defined in Figure 7):

- Constant load distribution (this value of $F_0$ is used in the following to normalise the resultant forces):

$$F_{0,\text{ref}} = q \cdot H = 4 \frac{t}{H} (G + 2P) \quad (3)$$

- Trapezoidal load distribution:

$$F_0 = q_b + \frac{q_b}{2} \cdot H = 4 \frac{t}{H} (G + 2P) \quad (4)$$

If the third hinge can form at an arbitrary height $b$ above the wall height, one obtains the following equations for the resultant force $F_0$:

- Constant load distribution:

$$F_0 = q \cdot H = \frac{2t}{\beta H} \left( G + \frac{P}{1 - \beta} \right) \quad (5)$$

- Trapezoidal load distribution:

$$F_0 = \frac{q_b}{2} \cdot H = \frac{3t}{\beta H} \left( \frac{1+r}{r(2-\beta)+(1+\beta)} \right) \left( G + \frac{P}{1 - \beta} \right) \quad (6)$$

where $\beta = b/H$. Figure 8 shows the variation of $F_0$ as a function of $\beta$ for a constant load distribution $q$. For walls that carry only their weight $G$ and no additional vertical load $P$, the third hinge forms close to the top hinge. Assuming that the third hinge forms at midheight leads for these cases to an overestimation by 100% of the lateral force $F_0$ that triggers the out-of-plane deformations. If the
wall carries, however, an additional axial load \( P \) that is larger than the weight of the wall \( (P>G) \), the assumption that the third hinge forms at midheight leads to a good estimation of the force \( F_0 \).

Note that \( \beta \) can only assume certain discrete values that correspond to the positions of the mortar joints. These values are shown in Figure 9 for the wall with 14 brick rows.

Figure 10 shows the variation of \( F_0 \) as a function of the additional vertical load \( P/G \) for a triangular load distribution \( (q_b/q_t = 0) \), a trapezoidal load distribution \( (q_b/q_t = 0.5) \) and a constant load distribution \( (q_b/q_t = 1.0) \). This figure confirms that for \( P>G \) the force \( F_0 \) can be well estimated by means of Equation (3). For smaller values of \( P \) Equation (3) overestimates \( F_0 \).

The constant load distribution reflects the load case “average acceleration” while the trapezoidal load distribution the “real acceleration” represents. If the third hinge forms at midheight, the load distribution does not influence the bilinear pushover curve. If the hinge can form in any of the mortar joints, the load distributions lead to slightly different values of \( F_0 \), which are however in general rather small. It is therefore unlikely that these explain the differences in the number of failures, which are represented in Figure 6.

5.2 Model to consider the relative displacements between wall head and base

In the following the relative displacements between wall head and base of the walls that were part of the parametric study of Section 3 are analysed: For each wall, the maximum relative displacement between wall head and base was computed (only the time until wall failure was considered) and the distribution of these maximum relative displacements plotted. If one compares the distribution of the relative displacements of the walls that failed to that of the walls that did not fail, one finds that walls that failed were subjected to larger relative displacement between wall head and base (Figure 11).

The effect of relative displacement between wall head and base will be again investigate by means of a pushover analysis of a wall consisting of two rigid bodies. To simplify matters, it is again assumed that the tensile strength of the mortar can be neglected. Based on the results of the previous section it is assumed that the third hinge forms at midheight and that the horizontal load can be represented by a constant load.

Figure 12 shows the mechanisms that develop if the wall is first subjected to a relative head...
displacement $u$ and then to a distributed load $q$. Figure 13 shows schematically the corresponding pushover curve. The force $F_1$ is the force that is required to initiate rocking of the wall. For a relative displacement $u$ it amounts to:

$$F_1 = q \cdot H = \frac{4}{H} \left( G \left( t - \frac{u}{4} \right) + P_1 \right)$$  \hspace{1cm} (7)$$

Figure 14 shows the force $F_1$ as a function of the relative displacement $u$ between wall head and base for different axial load ratios $P/G$. It is assumed that the third hinge forms at midheight and that the load distribution is constant.

6 DISCUSSION AND OUTLOOK

The response of walls that are loaded out-of-plane is strongly dependent on the static and kinematic boundary conditions. Previously it was generally assumed that walls are subjected to the same input accelerations at the top and bottom of the wall. This does however typically not apply since wall head and wall base are excited by different slabs and are therefore subjected to different acceleration time histories.

Parametric studies using nonlinear time-history analysis of discrete element models highlight the effect of different accelerations at the top and base of a wall: Walls that are subjected to different accelerations at the top and base fail significantly more frequently than walls that are subjected to the same average acceleration at the top and base.
This article investigated by means of simple static models two hypotheses, which could be the origin for this observation. The first one relates to the effect of a trapezoidal load distribution over the height of the wall when compared to a constant load distribution. The effect of the load distribution on the force that triggers out-of-plane rocking is however rather small.

The second hypothesis investigates the impact of a relative displacement between wall head and base. The results show that a relative displacement reduces the force that is required to trigger out-of-plane rocking. This relative displacement could therefore explain the difference in behaviour between walls that are subjected to the same excitation at the base and top and walls that are subjected to different excitations.

This study showed that the current assumption that wall head and base are subjected to the same acceleration time history might lead to conservative results when assessing the out-of-plane response of URM walls. Future studies should compare time-history results to those obtained with nonlinear time-history analysis and investigate also the dynamic properties of the input excitation at the wall head and base that might influence the wall response, i.e., frequency content and phase shift.

REFERENCES